Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.31, 2014

Construction of New Anisotropic Model of Ring Galaxies

Mariam Sultana* and Muhammad Khalid
Department of Mathematical Sciences,
Federal Urdu University of Arts, Sciences & Technology, Karachi-75300
*E-mail of corresponding author: marium.sultana@fuuast.edu.pk

Abstract
The theory of gravitational stability is necessary in order to understand physics of collision-less material of the Universe i.e. galaxies. Nuritdinov took a decisive step in this direction by constructing isotropic non-linear, non-stationary model which describes the role of gravitational instability in galaxy/galaxies formation process. Since, isotropic model is a somewhat as idealized picture, the question arises of also examining anisotropic model in the initial state with non-stationary condition. In this paper, we construct a new anisotropic model for ring galaxies. Ring galaxies are very important, as ring formation involves the entire history of galaxies. The stability of ring oscillation mode can be expressed in the form of critical value of pulsation, amplitude and degree of rotation for anisotropic model.
Key Words: Pulsation, Non-stationary, Asymmetric Oscillation, Virial Ratio, Critical Value

1. Introduction

Observations showed that many galaxies contain ring structures. Normally, they are spiral galaxies but the pure ring galaxies without spiral arms and the barred rings are also present. Many vague theories have been proposed on rings formation in galaxies. Earlier, scientists like Randers (1942), Icke (1979) and Lesceh (1990) proposed that rings are formed due to the viscous torques. It is interesting to note that rings may be formed at the turnover point of rotation curves where torque acts and collects matter. Renders claimed that accumulated collisions and scatterings were equivalent to a viscosity effect in stellar population. Cannon (1970) considered the formation of ring galaxy by the material of the disk galaxies separated from their nuclei due to an encounter with another galaxy, while Sersic (1970) suggested that the formation of transient annular structures is the material driven out from an exploding galaxy.

Dostal & Metlov (1978) stated that the mechanism of the creation of the ring must be inbuilt in the galaxy. Vorontsov (1962) considered that galaxies are continuously transforming, from ring galaxies to spiral galaxies and they embrace a family of subtypes as do spiral galaxies. Vorontsov' idea provided a motive for ring to be structural element of galaxies Vorontsov & Arkhipova (1974) noticed that the formation of ring galaxies is without any external influence. Danby (1965) argued that the existence of ring-like structures is related with the equi-potential surfaces in the rotating frame of the bar. Lin and Shu (1964) suggested wave theory which opposed the non-gravitational approaches. This theory's growing importance of N-body simulations. Schwarz (1981) made it clear that resonance ring are formed by gravitational torques on the gas. This version of theory decreases the artificial viscosity and increases the formation of distinguished rings. Buta (1995) also proposed that since the rings are relatively slim features therefore they are always associated with gas, and are usually the site of enhanced star formation. However, reliable mechanisms of their formation and possible non-linear non-stationary effects are not revealed till now.

To find out the corresponding formation criteria of ring structures in galaxies, the constructions of various non-stationary models comprising characteristic features of an early stage of global evolution of real systems are necessary. Therefore, in this paper, we construct new anisotropic model for ring structures in galaxies.

2. Model of Pulsating Disk

Let r_o and v_o denote the radius and velocity vector of a star that is in equilibrium state where Ω is the rotational parameter, σ is the surface density of the system. For the pulsating state of the system, the coordinates and the velocity of the star shall be written in vector form as

$$r = \alpha(t)\, r_\circ + \beta(t)\, v_\circ \quad \text{and} \quad v = \dot{\alpha}(t)\, r_\circ + \dot{\beta}(t)\, v_\circ \qquad (1)$$

Where $\alpha(t)$ and $\beta(t)$ are unknown function of time. Eq. (1) yields

$$r = \Pi(t)\, r_\circ \quad \text{and} \quad \Pi(t) = \sqrt{\alpha^2 + \Omega^2\beta^2} \qquad (2)$$

The equation (2) represents the elapsed time t, factor Π will expand the disk while maintaining similarity. On the other hand, surface density $\sigma(t)$ will vary in proportion to Π^{-3}, so that the following equation of motion will hold.

$$\ddot{\alpha}^2 = -\frac{\Omega^2\alpha}{\Pi^3} \quad \text{and} \quad \ddot{\beta}^2 = -\frac{\Omega^2\beta}{\Pi^3} \tag{3}$$

