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Theoretical study for the power distribution of the Fourier transform in the spatial frequency domain

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Abstract

Fourier transforms represent an important tool in applied mathmatics . The first yield of its application was in electrical engineering , communications and temporal signal processing . In this work the diffractive properties of a biconvex lens is utilized in a comparative study of the Fourier transform in two dimensional spatial domain .The work includes theoretical calculations of the (2-D) Fourier transforms for certain geometrical object function in the spatial domain . The theoretical calculations made use of the MATLAB® software in creating the aperture's geometry and then calculating its Fourier transform .The digital camera have been utilized to record the experimental results is a (CCD) camera to record the power distribution of the Fourier transform in the spatial frequency domain .

1- Introduction

The effect of diffraction is a general characteristic of wave phenomena occurring whenever a portion of a wavefront, be it sound, a matter wave, or light, is obstructed in some way[4] . The various segments of the wavefront that propagate beyond the obstacle interfere, causing the particular energy-density distribution referred to as the diffraction pattern [1].

The goal of this paper is to describe how the scientific analysis tool MATLAB® can be used to perform complex mathematical calculations with Fraunhofer diffraction domains and experimental implementation . Fraunhofer diffraction deals with the limiting cases where the light appoaching the diffracting object is parallel and monochromatic , and where the image plane is at a distance large compared to the size of the diffracting object [8] . Linear transforms , especially Fourier and Laplace transforms , are widely used in solving problems in science and engineering . The Fourier transform is used in linear systems analysis , antenna studies, optics , random process modeling , probability theory , quantum physics , and boundary - value problems [2,3] , and has been very successfully applied to restoration of astronomical data [1] .

2- Theory

Fraunhofer diffraction is the theory of transmission of light through apertures under the assumption that the incident wave is multiplied by the aperture function . Fraunhofer diffracton is far field approximation , where the observed pattern is located at the focal plane of a lens which usually called Fourier plane [5] .

When light is propagating (in positive $z-direction$) , the electric field in an arbitrary plane at (z) can be calculated from the field at any preceding plane at (z_0) applying Huygens's construction [6] .

In figure (3) the light is propagating from left to right $(z > z_0)$, for the field at $(r = (x, y, z))$ contributed by the point $(r_0 = (x_0, y_0, z_0))$ one may derive as

$$E(r)_{r_0} \propto \frac{E(r_0)}{|r-r_0|} \exp(i\,2\pi\upsilon |r-r_0|) \qquad .....(1)$$

Assuming monochromatic, coherent light beam . Furthermore , the scalar (E) which means that considered only one polarization component and light propagation approximately parallel to the $(z-axis)$. The total field is integrated over $(x_0\ and\ y_0)$ in the (z_0) plane [6].

$$E(r) \propto \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{E(r_0)}{|r-r_0|} \exp(i\,2\pi\,\upsilon\,|r-r_0|)\,dx_0\,dy_0 \qquad .....(2)$$

Here :

$$|r - r_0| = \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]^{1/2} \quad .....(3)$$

Eq. (3) can be approximated if assumed that

$$(z - z_0)^2 >> \ (x - x_0)^2 + (y - y_0)^2 \quad .....(4)$$

$$|r - r_0| \approx \ (z - z_0)\left[1 + \frac{(x - x_0)^2 + (y - y_0)^2}{2(z - z_0)^2}\right] \quad .....(5)$$

Eq. (2) can be written as :-

$$E(x, y) \propto \frac{P(x, y)}{z - z_0} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(x_0, y_0)\ P(x_0, y_0)\ \exp\left[-i2\pi\upsilon \frac{x\,x_0 + y\,y_0}{z - z_0}\right] dx_0 dy_0 \quad .....(6)$$

If the approximations leading to eq. (6) can be made , this is called *Fresnel diffraction* or *Fresnel approximation* . If in addition assumed that $\{P(x_0, y_0)\} \approx 1$ in the entire region considered (i.e. that $(z - z_0)$ is large enough , eq. (6) can be rewritten as :

$$E(x, y) \propto \frac{P(x, y)}{z - z_0} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(x_0, y_0)\ \ \exp\left[-i2\pi\,\upsilon \frac{x\,x_0 + y\,y_0}{z - z_0}\right] dx_0 dy_0 \quad .....(7)$$

In this system , $\{E(x, y)\}$ is just the two-dimensional Fourier transform of $\{E(x_0, y_0)\}$, except for a multiplicative phase factor which does not affect the intensity of the light . This system is called *Fraunhofer diffraction* or *Fraunhofer approximation* [7] .

