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The Velocity Profiles of HIV/AIDS In The Human Circulating Blood System: With Oscillating Time Dependent Total Phase Agle (*E*).

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ABSTACT
The human cyclic heart contraction generates pulsatile blood flow and latent vibration. The latent vibration is sinusoidal and central in character, that is, it flows along the middle of the vascular blood vessels and in the process it orients the active particles of the blood and sets them into oscillating motion with a unified frequency. We assume that it is the vibration of the HIV that interferes with the Human latent vibration. In this work, we utilized the known dynamical characteristics of the vibration of HIV and those of Man from a previous study to determine quantitatively the velocity profiles of HIV/AIDS in the human circulating blood system. We also used Fourier transform technique to determine the subsequent behaviour of the velocity of the constituted carrier wave (CCW) as it propagates away from the source which is the human heart. It is shown that the spectrum of the velocity profiles of the CCW become parasitically monochromatic with slow varying frequency beyond 77 months (6 years) and the velocity of the CCW finally fluctuate to zero after about 126 months (10 years).
Keywords: latent vibration, 'host wave', 'parasitic wave', CCW, HIV/AIDS, 'third world approximation'.

1. Introduction.

Since the advent of the human immunodeficiency virus (HIV) several theoretical and experimental approaches have been propounded by scientists in different fields of knowledge on how the deadly disease can be completely eradicated. However, none of these measures have been able to take Man beyond the threshold of this killer disease. It is therefore sufficient to say that the concepts advanced so far by scientists about the HIV/AIDS are inadequate and they lack proper understanding of the HIV/AIDS formation. Advancements in medical procedures and devices require a better understanding of the dynamical properties of HIV/AIDS and its formation.

HIV is the cause of the spectrum of disease known as the human immunodeficiency virus (HIV) and acquired immunodeficiency syndrome (AIDS). HIV is a retrovirus that primarily infects components of the human immune system. Accordingly, it directly and indirectly destroys the immune cells of the human system (Alimonti et al., 2003). During the initial infection a person may experience a brief period of influenza-like illness. This is typically followed by a prolonged period without symptoms. As the illness progresses it interferes more and more with the immune system, making people much more likely to get infections, including opportunistic infections, which do not usually affect people with immune systems. In the absence of specific treatment, around half of the people infected with HIV develop AIDS within 9 years and average survival time after infection with HIV is estimated to be 9 to 11 years [2][3]. (Mandel et al., 2010 ; UNIADS and WHO, 2007))

The human cyclic heart contraction generates pulsatile blood flow and latent vibration. The latent vibration is sinusoidal and central in character, that is, it flows along the middle of the vascular blood vessels and in the process it orients the active particles of the blood and sets them into oscillating motion with a unified frequency. Generally, it is the human blood that responds to the latent vibration from the heart with a specified wave form. The blood then propagates away from the region of the disturbance with certain velocity and in the process circulates oxygen and food nutrients to nourish the biological cells of the human system. Any alteration to this process, results into starvation, gradual weakening of the fundamental cells, and subsequent breakdown of the entire human biological system if uncontrolled.

In addition to the knowledge of the medical experts about HIV/AIDS, is the understanding that Man and the HIV are both active matter, as a result, they must have independent peculiar vibrations in order to exist. It is the vibration of the HIV that interferes with the vibration of Man (host) in the human blood circulating system after infection. The resultant interference of the two vibrations is parasitically destructive and it slows down or makes the biological system of Man to malfunction since the basic intrinsic parameters of the 'host wave' would have been altered.

Some waves in nature behave parasitically when they interfere with another one. Such waves as the name implies have the ability of transforming the initial characteristics and behaviour of the 'host wave' to its own form and quality after a period of time. Under this circumstance, all the active constituents of the 'host wave' would have been completely eroded and the resulting wave which is now parasitically monochromatic, will eventually attenuate to zero, since the 'parasitic wave' does not have its own physical parameters for sustaining a continuous independent existence (Enaibe et al., 2013).

In this work, we utilized the known dynamical characteristics of the vibration of HIV and those of Man from a previous study to determine quantitatively the velocity profiles of HIV/AIDS in the human circulating blood system (Enaibe and Idiodi, 2013). The HIV vibration is referred to as the 'parasitic wave' while that of Man as the 'host wave' and the combined effect of the two interfering waves is the constituted carrier wave (CCW).

Fourier series has long provided one of the principal methods of analysis for mathematical physics, engineering, and signal processing. It has spurred generalizations and applications that continue to develop right up to the present. While the original theory of Fourier series applies to periodic functions occuring in wave motion, such as with light and sound, its generalizations often relate to wider settings, such as the time-frequency analysis underlying the recent theories of wavelet analysis and local trigonometric analysis. Periodic functions arise in the study of wave motion, when a basic waveform repeats itself periodically (Walker, 1995).

In this study, we used a new method of approximation, otherwise, called the 'third world approximation' to derive the velocity of the CCW. The approximation has the advantage of easy convergence of results by direct analysis of the region of space of our interest and also it produces the expected minimized results in the microscale region. In this approximation, the first term in the series or 'first world' is usually a constant while the rest of the series is based on the choice of the parameter under evaluation.

In qualitative analysis, unlike numerical work, the number one is a fundamental number, an indiscriminate constant value which can only describe the neutral behaviour of a system of varying series. In consequence, the exact behaviour of a non-stationary system may not be studied in the indiscriminate region of a constant value. Thus the constant value term which is a non-zero-order approximation may therefore be neglected from the varying series solution by the 'third world approximation'. Thus the approximation has the advantage of fast convergence of result and high degree of minimization. It also help to control the complex anomalous behaviour of any square root displacement function which may produce imaginary and unuseful result.

Thus the aim of the present study was to characterize the mechanism of Fourier transform technique in determining the velocity profiles of HIV/AIDS in the Human circulating blood system. Also we want to establish the adequacy and effectiveness of the CCW with time dependent oscillating phase $E(t)$ in explaining the coexistence of HIV in the Human system. It is also our interest to show in this current study, the possible time taken for the HIV infection to degenerate to AIDS due to the alteration in the velocity process.

This paper is outlined as follows. Section 1, illustrates the basic concept of the work under study. The mathematical theory is presented in section 2. The results obtained are shown in section 3. While in section 4, we present the analytical discussion of the results obtained. The conclusion of this work is shown in section 5. This is immediately followed by appendix of some useful identities and a list of references.

2. Research Methodology.

In this current study, we first superposed a 'parasitic wave' on a 'host wave' and we used the 'third world approximation' to derive the velocity of the CCW, which is the combined effect of the superposition of the two waves. Finally, we applied the Fourier transform technique to study the behaviour of the CCW as it propagates between the time interval of 0 and 126 months.

2.1 Mathematical Theory.

Dynamical theory of superposition of two incoherent waves.

That the HIV kills slowly with time shows that the wave functions of the HIV and that of the host were initially incoherent. As a result, the amplitude, angular frequency, wave number and phase angle of the host wave which are the basic characteristics of vibration were initially greater than those of the HIV. The interference of one wave y_2 say 'parasitic wave' on another one y_1 say 'host wave', the interference could cause the 'host wave' to decay to zero if they are out of phase.

The decay process of y_1 can be gradual, over-damped or critically damped depending on the rate in which the amplitude is brought to zero. However, the general understanding is that the combination of y_1 and y_2 would first yield a third stage called the resultant wave y, before the process of decay sets in. Now let us consider two incoherent waves defined by the below non-stationary displacement vectors

$$y_1 = a\beta\cos\left(\vec{k}\beta.\vec{r} - n\beta t - \varepsilon\beta\right) \tag{2.1}$$

$$y_2 = b\lambda\cos\left(\vec{k}'\lambda.\vec{r} - n'\lambda t - \varepsilon'\lambda\right) \tag{2.2}$$

$$y = y_1 + y_2 = a\beta\cos\left(\vec{k}\beta.\vec{r} - n\beta t - \varepsilon\beta\right) + b\lambda\cos\left(\vec{k}'\lambda.\vec{r} - n'\lambda t - \varepsilon'\lambda\right) \tag{2.3}$$

where all the symbols have their usual meaning. In this study, (2.1) is regarded as the 'host wave' whose propagation depends on the inbuilt multiplier β (= 0, 1, 2, . . . $\beta_{\max}$). While (2.2) represents a 'parasitic wave' with an inbuilt multiplier λ(= 0, 1, 2, . . . $\lambda_{\max}$). The inbuilt multipliers are both dimensionless and they have the ability of gradually raising the basic characteristics of both waves respectively with time. We have established in a previous paper [5] that when (2.2) is superposed on (2.1) we get after some algebra that

$$y = \left\{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2\cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right\}^{\frac{1}{2}}\cos\left(\vec{k}.\vec{r} - (n - n'\lambda)t - E(t)\right) \tag{2.4}$$

In this study, equation (2.4) is regarded as the constituted carrier wave (CCW) and it is the equation that governs the dynamical behaviour of the coexistence of the HIV 'parasitic wave' in the human circulating blood system. Note that the variation of β with time is assumed to be very small and hence negligible (β =1). It is clear from the equation that once the multiplier λ raises the dynamical constituents of the HIV 'parasitic wave' to become equal to those of Man 'host wave', then the CCW goes to zero and the host biological system ceases to exist. However, for clarity of purpose we shall redefine in this work some of the parameters appearing in (2.4).

The total phase angle of the CCW is represented by $E(t)$, $\vec{k}.\vec{r}$ is the coordinate position vector. However, in this work we made the displacement vector represented by the CCW independent of space by simply using $\vec{k}.\hat{r}$ and $\hat{r} = \vec{r}/r$.

$$E(t) = \tan^{-1}\left(\frac{a\sin\varepsilon + b\lambda\sin\left(\varepsilon'\lambda - (n - n'\lambda)t\right)}{a\cos\varepsilon + b\lambda\cos\left(\varepsilon'\lambda - (n - n'\lambda)t\right)}\right) \tag{2.5}$$

The variation of the total phase angle $E(t)$ with time gives the characteristic angular velocity $Z(t)$ of the CCW.

$$\frac{dE(t)}{dt} = -Z(t) = -(n - n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos\left((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t\right)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos\left((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t\right)}\right) \tag{2.6}$$

We should understand at this point that $\vec{k}.\vec{r} = (k - k'\lambda)\,r\,(\cos\varphi + \sin\varphi)$ or $\vec{k}.\hat{r} = (k - k'\lambda)(\cos\varphi + \sin\varphi)$ is a two dimensional (2D) space vector and $\varphi = \pi - (\varepsilon - \varepsilon'\lambda)$ from the geometry of the two interfering waves and $\hat{r} = \vec{r}/r$ is a unit vector. By definition: $(n - n'\lambda)$ the modulation angular frequency, the modulation propagation constant $(k - k'\lambda)$, the phase difference δ between the two interfering waves is $(\varepsilon - \varepsilon'\lambda)$, the interference term is $2\left(a - b\lambda\right)^2\cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)$, while waves out of phase interfere destructively according to $\left(a - b\lambda\right)^2$ that is, if y comes out smaller than the larger of a and b; and waves in-phase interfere constructively according to $\left(a + b\lambda\right)^2$, that is, y comes out larger than both. Driving forces in antiphase $(\varepsilon - \varepsilon' = \pm\pi)$ provide full destructive superposition and the minimum possible amplitude; driving forces in phase $(\varepsilon = \varepsilon')$ provide full constructive superposition and the maximum possible amplitude.

In the regions where the amplitude of the wave is greater than either of the amplitude of the individual wave, we have constructive interference that means the path difference is $(\varepsilon + \varepsilon'\lambda)$, otherwise, it is destructive in which case the path difference is $(\varepsilon - \varepsilon'\lambda)$.

