Transient Analysis of Batch Arrival Feedback Retrial Queue with Starting Failure and Bernoulli Vacation

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Abstract
This paper discusses a batch arrival feedback retrial queue with Bernoulli vacation, where the server is subjected to starting failure. Any arriving batch finding the server busy, breakdown or on vacation enters an orbit. Otherwise one customer from the arriving batch enters a service immediately while the rest join the orbit. After the completion of each service, the server either goes for a vacation with probability β or may wait for serving the next customer with probability $(1-\beta)$. Repair times, service times and vacation times are assumed to be arbitrarily distributed. The time dependent probability generating functions have been obtained in terms of their Laplace transforms.
Keywords: Batch arrival, Time dependent solution, Retrial queues, Starting failures, Server vacation.

1. Introduction

Retrial queueing system are characterized by the fact that arriving customer who finds the server busy is obliged to leave the service area and repeat his demand after some time called retrial time. Between trials, the blocked customer joins a pool of unsatisfied customers called orbit.

One of the most important characteristic in the service facility of a queueing system is its starting failures. An arriving customer who finds the server idle must turn on the server. If the server is started successfully the customer gets the service immediately. Otherwise the repair for the server begins and the customer must join the orbit. The server is assumed to be reliable during service. Such systems with starting failures have been studied as queueing models by Yang and Li (1994), Mokaddis et al. (2007), Ke and Chang (2009), Sumitha and Udaya Chandrika (2012).

A retrial queueing system with FCFS discipline and general retrial times has been extensively discussed by Gomez-Corral (1999). Krishnakumar et al. (2002) discussed a retrial queue with feedback and starting failures. Yang and Templeton (1987), Kulkarni (1990), Artalejo (1999) have given explicit survey on retrial queueing systems.

In this paper we consider batch arrival feedback retrial queue, subject to starting failures and server vacation. After the completion of each service, the server either goes for a vacation or may wait for serving the next customer.

This paper is organized as follows. Model description is given in section 2. Definitions and equations governing the system are given in section 3. The time dependent solution have been obtained in section 4. Conclusion are given in section 5.

2. Model Description

We assume the following to describe the queueing model of our study.

a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a first come - first served basis. Let $\lambda c_i dt$ (i = 1, 2, . . .) be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t+dt]$, where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the arrival rate of batches.

b) We assume that there is no waiting space and therefore if an arriving customer finds the server busy or down, the customer leaves the service area and enters a group of blocked customers called "orbit" in accordance with an FCFS discipline. That is, only the customer at the head of the orbit queue is allowed for access to the server.

c) When the customer is served completely, he will decide either to join the retrial group again for another service with probability p or to leave the system forever with probability q. The successful commencement of service for a new customer who finds the server idle and sees no other customer in the orbit with probability δ and is α for all other new and returning customers. Interarrival times, retrial times, service times and breakdown times are assumed to be mutually independent. From this description, it is clear that at any service completion, the server becomes free and in such a case, a possible new (primary) arrival and the one (if any) at the head of the orbit, compete for service.

d) The retrial time follows a general (arbitrary) distribution with distribution function $A(s)$ and density function $a(s)$. Let $r(x)dx$ be the conditional probability density of retrial completion during the interval $(x, x+dx]$, given that the elapsed retrial time is x, so that

$$r(x) = \frac{a(x)}{1 - A(x)}$$

and therefore,

$$a(s) = r(s)e^{-\int_0^s r(x)dx}$$

e) The service follows a general (arbitrary) distribution with distribution function $B(x)$ and density function $b(x)$. Let $\mu(x)dx$ be the conditional probability of a completion of service during the interval $(x, x+dx]$ given that the elapsed service time is x, so that

$$\mu(x) = \frac{b(x)}{1 - B(x)}$$

and therefore,

$$b(t) = \mu(t)e^{-\int_0^t \mu(x)dx}.$$

f) The duration of repairs follows a general (arbitrary) distribution with distribution function $D(x)$ and density function $d(x)$. Let $\eta(x)dx$ be the conditional probability of a completion of repairs during the interval $(x, x+dx]$ given that the elapsed repair time is x, so that

$$\eta(x) = \frac{d(x)}{1 - D(x)}$$

and therefore,

$$d(t) = \eta(t)e^{-\int_0^t \eta(x)dx}$$

g) At the completion of each service the server may take a vacation with probability β or waits for the next customer with complementary probability $1-\beta$. After completing vacation, the server searches for customer in the orbit (if any) with probability θ or remains idle with probability $1-\theta$.

