Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol 4, 2012 www.iiste.org

Vibrational Characteristics of C-C-C-C Visco-Elstic Plate with Varying Thickness

Anupam Khanna*
Dept. Of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala, Haryana (INDIA)
*E-mail of corresponding author: rajieanupam@gmail.com

Abstract

The main objective of the present study is to analyze vibrational characteristics of visco-elastic rectangular plate whose thickness varies in two directions. Two dimensional variation of thickness is considered as cubically in x-direction and linearly in y-direction. 4 Rayleigh- Ritz technique is used to get convenient frequency equation. Time period (K) and deflection (w) i.e. amplitude of vibrating mode at different instants of time for the first two modes of vibration are calculated for various values of taper constant and aspect ratio.
Keywords: Visco-elastic, recatngular plate, variable thickness and vibration.

1. Introduction

In these days' researchers, scientists and technocrats are in search of material having less weight, size, low expenses, enhanced durability and strength. Now a days, plates of variable thickness commonly used in Modern technology such as in aerospace, nuclear plants, power plants etc. It may also used for construction of wings & tails of aero planes, rockets and missiles. There are different kinds of visco-elastic plates of variable thickness such as rectangular plates, square plates, circular plates, parallelogramic plates.

Recent development in technology, such as aeronautical field, nuclear reactor etc., study of vibrations of visco elastic isotropic plates with one directional varying thickness has great importance. But a very little work have been done in this field of two dimensional varying thickness. Few papers are available on the vibrations of uniform visco-elastic isotropic beams and plates.

It is assumed that rectangular visco-elastic plate is clamped support on all the four edges (C-C-C-C) and the visco-elastic properties of the plate are of the 'Kelvin' type. All the material constants used in numerical calculation have been taken for the alloy 'DURALIUM'. Assuming that deflection is small so it is refered to amplitude of vibrating mode.

Time period and deflection at different instant of time for first two modes of vibration for various values of aspect ratio (a/b) and taper constants (β_1 & β_2) are calculated. All the above results are also illustrated with graphs.

2. Analysis and Equation of Motion

The equation of motion of a visco-elastic isotropic plate of variable thickness is [4]:

$$[D_1(\partial^4W/\partial x^4+2\partial^4W/\partial x^2\partial y^2+\partial^4W/\partial y^4)+2D_{1,x}(\partial^3W/\partial x^3+\partial^3W/\partial x\partial y^2)+2D_{1,y}\,\partial^3W/\partial y^3+\partial^3W/\partial x^2\partial y)+D_{1,xx}(\partial^2W/\partial x^2+\nu\partial^2W/\partial y^2)+D_{1,yy}(\partial^2W/\partial y^2+\nu\partial^2W/\partial x^2)$$

$$+2(1-\nu)D_{1,xy}\partial^2W/\partial x\partial y] - \rho\, h\, p^2\, W = 0 \qquad \text{------(1)}$$

and

$$\ddot{T}+ p^2\,\tilde{D}T =0 \qquad \text{-------(2)}$$

where equations (1) and (2) are the differential equation of motion for isotropic plate of variable thickness and time function for visco-elastic plate for free vibration respectively with a constant p^2.

The expressions for Kinetic energy T and Strain energy V are [6]

$$T = (1/2)\,\rho\, p^2 \int_0^a\int_0^b h\, W^2\, dxdy \quad \text{----- (3)}$$

and

$$V=(1/2)\int_0^a\int_0^b D_1\{(\partial^2W/\partial x^2)^2+(\partial^2W/\partial y^2)^2+2\nu(\partial^2W/\partial x^2)(\partial^2W/\partial y^2)+2(1-\nu)(\partial^2W/\partial x\partial y)^2\}\, dx\, dy \quad \text{---(4)}$$

Assuming that the thickness variation of the plate in both directions as

$$h = h_0(1 +\beta_1 x^3/a^3)(1 + \beta_2 y/b) \quad \text{-------(5)}$$

The flexural rigidity of the plate can now be written as (assuming poisson's ratio ν is constant)

$$D_1= E\, h_0^3\, (1 +\beta_1 x^3/a^3)^3 (1 + \beta_2 y/b)^3/12(1-\nu^2) \quad \text{------- (6)}$$