3- Theoretical Results

When the distance away from the grating is large or a lens is used to focus the diffraction pattern to the image plane then the diffraction pattern becomes a Fourier transform as given by [9] .

$$E\,(x, y, z) = C\,\Im\{E\}\Big|_{\upsilon_x = \frac{x}{\lambda z}\ \upsilon_y = \frac{y}{\lambda z}} \quad ....(8)$$

Where : $E(x, y, z)$: is the electric field distribution .

C : is the phase factor .

υ_x and υ_y are spatial frequencies.

λ = wave length .

4-Experimental Results

The experimental work includes investigation simulation of Fraunhofer diffraction domains by using Fourier transform of Bitmap Images . When the (2-D) function or image is given with a bitmap file, we can use the m-file given in algorithm to find its Fourier transform . we have described a method to compute and generate far-field Fraunhofer diffraction pattern from image in gray scale. This simulation provide a simple and easy-to-use method to study the complex phenomenon of diffraction . Figures (1,4,6,8,10,12) is the bitmap image used when the image file of the size is (256×256) . It is easily generated with Microsoft® paint . Figures (2,5,7,9,11,13) is the diffraction pattern (or the Fourier transform) of the original image .

5- Conclusions

We conclude the diffraction pattern which we calculated with fft2 method relies on the size of aperture or element size , as shown in figures (14,15) . which recorded by a CCD camera ,the CCD camera record the intensity of its Fourier transform and this intensity was different in each figure and the reasoned is the noise .There are small differences between the experimental and theoretical results , and the intensity pattern observed varies with the distance from the aperture and hence obtained to Fraunhofer diffraction pattern . The reasoned is whenever smaller the aperture obtained to better pattern , we notice that the diffraction pattern relies on the diameter of aperture .

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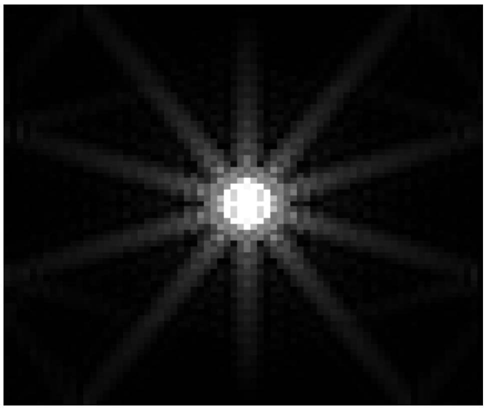


F.T.

Fig . 2 : The diffraction pattern .

Fig .1:The aperture function.

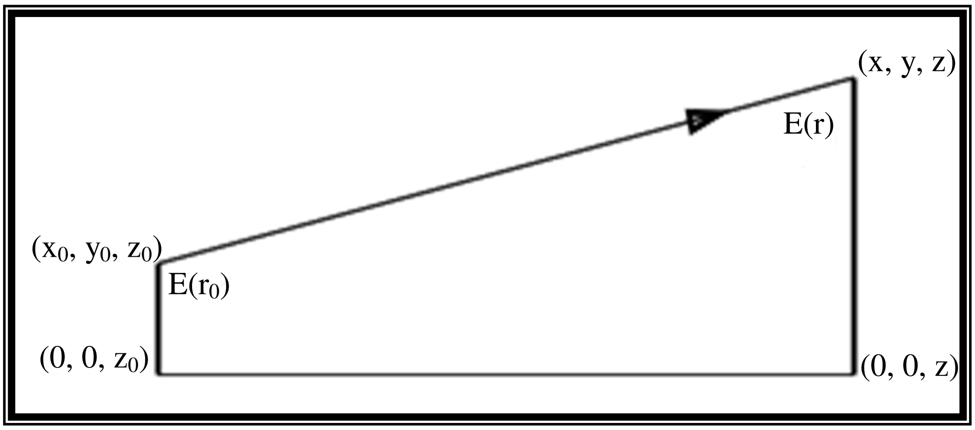
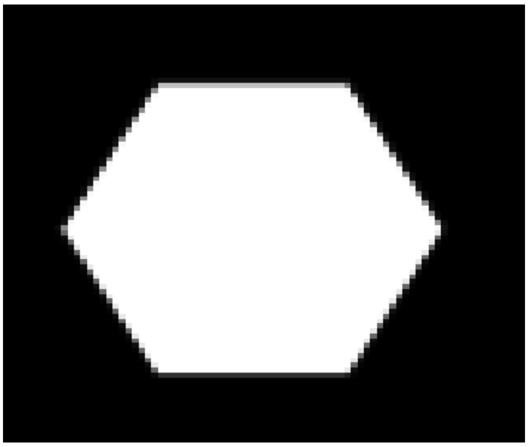


