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Surface Waves In Homogeneous Visco-Elastic Media Of Higher Order Under The Influence Of Gravity And Surface Stresses

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ABSTRACT
The aim of the present paper is to investigate the surface waves in a homogeneous, isotropic, visco-elastic solid medium of nth order, including time rate of strain under the influence of gravity and surface stresses. The theory of generalized surface waves is developed to investigate particular cases of waves such as the Stoneley, Rayleigh and Love waves. Corresponding equations have been obtained for different cases. These reduced to classical results, when the effects of gravity, surface stresses and viscosity are ignored.
Keywords: Gravity, Surface waves, Visco-elasticity, Surface stresses.

1 INTRODUCTION

The propagation of surface waves in elastic media is of considerable importance in earth-quake engineering and seismology due to the stratification in the earth's crust. As a result, the theory of surface waves has been developed by Stoneley [1], Bullen [2], Ewing et al. [3], Hunters [4] and Jeffreys [5].

The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich [6], who treated gravity as a type of body force. Love [7] extended the work of Bromwich [6] in investigating the influence of gravity on surface waves and showed that the Rayleigh wave velocity may be affected significantly by the gravity field. Sezawa [8] studied the dispersion of elastic waves propagated on curved surfaces.

The transmission of elastic waves through a stratified solid medium was first studied by Thomson [9]. Haskell [10] examined the dispersion of surface waves in multilayered media. Biot [11] studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. De and Sengupta [12] examined several problems of elastic waves and vibrations under the influence of gravity field. In another work, Sengupta and Acharya [13] studied the influence of gravity on the propagation of waves in a thermoelastic layer. Brunelle [14], meanwhile, analysed surface wave propagation under initial tension or compression. Roy [15] studied wave propagation in a thin two-layered laminated medium with couple under initial stress, while Datta [16] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. Goda [17] examined the effect of inhomogeneity and anisotropy on Stoneley waves. Abd-Alla and Ahmed [18] studied the Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. Bland [19], Flugge [20] and Voigt [21] all analysed the wave-propagation in viscoelastic media. Recently Sethi and Gupta [22] studied the surface waves in non-homogeneous, general visco-elastic media of higher order.

Gurtin & Murdoch [23], Chandrasekharaiah [24] and other authors [25-28] all reported that surface stress plays a vital role in the propagation of waves due to the fact that the surface of a body exhibits properties that are quite different than those associated with the interior of the medium. In fact, surface tension which is generally accounted for in the theory of liquids may be considered as a particular case of surface stress. The presence of surface stress on the boundary of bodies has been detected in some particular type of crystals where its order of magnitude agrees with the predictions made by molecular theory [23]. Compressive surface stress is involved in the case of short peening of ductile metals [23], and its knowledge is quite useful for the shaping of aircraft wing panels

A few problems on the propagation of plane waves in homogeneous and isotropic materials were considered [23]. Though the concept of surface stress is comparatively new, a few authors [24,25] investigated problems which are based on the effect of surface stress. Pal *et al* [26], in particular, investigated the effect of surface stress on the propagation of surface waves.

In the present paper, the problem of nth order visco-elastic surface waves involving time rates of strain in a homogeneous and isotropic medium under the influence of gravity, surface stresses, is studied. Biot's theory of incremental deformations is used to obtain the wave velocity equation for Stoneley, Rayleigh and Love waves. These equations are in complete agreement with the corresponding classical results in the absence of gravity, surface stresses, and viscosity.

2 FORMULATION OF THE PROBLEM

Consider M_1 and M_2 to be two homogeneous, viscoelastic, isotropic, semi-infinite media welded in contact to prevent any relative motion or sliding before or after the occurrence of any disturbance.

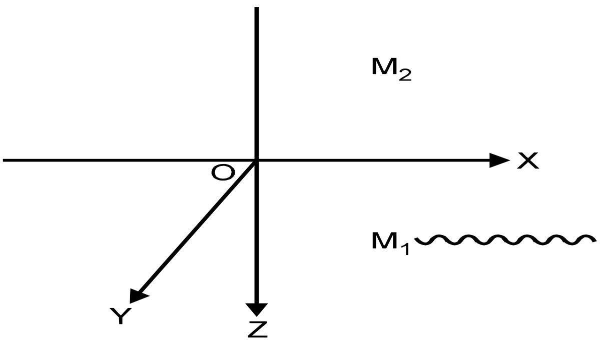

Figure 1 Two media, M_1 and M_2, in contact.