3. Solution and Frequency Equation

Rayleigh-Ritz technique is used to find the solution. This method requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V-T)=0 \quad \text{------ (7)}$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

For a rectangular plate clamped (c) along all the four edges, the boundary conditions are

$$W = W_{,x} = 0 \quad \text{at } x = 0, a \quad \text{and} \quad W= W_{,y} = 0 \quad \text{at } y = 0, b \quad \text{-------(8)}$$

and the corresponding two-term deflection function is taken as [4]

$$W=[(x/a)(y/b)(1-x/a)(1-y/b)]^2\,[A_1+A_2(x/a)(y/b)(1-x/a)(1-y/b)] \quad \text{----(9)}$$

which is satisfied equations (8).

Assuming the non-dimensional variables as

$$X = x/a,\ Y= y/a,\ \overline{W}=W/a,\ \overline{h} =h/a \quad \text{-------(10)}$$

and using equations (6) & (10) in equation (3) & (4), one obtains

$$T = (1/2)\,\rho\, p^2\, \overline{h_0}\, a^5 \int_0^1\int_0^{b/a} [\,(1 +\beta\ X^3)(1 + \beta_2 Ya/b)\overline{W}^2]\, dXdY \quad \text{------(11)}$$

and

$$V=Q\int_0^1\int_0^{b/a} [(1 +\beta\ X)^3(1+\beta_2 Ya/b)^3\{(\partial^2W/\partial X^2)^2+(\partial^2W/\partial Y^2)^2+2\nu(\partial^2W/\partial X^2)(\partial^2W/\partial Y^2) + 2(1-\nu)(\partial^2\overline{W}/\partial X\partial Y)^2\}\, dX\, dY \quad \text{-------(12)}$$

Substituting the values of T& V from (11) & (12) in equation (7), one obtains

$$(V^* -\lambda^2 p^2 T^*) = 0 \quad \text{------- (13)}$$

where

$$V^*=\int_0^1\int_0^{b/a} [(1 +\beta_1 X)^3 (1+\beta_2 Ya/b)^3\{(\partial^2\overline{W}/\partial x^2)^2+(\partial^2\overline{W}/\partial y^2)^2+2\nu(\partial^2\overline{W}/\partial x^2)(\partial^2\overline{W}/\partial y^2) +2(1-\nu)(\partial^2\overline{W}/\partial x\partial y)^2\}]dXdY \quad \text{------(14)}$$

and

$$T^{*} = \int_0^1 \int_0^{b/a} [(1+\beta_1 X)^3 (1+\beta_2 Y a/b)^3 W^2] dX\, dY \qquad \text{-------- (15)}$$

Here $\lambda^2 = 12\rho\,(1-\nu^2)\, a^2 / E h_0^2$ ----------(16)

Equation (13) having the unknowns A_1 & A_2 arising due to the substitution of W from equation (9).
These two constants are to be determined from equation (13) as

$$\partial(V^{*} - \lambda^2 p^2 T^{*}) / \partial A_n = 0, \qquad n = 1, 2 \qquad \text{--------(17)}$$

After simplifying equation (17), one gets

$$b_{n1} A_1 + b_{n2} A_2 = 0, \qquad n = 1, 2 \qquad \text{-------(18)}$$

where b_{n1}, b_{n2} (n=1,2) involve parametric constant and the frequency parameter.
For a non-trivial solution, the determinant of the coefficient of equation (18) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \qquad \text{--------- (19)}$$

From equation (19), one can obtains a quadratic equation in p^2 from which the two values of p^2 can found. After determining A_1 & A_2 from (18), one can obtain deflection function W as

$$W = [XY(a/b)(1-X)(1-Ya/b)]^2 [1 + (-b_{11}/b_{12}) X Y (a/b)(1-X)(1-Ya/b)] \qquad \text{--------(20)}$$