We can easily draw the expression for radial and transverse velocity component of star in the perturbed state of the system. The equation for the function Π is

$$\Pi^2\,\dot{\Pi}^2 + \Omega^2 = (\dot{\alpha}^2 + \Omega^2\beta^2)\Pi^2 \tag{4}$$

Hence, with the help of (3) and (4) the energy integral is

$$\frac{1}{2}\dot{\Pi}^2 + \Omega^2(\frac{1}{2}\Pi^{-2} - \Pi^{-1}) = E \tag{5}$$

Clearly, the constant $E < 0$ for otherwise the function $\Pi(t)$ would increase with time. The corresponding differential equation for $\Pi(t)$ is

$$\ddot{\Pi}^2(t) + \Omega^2(\Pi^{-2} - \Pi^{-3}) = 0 \tag{6}$$

As an analogy of two body problem in terms of eccentric anomaly, we may take

$$\psi = q\int_0^t \Pi^{-1}dt \tag{7}$$

We find, after transforming from argument, t to ψ, from equation (5), (6) and (7) that

$$\Pi''(\psi) + \Pi(\psi) = \frac{\Omega^2}{q^2} \tag{8}$$

where differentiation with respect to ψ is signified by prime. From (8) together with (5), we obtain the solution

$$\Pi(\psi) = \frac{\Omega^2}{q^2}(1 + \lambda\cos\psi) \quad \text{and} \quad \lambda = \sqrt{1 - \frac{q^2}{\Omega^2}} \tag{9}$$

provided that the state $t = 0$, $\dot{\Pi} = 0$ corresponding to the epoch of maximum expansion. Regarding this, we have evaluated dilation coefficient Π of the pulsating disk. Equation (2) with equation (9) yields

$$\alpha(\psi) = \frac{\Omega^2}{q^2}(\lambda + \cos\psi) \quad \text{and} \quad \beta(\psi) = \frac{1}{q}\sin\psi \tag{10}$$

3. Initial Non-Equilibrium State and Basic Equation of Motion

The structure of a galaxy is defined by a distribution function $f(\vec{r}, \vec{v})$ in phase-space with six dimensions (three spatial coordinates for positions and three corresponding velocity components) at each moment (time) t. The phase-space is given by the product of configuration space and velocity space. Such a system of bodies is subjected to the potential ϕ that is completely described by its phase-space distribution function $f(r_1, v_1; r_2, v_2; r_3, v_3, t)$. The dynamical theory of collision-less stellar systems is based on Boltzmann equation and Poisson's equation, which is given by

$$\frac{\partial\Psi}{\partial x} + \vec{v}\frac{\partial\Psi}{\partial\vec{r}} + \frac{\partial\Psi}{\partial\vec{r}}.\frac{\partial\Psi}{\partial\vec{v}} = 0 \tag{11a}$$

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = -4\pi G\int\int_{-\infty}^{+\infty} \Psi\,du\,dv \tag{11b}$$

where Ψ is the phase density and Φ is the potential energy. It is difficult to solve equation (11) analyticaly due to presence of non linear term in it and dependence on seven variables. Nuritdinov had developed generalization of Bisnovaty-kogan-Zeldovich's (1970) disk model with non-stationary background. The phase density of this non-stationary model [Nuitdinov (2008)] is given by

$$\Psi(r, v_r, v_\perp, t) = \frac{\sigma_0}{2\pi\Pi\sqrt{1-\Omega^2}} \times \left[\frac{1-\Omega^2}{\Pi^2}\left(1 - \frac{r^2}{\Pi^2}\right) - (v_r - v_a)^2 - (v_\perp - v_b)^2\right]^{-1/2} \chi(R - r) \tag{12}$$