Suppose that the media are separated by a plane horizontal boundary, which extends to an infinite large distance from the origin, M_2 is taken to be above M_1, and the mechanical properties of M_1 are different from those of M_2. As a reference co-ordinate system, we consider a set of orthogonal cartesian axes Oxyz, with the origin O being at an arbitrary point on the boundary, and Oz pointing outward normal to M_1 (figure 1). Consider the possibility of a type of wave traveling in the positive x-direction, in such a manner that the disturbance is largely confined to the neighborhood of the boundary and at any instant, all particles in any line parallel to Oy having equal displacement and all partial derivatives with respect to y are zero. These two assumptions suggest that the wave is a surface wave.

Further let us assume that "u, v, w" are the components of displacements at any point (x, y, z) at any time t. It is also assume that gravitational field produces a hydrostatic initial stress is produced by a slow process of creep where the shearing stresses tend to become small or vanish after a long period of time.

The equilibrium equation of the initial stress is in the form

$$\frac{\partial \tau}{\partial x} = 0, \ \frac{\partial \tau}{\partial z} + \rho g = 0.$$

The dynamical equations of motion for a three-dimensional isotropic, visco-elastic solid medium (e.g. Biot's [11]) are as follows:

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} + \rho g \frac{\partial w}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{1a}$$

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} + \rho g \frac{\partial w}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{1b}$$

$$\frac{\partial\tau_{13}}{\partial x}+\frac{\partial\tau_{23}}{\partial y}+\frac{\partial\tau_{33}}{\partial z}-\rho g\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\rho\,\frac{\partial^2 w}{\partial t^2}, \tag{1c}$$

where ρ be the density of the material medium, g be the acceleration due to gravity and $\tau_{ij} = \tau_{ji}$ are the stress components.

The stress-strain relations for a general isotropic, visco-elastic medium are assumed to be given by the following:

$$\tau_{ij} = D_\lambda\, \Delta\, \delta_{ij} + 2\, D_\mu\, e_{ij}, \tag{2}$$

where

$$D_\lambda = \sum_{K=0}^{n} \lambda_K \frac{\partial^K}{\partial t^K},\ D_\mu = \sum_{K=0}^{n} \mu_K \frac{\partial^K}{\partial t^K},$$

$$D'_\lambda = \sum_{K=0}^{n} \lambda'_K \frac{\partial^K}{\partial t^K},\ D'_\mu = \sum_{K=0}^{n} \mu'_K \frac{\partial^K}{\partial t^K}, \tag{3}$$

$$\Delta = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z},\ \nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

The coefficients λ_0, μ_0, λ'_0, μ'_0, are constants and λ_K, μ_K (K =1, 2..... n) are the parameters associated with Kth order visco-elasticity.

Introducing equations (2) and (3) to Eqs. (1a), (1b), (1c), we obtain:

$$(D_\lambda + D_\mu)\frac{\partial\Delta}{\partial x} + D_\mu \nabla^2 u + \rho_0 g\frac{\partial w}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{4a}$$

$$D_\mu \nabla^2 v = \rho\frac{\partial^2 v}{\partial t^2}, \tag{4b}$$

$$(D_\lambda + D_\mu)\frac{\partial\Delta}{\partial z} + D_\mu \nabla^2 w - \rho_0 g\frac{\partial u}{\partial x} = \rho\frac{\partial^2 w}{\partial t^2}. \tag{4c}$$

To investigate the propagation of a surface wave along the direction of Ox, we introduce the displacement potential ϕ (x, z, t) and ψ (x, z, t), which are related to the displacement components as follows:

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z},\ w = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x}. \tag{5}$$

The displacement potential ϕ and ψ in the above equation are two distinct "potentials", whose Laplacians specify the dilatation and rotation given by:

$$\nabla^2\phi = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \Delta, \qquad \nabla^2\psi = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = 2\Omega.$$

These are associated with P-waves and SV-waves.

Substituting equation (5) into equations (4a), (4b), (4c), we obtain:

$$H_T \nabla^2 \phi + g \frac{\partial \psi}{\partial x} = \frac{\partial^2 \phi}{\partial t^2}, \tag{6a}$$

$$H_\mu \nabla^2 v = \frac{\partial^2 v}{\partial t^2}, \tag{6b}$$

$$H_\mu \nabla^2 \psi - g \frac{\partial \phi}{\partial x} = \frac{\partial^2 \psi}{\partial t^2}, \tag{6c}$$

where

$$H_T = \sum_{K=0}^{n} U_{KT}^2 \frac{\partial^K}{\partial t^K}, \ H_\mu = \sum_{K=0}^{n} U_{KS}^2 \frac{\partial^K}{\partial t^K}, \tag{7}$$

with $U^2_{KT} = \frac{\lambda_K + 2\mu_K}{\rho_0}$, $U^2_{KS} = \frac{\mu_K}{\rho_0}$.