5. Time Functions of Visco-elastic Plates

Equation (2) is define as general differential equation of Time functions of free vibrations of visco-elastic plate. It depends on visco-elastic operator $\tilde{D}$.
For Kelvin's model, one have

$$\tilde{D} \equiv \{1 + (\eta/G)(d/dt)\} \qquad \text{------- (21)}$$

After using equation (21) in equation (2), one obtains

$$T_{,tt} + p^2 (\eta/G) T_{,t} + p^2 T = 0 \qquad \text{------(22)}$$

Equation (22) is a differential equation of second order for time function T.
Solution of equation (22) will be [11] :

$$T(t) = e^{a_1 t} [C_1 \cos b_1 t + C_2 \sin b_1 t] \qquad \text{------(23)}$$

where

$$a_1 = -p^2 \eta / 2G \quad \& \quad b_1 = p\sqrt{1 - (p\eta/2G)^2} \qquad \text{------ (24)}$$

and C_1, C_2 are constants which can be determined easily from initial conditions of the plate.
Assuming initial conditions as

$$T = 1 \quad \& \quad T_{,t} = 0 \quad \text{at} \quad t = 0 \qquad \text{------(25)}$$

Using equation (25) in equation (23), one obtains

$$C_1 = 1 \quad \& \quad C_2 = -a_1/b_1 \qquad \text{------ (26)}$$

After using equation (26) in equation (23), one gets

$$T(t) = e^{a_1 t} [\cos b_1 t + (-a_1/b_1) \sin b_1 t] \qquad \text{-------(27)}$$

Thus, deflection i.e. amplitude of vibrating mode w(x,y,t), which is equal to W(x,y)T(t), may be expressed as

$$w = [XY(a/b)(1-X)(1-Ya/b)]^2 [1 + (-b_{11}/b_{12}) XY(a/b)(1-X)(1-Ya/b)] \times [e^{a_1 t} \{\cos b_1 t + (-a_1/b_1) \sin b_1 t\}] \qquad \ldots(28)$$

by using equation (27) & equation (20).
Time period of the vibration of the plate is given by

$$K = 2\pi / p \qquad \text{-----(29)}$$

where p is frequency given by equation (19).

5. Result and Discussion

The values of time period (K) and deflection (w) for a isotropic visco-elastic rectangular plate for different values of aspect ratio a/b and taper constants β_{1} & β_2 at different points for first two modes of vibrations are calculated. The following material parameters are used for calculation which is for DURALIUM reported at [4]:

$E = 7.08 \times 10^{10}$ N/M^2, $G = 2.632 \times 10^{10}$ N/M^2, $\nu = 0.345$, $\eta = 14.612 \times 10^5$ N.S/M^2, $\rho = 2.80 \times 10^3$ Kg/M^3. The thickness of the plate at the centre is taken as h_o= 0.01 meter.

Computations have been made for calculating time period K and deflection w for different values of taper constants β_1 & β_2 and aspect ratio a/b for first two modes of vibration.

Fig. 1 shows a steady decrease in time period K with increase of taper constant β_1 for fixed aspect ratio a/b(=1.5) and for two values of β_2. It is simply seen that time period K decreases as taper constants increase for both the modes of vibration.

Fig. 2 shows time period K for different values of aspect ratio a/b for both the modes of vibration for uniform and non-uniform thickness having the following cases**:**

(i) $\beta_1 = \beta_2 = 0.0$ (ii) $\beta_1 = \beta_2 = 0.6$;

In both the above cases, one can note that time period K decreases as aspect ratio a/b increases for both the modes of vibration.

Figs. 3, 4, 5 and 6respectively show the numerical values of deflection w for fixed aspect ratio a/b (=1.5) for first two modes of vibration for different values of X for two values of Y i.e. Y=0.3 & Y=0.6 for the following cases**:**

(i) Fig. 3: $\beta_1 = \beta_2 = 0.0$ and Time is 0.K
(ii) Fig. 4: $\beta_1 = \beta_2 = 0.0$ and Time is 5.K
(iii) Fig. 5: $\beta_1 = \beta_2 = 0.6$ and Time is 0.K
(iv) Fig. 6: $\beta_1 = \beta_2 = 0.6$ and Time is 5.K

For first and second mode of vibration in figs. 4, 5 and 6; separate figs. are given. In all above four figs., deflection w for first mode of vibration first increases and then decreases as X increases for different values of Y (Y=0.3 & y=0.6) and for the second mode of vibration, deflection w first decreases and then increases for Y=0.3 & Y=0.6.