The CCW can be decomposed into two functions; function of the oscillating amplitude $f(A)$ and the function of the spatial oscillating phase $f(\theta)$. That is

$$f(A) = \left\{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right\}^{\frac{1}{2}} \tag{2.7}$$

$$f(\theta) = \cos\left(\vec{k}\,.\,\hat{r} - (n - n'\lambda)\,t - E(t)\right) \tag{2.8}$$

2.2 Application of the "third world approximation" on the oscillating amplitude $f(A)$ of the CCW.

Equation (2.7) is comprehensively valid in the macroscopic scale. However, if we implement the "third world approximation", then the function can be made valid for both macroscopic and microscopic scale. The "third world approximation" states that

$$\left(1 + \xi f(\phi)\right)^{\pm n} = \frac{d}{d\phi}\left(1 + n\,\xi f(\phi) + \frac{n(n-1)}{2!}\left(\xi f(\phi)\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\xi f(\phi)\right)^3 + \ldots\right) - n\frac{d}{d\phi}\left(\xi f(\phi)\right) \tag{2.9}$$

We should emphasize here that ϕ is a function of any variable which depends upon the dimension of the physical parameter we are investigating. However, in this study ϕ is taken as the time. In this approximation, the first term in the series or 'first world' is usually a constant while the rest of the series is based on the choice of the parameter under evaluation. For instance, the dimension of (2.7) is meters and if we apply (2.9) on it, then the first two terms, otherwise, the 'first world' and the 'second world' terms are both switched off leaving the third term or the 'third world' in m/s which is the dimension of velocity. Now let us rearrange (2.7) for the utilization of (2.9).

$$f(A) = \left(a^2 - b^2\lambda^2\right)^{\frac{1}{2}}\left\{1 - \frac{2\left(a - b\lambda\right)^2}{(a^2 - b^2\lambda^2)}\cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right\}^{\frac{1}{2}} \tag{2.10}$$

It can be shown that after a careful implementation of (2.9) in the parenthesis of (2.10) we obtain

$$\left\{1 - \frac{2\left(a - b\lambda\right)^2}{(a^2 - b^2\lambda^2)}\cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right\}^{\frac{1}{2}} = \frac{\left(a - b\lambda\right)^4(n - n'\lambda)}{2(a^2 - b^2\lambda^2)^{3/2}}\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \tag{2.11}$$

That is, we have used the fact that $\sin 2\theta = 2\sin\theta\cos\theta$ in the simplification to get the result. Hence

$$f(A) = \frac{(a - b\lambda)^4(n - n'\lambda)}{2(a^2 - b^2\lambda^2)^{3/2}}\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) = Q(n - n'\lambda)\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \tag{2.12}$$

For the purpose of linearity we have introduced a new parameter as

$$Q = \frac{(a - b\lambda)^4}{2(a^2 - b^2\lambda^2)^{3/2}} \tag{2.13}$$

2.3 Fourier series expansion of the oscillating amplitude $f(A)$ of the CCW.

The cornerstone of Fourier theory is a theorem which states that almost any periodic function can be analyzed into a series of harmonic functions with periods τ, $\tau/2$, $\tau/3$, ... , where τ is the period of the function under analysis (Lain, 1995). Expansion of an oscillating function by Fourier series gives all modes of oscillation (fundamental and all overtones) which is extremely useful in physics. In particular, astronomical phenomena are usually periodic, as are animal heartbeats, tides and vibrating strings, so it makes sense to express them in terms of periodic functions. Now, by expanding the oscillating term of (2.12) in terms of Fourier series we get

$$\begin{aligned} F\left[f(A)\right] = C_0 + C_1\left(\sin 2\left(1(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)_1\right)\right) + C_2\left(\sin 2\left(2(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)_2\right)\right) + \\ C_3\left(\sin 2\left(3(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)_3\right)\right) + \ldots + C_\alpha\left(\sin 2\left(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)_\alpha\right)\right) \end{aligned} \tag{2.14}$$

$$F\left[f(A)\right] = C_0 + \sum_{\alpha=1}^{\infty} C_\alpha \left(\sin 2\left(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)_\alpha\right)\right) \tag{2.15}$$

The constant term C_0 may be thought of as a harmonic with zero frequency. Each term in the series has amplitude and a phase constant; by adjusting these we can expand the various harmonics vertically, or shift them horizontally, to make the superposition fit the function $F\left[f(A)\right]$. Harmonic analysis consists essentially of finding C_α and $(\varepsilon - \varepsilon'\lambda)_\alpha$ for each value of α. From (2.15) it is however not always convenient to specify amplitude and phase (Lipson et al., 1996), we can decompose the last term as

$$C_\alpha \left(\sin 2\left(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) = A_\alpha \cos 2\alpha(n - n'\lambda)t + B_\alpha \sin 2\alpha(n - n'\lambda)t \tag{2.16}$$

We can specify the modulation amplitudes A_α and B_α as components of the variable phase angle as

$$\left.\begin{aligned} A_\alpha &= C_\alpha \cos 2(\varepsilon - \varepsilon'\lambda) \\ B_\alpha &= -C_\alpha \sin 2(\varepsilon - \varepsilon'\lambda) \end{aligned}\right\} \quad \Rightarrow \quad C_\alpha = \sqrt{A_\alpha^2 + B_\alpha^2} \tag{2.17}$$

The negative sign indicates complex conjugate of the real part and the inclusions will make the dynamic components of the phase angle real.

Thus (2.17) represents the amplitude of the nth harmonic. Where α is the Fourier index. From (2.16), if $\alpha = 0$

$$C_0 = -\frac{1}{\sin 2(\varepsilon - \varepsilon'\lambda)} A_0 \qquad \because \ (\sin(-x) = -\sin x) \tag{2.18}$$

2.4 Determination of the Fourier coefficients of the oscillating amplitude $f(A)$ of the CCW.

The Fourier components C_α in (2.15) which is specified in (2.17) and (2.18) are given by the Euler formulas

$$A_0 = \frac{1}{\tau}\int_0^\tau f(A)\,dt = \frac{1}{\tau}\int_0^\tau Q(n - n'\lambda)\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)dt \tag{2.19}$$

$$A_\alpha = \frac{1}{\tau}\int_0^\tau f(A)\cos 2\left(\alpha(n - n'\lambda)t\right)dt = \frac{1}{\tau}\int_0^\tau Q(n - n'\lambda)\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\cos 2\left(\alpha(n - n'\lambda)t\right)dt \tag{2.20}$$

$$B_\alpha = \frac{1}{\tau}\int_0^\tau f(A)\sin 2\left(\alpha(n - n'\lambda)t\right)dt = \frac{1}{\tau}\int_0^\tau Q(n - n'\lambda)\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\sin 2\left(\alpha(n - n'\lambda)t\right)dt \tag{2.21}$$

where τ is the period of the function under analysis, here it is the period of the latent vibration of the CCW generated by the beating of the human heart.

In this work, we define $\tau(n - n'\lambda) = 2\pi$. Let us now evaluate (2.19) for A_0. Direct integration and rearrangement gives

$$A_0 = \frac{Q}{2\tau}\left\{\cos 2(\varepsilon - \varepsilon'\lambda) - \cos 2\left((n - n'\lambda)\tau - (\varepsilon - \varepsilon'\lambda)\right)\right\} \tag{2.22}$$

$$A_0 = \frac{(a - b\lambda)^4 (n - n'\lambda)}{8\pi(a^2 - b^2\lambda^2)^{3/2}}\left\{\cos 2(\varepsilon - \varepsilon'\lambda) - \cos 2\left((2\pi - (\varepsilon - \varepsilon'\lambda)\right)\right\} \tag{2.23}$$

Thedimension of A_0 in m/s. Hence, when we substitute (2.23) into (2.18), we get

$$C_0 = -\frac{(a - b\lambda)^4 (n - n'\lambda)}{8\pi(a^2 - b^2\lambda^2)^{3/2}\sin 2(\varepsilon - \varepsilon'\lambda)}\left\{\cos 2(\varepsilon - \varepsilon'\lambda) - \cos 2\left((2\pi - (\varepsilon - \varepsilon'\lambda)\right)\right\} \tag{2.24}$$

By invoking the rule of compound angles in trigonometry, see appendix, we can further simplify (2.24) to yield

$$C_0 = \left(\frac{(a-b\lambda)^4 (n-n'\lambda)}{4\pi(a^2 - b^2\lambda^2)^{3/2} \sin 2(\varepsilon - \varepsilon'\lambda)} \right) \left\{ \sin 2(\pi) \sin 2\big((\varepsilon - \varepsilon'\lambda) - \pi\big) \right\} \tag{2.25}$$

Hence, C_0 has the dimension of velocity. Also from (2.20) we can solve for A_α as follows. Let us first use trigonometric identity $2\sin x \cos y = \sin(x+y) + \sin(x-y)$, to further reduce (2.20) so that

$$A_\alpha = \frac{Q(n-n'\lambda)}{2\tau} \int_0^\tau \left\{ \sin 2\big((1+\alpha)(n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\big) + \sin 2\big((1-\alpha)(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\big) \right\} dt \tag{2.26}$$

$$A_\alpha = \frac{Q(n-n'\lambda)}{2\tau} \left\{ -\left(\frac{1}{2(1+\alpha)(n-n'\lambda)} \Big(\cos 2\big((1+\alpha)(n-n'\lambda)\tau - (\varepsilon-\varepsilon'\lambda)\big) - \cos 2(-(\varepsilon-\varepsilon'\lambda)) \Big) \right) \right\} -$$
$$\frac{Q(n-n'\lambda)}{2\tau} \left\{ -\left(\frac{1}{2(1-\alpha)(n-n'\lambda)} \Big(\cos 2\big((1-\alpha)(n-n'\lambda)\tau - (\varepsilon-\varepsilon'\lambda)\big) - \cos 2(-(\varepsilon-\varepsilon'\lambda)) \Big) \right) \right\} \tag{2.27}$$

The second term on the right side of (2.27) is ignored since if $\alpha = 1$ according to the summation rule the expression in the parenthesis will otherwise be infinite and will not be useful in this work. So that

$$A_\alpha = \frac{(a-b\lambda)^4 (n-n'\lambda)}{16\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)} \left\{ \cos 2(\varepsilon-\varepsilon'\lambda) - \cos 2\big((1+\alpha)2\pi - (\varepsilon-\varepsilon'\lambda)\big) \right\} \tag{2.28}$$

$$A_\alpha = -\frac{(a-b\lambda)^4 (n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)} \left\{ \sin 2\big((1+\alpha)\pi\big) \sin 2\big((\varepsilon-\varepsilon'\lambda) - (1+\alpha)\pi\big) \right\} \tag{2.29}$$

where $\alpha = 1, 2, 3, \ldots, \infty$, and therefore leaving the dimension of A_α in m/s. Finally by following the same step and procedure that led to (2.29) we can solve for B_α as follows.

$$B_\alpha = \frac{Q(n-n'\lambda)}{2\tau} \int_0^\tau \left\{ \cos 2\big((1-\alpha)(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\big) - \cos 2\big((1+\alpha)(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\big) \right\} dt \tag{2.30}$$

$$B_\alpha = \frac{Q(n-n'\lambda)}{2\tau} \left\{ \frac{1}{2(1-\alpha)(n-n'\lambda)} \Big(\sin 2\big((1-\alpha)(n-n'\lambda)\tau - (\varepsilon-\varepsilon'\lambda)\big) - \sin 2(-(\varepsilon-\varepsilon'\lambda)) \Big) \right\} -$$
$$\frac{Q(n-n'\lambda)}{2\tau} \left\{ \frac{1}{2(1+\alpha)(n-n'\lambda)} \Big(\sin 2\big((1+\alpha)(n-n'\lambda)\tau - (\varepsilon-\varepsilon'\lambda)\big) - \sin 2(-(\varepsilon-\varepsilon'\lambda)) \Big) \right\} \tag{2.31}$$

The first term in (2.31) is also ignored since if $\alpha = 1$ according to the summation rule the expression in the parenthesis will be infinite.

$$B_\alpha = -\frac{(a-b\lambda)^4 (n-n'\lambda)}{16\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)} \left\{ \sin 2(\varepsilon-\varepsilon'\lambda) + \sin 2\big((1+\alpha)2\pi - (\varepsilon-\varepsilon'\lambda)\big) \right\} \tag{2.32}$$

$$B_\alpha = -\frac{(a-b\lambda)^4 (n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)} \left\{ \sin 2\big((1+\alpha)\pi\big) \cos 2\big((\varepsilon-\varepsilon'\lambda) - (1+\alpha)\pi\big) \right\} \tag{2.33}$$

$\alpha = 1, 2, 3, \ldots, \infty$. Where we have used the fact that $\sin(-\theta) = -\sin\theta$ (odd and antisymmetric function), thereby leaving the dimension of B_α the same as m/s.

Upon adding the squares of (2.29) and (2.33) then the Fourier coefficients C_α in (2.17) becomes

$$C_\alpha = \left(\frac{(a-b\lambda)^4 (n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)} \right) \sin 2\big((1+\alpha)\ \pi\big) \tag{2.34}$$

Then finally, we can now substitute (2.25) and (2.34) into (2.15) so that the Fourier analysis of the oscillating amplitude of the CCW becomes

$$F\big[f(A)\big]=\left(\frac{(a-b\lambda)^4(n-n'\lambda)\sin 2(\pi)\sin 2\big((\varepsilon-\varepsilon'\lambda)-\pi\big)}{4\pi(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)}\right)+\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}}\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\alpha)}\right)\times$$
$$\big\{\sin 2\big((1+\alpha)\ \pi\big)\times\sin 2\big(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\big\} \quad (2.35)$$

Equation (2.35) represents the Fourier transform of the oscillating amplitude of the CCW.