Similar relations for medium M_2 can be obtained by replacing λ_K, μ_K, ρ by λ'_K, μ'_K, ρ'.

3 SOLUTION OF THE PROBLEM

Now our main objective is to solve equations (6a), (6b) and (6c).

We seek a solution of the following form:

$$(\phi, \psi, v) = [f(z), V(z), h(z)]\, e^{i\eta(x-ct)} \tag{8}$$

Using equations (8) and (7) in equations (6a), (6b) and (6c), we obtain a set of differential equations for the medium M_1 as follows:

$$\frac{d^2 f}{dz^2} + h_1^2 f + i \eta g J_1^2 V = 0,$$

$$\frac{d^2 h}{dz^2} + K_1^2 h = 0, \tag{9}$$

$$\frac{d^2 V}{dz^2} + K_1^2 V - i \eta g N_1^2 f = 0,$$

where

$$h_1^2 = \frac{\eta^2 c^2}{\sum_{K=0}^{n} U_{KT}^2 (-i\eta c)^K} - \eta^2, \ K_1^2 = \frac{\eta^2 c^2}{\sum_{K=0}^{n} U_{KS}^2 (-i\eta c)^K} - \eta^2,$$

$$J_1^2 = \frac{1}{\sum_{K=0}^{n} U_{KT}^2 (-i\eta c)^K}, \ N_1^2 = \frac{1}{\sum_{K=0}^{n} U_{KS}^2 (-i\eta c)^K}. \tag{10}$$

Similar relations for medium M_2 can be obtained by replacing respective terms with primes.

The governing equations for medium M_1 and M_2 must have exponential solutions such that f, V and h will describes surface waves. They must become vanishingly small as $z \rightarrow \infty$.

Hence for the medium M_1, the desired solutions are given by the following expressions:

$$\phi\,(x, z, t) = \left\{A e^{-p_1 z} + B e^{-p_2 z}\right\}\, e^{i\eta(x-ct)},$$

$$\psi\,(x, z, t) = \left\{C e^{-p_1 z} + D e^{-p_2 z}\right\}\, e^{i\eta(x-ct)},$$

$$v\,(x, z, t) = E\, e^{-K_1 z + i\eta(x-ct)}. \tag{11}$$

Similarly for medium M_2 (for the region $0 \leq z <- \infty$) they are given by expressions:

$$\phi'\,(x, z, t) = \left\{A' e^{p'_1 z} + B' e^{p'_2 z}\right\}\, e^{i\eta(x-ct)},$$

$$\psi'\,(x, z, t) = \left\{C' e^{p'_1 z} + D' e^{p'_2 z}\right\}\, e^{i\eta(x-ct)},$$

$$v'\,(x, z, t) = E' e^{K'_1 z + i\eta(x-ct)}. \tag{12}$$

Likewise in equation (12) for finite disturbances as $z \rightarrow -\infty$ for medium M_2 must hold $\operatorname{Re}\left(p_i'\right) < 0$ for i = 1, 2, 3,

where p_j (j = 1, 2) are the real roots of the equation

$$P^4 + \xi_1 p^2 + \xi_2 = 0, \tag{13}$$

where

$$\xi_1 = K_1^2 + h_1^2,$$

$$\xi_2 = K_1^2 h_1^2 - \eta^2 g^2 J_1^2 N_1^2.$$

Similarly p'_j (j = 1, 2) are the real roots of the equation

$$p'^4 + \xi'_1 p'^2 + \xi'_2 = 0, \tag{14}$$

where ξ'_1, ξ'_2, are obtained by replacing corresponding terms in equation (13) with primes.

For the media M_1 and M_2 respectively, we take into considering the real roots of equation (13) and equation (14). The constants A, B and A', B' are related with C, D and C', D' in equations (11) and (12) by means of first equations in (9) and (10).