References

B. Singh and V. Saxena (1996), "Transverse vibration of rectangular plate with bi-directional thickness variation" Journal of Sound and Vibration 198, 51-65.

J.S. Tomar, and A. K. Gupta (1985), "Effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions", J. Sound and Vibration 98, 257-262.

A. W. Leissa (1987), "Recent studies in plate vibration 1981-1985 part II, complicating effects", The Shock and Vibration Dig. 19, 10-24.

A.K.Gupta and A. Khanna (2007) "Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions", J. Sound and Vibration.

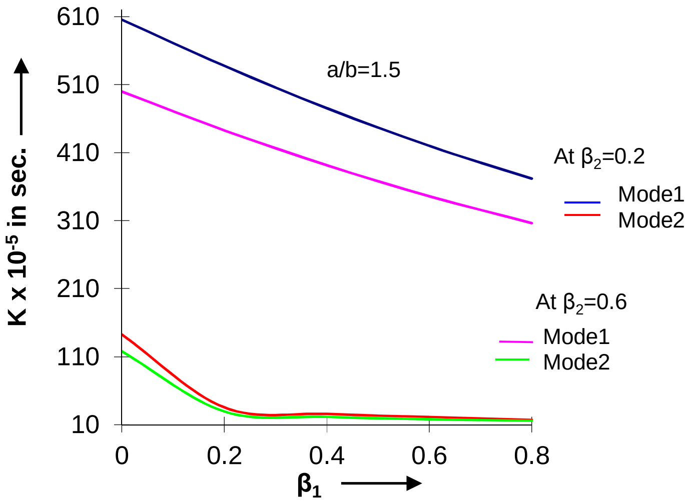
A. W. Leissa (1969), " Vibration of plate", NASA SP-160.

Sanjay Kumar (2003), "Effect of thermal gradient on some vibration problems of orthotropic visco- elastic plates of variable thickness", Ph.D.Thesis, C.C.S.University, Meerut (India).

P. A. A. Laura, R. O. Grossi, and G. I. Carneiro (1979)," Transverse vibrations of rectangular plates with thickness varying in two directions and with edges elastically restrained against rotation" , J.Sound and Vibration 63, 499-505.

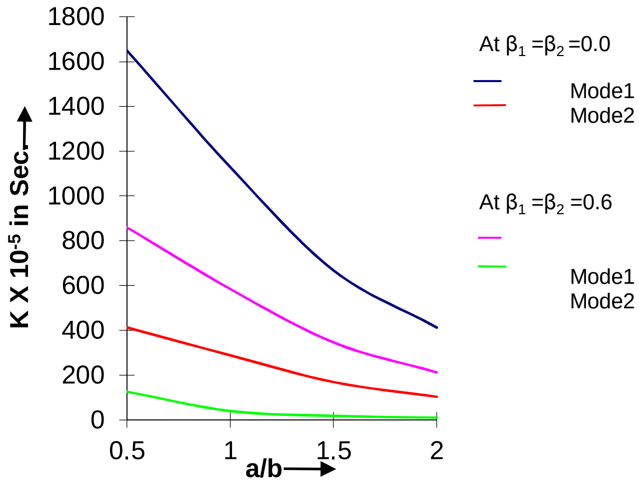
Gupta,A.K. and Khanna, Anupam (2008.), "Vibration of clamped visco-elastic rectangular plate with parabolic thickness variations", J. Shock and Vibration, USA, Gupta, A.K. and Kumar, Lalit (2008), "Thermal effect on vibration of non-homogenous visco-elastic rectangular plate of linear varying thickness", MECCANICA, ITALY,.