2.5 Fourier series expansion of the spatial oscillating phase $f(\theta)$ of the CCW.

$$F\big[f(\theta)\big]=C_0+C_1\cos\big(\vec{k}.\hat{r}-\big((n-n'\lambda)t+E(t)\big)\big)+C_2\cos\big(\vec{k}.\hat{r}-2\big((n-n'\lambda)t+E(t)\big)\big)+$$
$$C_3\cos\big(\vec{k}.\hat{r}-3\big((n-n'\lambda)t+E(t)\big)\big)+\ldots+C_\beta\cos\big(\vec{k}.\hat{r}-\beta\big((n-n'\lambda)t+E(t)\big)\big) \quad (2.36)$$

$$F\big[f(\theta)\big]=C_0+\sum_{\beta=1}^{\infty}C_\beta\cos\big(\vec{k}.\hat{r}-\beta\big((n-n'\lambda)t+E(t)\big)\big) \quad (2.37)$$

However, there is need to separate the function in the summation sign into two components.

$$C_\beta\cos\big(\vec{k}.\hat{r}-\beta\big((n-n'\lambda)t+E(t)\big)\big)=A_\beta\cos\beta\big((n-n'\lambda)t+E(t)\big)+B_\beta\sin\ \beta\big((n-n'\lambda)t+E(t)\big) \quad (2.38)$$

With the assumption that

$$\left.\begin{aligned}A_\beta&=C_\beta\cos\big(\vec{k}..\hat{r}\big)\\B_\beta&=-C_\beta\sin\big(\vec{k}..\hat{r}\big)\end{aligned}\right\}\quad\Rightarrow\quad C_\beta=\sqrt{A_\beta^2+B_\beta^2} \quad (2.39)$$

When we make the assumption that $\beta=0$, then (2.38) approximates to

$$C_0=\frac{1}{\cos\big(\vec{k}.\hat{r}\big)}A_0 \quad (2.40)$$

where A_0 , A_β and B_β are the Fourier coefficients of the series expansion of the CCW to be determined.

2.6 Determination of the Fourier coefficients of the spatial oscillating phase $f(\theta)$ of the CCW.

The Fourier coefficients of $F\big[f(\theta)\big]$ in (2.39) are given by the Euler formulas

$$A_0=\frac{1}{\tau}\int_0^\tau f(\theta)\,dt\quad=\frac{1}{\tau}\int_0^\tau\cos\big(\vec{k}.\hat{r}-(n-n'\lambda)t-E(t)\big)dt \quad (2.41)$$

$$A_\beta=\frac{1}{\tau}\int_0^\tau f(\theta)\cos\beta\big((n-n'\lambda)t+E(t)\big)dt=\frac{1}{\tau}\int_0^\tau\cos\big(\vec{k}.\hat{r}-(n-n'\lambda)t-E(t)\big)\cos\beta\big((n-n'\lambda)t+E(t)\big)dt$$
$$(2.42)$$

$$B_\beta=\frac{1}{\tau}\int_0^\tau f(\theta)\sin\beta\big((n-n'\lambda)t+E(t)\big)dt\ =\frac{1}{\tau}\int_0^\tau\cos\big(\vec{k}.\hat{r}-(n-n'\lambda)t-E(t)\big)\sin\beta\big((n-n'\lambda)t+E(t)\big)dt$$
$$(2.43)$$

To integrate (2.41) we should know that the total phase angle E is also a function of time t .
Thus by substitution method we simply write

$$u=\vec{k}.\hat{r}-(n-n'\lambda)t-E(t)\ \Rightarrow\ \frac{du}{dt}=-(n-n'\lambda)+Z(t)\ \Rightarrow dt=-\left(\frac{1}{(n-n'\lambda)-Z(t)}\right)du \quad (2.44)$$

Then

$$A_0=-\frac{1}{\tau}\left\{\frac{\sin\big(\vec{k}.\hat{r}-(n-n'\lambda)\tau-E(\tau)\big)}{\big((n-n'\lambda)-Z(\tau)\big)}-\frac{\sin\big(\vec{k}.\hat{r}-E(0)\big)}{\big((n-n'\lambda)-Z(0)\big)}\right\} \quad (2.45)$$

$$A_0=\frac{(n-n'\lambda)}{2\pi}\left\{\frac{\sin\big(\vec{k}.\hat{r}-E(0)\big)}{\big((n-n'\lambda)-Z(0)\big)}-\frac{\sin\big(\vec{k}.\hat{r}-2\pi-E(\tau)\big)}{\big((n-n'\lambda)-Z(\tau)\big)}\right\} \quad (2.46)$$

$$C_0=\frac{(n-n'\lambda)}{2\pi\cos\big(\vec{k}.\hat{r}\big)}\left\{\frac{\sin\big(\vec{k}.\hat{r}-E(0)\big)}{\big((n-n'\lambda)-Z(0)\big)}-\frac{\sin\big(\vec{k}.\hat{r}-2\pi-E(\tau)\big)}{\big((n-n'\lambda)-Z(\tau)\big)}\right\} \quad (2.47)$$

Also upon using trigonometric relations we can further reduce (2.42) as

$$A_\beta = \frac{1}{2\tau}\left\{\int_0^\tau \cos\left(\vec{k}.\hat{r} - (1-\beta)(n-n'\lambda)t - (1-\beta)E(t)\right)dt + \int_0^\tau \cos\left(\vec{k}.\hat{r} - (1+\beta)(n-n'\lambda)t - (1+\beta)E(t)\right)dt\right\} \quad (2.48)$$

$$A_\beta = \frac{1}{2\tau}\left\{-\left(\frac{\sin\left(\vec{k}.\hat{r} - (1-\beta)\left((n-n'\lambda)\tau + E(\tau)\right)\right)}{(1-\beta)\left((n-n'\lambda) - Z(\tau)\right)}\right) + \left(\frac{\sin\left(\vec{k}.\hat{r} - (1-\beta)E(0)\right)}{(1-\beta)\left((n-n'\lambda) - Z(0)\right)}\right)\right\} +$$
$$\frac{1}{2\tau}\left\{-\left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)\left((n-n'\lambda)\tau + E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda) - Z(\tau)\right)}\right) + \left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda) - Z(0)\right)}\right)\right\} \quad (2.49)$$

The first term on the right side of (2.49) is ignored since it becomes infinite if $\beta = 1$. As a result,

$$A_\beta = \frac{(n-n'\lambda)}{4\pi}\left\{\left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda) - Z(0)\right)}\right) - \left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)\left(2\pi + E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda) - Z(\tau)\right)}\right)\right\} \quad (2.50)$$

Finally, when we follow the same arithmetic method that led to (2.50) we can solve for B_β in (2.43).

$$B_\beta = \frac{1}{2\tau}\left\{\int_0^\tau \sin\left(\vec{k}.\hat{r} - (1-\beta)\left((n-n'\lambda)t - E(t)\right) - (1-\beta)E(t)\right) - \int_0^\tau \sin\left(\vec{k}.\hat{r} - (1+\beta)(n-n'\lambda)t - (1+\beta)E(t)\right)\right\}dt \quad (2.51)$$

$$B_\beta = \frac{(n-n'\lambda)}{4\pi}\left\{\left(\frac{\cos\left(\vec{k}.\hat{r} - (1+\beta)\left(2\pi + E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda) - Z(\tau)\right)}\right) - \left(\frac{\cos\left(\vec{k}.\hat{r} - (1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda) - Z(0)\right)}\right)\right\} \quad (2.52)$$

Eventually upon adding the squares of (2.50) and (2.52) we obtain

$$C_\beta^2 = \left\{\frac{(n-n'\lambda)}{4\pi}\left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda) - Z(0)\right)}\right) - \left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)\left(2\pi + E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda) - Z(\tau)\right)}\right)\right\}^2 +$$
$$\left\{\frac{(n-n'\lambda)}{4\pi}\left(\frac{\cos\left(\vec{k}.\hat{r} - (1+\beta)\left(2\pi + E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda) - Z(\tau)\right)}\right) - \left(\frac{\cos\left(\vec{k}.\hat{r} - (1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda) - Z(0)\right)}\right)\right\}^2 \quad (2.53)$$

$$C_\beta = \left(\frac{(n-n'\lambda)}{4\pi(1+\beta)}\right)\left\{\frac{\left((n-n'\lambda) - Z(\tau)\right)^2 + \left((n-n'\lambda) - Z(0)\right)^2}{\left((n-n'\lambda) - Z(0)\right)^2\left((n-n'\lambda) - Z(\tau)\right)^2} - \frac{2\cos\left((1+\beta)(2\pi + E(\tau) - E(0)\right)}{\left((n-n'\lambda) - Z(0)\right)\left((n-n'\lambda) - Z(\tau)\right)}\right\}^{\frac{1}{2}} \quad (2.54)$$

Consequently, when we substitute the values of C_0 and C_β into (2.37) and after some lengthy algebra it can be shown that the Fourier transform of the spatial oscillating phase of the CCW is given by

$$F\left[f(\theta)\right] = \frac{(n-n'\lambda)}{2\pi\cos\left(\vec{k}.\hat{r}\right)}\left\{\frac{\sin\left(\vec{k}.\hat{r} - E(0)\right)}{\left((n-n'\lambda) - Z(0)\right)} - \frac{\sin\left(\vec{k}.\hat{r} - 2\pi - E(\tau)\right)}{\left((n-n'\lambda) - Z(\tau)\right)}\right\} +$$
$$\left(\frac{(n-n'\lambda)}{4\pi}\right)\sum_{\beta=1}^{\infty}\left(\frac{1}{1+\beta}\right)\left\{\frac{\left((n-n'\lambda) - Z(\tau)\right)^2 + \left((n-n'\lambda) - Z(0)\right)^2}{\left((n-n'\lambda) - Z(0)\right)^2\left((n-n'\lambda) - Z(\tau)\right)^2} - \right.$$
$$\left.\frac{2\cos\left((1+\beta)\,2\pi + E(\tau) - E(0)\right)}{\left((n-n'\lambda) - Z(0)\right)\left((n-n'\lambda) - Z(\tau)\right)}\right\}^{\frac{1}{2}}\cos\left(\vec{k}.\hat{r} - \beta\left((n-n'\lambda)t + E(t)\right)\right) \quad (2.55)$$

It is clear that (2.55) has no unit of dimension. We have from (2.5) and (2.6) that

$$E(\tau) = \tan^{-1}\left(\frac{a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda - 2\pi)}{a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - 2\pi)}\right) \quad ; \quad E(0) = \tan^{-1}\left(\frac{a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda)}{a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda)}\right) \tag{2.56}$$

$$Z(t) = (n - n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos\big((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t\big)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos\big((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t\big)}\right) \tag{2.57}$$

$$Z(0) = (n - n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos(\varepsilon - \varepsilon'\lambda)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos(\varepsilon - \varepsilon'\lambda)}\right) ; Z(\tau) = (n - n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos\big((\varepsilon - \varepsilon'\lambda) + 2\pi\big)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos\big((\varepsilon - \varepsilon'\lambda) + 2\pi\big)}\right) \tag{2.58}$$

2.7 Convolution theory of the Fourier transform of the amplitude and the spatial oscillating phase of the CCW.

Now that we have separately determined the Fourier transform of the oscillating amplitude $F\big[f(A)\big]$ and the spatial oscillating phase $F\big[f(\theta)\big]$ respectively the necessary requirement now is to convolute them in order to obtain a concise equation of the CCW in the frequency time domain. Convolution here means multiplying (2.35) by (2.55) term by term. Let us represent the result of the convolution of these functions by H and then with the same displacement vector y which represents the CCW.