Fig .3: Geometry and parameters used for the paraxial approximation [7].



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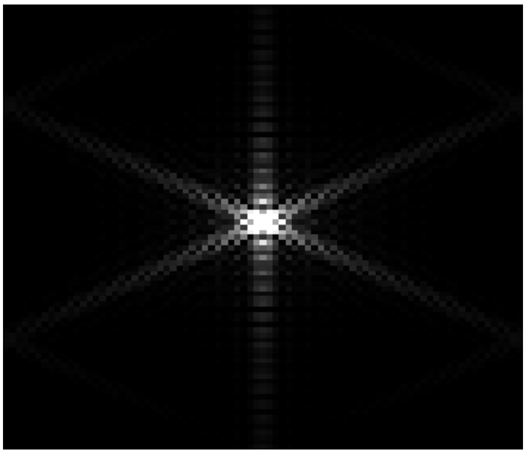
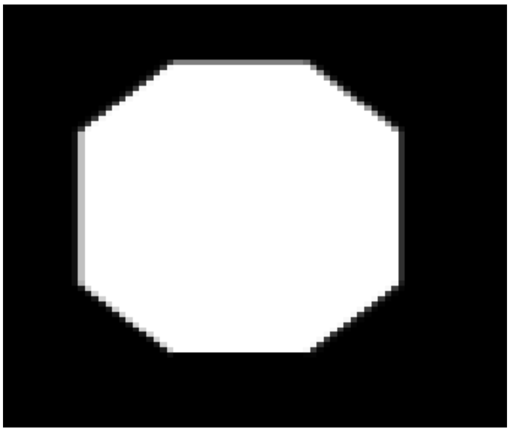


Fig .4:The aperture function.

Fig. 5: The diffraction pattern.



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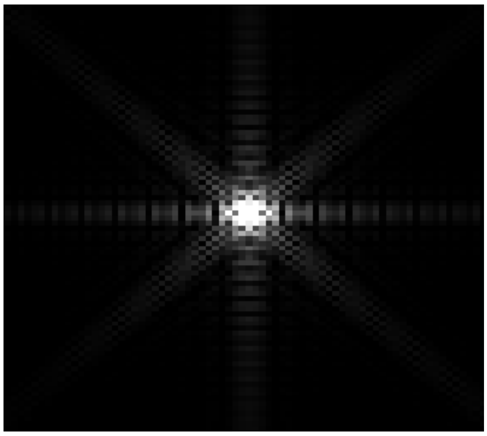
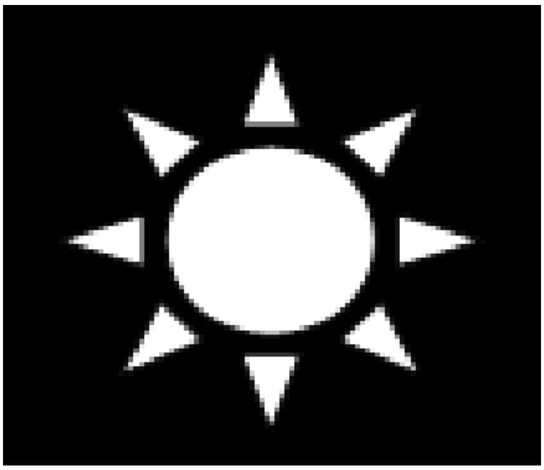


Fig .6:The aperture function.