h) The server's vacation time follows a general (arbitrary) distribution with distribution function $V(t)$ and density function $v(t)$. Let $\gamma(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x+dx]$ given that the elapsed vacation time is x, so that

$$\gamma(x) = \frac{v(x)}{1 - V(x)},$$

and therefore,

$$v(t) = \gamma(t)e^{-\int_0^t \gamma(x)dx}$$

various stochastic processes involved in the system are assumed to be independent of each other.

3. Definitions and Equations Governing the System

We define

$P_n(x,t)$ = Probability that at time t, the server is idle and there are n ($n \geq 0$) customers in the orbit and the elapsed retrial time for this customer is x.

$Q_n(x,t)$ = Probability that at time t, the server is busy and there are n ($n \geq 0$) customers in the orbit and the elapsed service time for this customer is x.

$R_n(x,t)$ = probability that at time t, there are n ($n \geq 0$) customers in the orbit and the server is inactive due to system repair and waiting for repairs to start with elapsed repair time x.

$V_n(x,t)$ = Probability that at time t, the server is under vacation with elapsed vacation time x and there are n ($n \geq 0$) customers in the orbit.

$P_0(t)$ = Probability that at time t, there are no customers in the orbit and the server is idle but available in the system.

The model is then, governed by the following set of differential-difference equations:

$$\frac{d}{dt}P_0(t) = -\lambda P_0(t) + (1-\beta)q\int_0^\infty Q_0(x,t)\mu(x)dx + \int_0^\infty V_0(x,t)\gamma(x)dx \qquad (1)$$

$$\frac{\partial}{\partial x}P_n(x,t) + \frac{\partial}{\partial t}P_n(x,t) = -[\lambda + r(x)]P_n(x,t), n \geq 1 \qquad (2)$$

$$\frac{\partial}{\partial x}Q_0(x,t) + \frac{\partial}{\partial t}Q_0(x,t) = -[\lambda + \mu(x)]Q_0(x,t) \qquad (3)$$

$$\frac{\partial}{\partial x}Q_n(x,t) + \frac{\partial}{\partial t}Q_n(x,t) = -[\lambda + \mu(x)]Q_n(x,t) + \lambda\sum_{k=1}^{n}c_k Q_{n-k}(x,t), \quad n \geq 1 \qquad (4)$$

$$\frac{\partial}{\partial x}R_1(x,t) + \frac{\partial}{\partial t}R_1(x,t) = -[\lambda + \eta(x)]R_1(x,t) \qquad (5)$$

$$\frac{\partial}{\partial x}R_n(x,t) + \frac{\partial}{\partial t}R_n(x,t) = -[\lambda + \eta(x)]R_n(x,t) + \lambda\sum_{k=1}^{n}c_k R_{n-k}(x,t), \quad n \geq 2 \qquad (6)$$

$$\frac{\partial}{\partial x}V_0(x,t) + \frac{\partial}{\partial t}V_0(x,t) = -[\lambda + \gamma(x)]V_0(x,t) \qquad (7)$$

$$\frac{\partial}{\partial x}V_n(x,t) + \frac{\partial}{\partial t}V_n(x,t) = -[\lambda + \gamma(x)]V_n(x,t) + \lambda\sum_{k=1}^{n}c_k V_{n-k}(x,t), \quad n \geq 1 \qquad (8)$$