Fig. 1



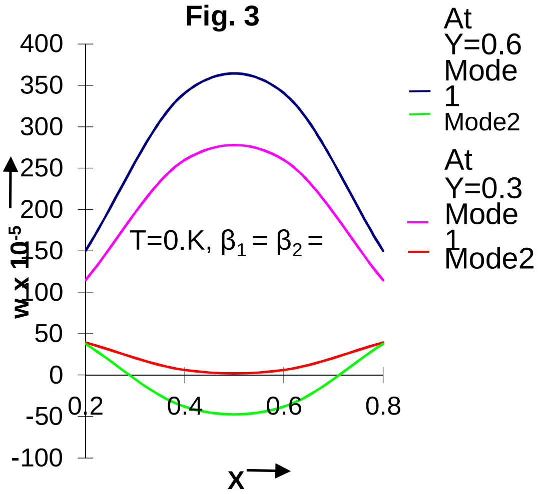
TIME PERIOD K Vs TAPER CONSTANT β_1 OF
A CLAMPED VISCO-ELASTIC RECTANGULAR PLATE

Fig. 2



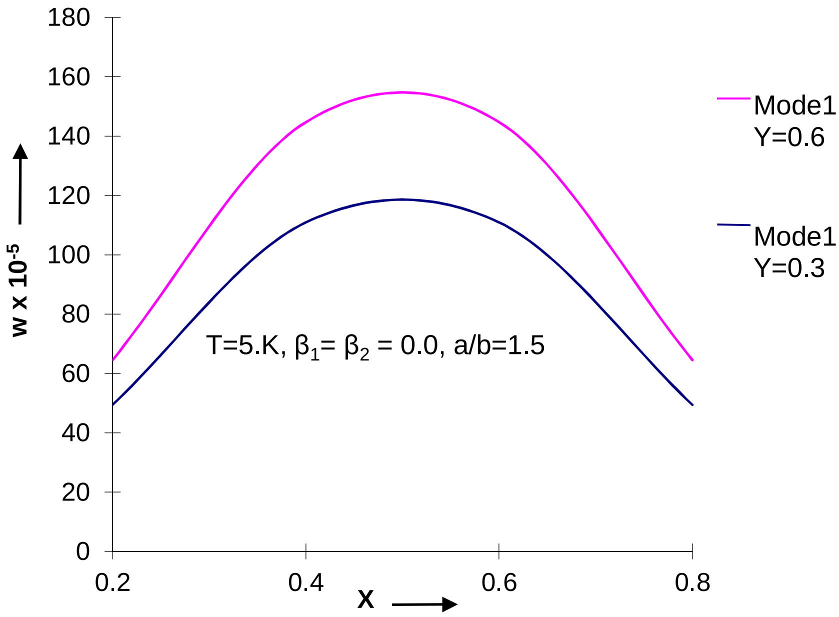
TIME PERIOD K Vs ASPECT RATIO a/b OF A
CLAMPED VISCO-ELASTIC RECTANGULAR PLATE

Fig. 3



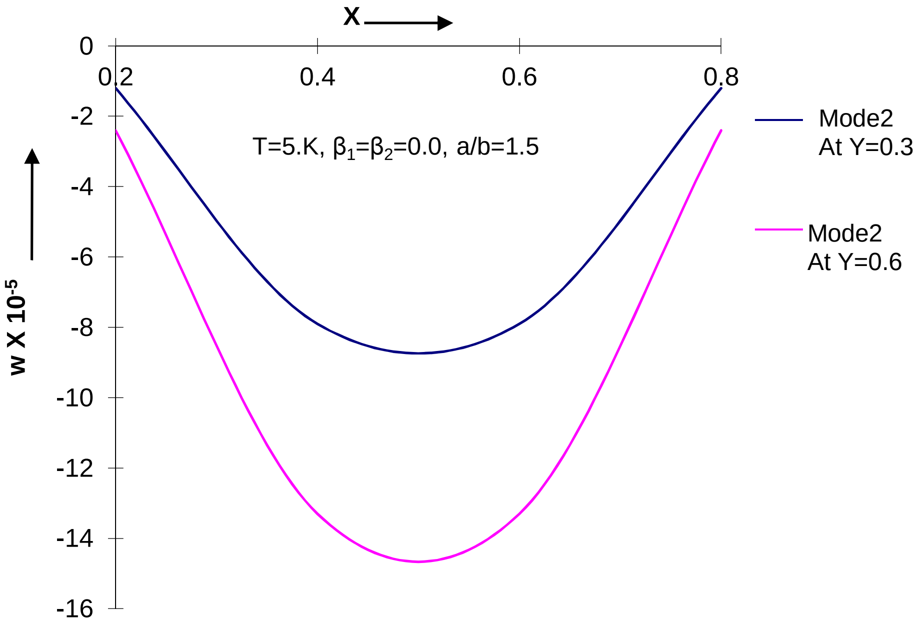
DEFLECTION w Vs X OF A VISCO-ELASTIC RECTANGULAR PLATE

