Effect of couple sterss fluid on MHD peristaltic motion and heat transfer with partial slip

Ghusoon G.mhammad ,Ahmed M. Abdulhadi Department of Mathematic, College of Science, University of Baghdad, Baghdad, Iraq. email: ghusoon.ghazi@yahoo.com , ahm6161@yahoo.com

The research is financed by Asian Development Bank. No. 2006-A171(Sponsoring information)

Abstract

In this investigation we have analyzed the effect of variable couple stress fluid on the peristaltic flow of non-Newtonian fluid in an asymmetric channel. The motion and energy equations have been calculated under the assumptions of long wave length approximation and the expression for pressure rise is obtained by using analytical integration. The contour plots for the stream lines obtained and trapping phenomena for the flow field is discussed. The computational results presented in graphical form, it is found that the temperature field decreases with the increase in slip parameter L ,magnetic field M and couple stress α while with the increase in Pr and Ec the temperature field increases. This study is done through drawing many graphs by using the MATHEMATICA package.

Keywords: Heat transfer ,Non - Newtonian fluid, Couple stress, magnetic field, Partial slip.

1.Introduction

Many applications of non-Newtonian fluids in engineering and industry have led to renewed interest among the researchers. Such applications include extraction of crude oil from petroleum products, food mixing and chime movement in the intestine, flow of plasma, flow of blood, flow of nuclear fuel slurries, flow of liquid metals and alloys, and flow of mercury amalgams. In non-Newtonian fluid models, couple stress fluid model has distinct features, such as polar effects in addition to possessing large viscosity. The theory of couple stress was first developed by Stokes [1] and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples. A number of studies containing couple stress have been investigated in Refs. [2-4]. The couple-stress fluid may be considered as a special case of a non-Newtonian fluid which is intended to take into account the particle size effects. Moreover, the couple stress fluid model is one of the numerous models that proposed to describe response characteristics of non-Newtonian fluids. The constitutive equations in these fluid models can be very complex and involving a number of parameters, also the out coming flow equations lead to boundary value problems in which the order of differential equations is higher than the Navier-Stokes equations. Some recent investigations regarding such fluids are mentioned in the studies [5-13]. Recently, Peristaltic problems have gained a considerable importance because of it applications in physiology, engineering, and industry. Such applications include urine transport from kidney to bladder, swallowing food through the esophagus, movement of chime in the gastrointestinal tract, transport of spermatozoa in the ducts efferent of the male reproductive tract, movement of ovum in the female fallopian tubes, vasomotor of small blood vessels, transport of slurries, corrosive fluids, sanitary fluids, and noxious fluids in nuclear industry. In view of these applications, a number of researchers have discussed the peristaltic flows involving Newtonian and non-Newtonian fluids with different kinds of geometries [14-26]. Mekheimer[27] has discussed the effects of the induced magnetic field on peristaltic flow of a couple stress fluid in a slit channel. Magneto hydrodynamics (MHD) is the science which deals with the motion of a highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid[28]. It has now been accepted that most of the physiological fluids behave like a non-Newtonian fluids. This approach provides a satisfactory understanding of the peristaltic mechanism involved in small blood vessels, lymphatic vessels, intestine, duct us efferent of the male reproductive tract and in transport of spermatozoa in the cervical canal. Some recent studies [29-40] have considered the effect of a magnetic field on peristaltic flow of a Newtonian and non-Newtonian fluids, and in all of these studies the effect of the induced magnetic field have been neglected.

With the above discussion in mind, the goal of this investigation is to study the effect of couple stress fluid on MHD peristaltic motion and heat transfer with partial slip. The governing equations are simplified using long wavelength approximation. An exact solutions of velocity, stream function, energy equation and pressure gradient has found. The expressions for pressure rise has been calculated using numerical integration by software Mathematica. The effects of pertinent parameters on the velocity, energy equation, pressure gradient, and stream functions are presented graphically.

2. Mathematical formulation and analysis

Consider MHD flow of an electrically conducting viscous fluid in asymmetric channal through porous medium. The lower wall of the channal is maintained at temperature T1 while the upper wall has temperature T0 as shown in the Fig.1.