The above equations are to be solved subject to the following boundary conditions:

$$P_n(0,t) = (1-\beta)q\int_0^\infty Q_n(x,t)\mu(x)dx + (1-\beta)p\int_0^\infty Q_{n-1}(x,t)\mu(x)dx + \int_0^\infty R_n(x,t)\eta(x)dx + (1-\theta)\int_0^\infty V_n(x,t)\gamma(x)dx, n \geq 1 \qquad (9)$$

$$Q_0(0,t) = \delta\lambda c_1 P_0(t) + \alpha\int_0^\infty P_1(x,t)r(x)dx \qquad (10)$$

$$Q_n(0,t) = \alpha\lambda\int_0^\infty \sum_{k=1}^{n}c_k P_{n-k+1}(x,t)dx + \alpha\int_0^\infty P_{n+1}(x,t)r(x)dx + \delta\lambda c_{n+1}P_0(t) \qquad (11)$$

$$R_1(0,t) = \bar{\delta}\lambda P_0(t) + \bar{\alpha}\int_0^\infty P_1(x,t)r(x)dx \qquad (12)$$

$$R_n(0,t) = \bar{\alpha}\lambda\int_0^\infty P_{n-1}(x,t)dx + \bar{\alpha}\int_0^\infty P_n(x,t)r(x)dx, \ n \geq 2 \qquad (13)$$

$$V_n(0,t) = \beta\int_0^\infty Q_n(x,t)\mu(x)dx, n \geq 0 \quad (14)$$

We assume that initially there are no customers in the system and the server is idle. So the initial conditions are

$$V_n(0) = R_n(0) = 0, n \geq 0,\ Q_n(0) = 0 \text{ and}$$

$$P_0(0) = 1, P_n^{(i)}(0) = 0 \text{ for } n \geq 1. \quad (15)$$

4. Generating Functions of the Queue Length: The time-dependent solution

Now we shall find the transient solution for the above set of dfferential-difference equations.
First, let us define the probability generating functions.

$$P(x,z,t) = \sum_{n=0}^\infty z^n P_n^{(i)}(x,t);\ P(z,t) = \sum_{n=0}^\infty z^n P_n(t)$$

$$Q(x,z,t) = \sum_{n=0}^\infty z^n Q_n(x,t);\ Q(z,t) = \sum_{n=0}^\infty z^n Q_n(t)$$

$$R(x,z,t) = \sum_{n=0}^\infty z^n R_n(x,t);\ R(z,t) = \sum_{n=0}^\infty z^n R_n(t)$$

$$V(x,z,t) = \sum_{n=0}^\infty z^n V_n(x,t); V(z,t) = \sum_{n=0}^\infty z^n V_n(t), C(z) = \sum_{n=1}^\infty c_n z^n \quad (16)$$

which are convergent inside the circle given by $z \leq 1$ and define the Laplace transform of a function $f(t)$ as

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt,\ \Re(s) > 0. \quad (17)$$

Taking the Laplace transform of equations (1) to (14) and using (15), we obtain

$$(s+\lambda)\overline{P}_0(s) = 1 + (1-\beta)q\int_0^\infty \overline{Q}_0(x,s)\mu(x)dx + \int_0^\infty \overline{V}_0(x,s)\gamma(x)dx \quad (18)$$

$$\frac{\partial}{\partial x}\overline{P}_n(x,s) + [s+\lambda+r(x)]\overline{P}_n(x,s) = 0, n \geq 1 \quad (19)$$

$$\frac{\partial}{\partial x}\overline{Q}_0(x,s) + [s+\lambda+\mu(x)]\overline{Q}_0(x,s) = 0 \quad (20)$$

$$\frac{\partial}{\partial x}\overline{Q}_n(x,s) + [s+\lambda+\mu(x)]\overline{Q}_n(x,s) = \lambda\sum_{k=1}^n c_k \overline{Q}_{n-k}(x,s), n \geq 1 \quad (21)$$

$$\frac{\partial}{\partial x}\overline{R}_1(x,s) + [s+\lambda+\eta(x)]\overline{R}_1(x,s) = 0 \quad (22)$$

$$\frac{\partial}{\partial x}\overline{R}_n(x,s) + [s+\lambda+\eta(x)]\overline{R}_n(x,s) = \lambda\sum_{k=1}^n c_k \overline{R}_{n-k}(x,s), n \geq 2 \quad (23)$$

$$\frac{\partial}{\partial x}\overline{V}_0(x,s) + [s+\lambda+\gamma(x)]\overline{V}_0(x,s) = 0 \quad (24)$$

$$\frac{\partial}{\partial x}\overline{V}_n(x,s) + [s+\lambda+\gamma(x)]\overline{V}_n(x,s) = \lambda\sum_{k=1}^n c_k \overline{V}_{n-k}(x,s), n \geq 0 \quad (25)$$