$$y = H\big\{F\big[f(A)\big]; F\big[f(\theta)\big]\big\} = F\big[f(A)\big] \otimes F\big[f(\theta)\big] \tag{2.59}$$

$$y = H\big\{F\big[f(A)\big]; F\big[f(\theta)\big]\big\} = \left(\frac{(a - b\lambda)^4 (n - n'\lambda)^2 \sin 2(\pi)\sin 2\big((\varepsilon - \varepsilon'\lambda) - \pi\big)}{8\pi^2 (a^2 - b^2\lambda^2)^{3/2} \sin 2(\varepsilon - \varepsilon'\lambda)\cos\big(\vec{k}.\hat{r}\big)}\right) \times$$
$$\left\{\frac{\sin\big(\vec{k}.\hat{r} - E(0)\big)}{\big((n - n'\lambda) - Z(0)\big)} - \frac{\sin\big(\vec{k}.\hat{r} - 2\pi - E(\tau)\big)}{\big((n - n'\lambda) - Z(\tau)\big)}\right\} + \left(\frac{(a - b\lambda)^4 (n - n'\lambda)^2 \sin 2(\pi)\sin 2\big((\varepsilon - \varepsilon'\lambda) - \pi\big)}{16\pi^2 (a^2 - b^2\lambda^2)^{3/2} \sin 2(\varepsilon - \varepsilon'\lambda)}\right) \times$$
$$\sum_{\beta=1}^{\infty}\left(\frac{1}{1+\beta}\right)\left\{\frac{\big((n - n'\lambda) - Z(\tau)\big)^2 + \big((n - n'\lambda) - Z(0)\big)^2}{\big((n - n'\lambda) - Z(0)\big)^2\big((n - n'\lambda) - Z(\tau)\big)^2} - \frac{2\cos\big((1+\beta)2\pi + E(\tau) - E(0)\big)}{\big((n - n'\lambda) - Z(0)\big)\big((n - n'\lambda) - Z(\tau)\big)}\right\}^{\frac{1}{2}} \times$$
$$\cos\big(\vec{k}.\hat{r} - \beta\big((n - n'\lambda)t + E(t)\big)\big) + \left(\frac{(a - b\lambda)^4 (n - n'\lambda)^2 \sin 2(\pi)\sin 2\big((\varepsilon - \varepsilon'\lambda) - \pi\big)}{16\pi^2 (a^2 - b^2\lambda^2)^{3/2} \cos\big(\vec{k}.\hat{r}\big)}\right) \times$$
$$\left\{\frac{\sin\big(\vec{k}.\hat{r} - E(0)\big)}{\big((n - n'\lambda) - Z(0)\big)} - \frac{\sin\big(\vec{k}.\hat{r} - 2\pi - E(\tau)\big)}{\big((n - n'\lambda) - Z(\tau)\big)}\right\} \times \sum_{\beta=1}^{\infty}\left(\frac{1}{(1+\beta)}\right) \big\{\sin 2\big((1+\beta)\ \pi\big)\sin 2\big(\beta(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\big)\big\} +$$
$$\left(\frac{(a - b\lambda)^4 (n - n'\lambda)^2}{32\pi^2 (a^2 - b^2\lambda^2)^{3/2}}\right) \sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)^2 \big\{\sin 2\big((1+\beta)\ \pi\big)\sin 2\big(\beta(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\big)\big\} \times$$
$$\left\{\frac{\big((n - n'\lambda) - Z(\tau)\big)^2 + \big((n - n'\lambda) - Z(0)\big)^2}{\big((n - n'\lambda) - Z(0)\big)^2\big((n - n'\lambda) - Z(\tau)\big)^2} - \frac{2\cos\big((1+\beta)2\pi + E(\tau) - E(0)\big)}{\big((n - n'\lambda) - Z(0)\big)\big((n - n'\lambda) - Z(\tau)\big)}\right\}^{\frac{1}{2}} \times$$
$$\cos\big(\vec{k}.\hat{r} - \beta\big((n - n')t + E(t)\big)\big) \tag{2.60}$$

In most cases, equation (2.60) may yield imaginary values and not absolute values due to the expression in the square root sign. In order to avoid such unnecessary complications there is need for us to use Binomial expansion to find an approximation to the expression in the square root and once this is done we get

$$y = H\big\{F\big[f(A)\big]; F\big[f(\theta)\big]\big\} = \left(\frac{(a - b\lambda)^4 (n - n'\lambda)^2 \sin 2(\pi)\sin 2\big((\varepsilon - \varepsilon'\lambda) - \pi\big)}{8\pi^2 (a^2 - b^2\lambda^2)^{3/2} \sin 2(\varepsilon - \varepsilon'\lambda)\cos\big(\vec{k}.\hat{r}\big)}\right) \times$$

$$\left\{\frac{\sin\left(\vec{k}.\hat{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\hat{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}+\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)}\right)\times$$

$$\sum_{\beta=1}^{\infty}\left(\frac{1}{1+\beta}\right)\left(\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}\right)^{\frac{1}{2}}\times$$

$$\left\{1-\frac{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)\cos\left((1+\beta)\,2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}\right\}\times$$

$$\cos\left(\vec{k}.\hat{r}-\beta\left((n-n'\lambda)t+E(t)\right)\right)\;+\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^{3/2}\cos\left(\vec{k}.\hat{r}\right)}\right)\times$$

$$\left\{\frac{\sin\left(\vec{k}.\hat{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\hat{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}\times\sum_{\beta=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)\;\left\{\sin 2\left((1+\beta)\;\pi\right)\times\right.$$

$$\left.\sin 2\left(\beta\,(n-n'\lambda)\,t-(\varepsilon-\varepsilon'\lambda)\right)\right\}+$$

$$\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2}{32\pi^2(a^2-b^2\lambda^2)^{3/2}}\right)\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)^2\;\left(\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}\right)^{\frac{1}{2}}\times$$

$$\left\{1-\frac{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)\cos\left((1+\beta)\,2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}\right\}\times$$

$$\cos\left(\vec{k}.\hat{r}-\beta\left((n-n'\lambda)t+E(t)\right)\right)\;\times\;\left\{\sin 2\left((1+\beta)\;\pi\right)\sin 2\left(\beta\,(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}\qquad(2.61)$$

In this work we only considered situation where the constraints are of equal weight, that is $\alpha = \beta$. Otherwise, if we apply the double summation rule as it stands, that means, we shall first allow α take the value of one and let β run from one to infinity, again we allow α take the value of two and let β run from one to infinity and the process is repeated. However, since both constraints are of the same source function we can equate them so as to save us computation time and unnecessary difficult task.

2.8 Calculated values of the dynamical characteristics of the latent vibration of Man represented by the 'host wave' and those of the HIV represented by the 'parasitic wave'.

We have in a previous study (Enaibe and Idiodi, 2013) presented a model for determining the dynamical characteristics of HIV/AIDS in the human blood circulating system. Our work assumes that the physical dynamic components of the HIV responsible for their destructive tendency are $b\lambda$, $n'\lambda$, $\varepsilon'\lambda$ and $k'\lambda$ been influenced by the multiplicative factor λ whose physical range of interest is $0 \le \lambda \le 13070$. In this study, we calculated values of the amplitude $b = 1.60\times10^{-10} m$, angular frequency $n' = 1.91\times10^{-11} rad./s$, phase angle $\varepsilon' = 0.0000466 rad$, the wave number or the spatial frequency $k' = 0.0127 rad./m$ of the HIV parameters and with a slow varying interval of the multiplier $\lambda = 0, 1, 2, 3, \ldots, 13070$. While the dynamical characteristics of the latent vibration of the human blood circulating system caused by the beating of the human heart also from the calculation are; amplitude $a = 2.1\times10^{-6} m$, angular frequency $n = 2.51\times10^{-7} rad./s$, phase angle $\varepsilon = 0.6109 rad$, wave number $k = 166 rad./m$. We also established in the study that the average survival time for HIV/AIDS patient is about 10 years (126 months) counting from the moment the HIV is contacted. However we classified the time interval in seconds as $0 \le t \le 328479340 s$, with a slow varying time interval $t = 0, 1, 2, 3, \ldots, 328479340 s$.

3. Results.

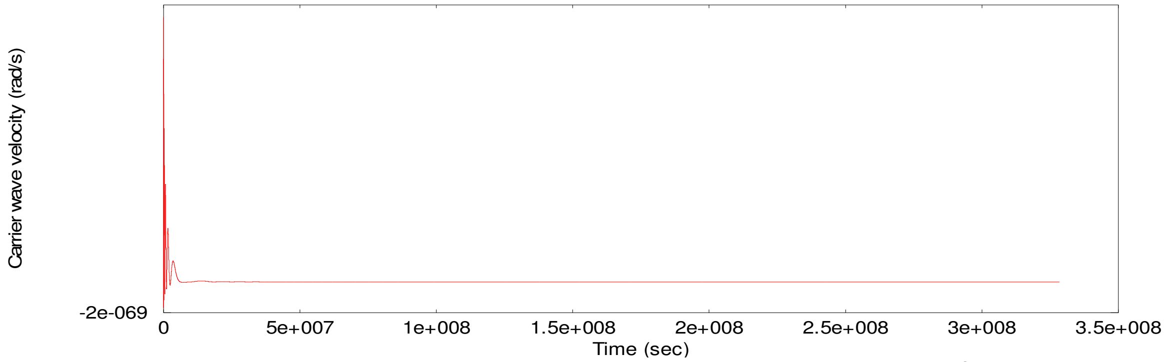


Fig. 3.1: Represents the multiplier λ [0 – 13070] and time [0 – 126 months], β =0 .

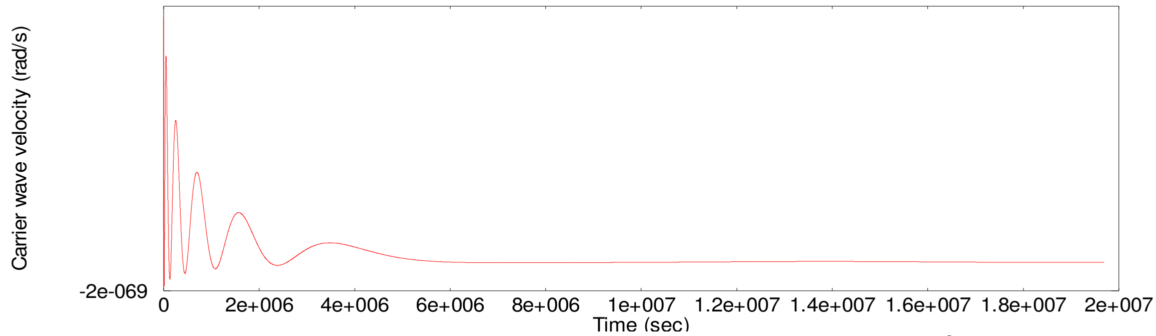


Fig. 3.2: Represents the multiplier λ [0 – 6535] and time [0 – 7 months], β =0 .

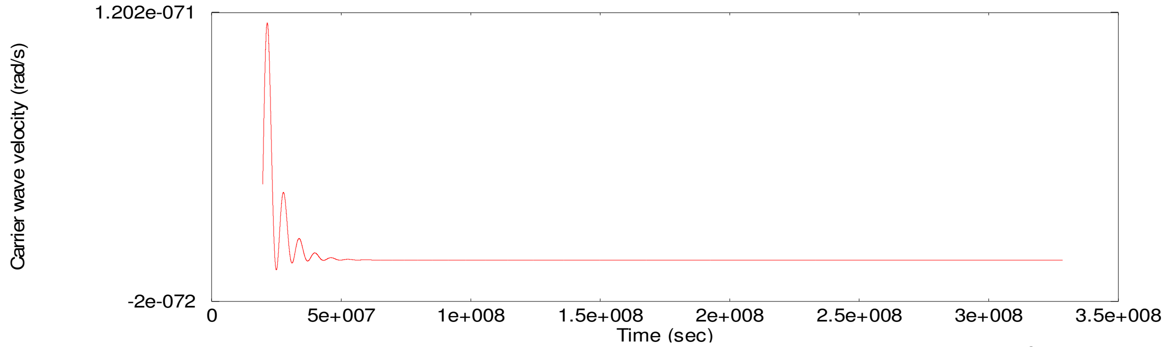


Fig. 3.3: Represents the multiplier λ [6536 – 13070] and time [7 months – 126 months], β =0

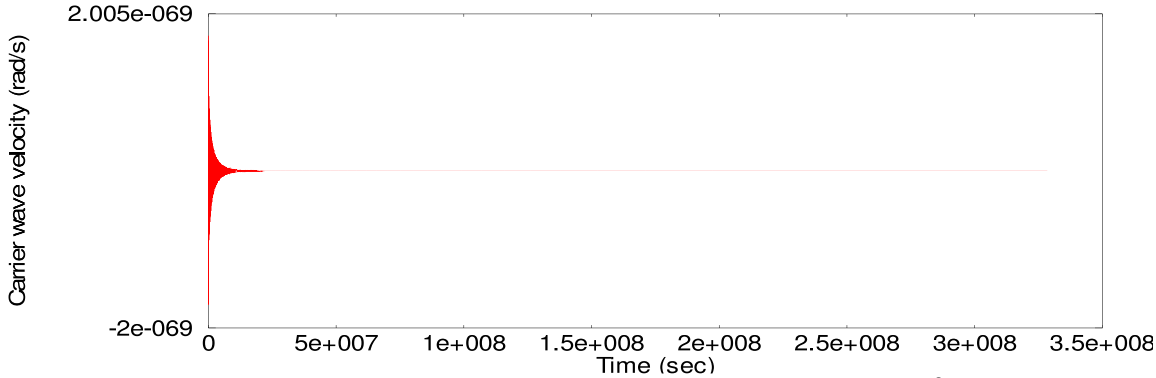


Fig. 3.4: Represents the multiplier λ [0 – 13070] and time [0 – 126 months], β =13070.