The geometry of the wallsurfase is define as:

$$Y = H_1 = d_1 + a_1 \cos[\frac{2\pi}{\lambda}(X - ct)], \quad Y = H_2 = -d_2 - b_1 \cos[\frac{2\pi}{\lambda}(X - ct) + \phi]$$
(1)

Where a_1 and b_1 are amplitudes of the waves, λ is the wave length, $d_1 + d_2$ is the width of the channel, c is the velocity of propagation, t is the time and X is the direction of wave propagation. the phase difference ϕ varies in the range $0 \le \phi \le \pi$, in which $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, and further a_1, b_1, d_1, d_2 and ϕ satisfies the condition $a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \le (d_1 + d_2)^2$

For the two dimensional incompressible flow, the govering equations of motion and energe are $\partial U = \partial V$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial X} + v(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}) - \frac{v}{K}U - \frac{\sigma B_0^2 U}{\rho} - \frac{\eta}{\rho} \nabla^4 U,$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial Y} + v(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) - \frac{v}{K}V,$$

$$c'[\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y}] = \frac{K'}{\rho} \nabla^2 T + v\phi.$$
(2)
Where $\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}, \quad \nabla^4 = \frac{\partial^4}{\partial X^4} + \frac{\partial^4}{\partial Y^4} + 2\frac{\partial^4}{\partial X^2 \partial Y^2}, \quad \phi = [2(\frac{\partial U}{\partial X})^2 + 2(\frac{\partial V}{\partial X})^2 + (\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X})^2],$

U, V are the velocities in X and Y directions in fixed frame, ρ is constant density, P is the pressure, v is the kinematics viscosity, σ is the electrical conductivity, K is the permeability parameter, K' is the thermal conductivity, c' is the specific heat and T is the temperature.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (x, y) by the transformations: x = X - ct, y = Y, u = U - c, v = V, p(x) = P(X, t)(3)
Defining

$$\overline{x} = \frac{x}{\lambda}, \quad \overline{y} = \frac{y}{d_1}, \quad \overline{u} = \frac{u}{c}, \quad \overline{v} = \frac{v}{c}, \quad \delta = \frac{d_1}{\lambda}, \quad d = \frac{d_2}{d_1}, \quad \overline{p} = \frac{d_1^2 p}{\mu c \lambda}, \quad \overline{t} = \frac{ct}{\lambda}, \quad h_1 = \frac{H_1}{d_1}, \quad h_2 = \frac{H_2}{d_2}, \quad a = \frac{a_1}{d_1}$$

$$b = \frac{b_1}{d_1}, \operatorname{Re} = \frac{cd_1}{v}, \overline{\psi} = \frac{\psi}{cd_1}, \overline{K} = \frac{K}{d_1^2}, \theta = \frac{T - T_0}{T_1 - T_0}, Ec = \frac{c^2}{c'(T_1 - T_0)}, \operatorname{Pr} = \frac{\rho v c'}{K'}, M = \sqrt{\frac{\sigma}{\mu}} B_0 d_1,$$

$$\alpha = d_1 \sqrt{\frac{\mu}{\eta}}$$

$$(4)$$

Using the above non-dimensional quantities and neglecting the terms of order δ and higher, the resulting equations in terms of stream function $\psi(u = \frac{\partial \psi}{\partial v}, v = -\delta \frac{\partial \psi}{\partial x})$ can be written as

$$\psi_{yyyy} - N^2 \psi_{yy} - \frac{1}{\alpha^2} \psi_{yyyyyy} = 0,$$

$$\frac{1}{\Pr} \theta_{yy} + Ec \psi_{yy}^2 = 0$$
(5)
(5)

Since we are considering the partial slip on the wall, therefore, the corresponding boundary conditions for the present problem can be written as

$$\psi = \frac{q}{2} \text{ at } y = h_1 = 1 + a \cos 2\pi x, \quad \psi = -\frac{q}{2} \text{ at } y = h_2 = -d - b \cos(2\pi x + \phi),$$
$$\frac{\partial \psi}{\partial y} + L \frac{\partial^2 \psi}{\partial y^2} = -1 \text{ at } y = h_1, \quad \frac{\partial \psi}{\partial y} - L \frac{\partial^2 \psi}{\partial y^2} = -1 \text{ at } y = h_2,$$

The finishing couple stress boundary condition are:

$$\frac{\partial^3 \psi}{\partial y^3} = 0 \text{ at } y = h_1, \quad \frac{\partial^4 \psi}{\partial y^4} = 0 \text{ at } y = h_2.$$

And the boundary condition for heat transfer are :

$$\theta = 0$$
 on $y = h_1$, $\theta = 1$ on $y = h_2$. (7)

Where q is the flux in the wave frame, a, b, ϕ and d satisfy the relation $a^2 + b^2 + 2ab\cos\phi \le (1+d)^2$ The solution of the momentum equation straight forward can be written as

$$\Psi = f_0 + f_1 y + f_2 \cosh m_1 y + f_3 \sinh m_1 y + f_4 \cosh m_2 y + f_5 \sinh m_2 y$$
(8)
Where

$$m_{1} = \sqrt{\frac{\alpha^{2} - \sqrt{\alpha^{2}(-4N^{2} + \alpha^{2})}}{2}}, m_{2} = \sqrt{\frac{\alpha^{2} + \sqrt{\alpha^{2}(-4N^{2} + \alpha^{2})}}{2}}, N^{2} = \frac{1}{K} + M^{2}$$
(9)