$$\overline{P}_n(0,s) = q(1-\beta)\int_0^\infty \overline{Q}_n(x,s)\mu(x)dx + p(1-\beta)\int_0^\infty \overline{Q}_{n-1}(x,s)\mu(x)dx$$
$$+ \int_0^\infty \overline{R}_n(x,s)\eta(x)dx + (1-\theta)\int_0^\infty \overline{V}_n(x,s)\gamma(x)dx, n \geq 0 \quad (26)$$

$$\overline{Q}_0(0,s) = \delta\lambda c_1 \overline{P}_0(s) + \alpha\int_0^\infty \overline{P}_1(x,s)r(x)dx \quad (27)$$

$$\overline{Q}_n(0,s) = \alpha\lambda\int_0^\infty \sum_{k=1}^{n} c_k \overline{P}_{n-k+1}(x,s)dx + \alpha\int_0^\infty \overline{P}_{n+1}(x,s)r(x)dx$$

$$+\delta\lambda c_{n+1}\overline{P}_0(s),\ n\geq 1 \tag{28}$$

$$\overline{R}_1(0,s) = \bar{\delta}\lambda\overline{P}_0(s) + \bar{\alpha}\int_0^\infty \overline{P}_1(x,s)r(x)dx, \tag{29}$$

$$\overline{R}_n(0,s) = \bar{\alpha}\lambda\int_0^\infty \overline{P}_{n-1}(x,s)dx + \bar{\alpha}\int_0^\infty \overline{P}_n(x,s)r(x)dx,\ n\geq 2 \tag{30}$$

$$V_n(0,s) = \beta\int_0^\infty \overline{Q}_n(x,s)\mu(x)dx, n\geq 0 \tag{31}$$

Now multiplying equations (19) to (31) by z^n and summing over n, using the generating functions defined in (16), we get

$$\frac{\partial}{\partial x}\overline{P}(x,z,s) + [s+\lambda+r(x)]\overline{P}(x,z,s) = 0 \tag{32}$$

$$\frac{\partial}{\partial x}\overline{Q}(x,z,s) + [s+\lambda-\lambda C(z)+\mu(x)]\overline{Q}(x,z,s) = 0 \tag{33}$$

$$\frac{\partial}{\partial x}\overline{R}(x,z,s) + [s+\lambda-\lambda C(z)+\eta(x)]\overline{R}(x,z,s) = 0 \tag{34}$$

$$\frac{\partial}{\partial x}\overline{V}(x,z,s) + [s+\lambda-\lambda C(z)+\gamma(x)]\overline{V}(x,z,s) = 0 \tag{35}$$

$$\overline{P}(0,z,s) = (q+pz)(1-\beta)\int_0^\infty \overline{Q}(x,z,s)\mu(x)dx + \int_0^\infty \overline{R}(x,z,s)\eta(x)dx$$

$$+(1-\theta)\int_0^\infty \overline{V}(x,z,s)\gamma(x)dx - q(1-\beta)\int_0^\infty \overline{Q}_0(x,s)\mu(x)dx$$

$$-(1-\theta)\int_0^\infty \overline{V}_0(x,s)\gamma(x)dx \tag{36}$$

$$z\overline{Q}(0,z,s) = \delta\lambda C(z)\overline{P}_0(s) + \alpha\int_0^\infty \overline{P}(x,z,s)r(x)dx + \alpha\lambda C(z)\int_0^\infty \overline{P}(x,z,s)dx \tag{37}$$

$$\overline{R}(0,z,s) = \lambda z\bar{\delta}\overline{P}_0(s) + \bar{\alpha}\lambda z\int_0^\infty \overline{P}(x,z,s)dx + \bar{\alpha}\int_0^\infty \overline{P}(x,z,s)r(x)dx, \tag{38}$$

$$\overline{V}(0,z,s) = \beta\int_0^\infty \overline{Q}(x,z,s)\mu(x)dx, \tag{39}$$

Using equation (18) in (36), we get

$$\overline{P}(0,z,s) = [1-(s+\lambda)\overline{P}_0(s)] + (q+pz)(1-\beta)\int_0^\infty \overline{Q}(x,z,s)\mu(x)dx$$

$$+\int_0^\infty \overline{R}(x,z,s)\eta(x)dx + (1-\theta)\int_0^\infty \overline{V}(x,z,s)\gamma(x)dx \tag{40}$$

Integrating equation (32) between 0 and x, we get

$$\overline{P}(x,z,s) = \overline{P}(0,z,s)e^{-[s+\lambda]x-\int_0^x r(t)dt} \tag{41}$$

where $P(0,z,s)$ is given by equation (40).