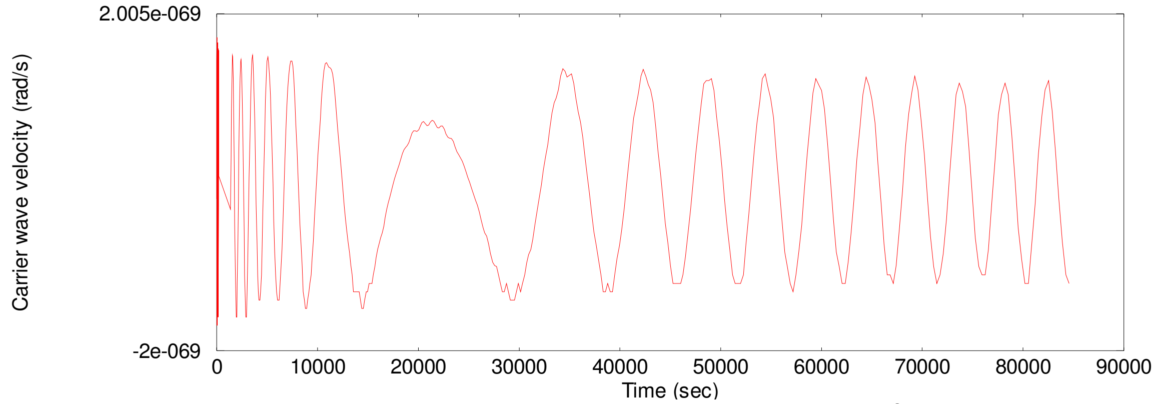


Fig. 3.5: Represents the multiplier λ [0 – 500] and time [0 – 1 day], β =13070.

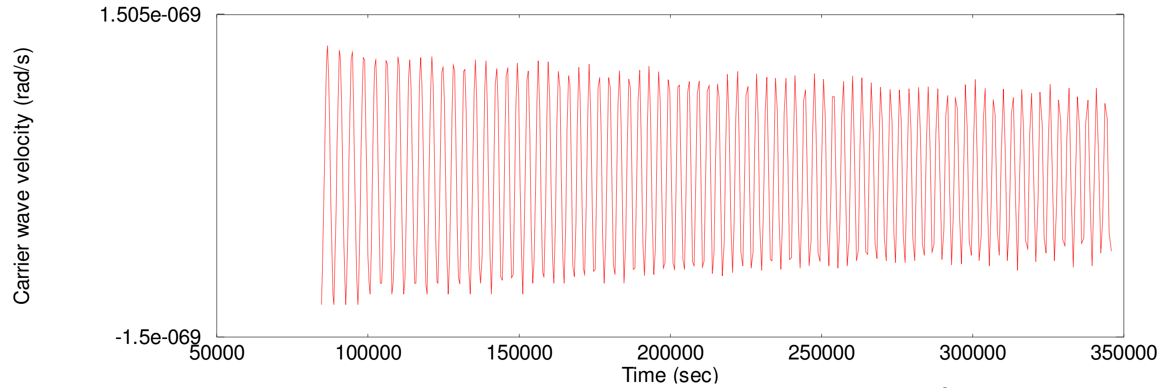


Fig. 3.6: Represents the multiplier λ [500 – 1000] and time [1 day – 4 days], β =13070.

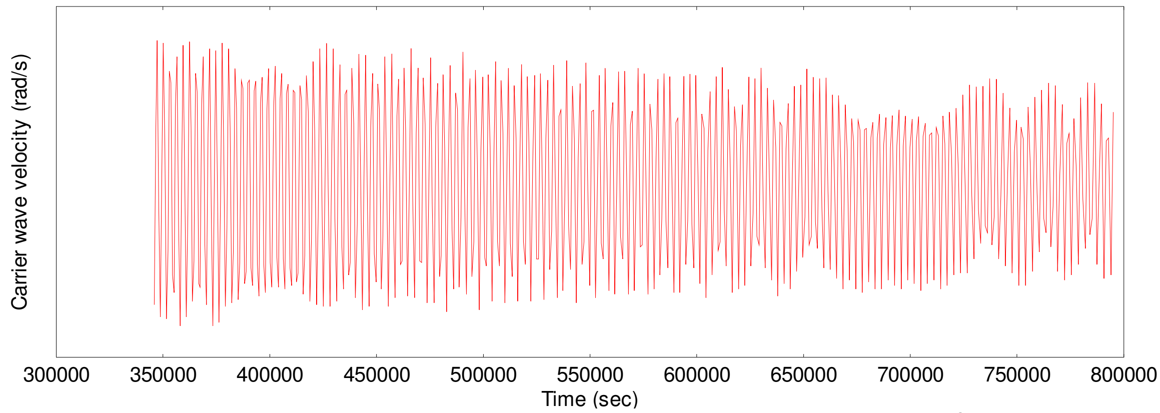


Fig. 3.7: Represents the multiplier λ [1000 – 1500] and time [4 days – 9 days], β =13070.

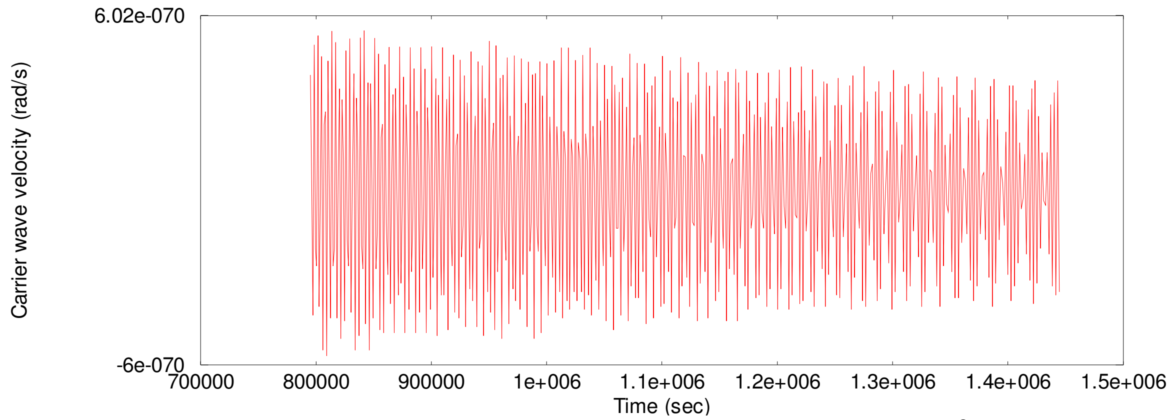


Fig. 3.8: Represents the multiplier λ [1500 – 2000] and time [9 days – 16 days], β =13070.

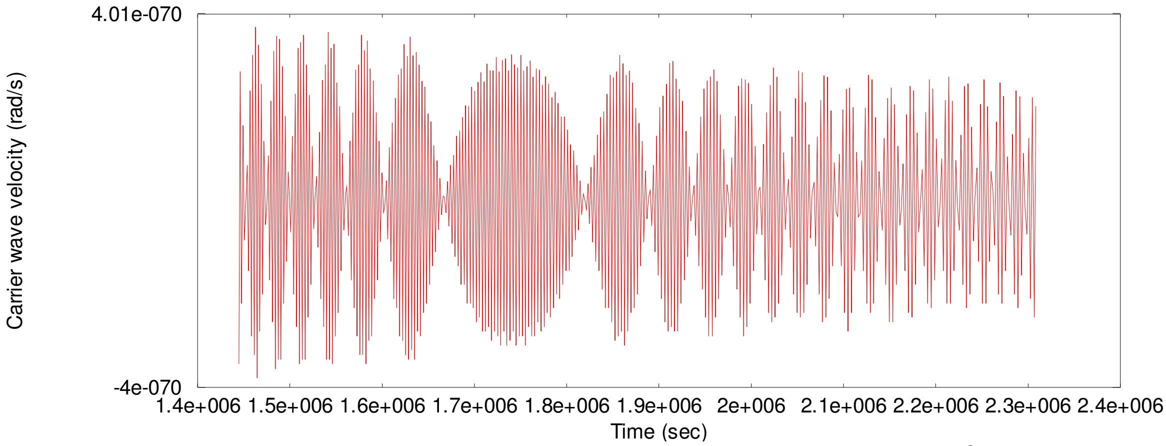


Fig. 3.9: Represents the multiplier λ [2000 – 2500] and time [16 days – 26 days], β =13070.

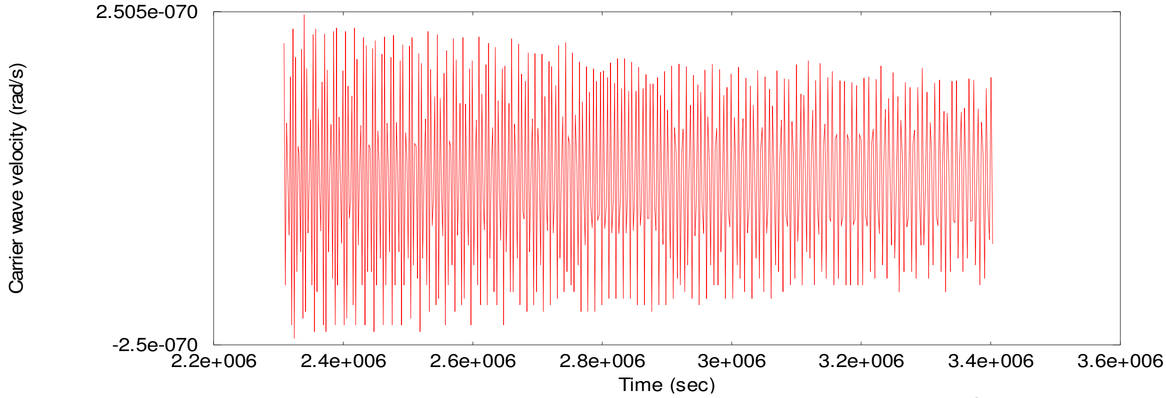


Fig. 3.10: Represents the multiplier λ [2500 – 3000] and time [26 days – 1 month], β =13070.

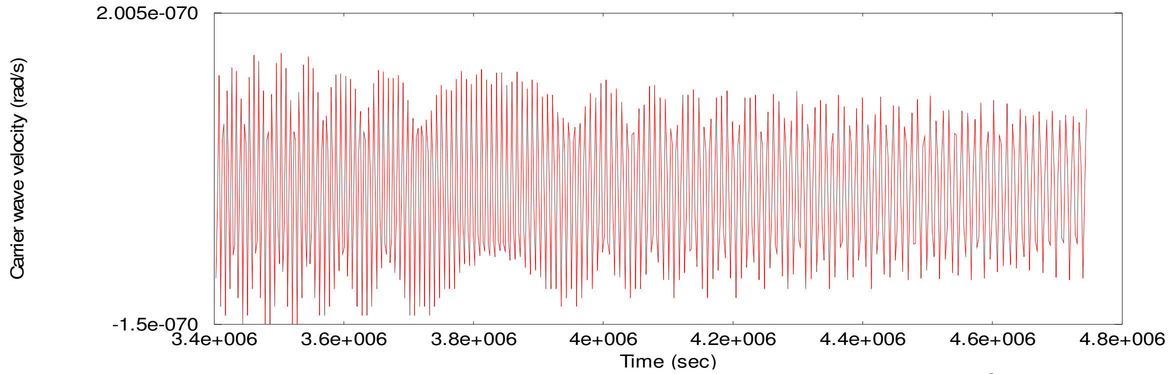


Fig. 3.11: Represents the multiplier λ [3000 – 3500] and time [1 month – 1.8months], β =13070.