Fig .7:The diffraction pattern .



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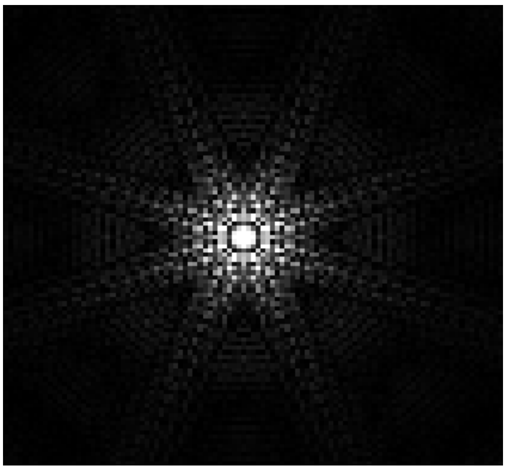
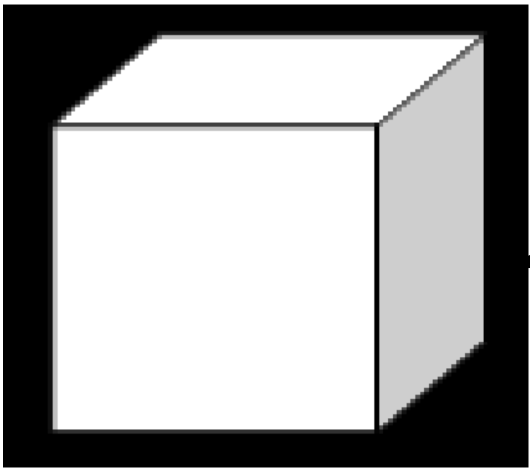


Fig .8:The aperture function.

Fig .9:The diffraction pattern.



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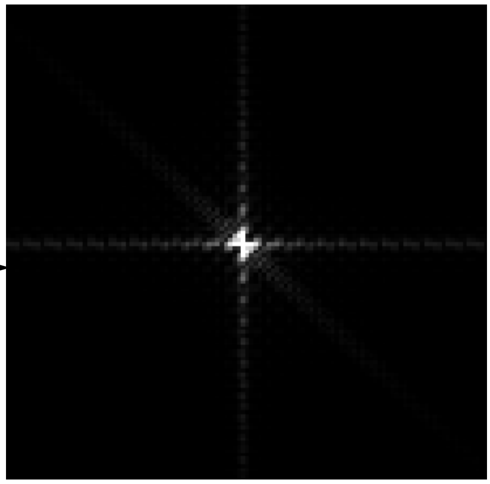
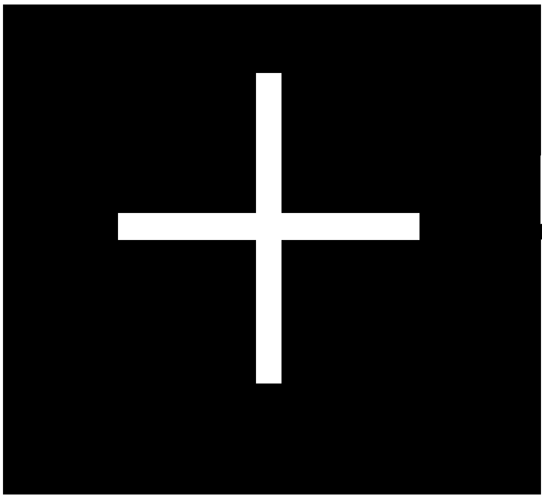


Fig .10:The aperture function

Fig .11:The diffraction pattern .



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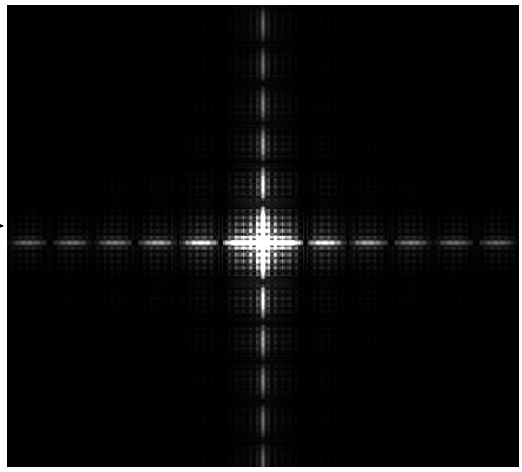

Fig .13:The diffraction pattern .