Equating the co-efficients of $e^{-p_1 z}, e^{-p_2 z}, e^{p'_1 z}, e^{p'_2 z}$ to zero after substituting equations (11) and (12) in the first equations in (9) and (10) respectively, we obtain

$C = \gamma_1 A$, $D = \gamma_2 B$, $C' = \gamma'_1 A'$, $D' = \gamma'_2 B'$, where

$$\gamma_j = \frac{i}{\eta\, g\, J_1^{\,2}}\ [p_j^{\,2} + h_1^{\,2}],$$

$$\gamma_j' = \frac{i}{\eta\, g\, J_1'^{2}}\ [p_j'^{2} + h_1'^{2}],\ \ j = 1,2. \tag{15}$$

4 BOUNDARY CONDITIONS

We assume that the plane z = 0 is a material layer that adheres to its neighboring layer without slipping. The layer is capable of supporting its own stress represented by a surface stress tensor $\sum_{i\alpha}$ that obeys the equation given by Chandrashekraiah [24], i.e.,

$$\sum_{i\alpha} = [\delta_{i\alpha}\{\sigma + (\lambda_d + \sigma)u_{\gamma,\gamma}\} + \mu_d u_{i,\alpha} + (\mu_d - \sigma)u_{\alpha,i}] \quad \text{for i, } \alpha, \gamma = 1,2,$$

$$= \sigma u_{3,\alpha} \quad \text{for i = 3.} \tag{16}$$

Here $\lambda_{d,}$ μ_d are the Lame's moduli of the material boundary and σ is the residual surface tension on the layer z = 0. The forces on the bounding surface are governed by surface stress tensor $\sum_{i\alpha}$. The dimensions of $\lambda_{d,}$ μ_d and σ are N/m.

(i) The displacement components at the boundary surface between the media M_1 and M_2 must be continuous at all times and positions.

i.e. [u, v, w] M_1 = [u, v, w] M_2 at z = 0 respectively.

(ii) $\tau_{i3} + \sum_{i\alpha,\alpha} - \rho_1 \dfrac{\partial^2 u_i}{\partial t^2} = \tau'_{i3,}$ at z = 0, (u_1= u, u_2 = v, u_3 = w).

Here, ρ_1 is the mass per unit area of the layer and τ_{ij} and τ'_{ij} are the stress tensor in the interior of the medias M_1 and M_2. Dimensions of conventional stress tensor τ_{i3} are force per unit area and stress tensor $\sum_{i\alpha,\alpha}$ are force per unit length, and these further obeys the law given by Gurtin and Murdoch [23]:

$$\tau_{ij} = D_\lambda \delta_{ij}\, u_{k,k} + D_\mu (u_{i,j} + u_{j,i}).$$

The boundary conditions become:

$$D_\mu\left(2\frac{\partial^2\varphi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right) + \left((\lambda_d + 2\mu_d)\frac{\partial^2}{\partial x^2} - \rho_1\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}\right)$$
$$= D_\mu'\left(2\frac{\partial^2\varphi'}{\partial x\partial z} + \frac{\partial^2\psi'}{\partial x^2} - \frac{\partial^2\psi'}{\partial z^2}\right)$$

$$\left(\mu_d \frac{\partial^2}{\partial x^2} - \rho_1 \frac{\partial^2}{\partial t^2} + \mathrm{D}_\mu \frac{\partial}{\partial z} \right) v = \mathrm{D}_\mu' \frac{\partial v}{\partial z},$$

$$2\mathrm{D}_\mu \left(\frac{\partial^2 \psi}{\partial x \partial z} - \frac{\partial^2 \varphi}{\partial x^2} \right) + \frac{\mathrm{D}_\mu}{H_\mu} \frac{\partial^2 \varphi}{\partial t^2} + \left(\sigma \frac{\partial^2}{\partial x^2} - \rho_1 \frac{\partial^2}{\partial t^2} \right) \left(\frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \right)$$

$$= 2\mathrm{D}_\mu' \left(\frac{\partial^2 \psi'}{\partial x \partial z} - \frac{\partial^2 \varphi'}{\partial x^2} \right) + \frac{\mathrm{D}_\mu'}{H_\mu'} \frac{\partial^2 \varphi'}{\partial t^2}. \qquad (17)$$

Applying boundary conditions (17) to (11) and (12) the following system of equations is obtained,

$$A(i + \beta_1\gamma_1) + B(i + \beta_2\gamma_2) - A'(i - \beta'_1\gamma'_1) - B'(i - \beta'_2\gamma'_2) = 0, \qquad (18a)$$

$$E = E', \qquad (18b)$$

$$A(i\gamma_1 - \beta_1) + B(i\gamma_2 - \beta_2) - A'(i\gamma'_1 + \beta'_1) - B'(i\gamma'_2 + \beta'_2) = 0, \qquad (18c)$$

$$\mu_K^* \left[A\{(2i\beta_1 + \gamma_1 + \beta_1^2\gamma_1) + F\eta(i + \beta_1\gamma_1)\} + B\{(2i\beta_2 + \gamma_2 + \beta_2^2\gamma_2) + F\eta(i + \beta_2\gamma_2)\}\right]$$