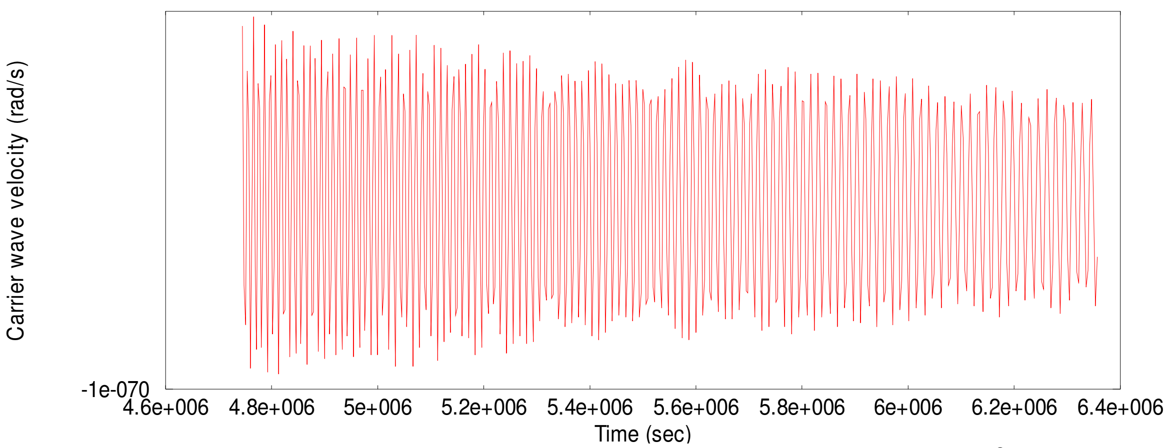


Fig. 3.12: Represents the multiplier λ [3500 – 4000] and time [1.8 months – 2months], β =13070.

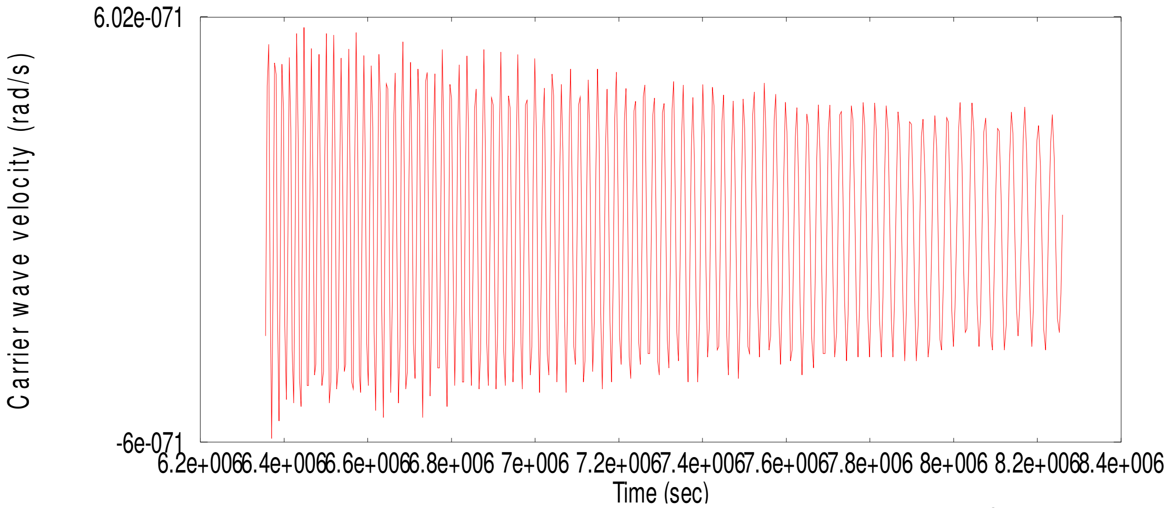


Fig. 3.13: Represents the multiplier λ [4000 – 4500] and time [2 months – 3months], β =13070.

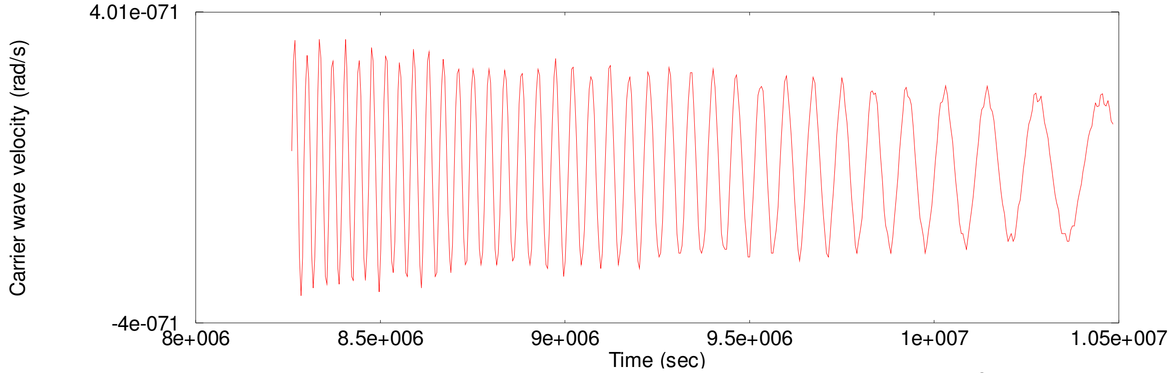


Fig. 3.14: Represents the multiplier λ [4500 – 5000] and time [3 months – 4months], β =13070.

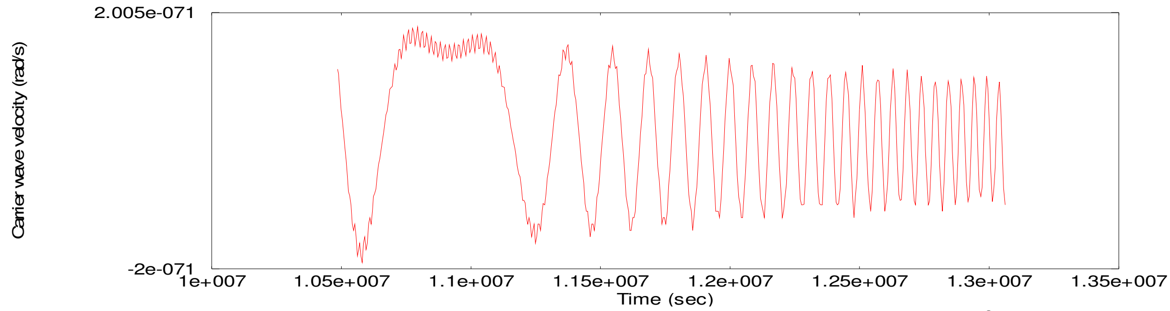


Fig. 3.15: Represents the multiplier λ [5000 – 5500] and time [4 months – 5 months], β =13070.

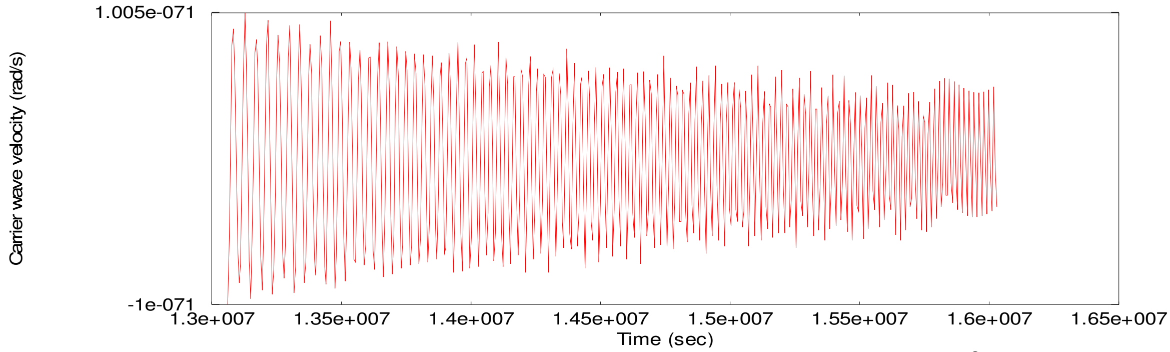


Fig. 3.16: Represents the multiplier λ [5500, 6000] and time [5 months, 6 months], β =13070.

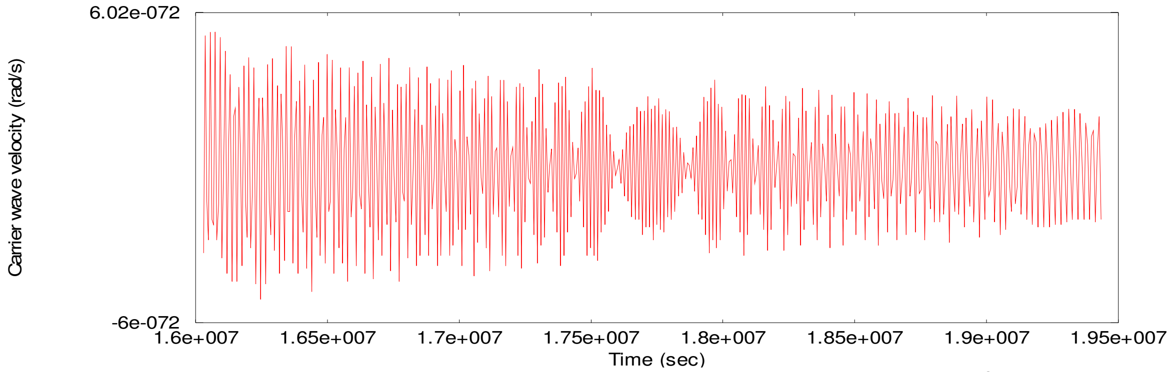


Fig. 3.17: Represents the multiplier λ [6000 – 6500] and time [6 months – 7months], β =13070.

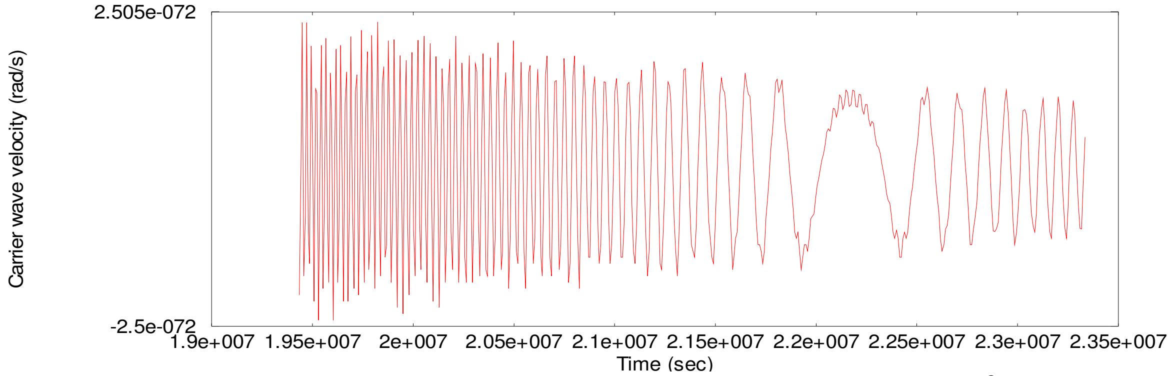


Fig. 3.18: Represents the multiplier λ [6500 – 7000] and time [7 months – 9 months], β =13070.

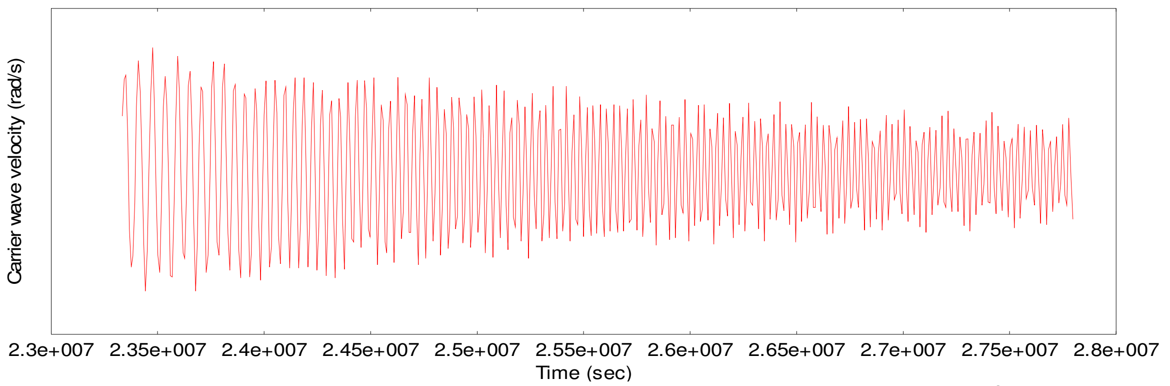


Fig. 3.19: Represents the multiplier λ [7000 – 7500] and time [9 months – 10 months], β =13070.