Fig.4a



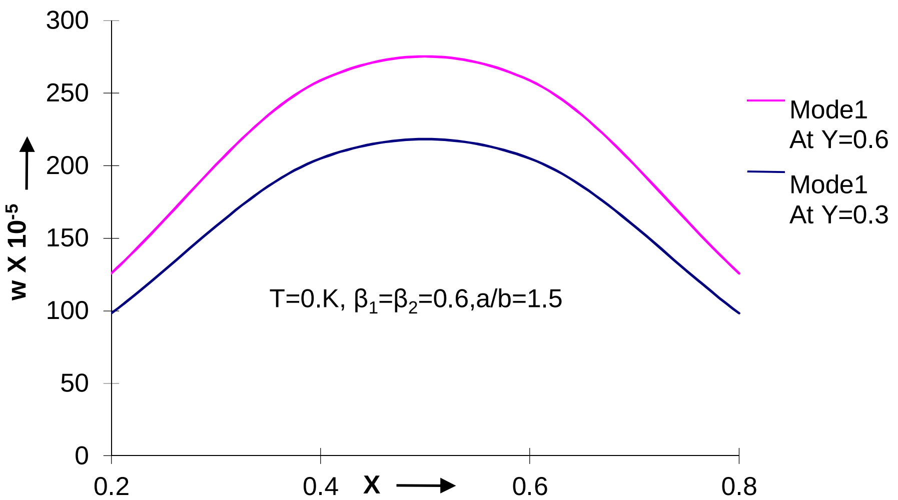
DEFLECTION w Vs X OF A CLAMPED VISCO-ELASTIC RECTANGULAR PLATE

Fig.4b



DEFLECTION w Vs X OF A CLAMPED VISCO-ELASTIC RECTANGULAR PLATE

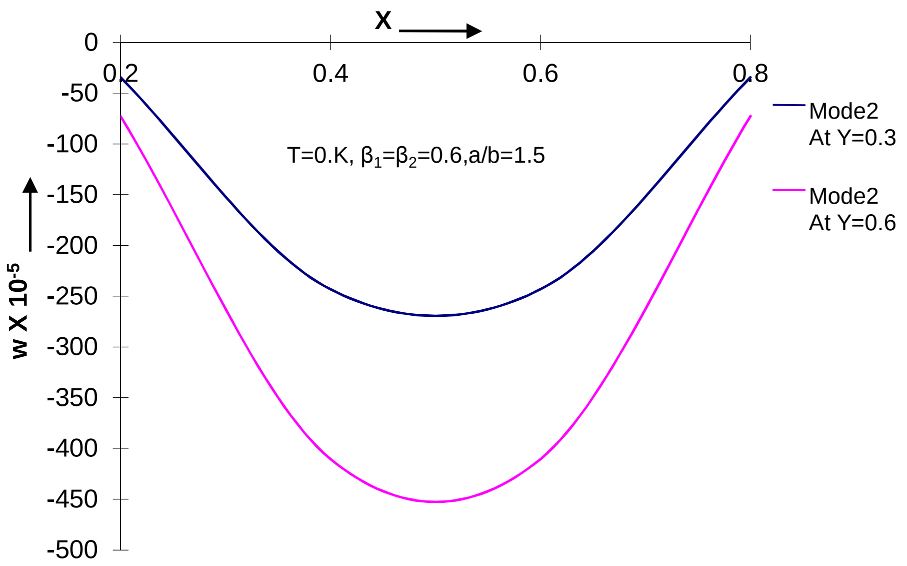
Fig.5a



DEFLECTION w Vs X OF A CLAMPED VISCO-ELASTIC RECTANGULAR PLATE

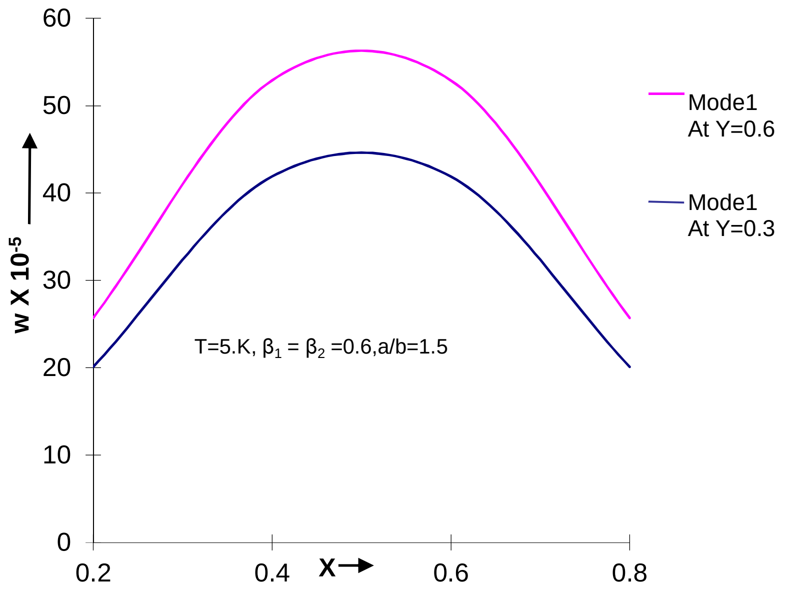