The functions $f_0, ..., f_5$ are large expressions will not mentioned here for sake of simplify. The flux and average volume flow rate is defined

$$Q = \frac{1}{T} \int_{0}^{T} \overline{Q} dt = \frac{1}{T} \int_{0}^{T} (q + h_1 - h_2) dt = q + 1 + d, \qquad (10)$$

$$\frac{dp}{dx} = \psi_{yyy} - N^2 \psi_y - \frac{1}{\alpha^2} \psi_{yyyyy}$$

$$\Delta \mathbf{P} = \int_{0}^{1} \frac{dp}{dx} dx \qquad (11)$$

The axial velocity component in the fixed frame is given as

 $U(X,Y,t) = 1 + \psi_y = 1 + f_1 + f_2 m_1 \sinh m_1 y + f_3 m_1 \cosh m_1 y + f_4 m_2 \sinh m_2 y + f_5 m_2 \cosh m_2 y$ (12) Where $h_1 = 1 + a \cos[2\pi (X - t)]$ And $h_2 = -d - b \cos[(2\pi (X - t) + \phi)]$

By using Eq. (8) the solution of Eq. (6) satisfying the boundary conditions (7) can be written as:

www.iiste.org IISTE

$$\theta = \theta(f_0, f_1, f_2, f_3, f_4, f_5, m_1, m_2, Ec, \Pr, y)$$
(13)

where c_1 and c_2 are constant can be determined from the boundary conditions (7).

3. Result and discussion

In this section, the results are discussed through the graphical illustrations for different physical quantities. Figs.2-6 show the pressure gradient for different value of K, couple stress α , magnetic field M, amplitude ratio ϕ and partial slip L It is noticed that pressure gradient is maximum at X=0.5 for α =1, K=.1, M=4 and L=1, and the pressure gradient increase when the parameter M increase as shown in Fig.4 and the pressure gradient decrease when the other parameters increase.



Fig.2.variation of dp/dx with x for different values of $\, lpha \,$ at , Fig.3. variation of dp/dx with x for different values of K at, k=1000, M=0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, φ =0.001

 α =1, M=0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, ϕ =0.001



Fig.4. variation of dp/dx with x for different values of M at α =1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, ϕ =0.001









Fig.6. variation of dp/dx with x for different values of ϕ at $\alpha = 1$, K = 0.1, d=2, Q=-1, a=0.7, b=1.2, M=1, L = 0.001 Pressure rise is important physical measures in peristaltic mechanism, so Figs.7-10 show the effect of L, α , M and ϕ on pressure rise. We noticed that increases in α , M and K the Δp increases and increases in L the Δp will be decreases. In Figs.7 and 9 Δp increases for Q> -0.4 and for Q< -0.4, Δp has an opposite behavior. In Fig.8 Δp increases for Q> 0, Δp has an opposite behavior. in fig.10 Δp increases for Q > 6.5 and for Q < 6.5, Δp has an opposite behavior.



$$\phi = \frac{\pi}{6}$$
, k=.2, M=2, d=2, a=0.7, b=1.2, L=0.02, y=1.4 $\alpha = 2$, $\phi = \frac{\pi}{6}$, k=.1000, M=2, d=2, a=0.7, b=1.2, y=1.4



Fig.9. variation of Q with Δp for different values of K at Fig.10. variation of Q with Δp for different values of M at

$$\alpha = 2$$
, M=2, d=2, a=0.7, b=1.2, L=0.04, $\phi = \frac{\pi}{6}$, y=1.4. $\alpha = 1$, K=10, d=2, a=0.7, b=1.2, L=0.04, $\phi = \frac{\pi}{6}$, y=1.4.

Figs.11-15 illustrate the velocity field for different values of α ,K,M and L.It is observed that the velocity field increase when K increase and velocity field decrease when the other parameters increase and finally the shapes looks like a parabola and it can be noticed that the velocity take the maximum value in the middle.





The temperature field for different value of L, M, K, Pr, EC, α are shown in Figs.16 – 20.It is noticed from the figures that the increase in L,M and α the temperature filed decreases while the increase in Pr, Ec, the temperature filed increases.