Again integrating equation (41) by parts with respect to x yields,

$$\overline{P}(z,s) = \overline{P}(0,z,s)\left[\frac{1-\overline{A}(s+\lambda)}{s+\lambda}\right] \tag{42}$$

where $\overline{A}(s+\lambda)=\int_0^\infty e^{-[s+\lambda]x}dA(x)$

is the Laplace-Stieltjes transform of the retrial time $A(x)$. Now multiplying both sides of equation (41) by $r(x)$ and integrating over x we obtain

$$\int_0^\infty \overline{P}(x,z,s)r(x)dx=\overline{P}(0,z,s)\overline{A}(s+\lambda) \tag{43}$$

Similarly, on integrating equations (33) to (35) from 0 to x, we get

$$\overline{Q}(x,z,s)=\overline{Q}(0,z,s)e^{-[s+\lambda-\lambda C(z)]x-\int_0^x \mu(t)dt} \tag{44}$$

$$\overline{R}(x,z,s)=\overline{R}(0,z,s)e^{-[s+\lambda-\lambda C(z)]x-\int_0^x \eta(t)dt} \tag{45}$$

$$\overline{V}(x,z,s)=\overline{V}(0,z,s)e^{-[s+\lambda-\lambda C(z)]x-\int_0^x \gamma(t)dt} \tag{46}$$

where $\overline{Q}(0,z,s), \overline{R}(0,z,s)$ and $\overline{V}(0,z,s)$ are given by equations (37) to (39). Again integrating equations (44) to (46) by parts with respect to x yields,

$$\overline{Q}(z,s)=\overline{Q}(0,z,s)\left[\frac{1-\overline{B}(s+\lambda-\lambda C(z))}{s+\lambda-\lambda C(z)}\right] \tag{47}$$

$$\overline{R}(z,s)=\overline{R}(0,z,s)\left[\frac{1-\overline{D}(s+\lambda-\lambda C(z))}{s+\lambda-\lambda C(z)}\right] \tag{48}$$

$$\overline{V}(z,s)=\overline{V}(0,z,s)\left[\frac{1-\overline{V}(s+\lambda-\lambda C(z))}{s+\lambda-\lambda C(z)}\right] \tag{49}$$

where $\overline{B}(s+\lambda-\lambda C(z))=\int_0^\infty e^{-[s+\lambda-\lambda C(z)]x}dB(x)$

$$\overline{D}(s+\lambda-\lambda C(z))=\int_0^\infty e^{-[s+\lambda-\lambda C(z)]x}dD(x)$$

$$\overline{V}(s+\lambda-\lambda C(z))=\int_0^\infty e^{-[s+\lambda-\lambda C(z)]x}dV(x)$$

are the Laplace-Stieltjes transform of the service time $B(x)$, repair time $D(x)$ and vacation time $V(x)$ respectively. Now multiplying both sides of equation (44) to (46) by $\mu(x)$, $\eta(x)$ and $\gamma(x)$ and integrating over x, we obtain

$$\int_0^\infty \overline{Q}(x,z,s)\mu(x)dx=\overline{Q}(0,z,s)\overline{B}[s+\lambda-\lambda C(z)] \tag{50}$$

$$\int_0^\infty \overline{R}(x,z,s)\eta(x)dx=\overline{R}(0,z,s)\overline{D}[s+\lambda-\lambda C(z)] \tag{51}$$

$$\int_0^\infty \overline{V}(x,z,s)\gamma(x)dx=\overline{V}(0,z,s)\overline{V}[s+\lambda-\lambda C(z)] \tag{52}$$

Using equations (50) in (39), we can write as

$$\overline{V}(0,z,s)=\beta\overline{Q}(0,z,s)\overline{B}(s+\lambda-\lambda C(z)) \tag{53}$$