cccc

Fig.5b



DEFLECTION w Vs X OF A CLAMPED VISCO-ELASTIC RECTANGULAR PLATE

Fig.6a



DEFLECTION w Vs X OF A CLAMPED VISCO-ELASTIC RECTANGULAR

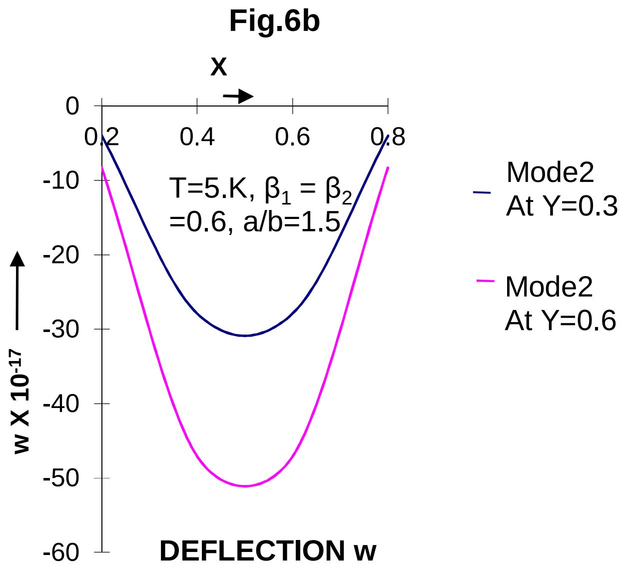


Fig.6b
X
0
0.2
0.4
0.6
0.8
-10
-20
-30
-40
-50
-60
T=5.K, β1 = β2
=0.6, a/b=1.5
w X 10-17
DEFLECTION w
Mode2
At Y=0.3
Mode2
At Y=0.6