Where ψ is an auxiliary variable, Ω represent the rotation of disk with $0 \le \Omega \le 1$, and $\lambda = 1 - \left(\frac{2T}{|U|}\right)_o$ define as virial ratio at the time of collapse. Also λ varies from $0 \le \lambda \le 1$, v_r and $v_\perp$ are the radial and tangential velocity of the "particle" with coordinates $\vec{r} = (x, y)$. The function $\Pi(t)$ is the expansion coefficient of gravitating system such that

$$\Pi(t) = \frac{1 + \lambda \cos\psi}{1 - \lambda^2} \quad \text{and} \quad t = \frac{\psi + \lambda \sin\psi}{(1 - \lambda^2)^{3/2}} \tag{13}$$

and

$$V_a = -\lambda\sqrt{1 - \lambda^2}\,\frac{r \sin\psi}{\Pi^2} \tag{14}$$

$$v_b = \frac{\Omega r}{\Pi^2} \tag{15}$$

4. Small Asymmetric Oscillation

By considering a small potential perturbation against the background of a non equilibrium model (12), we have the following equation of motion for the displacement vector of centroid δr

$$(1 + \lambda \cos\psi)\frac{d^2 \delta r}{d\psi^2} + \lambda \sin\psi \frac{d\delta r}{d\psi} + \delta r = \Pi^3 \frac{\partial(\delta\Phi)}{\partial r} \tag{16}$$

Where Φ = potential perturbation. The integral representation of the solution of (16) is as follows.

$$\overline{\delta\vec{r}} = \frac{\Omega^6}{q^6}\int_{-\infty}^{\psi} (1 + \lambda \cos\psi)^3 \times S(\psi, \psi_1)\left[\frac{\partial\overline{\delta\Phi}}{\partial r}\right] d\psi_1 \tag{17}$$

Where $S(\psi, \psi_1)$ is the green function for the homogenous equation corresponding to (16). The potential perturbation $\delta\Phi$ in accordance with the spatial homogeneity of the model is given by

$$\delta\Phi = r^m a_\circ(\psi) r^{N-m} \tag{18}$$

Here m is the azimuthal wave number, N is the primary index of the harmonic. Here for ring mode $N = 4$, $m = 0$. According to Eq (1), the coordinates and velocities in the normal state are

$$r_\circ = \dot{\beta} r(\psi) - \beta v(\psi) \quad ; \quad v_\circ = -\dot{\alpha} r(\psi) + \alpha v(\psi) \quad \text{and} \quad r(\psi_1) = \alpha(\psi_1)\, r_\circ + \beta(\psi_1)\, v_\circ \tag{19}$$

One of the authors of this paper with Nuritdinov (2008) constructed isotropic non-stationary model for ring galaxy. We proceed in the same way as in case of isotropic model. We derive dispersion relation by writing equation (18) in terms of its components.

$$\overline{\partial\,\vec{x}} = \frac{\Omega^6}{q^6}\int_{-\infty}^{\psi} (1 + \lambda \cos\psi_1)^3\, S(\psi, \psi_1)\left[\frac{\partial\overline{\delta\Phi}}{\partial x_1}\right] d\psi_1 \tag{20a}$$

$$\overline{\partial\,\vec{y}} = \frac{\Omega^6}{q^6}\int_{-\infty}^{\psi} (1 + \lambda \cos\psi_1)^3\, S(\psi, \psi_1)\left[\frac{\partial\overline{\delta\Phi}}{\partial y_1}\right] d\psi_1 \tag{20b}$$

Where $\vec{r}_1 = H_\alpha \vec{r} + H_\beta \vec{v}$ (21)

$$H_\alpha = \frac{1}{(1 + \lambda \cos\psi)}\left[\cos\psi(\cos\psi_1 + \lambda) + \sin\psi \sin\psi_1\right] \tag{22a}$$

$$H_\beta = \frac{\Omega_\circ^2}{q^3}\left[\sin\psi_1(\cos\psi_1 + \lambda) - \sin\psi(\cos\psi_1 + \lambda)\right] \tag{22b}$$

The density perturbation can be calculated by the equation

$$\delta\sigma = -\frac{\partial\left(\sigma\overline{\delta x}\right)}{\partial x} - \frac{\partial\left(\sigma\overline{\delta y}\right)}{\partial y} \tag{23}$$

5. Construction Of New Anisotropic Model

Nuritdinov (1991), Antonov & Nuritdinov (1981) and Nuritdinov (1985) constructed non-stationary models for non-linearly pulsating spherical Einstein and Camm model. We used Nuritdinov's weight function

$$\rho\left(\Omega''\right) = \frac{2}{\pi}\sqrt{1-\Omega'^2}\left(1-\Omega\Omega'\right)\rho\left(\Omega'\right) \tag{24}$$