$$= \mu'^*_K \left[(-2i\beta'_1 + \gamma'_1 + \beta_1'^2\gamma'_1)A' + (-2i\beta'_2 + \gamma'_2 + \beta_2'^2\gamma'_2)B'\right] \qquad (18d)$$

$$E\left[\mu_d - c^2\rho_1 + \frac{\mu_K^*}{\eta}\beta_1\right] = \frac{\mu'^*_K}{\eta}[\beta'_1 E'], \qquad (18e)$$

$$\mu_K^* \left[A\{(2 - S^2 - 2i\beta_1\gamma_1) - H\eta(i\gamma_1 - \beta_1)\} + B\{(2 - S^2 - 2i\beta_2\gamma_2) - H\eta(i\gamma_2 - \beta_2)\}\right]$$

$$- \mu'^*_K \left[A'(2 - S'^2 + 2i\beta'_1\gamma'_1) + B'(2 - S'^2 + 2i\beta'_2\gamma'_2)\right] = 0, \qquad (18f)$$

where we have taken,

$$F = \frac{\lambda_d + 2\mu_d - \rho_1 c^2}{\mu_K^*}, \quad H = \frac{\sigma - \rho_1 c^2}{\mu_K^*}, \quad \beta_j = \frac{p_j}{\eta}, \quad \beta'_j = \frac{p_j'}{\eta}, \quad j = 1, 2, \qquad (19)$$

and

$$\mu_K^* = \sum_{K=0}^{n} \mu_K (-i\eta c)^K, \quad \mu'^*_K = \sum_{K=0}^{n} \mu'_K (-i\eta c)^K,$$

$$S^2 = \frac{c^2}{\sum_{K=0}^{n} U_{KS}^2 (-i\eta c)^K}, \quad S'^2 = \frac{c^2}{\sum_{K=0}^{n} U'^2_{KS} (-i\eta c)^K}.$$

From equations (18b) and (18e), we have E = E' = 0. Thus there is no propagation of displacement v. Hence SH-waves are decoupled in this case.

Finally, eliminating the constants A, B, A', B' from equations (18a), (18c), (18d) and (18f), we obtain:

$$\det(a_{ij}) = 0, \quad i, j = 1, 2, 3, 4, \qquad (20)$$

where

$a_{11} = (i + \beta_1\gamma_1)$, $a_{12} = (i + \beta_2\gamma_2)$, $a_{13} = -(i - \beta'_1\gamma'_1)$, $a_{14} = -(i - \beta'_2\gamma'_2)$,

$a_{21} = (i\gamma_1 - \beta_1)$, $a_{22} = (i\gamma_2 - \beta_2)$, $a_{23} = -(i\gamma'_1 + \beta'_1)$, $a_{24} = -(i\gamma'_2 + \beta'_2)$,

$a_{31} = \mu^*_K [(2i\beta_1 + \gamma_1 + \beta_1^2\gamma_1) + F\eta(i + \beta_1\gamma_1)]$, $a_{32} = \mu^*_K [(2i\beta_2 + \gamma_2 + \beta_2^2\gamma_2) + F\eta(i + \beta_2\gamma_2)]$,

$a_{33} = -\mu'^*_K (-2i\beta'_1 + \gamma'_1 + \beta_1'^2\gamma'_1)$, $a_{34} = -\mu'^*_K (-2i\beta'_2 + \gamma'_2 + \beta_2'^2\gamma'_2)$,

$a_{41} = \mu^*_K [(2 - S^2 - 2i\beta_1\gamma_1) - H\eta(i\gamma_1 - \beta_1)]$, $a_{42} = \mu^*_K [(2 - S^2 - 2i\beta_2\gamma_2) - H\eta(i\gamma_2 - \beta_2)]$,
$a_{43} = -\mu'^*_K (2 - S'^2 + 2i\beta'_1\gamma'_1)$, $a_{44} = -\mu'^*_K (2 - S'^2 + 2i\beta'_2\gamma'_2)$.

Here equation (20) represents the wave velocity dispersion equation for interface waves in homogeneous, visco-elastic solid media under the influence of gravity and surface stresses, where the viscosity is of general nth order involving time rates of change of strain.

5 PARTICULAR CASES

5.1 Stoneley Waves

Stoneley waves are a generalized form of Rayleigh waves propagating along the common boundary of two semi-infinite media M_1 and M_2. Therefore equation (20) determines the wave velocity equation for Stoneley waves in homogeneous, visco-elastic, solid media of nth order involving time rates of strain under the influence of gravity and surface stresses.

Clearly from equation (20), It follows that wave velocity of the Stoneley waves depends upon the gravity, surface stresses and viscosity.