Finally ,the Figs.21-25 describe the stream line and trapping phenomena and the effect of α, L, M, Q , and ϕ we noticed that all diagram were not symmetric and the trapping is about the center line and the trapped bolus decrease in size as α, M, L increase and slowly disappear for the large value while increase of the parameter Q the trapping will be increase.





Fig.20.variation of temperature θ with Y for different value of Ec at, α =1, k=1000, M=1, d=1.5, Q=1.4, a=0.5, b=1.2, L=.01, ϕ =Pi/2, x=1, Pr=1



Fig.21.stream line for different values of M.(a) M=.1,(b) M=.5(c)M=1(d)M=1.5,(e)M=1.85,(f)M=2 and the other parameters are



Advances in Physics Theories and Applications ISSN 2224-719X (Paper) ISSN 2225-0638 (Online) Vol.31, 2014



Fig.24.stream line for different values of ϕ .(a) ϕ =0,(b) ϕ =Pi, (c) ϕ =Pi/2,(d) ϕ = Pi/3,(e) ϕ = Pi/4,(f)= ϕ Pi/5 and the other parameters are α =1,k=0.2,d=1,Q=1.5,a=0.5,b=0.5,L=0.02, M=2.



 $\label{eq:Fig.25.stream line for different values of \mathcal{X} .(a) $\mathcal{X}=.5$,(b) $\mathcal{X}=1$,(c) $\mathcal{X}=1$,(d) $\mathcal{X}=2$,(e) $\mathcal{X}=2$,(f)=2.4$, and the other parameters are $k=0.2$,d=1,Q=1.5,a=0.5,b=0.5,L=0.02$, M=2.}$

4. Conclusion

we have discussed the influence of couple stress with heat transfer and magnetic field on the peristaltic flow of a non-Newtonian fluid with partial slip. the governing equations of motion and energy equation have been calculated under the assumptions of long wave length approximation The results are discussed through graphs. We have concluded the following observations:

- 1. The pressure gradient decreases with the increases in M.
- 2. The pressure rise decreases with the increases in Land increases when α ,M and K increases
- 3. The velocity field increases with the increase in k and decreases with the increase in M, α , L, ϕ .
- 4. The temperature field decreases with the increase in L,M and α , while with the increase in Pr and Ec, the temperature field increases.
- 5. The size of the trapping bolus decreases by increasing in L, α , M.
- 6. The size of the trapping bolus increases by increasing Q.
- 7. If the couple stress $\alpha \to \infty$ then the solution of S. Nadeem et al.[41] is recovered as a special case of our analysis.

References

- 1. Stokes, V.K.: Couple stress fluid. Phys. Fluids. 9, 1709-1715 (1966)
- 2. Srivastava, L.M.: Peristaltic transport of a couple stress fluid. Rheol. Acta. 25, 638-641 (1986)

3. Mekheimer, Kh.S., Abd Elmaboud, Y.: Peristaltic flow of a couple stress fluid in an annulus: Application of an endoscope.Phys. A 387, 2403–2415 (2008)

4. Valanis, K.C., Sun, C.T.: Poiseuille flow of a fluid with couple stress with application to blood flow. Biorheology 6, 85–97 (1969)

- 5. Abd El Hakeem, Abd El Naby, A.E.M. El Misiery, Appl. Math. Comput. 128 (2002) 19.
- 6. T. Hayat, Y. Wang, A.M. Siddiqui, K. Hutter, S. Asghar, Math. Models Methods Appl. Sci. 12 (2002) 1691.
- 7. T. Hayat, A.H. Kara, E. Momoniat, Int. J. Non-Linear Mech. 38 (2003) 1533.
- 8. C. Fetecau, C. Fetecau, Z. Angew. Math. Phys. 56 (2005) 1098.
- 9. C. Fetecau, C. Fetecau, Int. J. Eng. Sci. 43 (2005) 781.
- 10. T. Hayat, A.H. Kara, Comput. Model. 43 (2006) 132.
- 11. T. Hayat, N. Ali, Physica A 371 (2006) 188.
- 12. T. Hayat, N. Ali, S. Asghar, Appl. Math. Comput. 186 (1) (2007) 309.
- 13. T. Hayat, N. Ali, Appl. Math. Comput. 193 (2) (2007) 535.