Where Ω has been considered as a new parameter. We multiply this weight function with isotropic non-stationary model (12) in order to get anisotropic model. Then, after multiplying equation (12), one obtains the corresponding new phase density function as

$$\psi_{Anis} = \frac{\sigma_o}{\pi^2}\int_{-1}^{1}\frac{\left(1+\Omega\Omega'\right)}{\sqrt{K\left(\Omega'\right)}}d\Omega' \tag{25}$$

where the function

$$K\left(\Omega'\right) = 1 - \frac{r^2}{\Pi^2} - \Pi^2\left[v_\perp^2 + \left(v_r - v_a\right)^2\right] + 2rV_\perp\Omega' - \Omega'^2 \tag{26}$$

which requires that the condition

$$D = \left(1 - \frac{r^2}{\Pi^2}\right)\left(1 - \Pi^2 V_\perp^2\right) - \Pi^2\left(v_r - v_a\right)^2 \tag{27}$$

should be valid. After some transformation which we already discussed in our paper [Sultana & Khalid (2012)], we can easily obtain the following anisotropic model of non-stationary self-gravitating system as

$$\psi_{Anis} = \frac{\sigma_o}{\pi}\left(1 + \Omega r v_\perp\right)\chi(D) \tag{28}$$

where $\chi(D)$ is a Heaviside function. Ω again plays a role of a rotation parameter. The non-stationary analog of the dispersion relation is easily obtained by averaging (12) over Ω with the weighting function (24). Actually, we substitute $\Omega = \Omega'$ in equation (24). Then we integrate the result with respect to Ω' with integration limit -1 to 1 and finally get the required non-stationary analog equation for ring mode of the anisotropic model (28) for non-stationary gravitating systems.

6. Derivation Of Dispersion Relation Of Anisotropic Model

The non-stationary analog of dispersion relation has been achieved by comparing theoretical values of potential perturbation and density perturbation with their values calculated by non-stationary model .This method is already described and used in [Nuritdinov Sultana & Khalid (2008) and Sultana & Khalid (2013)].We calculated non-stationary analog of dispersion relation for ring mode in the same manner, which is given by

$$\Lambda F_\tau = \left(\Lambda + \cos\psi\right)^{k-\tau}\sin^\tau\psi\,\Pi^3(\psi)Y(\psi)\ ;\ \tau = 0,1,2,3 \tag{29}$$

where Λ is differential operator ,defined as

$$\Lambda = \left(1 + \lambda\cos\psi\right)\frac{d^2}{d\psi^2} + \lambda\sin\psi\frac{d}{d\psi} + 1 \tag{30}$$

With

$$Y(\psi) = F_o(\psi)M_o + F_1(\psi)M_1 + F_2(\psi)M_2 + F_3(\psi)M_3 \tag{31}$$

Where

$$M_o = 8k_1\cos^3\psi + k_2\left(-\sin\psi\cos^2\psi\right) + k_3\left(\cos\psi\sin^2\psi\right) + k_4\left(-\sin^3\psi\right) \tag{32}$$

$$\begin{aligned} M_1 = {} & 24k_1\sin\psi\cos^2\psi + k_2\left[\cos^2\psi\left(\cos\psi + \lambda\right) - 2\sin^2\psi\cos\psi\right] + \\ & k_3\left[\sin^3\psi - 2\sin\psi\cos\psi\left(\cos\psi + \lambda\right)\right] + k_4\left[3\sin^2\psi\left(\cos\psi + \lambda\right)\right] \end{aligned} \tag{33}$$

$$M_2 = 24k_1 \sin^2\psi\cos\psi + k_2\left[2\cos\psi(\cos\psi+\lambda)\sin\psi - \sin^3\psi\right] + k_3\left[\cos\psi(\cos\psi+\lambda)^2 - 2\sin^2\psi(\cos\psi+\lambda)\right] + k_4\left[-3\sin\psi(\cos\psi+\lambda)^2\right] \quad (34)$$

$$M_3 = 8k_1\sin^3\psi + k_2\left[\sin^2\psi(\cos\psi+\lambda)\right] + k_3\left[\sin\psi(\cos\psi+\lambda)^2\right] + k_4\left[(\cos\psi+\lambda)^3\right] \quad (35)$$