Fig .12:The aperture function

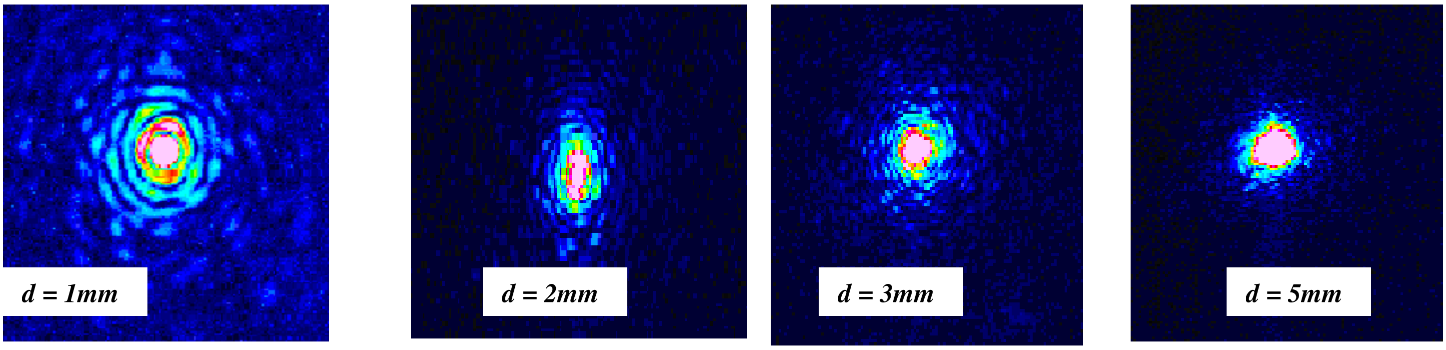

Fig .14: Different size of circular aperture functions in a CCD camera .

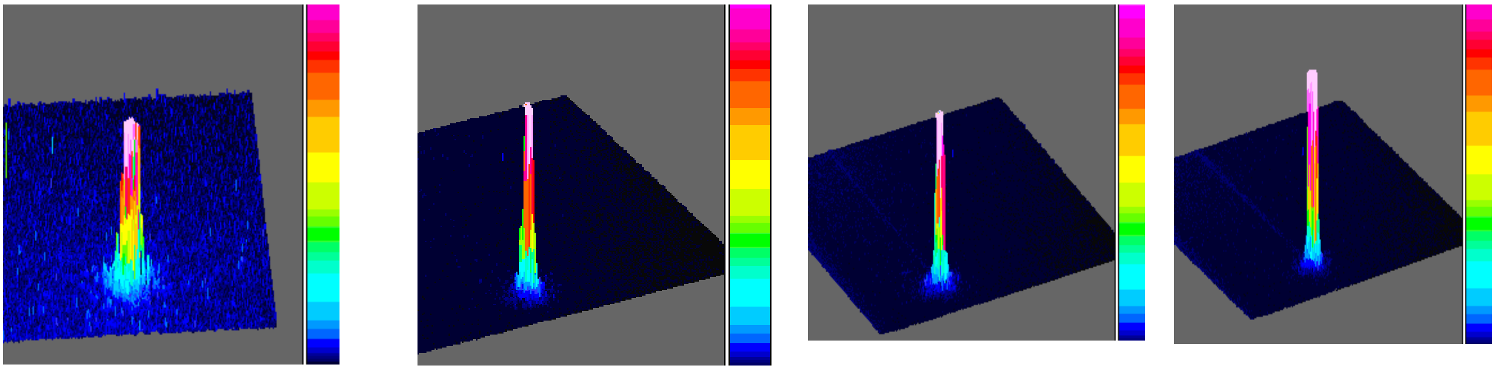

Fig .15: The power distribution of circular aperture functions by (3-D) in a CCD camera .