14. Vajravelu, K., Sreenadh, S.: Peristaltic transport of a Herschel-Bulkley fluid in an inclined tube. Int. J. Non-LinearMech. 40, 83 (2005)

15. Nadeem, S., Akbar, N.S.: Effects of heat transfer on the peristaltic transport of MHD Newtonian fluid with variable viscosity: Application of Adomian decomposition method. Commun. Nonlinear Sci. Numer. Simul. 14, 3844–3855 (2009)

16. Nadeem, S., Akram, S.: Heat transfer in a peristaltic flow of MHD fluid with partial slip. Commun. Nonlinear Sci. Numer.Simul. 15, 312–321 (2010)

17. Ali, N., Hayat, T., Sajid, M.: Peristaltic flowof a couple strees fluid in an asymmetric channel. Biorheology 44, 125–138 (2007)

18. Haroun, M.H.: Non-linear peristaltic flow of a fourth grade fluid in an inclined asymmetric channel. Comput. Mater.Sci. 39, 324–333 (2007)

19. El Hakeem, A., El Naby, A., El Misiery, A.E.: Efects of an endoscope and generalized Newtonian fluid on peristaltic motion. Appl. Math. Comput. 128, 19–35 (2002)

20. Haroun, M.H.: Effects of Deborah number and phase difference on peristaltic transport of a third order fluid in an asymmetric channel. Commun. Nonlinear Sci. Numer. Simul. 12, 1464–1480 (2007)

21. Hayat, T., Ali, N., Asghar, S.: Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Phys. Lett. A 363, 397–403 (2007)

22. Haroun, M.H.: J. Appl. Mech. Tech. Phys. 46, 842-850 (2005)

23. Mekheimer, Kh.S.: Nonlinear peristaltic transport through a porous medium in an inclined planar channel. J. PorousMedia. 6, 189–201 (2003)

24. Elshehawey, E.F., Eldabe, N.T., Elghazy, E.M., Ebaid, A.: Peristaltic transport in an asymmetric channel through porousmedium. Appl. Math. Comput. 182, 140–150 (2006)

25. El Hakeem, A., El Naby, A., El Misiery, A.E.: Hydromagnetic flow of generalized Newtonian fluid through a uniform tube through peristalsis. Appl. Math. Comput. 173, 856–871 (2006)

26. Hayat, T., Ali, N.: A mathematical decription of peristaltic hydromagnetic flow in a tube. Appl. Math. Comput. 188, 1491–1502 (2007)

27. Mekheimer, Kh.S.: Effect of induced magnetic field on peristaltic flow of a couple stress fluid. Phys. Lett. A. 372, 4271–4278 (2008)

28. V.C.A. Ferraro, An Introduction to Magneto-Fluid Mechanics, Clarendon Press, Oxford, 1966.

29. Kh.S. Mekheimer, Arab. J. Sci. Eng. 28 (2A) (2003) 183.

30. Kh.S. Mekheimer, T.H. Al-Arabi, Int. J. Math. Math. Sci. 26 (2003) 1663.

31. T. Hayat, F.M. Mahomed, S. Asghar, Nonlinear Dynam. 40 (2005) 375.

32. Abd El Hakeem, Abd El Naby, A.E.M. El Misery, M.F. Abd El Kareem, Physica A367(2006)79.

33. T. Hayat, N. Ali, Physica A 370 (2006) 225.

34. T. Hayat, O. Masood Khan, A.M. Siddiqui, S. Asghar, Commun. Nonlinear Sci.Numer. Simul. 12 (2007) 910.

35. Kh.S. Mekheimer, Y. Abd el Mabod, Phys. Lett. A 372 (2007) 225.

- 36. T. Hayat, N. Ali, Phys. Lett. A 363 (2007) 397.
- 37. T. Hayat, A. Afsar, M. Khan, S. Asghar, Comput. Math. Appl. 53 (7) (2007) 1074.
- 38. T. Hayat, N. Ali, Appl. Math. Comput. 188 (2) (2007) 1491.

39. Y. Wang, T. Hayat, N. Ali, M. Oberlack, Physica A 387 (2–3) (2008) 347.

40. N. Ali, Q. Hussain, T. Hayat, S. Asghar, Phys. Lett. A 372 (9) (2008) 1477.

41.S.Nadeem, Safia Akram: Heat transfer in a peristaltic flow of MHD fluid with partial slip.commun NonlinearSci Numer Simulat 15(2010)312-321.

The IISTE is a pioneer in the Open-Access hosting service and academic event management. The aim of the firm is Accelerating Global Knowledge Sharing.

More information about the firm can be found on the homepage: <u>http://www.iiste.org</u>

CALL FOR JOURNAL PAPERS

There are more than 30 peer-reviewed academic journals hosted under the hosting platform.

Prospective authors of journals can find the submission instruction on the following page: <u>http://www.iiste.org/journals/</u> All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Paper version of the journals is also available upon request of readers and authors.

MORE RESOURCES

Book publication information: <u>http://www.iiste.org/book/</u>

Recent conferences: http://www.iiste.org/conference/

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library, NewJour, Google Scholar