$$k_1 = \frac{1}{(1+\lambda\cos\psi)^3} \quad (36)$$

$$k_2 = \frac{-24\lambda\sin\psi}{(1+\lambda\cos\psi)^4} \quad (37)$$

$$k_3 = \frac{24\lambda^2\sin^2\psi - 6(1-\lambda^2)}{(1+\lambda\cos\psi)^5} \quad (38)$$

$$k_4 = \frac{-8\lambda^3\sin^3\psi + 6\lambda\sin\psi(1-\lambda^2)}{(1+\lambda\cos\psi)^6} \quad (39)$$

Equation (29) is system of eight second order non-linear differential equations which is not easy to solve analytically. We used method of periodic solution, which is well defined in [Malkin (1967)] to solve this system of differential equation. Method of Periodic solution gives eigen values of differential equations, on the basis of which we examine the stability of the system (As shown in Fig 1)

This solution of system of differential equation has been used to construct a critical diagram for the initial virial ratio $\left(\frac{2T}{|U|}\right)_o$ as a function of Ω for the system .This diagram shows that a non-linearly pulsating disk with random value of Ω can remain unstable with $\left(\frac{2T}{|U|}\right)_o < 0.445$. There is also a narrow band of instability between $0.480 < \left(\frac{2T}{|U|}\right)_o < 0.520$ for any value of Ω. For region $\left(\frac{2T}{|U|}\right)_o \geq 0.520$ there is perfect stability which does not depend upon Ω. In other words, ring mode remains constant with respect to rotation parameter Ω.

7. Conclusion

In this study, we construct and examine the role of anisotropic model in the development of instability in ring mode. This model is formed against the background of specified non-stationary and non-equilibrium initial conditions. Non-stationary dispersion relation has been obtained for ring mode imposed on the non-stationary anisotropic model. Critical diagram between initial virial ratio $\left(\frac{2T}{|U|}\right)_o$ and degree of rotation Ω of the model for ring oscillations mode of the pulsating disk shows that ring oscillation mode does not depend on rotation parameter Ω for anisotropic model.

8. Acknowledgement

We wish to thank Prof. S.N. Nuritdinov for the discussion on present results and also Ms Erica for her help. This study is part of first author's doctoral dissertation.

References

Antonov, V.A. & Nuritdinov, S.N (1981), "Instability of a non-linearly Pulsating Model Stellar System. An Einstein Sphere", *Astron. Zh.* 58: 1158-1166

Bisnovatyai-Kogen, G.S. & Zel'Dovich, A.B. (1970), "Models of Point Mass Clusters with Quadratic Gravitational Potential", *Astrofizika Journal*. 6: 387-396

Buta, R.B. & Purcel, G.B. (1995), "Intrinsic Bar/Ring Misalignment and Starburst Nuclear Ring in the Peculiar

Spiral Galaxy", *Astrophysical Journal*. 110: 1588-1601

Cannon, R.D. & Lloyd, C. (1970), "Ring Galaxies", *The Observatory Journal*. 90: 153-154

Danby, J.M. (1965), "The formation of Arms in Barred spirals", *Astronomical Journal*. 70: 501-512

Dostal, V.A. & Metlov, V.G. (1978), "The Large Scale Structure of the Universe", *IAU Symp.79*, 117

Icke, V. (1979), "Disk accretion in a Soft Potential Well.", *Astronomy & Astrophysics Journal.* 78: 21-24

Lesceh, H. & Biermann, P.L. (1990), "The Formation of Molecular Gas Rings in Galactic Central Regions", *Astronomy and Astrophysics Journal*, 242: 194-199

Lin, C.C. & Shu, F.H. (1964), "On the Spiral Structure of Disk Galaxies", *Astrophysical Journal*. 140: 646-655

Malkin, I.G. The Theory of Stability of Oscillation Motion. *Nauka, Alma-Ata, Moscow*, (1967)

Nuritdinov, S.N., Mirtadjieva, K.T. & Sultana, M. (2008), "Instabilities in a Non-stationary Model of Self-Gravitating Disks. I, Bar and Ring Perturbation Modes", *Astrophysics Journal*. 51: 410-423