Using equation (43) in (37) and (38), we get

$$z\overline{Q}(0,z,s)=\delta\lambda C(z)\overline{P}_0(s)+\alpha\overline{A}(s+\lambda)\overline{P}(0,z,s)+\alpha\lambda C(z)\frac{[1-\overline{A}(s+\lambda)]}{s+\lambda}\overline{P}(0,z,s) \quad (54)$$

$$\overline{R}(0,z,s)=z\lambda\overline{\delta}\overline{P}_0(s)+\overline{\alpha}\overline{A}(s+\lambda)\overline{P}(0,z,s)+\overline{\alpha}\lambda z\frac{[1-\overline{A}(s+\lambda)]}{s+\lambda}\overline{P}(0,z,s) \quad (55)$$

Using equation (50) to (52) in (40), we get

$$\begin{aligned}\overline{P}(0,z,s)&=[1-(s+\lambda)\overline{P}_0(s)]+\overline{D}(s+\lambda-\lambda C(z))\overline{R}(0,z,s)\\&+(q+pz)(1-\beta)\overline{B}(s+\lambda-\lambda C(z))\overline{Q}(0,z,s)\\&+(1-\theta)\overline{V}(s+\lambda-\lambda C(z))\overline{V}(0,z,s)\end{aligned} \quad (56)$$

Similarly using equations (53), (54), (55) in (56), we get

$$\overline{P}(0,z,s)=\frac{Nr}{Dr} \quad (57)$$

where

$$\begin{aligned}Nr=\overline{P}_0(s)[&\delta\lambda C(z)\overline{B}(a)((q+pz)(1-\beta)+\beta(1-\theta)\overline{V}(a))\\&+\lambda z^2\overline{\delta}\overline{B}(a)]+z[1-s\overline{Q}(s)]-z\lambda\overline{P}_0(s)\end{aligned} \quad (58)$$

$$\begin{aligned}Dr=z-\alpha\overline{B}(a)&[(q+pz)(1-\beta)+\beta(1-\theta)\overline{V}(a)][\overline{A}(s+\lambda)\\&+\lambda C(z)\frac{(1-\overline{A}(s+\lambda))}{s+\lambda}]-\overline{\alpha}z\overline{B}(a)[\lambda z\frac{(1-\overline{A}(s+\lambda))}{s+\lambda}+\overline{A}(s+\lambda)]\end{aligned} \quad (59)$$

$a=s+\lambda-\lambda C(z)$. Substituting the value of $\overline{P}(0,z,s)$ from equation (57) into equations (54) and (55), we get

$$\overline{R}(0,z,s)=\lambda z\overline{\delta}\overline{P}_0(s)+\overline{\alpha}[\lambda z\frac{(1-\overline{A}(s+\lambda))}{s+\lambda}+\overline{A}(s+\lambda)]\frac{Nr}{Dr} \quad (60)$$

$$\overline{Q}(0,z,s)=\frac{\delta\lambda C(z)}{z}\overline{P}_0(s)+\frac{\alpha}{z}[\lambda C(z)\frac{(1-\overline{A}(s+\lambda))}{s+\lambda}+\overline{A}(s+\lambda)]\frac{Nr}{Dr} \quad (61)$$

Using equation (61) in (53), we get

$$\overline{V}(0,z,s)=\beta\overline{B}(a)[\frac{\delta\lambda C(z)}{z}\overline{P}_0(s)+\frac{\alpha}{z}(\lambda C(z)\frac{(1-\overline{A}(s+\lambda))}{s+\lambda}+\overline{A}(s+\lambda))\frac{Nr}{Dr}] \quad (62)$$

Substituting equations (57), (60) to (62) in (42), (47) to (49), we get

$\overline{P}(z,s),\overline{Q}(z,s),\overline{R}(z,s)$ and $\overline{V}(z,s)$. Thus the transient solution is obtained.

5. Conclusion

In this paper we have studied a batch arrival feedback retrial queue with Bernoulli vacation, where the server is subjected to starting failure. This paper clearly analyzes the transient solution of our queueing system. As a future work busy period analysis and reliability analysis will be discussed.

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