Thus, after simplification, equation (20) reduces to:

$$\mu^*_K \eta^2 (N_1R_2 - N_2R_1)(Q_1M_2 - Q_2M_1) + \eta[\mu^*_K (FM_1P_2 - HL_1Q_2)(N_1R_2 - N_2R_1) + FM_1\mu'^*_K \{P_1'(M_2R_2 - N_2Q_2) - P_2'(M_2R_1 - N_1Q_2)\} + \mu^*_K (HL_2Q_1 - FM_2P_1)(N_1R_2 - N_2R_1) - FM_1\mu'^*_K \{P_1'(M_1R_2 - N_2Q_1) - P_2'(M_1R_1 - N_1Q_1)\} + \mu'^*_K L_1'\{HQ_1(M_2R_2 - N_2Q_2) - HQ_2(M_1R_2 - N_2Q_1)\} + \mu'^*_K L_2'\{HQ_2(M_1R_1 - N_1Q_2) - HQ_1(M_1R_1 - N_1Q_1)\}] + \Delta(s) = 0, \tag{21}$$

where

$$\begin{aligned}\Delta(s) = {} & \mu^*_K L_1P_2(N_1R_2 - N_2R_1) + L_1\mu'^*_K \{P_1'(M_2R_2 - N_2Q_2) - P_2'(M_2R_1 - N_1Q_2)\} \\ & - \mu^*_K L_2P_1(N_1R_2 - N_2R_1) - L_2\mu'^*_K \{P_1'(M_1R_2 - N_2Q_1) - P_2'(M_1R_1 - N_1Q_1)\} \\ & + \mu'^*_K L_1'\{P_2(M_1R_2 - N_2Q_1) - P_1(M_2R_2 - N_2Q_2) + \frac{\mu'^*_K}{\mu^*_K} P_2'(M_1Q_2 - Q_1M_2)\} \\ & + \mu'^*_K L_2'\{P_1(M_2R_1 - N_1Q_2) - P_2(M_1R_1 - N_1Q_1) - \frac{\mu'^*_K}{\mu^*_K} P_1'(M_1Q_2 - Q_1M_2)\},\end{aligned} \tag{22}$$

and $M_1 = (i + \beta_1\gamma_1)$, $M_2 = (i + \beta_2\gamma_2)$, $N_1 = -(i - \beta'_1\gamma'_1)$, $N_2 = -(i - \beta'_2\gamma'_2)$,

$Q_1 = (i\gamma_1 - \beta_1)$, $Q_2 = (i\gamma_2 - \beta_2)$, $R_1 = -(i\gamma'_1 + \beta'_1)$, $R_2 = -(i\gamma'_2 + \beta'_2)$,

$L_1 = (2i\beta_1 + \gamma_1 + \beta_1^2\gamma_1)$, $L_2 = (2i\beta_2 + \gamma_2 + \beta_2^2\gamma_2)$,

$L_1' = (-2i\beta'_1 + \gamma'_1 + \beta_1'^2\gamma'_1)$, $L_2' = (-2i\beta'_2 + \gamma'_2 + \beta_2'^2\gamma'_2)$,

$P_1 = (2 - S^2 - 2i\beta_1\gamma_1)$, $P_2 = (2 - S^2 - 2i\beta_2\gamma_2)$,

$P_1' = (2 - S'^2 + 2i\beta'_1\gamma'_1)$, $P_2' = (2 - S'^2 + 2i\beta'_2\gamma'_2)$.

Due to the presence of wave number k in equation (21), it follows that Stoneley waves are dispersive in nature.

In case of absence of gravity and surface stresses, we take λ_d, μ_d, σ and ρ_1 are equal to 0.
Then equation (21) reduces to,

$$(1-R'T')\{(2-S^2)^2 - 4RT - 4\frac{\mu'^*_K}{\mu^*_K}(1-S^2-RT)\} + (1-RT)\{(2-F^2S^2)^2 - 4R'T' - 4\frac{\mu'^*_K}{\mu^*_K}(1-F^2S^2-R'T')\} - F^2S^4\frac{\mu_0'}{\mu_0}(2+TR'+T'R) = 0, \tag{23}$$

where

$T^2 = 1 - S^2$, $R^2 = (1 - E^2S^2)$, $T'^2 = (1 - F^2S^2)$, $R'^2 = (1 - J^2S^2)$,

$$E^2 = \frac{\sum_{K=0}^{n} U_{KT}^2\,(-i\eta c)^K}{\sum_{K=0}^{n} U_{KS}^2\,(-i\eta c)^K},\quad F^2 = \frac{\sum_{K=0}^{n} U_{KS}^2\,(-i\eta c)^K}{\sum_{K=0}^{n} U'^2_{KS}\,(-i\eta c)^K},\quad J^2 = \frac{\sum_{K=0}^{n} U_{KS}^2\,(-i\eta c)^K}{\sum_{K=0}^{n} U'^2_{KT}\,(-i\eta c)^K}.$$

Thus, equation (23) represents the wave velocity equation of Stoneley waves in a homogeneous visco-elastic media which is completely in agreement with corresponding classical result.