Nuritdinov, S.N. (1991), "Instability of a Non-linearly Pulsating Model of Stellar System: Camm's Model", (Russian). *Astron. Zh.* 68: 763-775

Nuritdinov, S.N.(1985), "Stability a Contracting Flat Model Stellar System of Finite Radius", (Russian), *Pis'ma Astron. Zh.* 11: 89-93

Nuritdinov, S.N., Mirtadijieva, K.T., Sultana, M. & Khalid, M. (2008), "Towards the Formation Theory of Large-Scale Structures of Galaxies. *Experimental & Theoritical Physics Journal*. 3: 201-208

Randers, G.. (1942), "The Equilibrium and Stability of Ringed Shaped Barred Spiral", *Astrophysical Journal.* 95: 88-111

Schwarz, M.P. (1981), "The Response of gas in a Galactic Disk to Bar Forming", *Astrophysical Journal*. 247: 77-88

Sersic, J.L. (1970), "Periodic Orbits, stability and Resonance", *The D. Reidel Publication Co. Dordrecht, Holland* , First Ed.

Sultana, M. & Khalid, M. (2013), "Formation of Lopsided and Bar structures in non-stationary gravitating systems: II Anisotropic case", *The Nucleus,* 50(4): 411-414

Sulatan, M. & Khalid, M. (2012), "Anisotropic Model of Self-Gravitating Systems of Universe", *The Research Journal of Science & Technology.* 3: 20-24

Vorontsov, B.A., Arkhipova, V.P. & Krasnogorskaya, A.A. (1962), "Morphological Catalogue of Galaxies", *Moscow University Press, Moscow*. Part 1

Vorontsov, B.A. & Arkhipova, V.P. (1974), "Morphological Catalogue of Galaxies", *Moscow University Press, Moscow.* Part 5

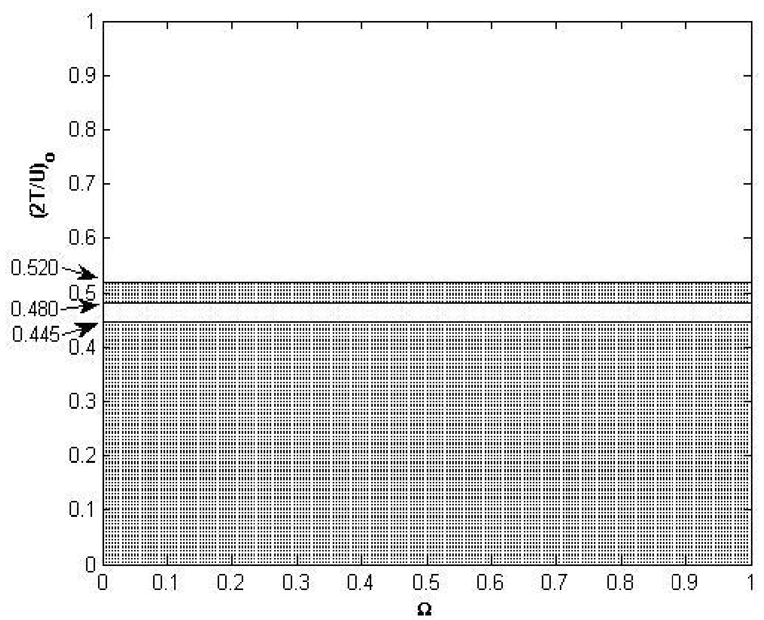

Fig. 1: Diagram showing the relation between initial virial ratio $\left(\frac{2T}{|U|}\right)_o$ and degree of rotation Ω of the model for ring oscillations mode.

The IISTE is a pioneer in the Open-Access hosting service and academic event management. The aim of the firm is Accelerating Global Knowledge Sharing.

More information about the firm can be found on the homepage:
http://www.iiste.org

CALL FOR JOURNAL PAPERS

There are more than 30 peer-reviewed academic journals hosted under the hosting platform.

Prospective authors of journals can find the submission instruction on the following page: http://www.iiste.org/journals/ All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Paper version of the journals is also available upon request of readers and authors.

MORE RESOURCES

Book publication information: http://www.iiste.org/book/

Recent conferences: http://www.iiste.org/conference/

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library , NewJour, Google Scholar