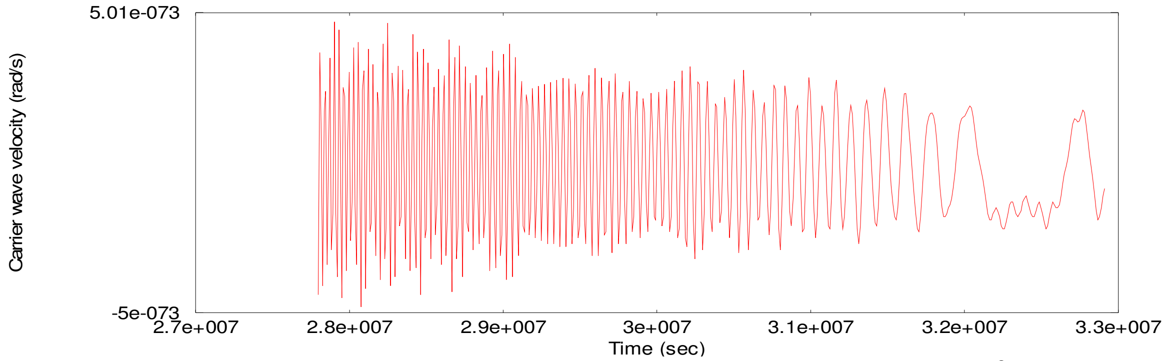


Fig. 3.20: Represents the multiplier λ [7500 – 8000] and time [10 months – 12 months], β =13070,

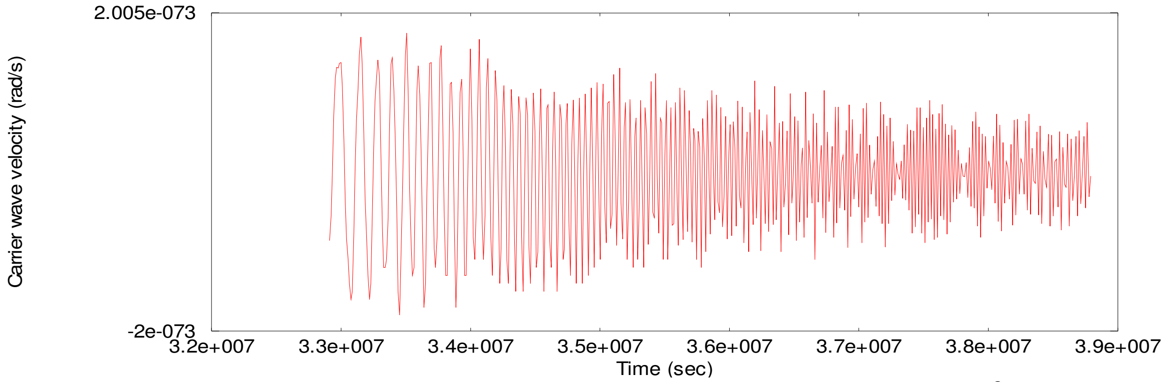


Fig. 3.21: Represents the multiplier λ [8000 – 8500] and time [12 months – 14 months] β =13070.

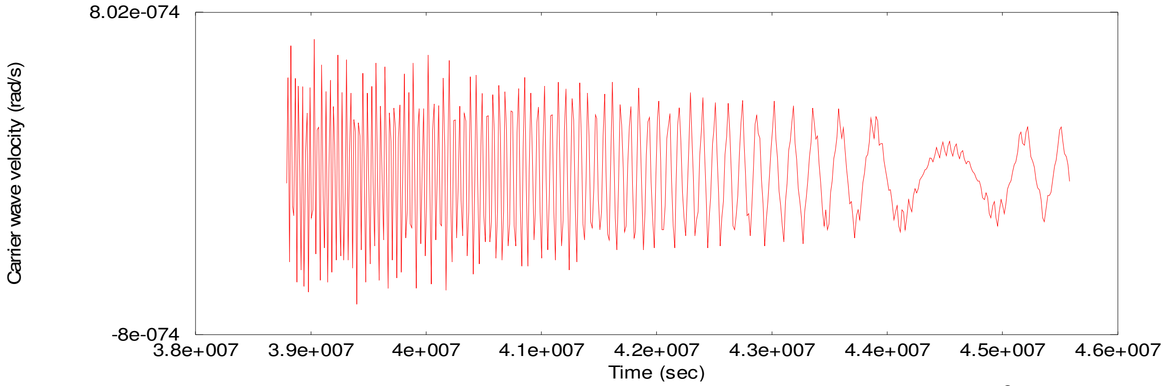


Fig. 3.22: Represents the multiplier λ [8500 – 9000] and time [14 months – 17 months], β =13070.

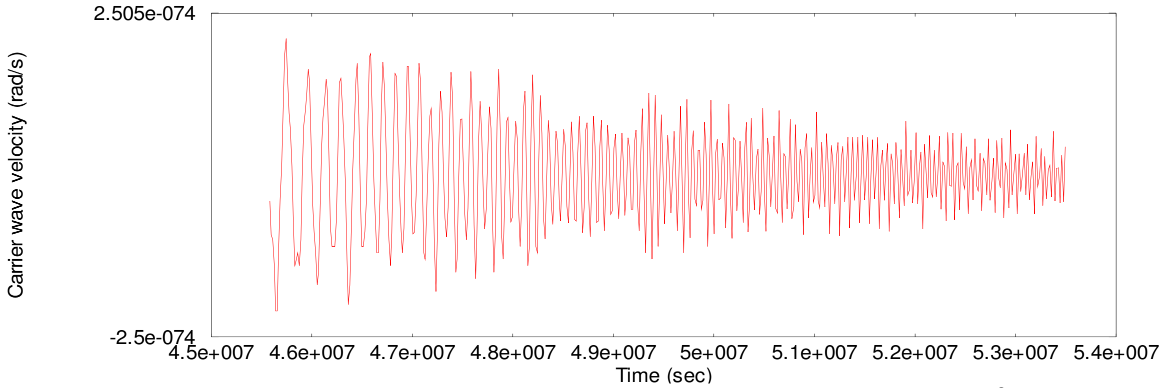


Fig. 3.23: Represents the multiplier λ [9000 – 9500] and time [17 months – 20 months], β =13070.

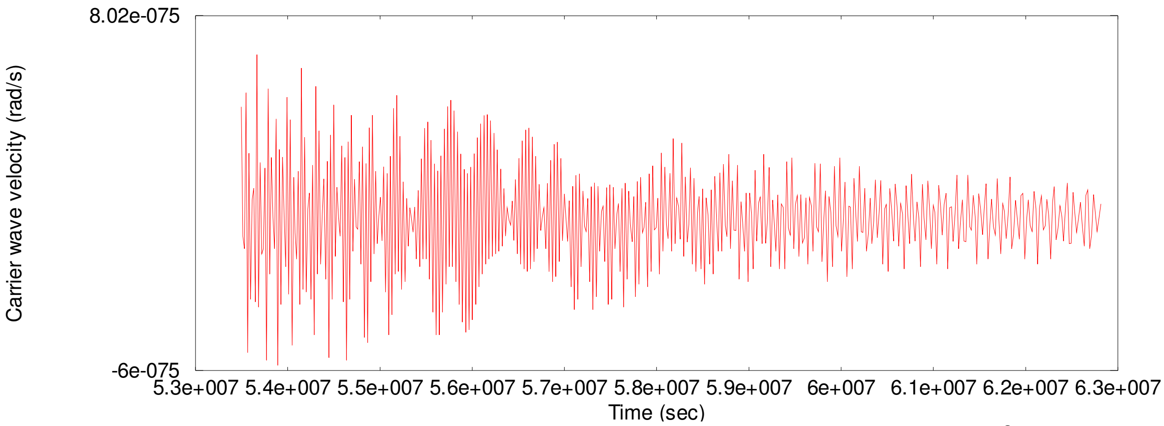


Fig. 3.24: Represents the multiplier λ [9500 – 10000] and time [20 months – 24 months], β =13070.

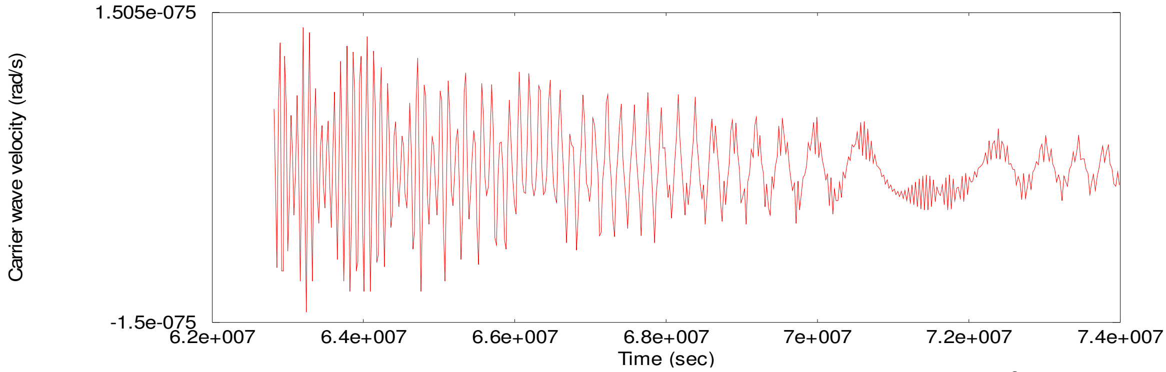


Fig. 3.25: Represents the multiplier λ [10000 – 10500] and time [24 months – 28 months], β =13070.

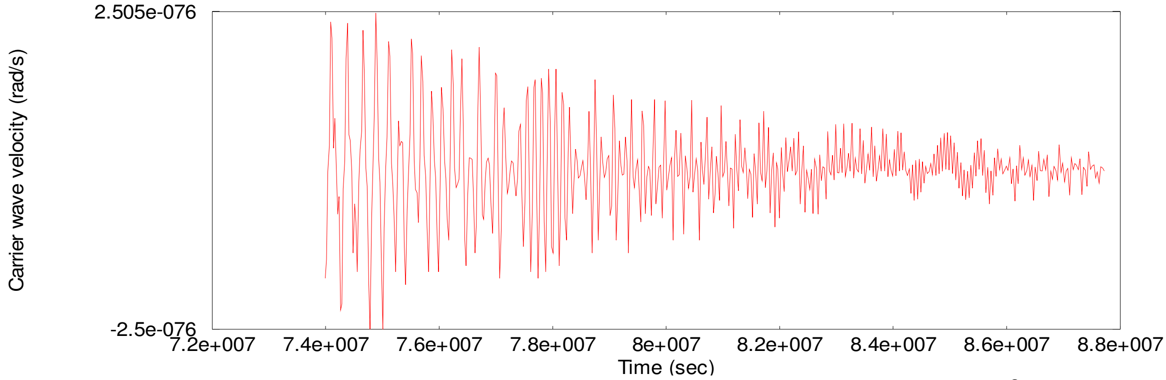


Fig. 3.26: Represents the multiplier λ [10500 – 11000] and time [28 months – 33 months], β =13070.

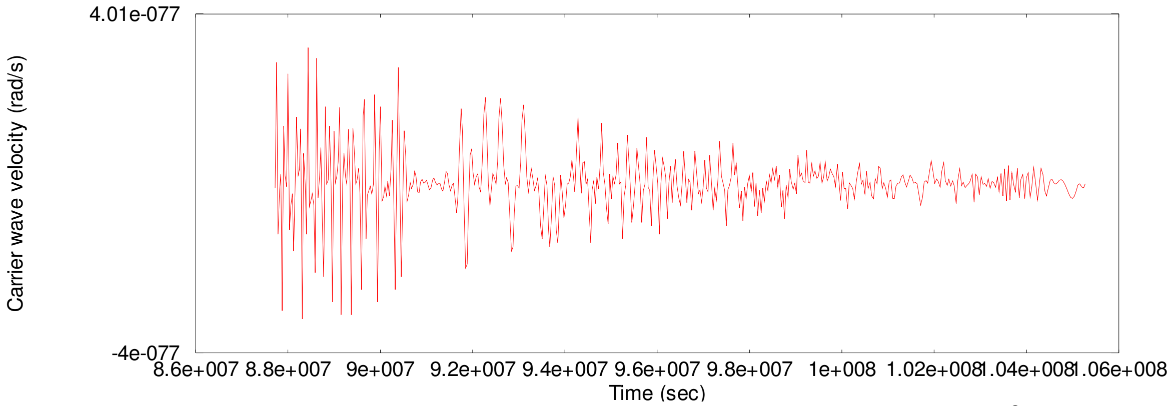


Fig. 3.27: Represents the multiplier λ [11000 – 11500] and time [33 months – 40 months], β =13070.