In case of absence of viscous field, then equation (23) reduces to,

$$(1-R'S')\{(2-s^2)^2 - 4RS - 4\frac{\mu_0'}{\mu_0}(1-s^2-RS)\} + (1-RS)\{(2-q^2s^2)^2 - 4R'S' - 4\frac{\mu_0'}{\mu_0}(1-q^2s^2-R'S')\} - q^2s^4\frac{\mu_0'}{\mu_0}(2+SR'+S'R) = 0, \tag{24}$$

where

$s = c/b$, $S = (1-s^2)^{1/2}$, $R = (1-e^2s^2)^{1/2}$, $S' = (1-q^2s^2)^{1/2}$, $R' = (1-t^2s^2)^{1/2}$,

$e = b/a$, $q = b/b'$, $t = b/a'$, $a^2 = \dfrac{\lambda_0 + 2\mu_0}{\rho_0}$, $b^2 = \dfrac{\mu_0}{\rho_0}$, $a'^2 = \dfrac{\lambda_0' + 2\mu_0'}{\rho_0'}$, $b'^2 = \dfrac{\mu_0'}{\rho_0'}$.

Thus, equation (24) represents the wave velocity equation of Stoneley waves in a elastic media which is completely in agreement with corresponding classical result.

5.2 Rayleigh Waves

To investigate the possibility of Rayleigh waves in a homogeneous, semi-infinite visco-elastic, media, we replace medium M_2 by vacuum, in the proceeding problem.We also note that SH-wave is decoupled in this case.

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By applying the boundary conditions:

$$\tau_{13} + \sum_{1\alpha,\alpha} -\rho_1 \frac{\partial^2 u}{\partial t^2} = 0,$$

$$\tau_{33} + \sum_{3\alpha,\alpha} -\rho_1 \frac{\partial^2 w}{\partial t^2} = 0. \tag{25}$$

Thus equations (18d) and (18f), reduced to:

$$[(2i\beta_1 + \gamma_1 + \beta_1^2 \gamma_1) + F\eta\,(i + \beta_1\gamma_1)]\,A + [(2i\,\beta_2 + \gamma_2 + \beta_2^2\,\gamma_2) + F\,\eta\,(i + \beta_2\,\gamma_2)]\,B = 0, \quad (26a)$$

$$[(2 - s^2 - 2i\beta_1\,\gamma_1) - H\,\eta\,(i\gamma_1 - \beta_1)]\,A + [(2 - s^2 - 2i\beta_2\,\gamma_2) - H\,\eta\,(i\gamma_2 - \beta_2)]\,B = 0. \quad (26b)$$

Eliminating A and B from equations, (26a) and (26b), we obtain:

$$[(2i\beta_1 + \gamma_1 + \beta_1^2\,\gamma_1) + F\,\eta\,(i + \beta_1\gamma_1)]\,[(2 - S^2 - 2i\beta_2\,\gamma_2) - H\,\eta\,(i\gamma_2 - \beta_2)]$$

$$- [(2i\,\beta_2 + \gamma_2 + \beta_2^2\,\gamma_2) + F\,\eta\,(i + \beta_2\,\gamma_2)]\,[(2 - S^2 - 2i\beta_1\,\gamma_1) - H\,\eta\,(i\gamma_1 - \beta_1)] = 0 \quad (27)$$

Here, equation (27) represents wave velocity equation for Rayleigh waves in a homogeneous, visco-elastic solid medium of nth order involving time rate of strain under the influence of gravity and surface stresses. Thus, from equation (27), we conclude that Rayleigh waves depends on the residual surface tension, surface stresses, viscosity and gravity.

This equation, of course, is in complete agreement with the corresponding classical result, when the effects of viscosity, gravity and surface stresses are neglected.

5.3 Love Waves

To investigate the possibility of Love waves in a homogeneous, visco-elastic solid media, we restrict medium M_2 by two horizontal plane surface at a distance H-apart, while M_1 remains infinite (figure 2). For medium M_1, the displacement component "v" remains same as in general case given by equation (11).