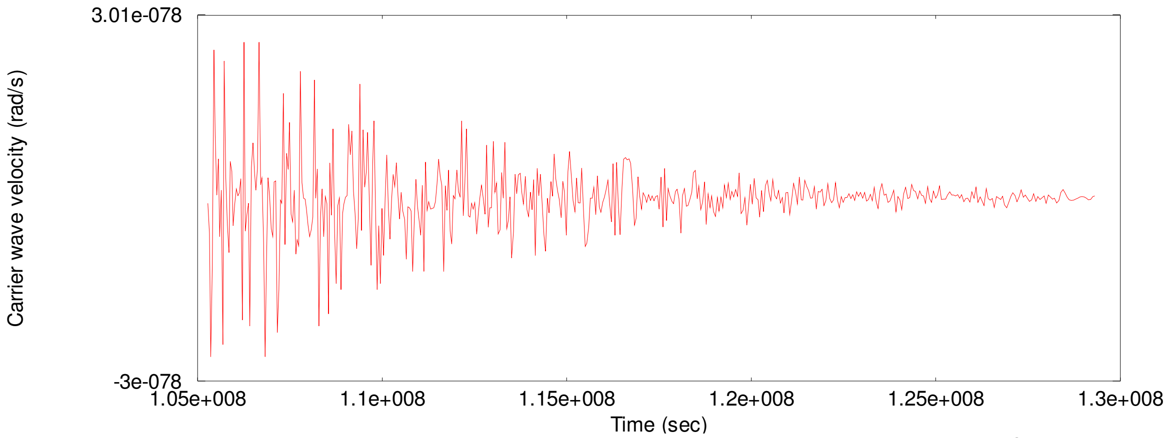
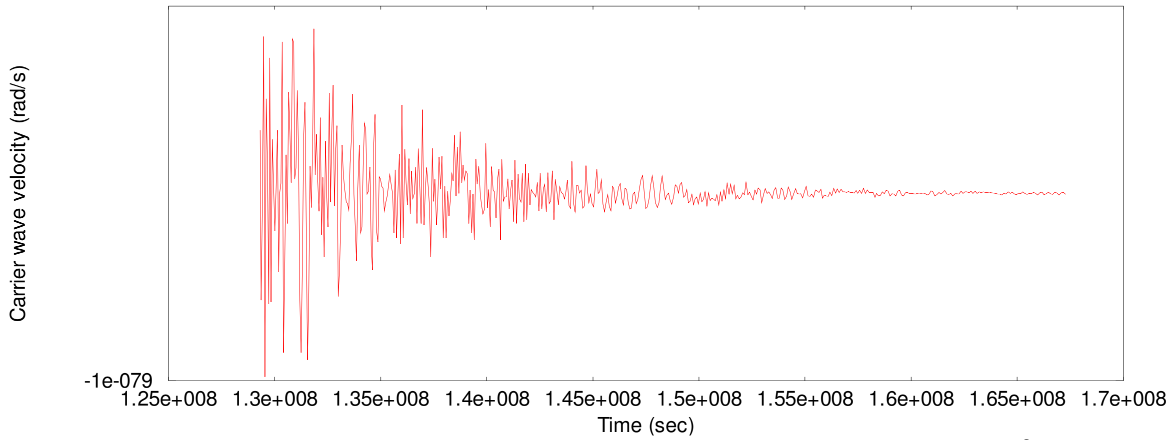


Fig. 3.28: Represents the multiplier λ [11500 – 12000] and time [40 months – 49 months], β =13070.



dFig. 3.29: Represents the multiplier λ [12000 – 12500] and time [49 months – 64 months], β =13070.

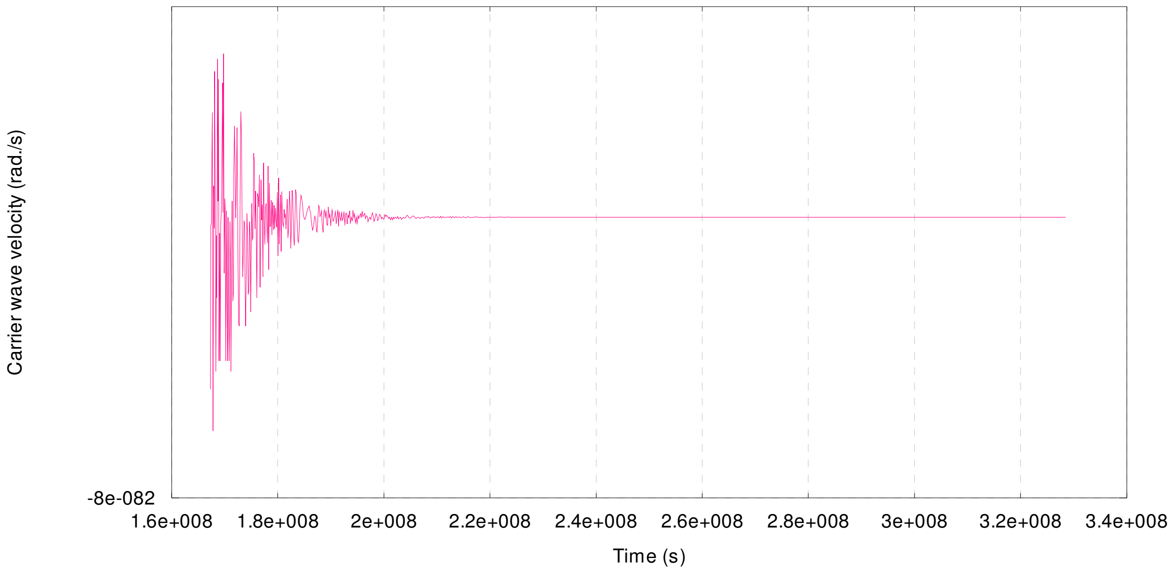


Fig. 3.30: Represents the multiplier λ [12500 – 13070] and time [64 months – 126 months], β =13070.

4. Discussion.

The graph of the velocity gradient of the constituted carrier wave is represented by figs 3.1 – 3.30. It is clear from figs. 3.1 and 3.5, that because of the numerous waveforms involved when the Fourier index β =13070 for every value of the multiplier λ, these figures could not really reflect all the possible waveforms available to the period of time 0 – 126 months that the CCW lasted, as a result, the figures almost displayed a straight line. Consequently, we classified our work based on the interval of the multiplier [0 – 500]. Although, our work was confined to only when the Fourier index was 13070, since we believe that this is the region of most relevant interest to our work. Note that fig. 3.1 which is the first term of equation (2.62) is the harmonic analysis and it does not contain the Fourier index β .

Generally, all the figures show sinusoidal waves which reveal the velocity amplitude fluctuations of the CCW resulting from a spread in the component frequencies. At time t = 0, λ =0 the 'host wave' has initial possible maximum positive radial velocity of 2.005 x 10^{-69} rad. /s and a final minimum positive radial velocity of 8.02 x 10^{-88} rad. /s at time t = 126 months, while an initial maximum negative radial velocity of -2.0 x 10^{-69} rad. /s at time t = 0 and a final minimum negative radial velocity of -6.0 x 10^{-88} rad. /s at time t = 126 months. Positive radial velocity means attraction and hence constructive interference between the 'host wave' and the 'parasitic wave', while negative radial velocity means repulsion and hence destructive interference between them. This information is shown in figs. 3.5 and 3.30. It is the reduction in the radial velocity of the CCW that causes a delay or a slow down process in the energy transfer mechanism which eventually leads to energy attenuation in a HIV/AIDS patient.

It is obvious from fig. 3.5, that between 2 – 8 hours after the HIV infection, that is, immediately when one contacts HIV, the velocity profile of the CCW show something different from usual which indicates the presence of strange manifestations of a velocity-like body. However, this situation is renormalized to a continuous group velocity with high component frequencies after this time. This phenomenon is reflected in figs 3.6 and 3.7. Clearly, the components of the CCW regroup into a continuum velocity with rapid frequencies as indicated in figs. 3.8 – 3.14. This is synonymous with the fact that the process of constant degeneracy in the host system after the HIV infection is not immediate, and that the host system would by itself tends to annul the destructive effect of the interfering HIV.

It is observed that there are certain regions of discontinuity in the velocity profiles of the CCW as shown in figs. 3.15, 3.18, 3.20, 3.22 and 3.25. These regions are remarkably characterised by a reduced frequencies of the components of the CCW, also this formation indicates certain advanced stage of the effect of the HIV vibration in the human system. The time for these remarkable discontinuities in the velocity profiles are 4, 8, 12, 17, and 27 months respectively. To be consistent with the medical requirements these times are regarded as the window periods. The window period signifies the time when the human biological system is now reacting fully to the presence of the HIV due to the noticeable damage it would have done to the velocity of the CCW. Consequently, the window period differs from one individual to another due to different human immune system. However, while it may appear in some individual after 4 months, others could be 8 months or so.

The frequencies and the bandwidth of the velocity profiles of the CCW decrease from figs. 3.26 – 3.30. The spectrum of the velocity profiles becomes parasitically monochromatic beyond 2×10^8 sec or about 77 months (6 years) as shown in fig. 3.30. This however, indicates the prominence of the HIV active components in the CCW. Thus within this region all the active components of the 'host wave' containes in the CCW would have been completely eroded by the interfering HIV 'parasitic wave' thereby rendering the immune system of the host ineffective and non restorable. This situation depicts the possible period of time when the HIV infection degenerates to AIDS. Finally, the velocity of the CCW is brought to zero or rest after 126 months (10 years) as shown in fig. 3.30 and once this stage is reached the phenomenon called death of the host occurs.

5. Conclusion.

The cessation of the velocity of the CCW which describes the coexistence of HIV/AIDS in the system of Man is not instantaneous but gradual. Initially, the biological system of Man tends to annul the destructive influence of the HIV starting from the moment an individual contacted it. In the absence of specific treatment, the HIV infection degenerates to AIDS after about 77 months (6 years). This period involves a steady decay process in the velocity spectrum of the CCW and this result to a rapid weakening in the initial strength of the intrinsic parameters of the host biological system. The velocity of the CCW that describes the biological system of Man finally goes to zero - a phenomenon called death, when the multiplier approaches the critical value of 13070 and the time it takes to attain this value is about 126 months (10 years). It is the reduction in the radial velocity of the CCW that causes a delay or a slow down process in the energy transfer mechanism which eventually leads to energy attenuation in a HIV/AIDS patient. Thus this study has to some extent provided the means of determining the basic activity and performance of HIV in the human blood circulating system. As a result, the HIV can be selectively and discriminately destroyed from the human biological system by anti-vibrating component without causing the slightest harm to the mechanism of the Human system. This work thus identifies the matrix of scientific priorities that should bring us measurably closer to our vision of developing a cure to HIV/AIDS infection.

Suggestions for further work

This study in theory and practice can be extended to investigate wave interference and propagation in two- and three- dimensional systems. The constituted carrier wave CCW that we have developed can be utilized in the deductive and predictive study of wave attenuation in exploration geophysics and telecommunication engineering. This work can also be extended to investigate energy attenuation in HIV/AIDS patients, and most importantly to study wave destruction technique for the control and possible eradication of HIV/AIDS.

APPENDIX

The following is the list of some useful identities which we implemented in the study.

(1) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$; (2) $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

(3) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$; (4) $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

(5) $2\sin x\cos y = \sin(x+y) + \sin(x-y)$; (6) $2\cos x\sin y = \sin(x+y) - \sin(x-y)$

(7) $2\cos x\cos y = \cos(x+y) + \cos(x-y)$; (8) $2\sin x\sin y = \cos(x-y) - \cos(x+y)$

(9) $\sin(x \pm y) = \sin x\ \cos y\ \pm \cos x\ \sin y$; (10) $\cos(x \pm y) = \cos x\ \cos y\ \mp \sin x\ \sin y$

(11) $\sin 2x = 2\sin x\cos x$; (12) $\sin(-x) = -\sin x$; (13) $\cos(-x) = \cos x$

REFERENCES

Alimonti J .B., Ball T. B., Fowke K. R. (2003). "Mechanics of CD4+T lymphocytes cell death in human immunodeficiency virus infection and AIDS". J. Gen. virol 84 (7): 1649 – 1661.
Edison A. Enaibe and John O. A. Idiodi (2013). The biomechanics of HIV/AIDS and the prediction of Lambda λ. The International Journal of Engineering and Science (IJES), Vol. 2, Issue 7, pp; 43 – 57
Enaibe A. Edison, Osafile E. omosede and John O. A. Idiodi (2013). Quantitative treatment of HIV/AIDS in the human microvascular circulating blood system. International Journal of Computational Engineering Research (IJCER). Vol. 03, Issue 7, pp; 1 – 13.
Lain G. Main (1995). Vibrations and waves in Physics. Cambridge University Press, third edition.
Lipson S. G., Lipson H. and Tannhauser (1996). Optical physics. Cambridge University press third edition.
Mandel, Bennet and Dolan (2010). HIV/AIDS – Wikipendia, chapter 118.
UNAIDS, WHO (December 2007). '2007 AIDS Epidemic Update'
Walker, J .S. (1988). Fourier Analysis. Oxford Univ. Press, Oxford.