For the medium M_2, we preserve the full solution, since the displacement component along y-axis (i.e., v) no longer diminishes with increasing distance from the boundary surface of two media.

Thus,

$$v' = E_1\, e^{-K'_1\, z + i\eta(x - ct)} + E_2\, e^{K'_1 z + i\eta(x-ct)}. \tag{28}$$

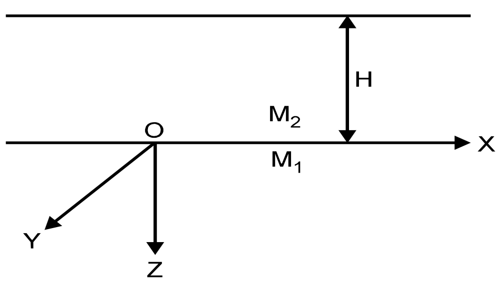

Figure 2 Configuration for Love waves.

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In this case, the boundary conditions are given by:

(i) v and τ_{32} are continuous at $z = 0$,

(ii) $\tau'_{32} = 0$ at $z = -H$,

where we have taken

$$\tau_{23} + \sum_{2\alpha,\alpha} -\rho_1 \frac{\partial^2 v}{\partial t^2} = 0.$$

Applying boundary conditions (i) and (ii) and using equations (11), (17) and equation (28), we obtain:

$$E = E_1 + E_2, \tag{29}$$

$$-K_1\, \mu^*_K\, E = \mu'^*_K\, [-K'_1 E_1 + K'_1 E_2], \tag{30}$$

$$[\mu'^*_K\, K'_1 + \mu_d - \rho_1 c^2]\, e^{K'_1 H}\, E_1 - [\mu'^*_K\, K'_1 - (\mu_d - \rho_1 c^2)]\, e^{-K'_1 H}\, E_2 = 0. \tag{31}$$

On eliminating the constant E, E_1 and E_2 from equations (29), (30) and (31), we obtain:

$$\tan(i K'_1 H) = -i \frac{[\mu^*_K\, K_1 ? \mu^*_K\, K'_1 ? + \mu^*_K\, K'_1 (\mu_d - \rho_1 c^2 ?]}{[? \mu^{*2}_K\, K'^2_1 ? + \mu^*_K\, K_1 (\mu_d - \rho_1 c^2 ?]} \tag{32}$$

Thus equation (32) represents the wave velocity equation for Love waves in a homogeneous, visco-elastic solid medium of nth order involving time rates of strain under the influence of gravity and surface stresses. Clearly it depends upon the viscous field, $\mu_{d,}$ ρ_1 and independent of gravity and residual surface tension σ.

Further when surface stresses, gravity and effect of viscous field are ignored, this equation, of course, is in complete agreement with the corresponding classical result of Love.

6 DISCUSSION AND CONCLUSIONS

The present study reveals the effects of gravity, surface stresses, residual surface tension and viscous field, on the wave velocity equations corresponding to Stoneley waves, Rayleigh waves and Love waves. Further it is investigated that visco-elastic surface waves are affected by the time rates of strain parameters. These parameters influence the wave velocity to an extent depending on the corresponding constants characterizing the visco-elastic behavior of the material. So the results of this analysis become useful in circumstances where these effects cannot be neglected. Some special cases of this study in homogeneous elastic medium are discussed by several authors, including Chandrasekharaiah [24], Gurtin and Murdoch [23, 27, 28].

The wave velocity equation for Rayleigh waves under the influence of gravity and surface stresses is dispersive due to the presence of wave number. It also depends on gravity, viscosity, residual surface tension and surface stresses.
Our results are in complete agreement with the corresponding classical results when gravity, surface stresses and other effects are neglected.

By contrast, Love waves do not depend on gravity, residual surface tension σ, these are only affected by such factors as viscous field, Lame moduli of material boundary and surface stresses. In the absence of surface stresses and other effects, the dispersion equation is in complete agreement with the corresponding classical result.

Further it is noted that the wave velocity equation of Stoneley waves is very similar to the corresponding problem in the classical theory of elasticity. Here we also observed the dispersion of waves due to the presence of wave number, gravity, surface stresses and visco-elastic nature of the solid. Moreover, the wave velocity equation of this generalized type of surface waves in homogeneous visco-elastic solid media of higher order under the influence of gravity and surface stresses is in complete agreement with the corresponding classical results when gravity, surface stresses and viscous field effects are neglected.

Finally, the solution of wave velocity equation for Stoneley waves cannot be determined by easy analytical methods. One needs to apply numerical techniques to solve the relevant detrimental equation by choosing suitable values of physical constants for both media M_1 and M_2.

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