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The Influence of the Angle of Inclination on Laminar Natural Convection in a Square Enclosure

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Abstract

This paper discusses the results obtained by the numerical modeling of natural convection in a water-filled two-dimensional square enclosure inclined to the horizontal. Here, the top and bottom walls of the cavity are considered adiabatic, left vertical wall is maintained at a constant low temperature and the right vertical wall is maintained at a constant high temperature. The aim is to investigate the effects of angle of inclination on the flow patterns. We use the Krylov subspace method, GMRES, to solve the discretized formulation of the governing equations. At the validation stage, our results are in good agreement with those reported in the literature. Results are presented in the form of velocity vector and isotherm plots as well as the variation of the average Nusselt number for different angles of inclination.

Keywords: natural convection, GMRES, Nusselt number

1. Introduction

Natural convection is governed by the conservation laws of mass, momentum and energy. For a two-dimensional unsteady flow in rectangular coordinates (x, y), these conservation laws can be expressed in the dimensionless form as (Bejan 1993):

$$\partial u/\partial x+\partial v/\partial x= 0, \tag{1}$$

$$\partial u/\partial t+\partial u^2/\partial x+\partial uv/\partial y = -\partial p/\partial x+ (1/Re)(\partial^2 u/\partial x^2+\partial^2 u/\partial y^2)+(Gr/Re^2)T\sin\theta, \tag{2}$$

$$\partial v/\partial t+\partial v^2/\partial y+\partial uv/\partial x = -\partial p/\partial y+ (1/Re)(\partial^2 v/\partial x^2+\partial^2 v/\partial y^2)+(Gr/Re^2)T\cos\theta, \tag{3}$$

$$\partial T/\partial t+u\partial T/\partial x+v\partial T/\partial y = (1/Re\,Pr)(\partial^2 T/\partial x^2+\partial^2 T/\partial y^2). \tag{4}$$

In the above equations, u and v are fluid velocity components along x and y axes respectively, p is the pressure, T is the temperature and θ is the inclination angle. Re, Gr and Pr are the Reynolds, Grashof and Prandtl numbers. The system we study is illustrated in Fig. 1. It is a square cavity $\Omega(x, y) = [0, 1] \times [0, 1]$, which is completely filled by the fluid, with no-slip boundary condition for velocity components ($u = v = 0$) on all its walls and adiabatic conditions for temperature on its top and bottom walls. The left vertical wall of the cavity is maintained at a constant low temperature (T_c) and the right vertical wall is maintained at a constant high temperature (T_h). The cavity is tilted from the horizontal with angle θ and the working fluid is chosen as water with Prandtl number, $Pr = 5.96$.

2. Procedure

We employ the finite difference method (Ozisik 1994) on a uniform staggered grid to discretize (1)-(4). The resulting set of algebraic equations can be cast as the linear system

$$Ax = b,$$

which is solved using the Krylov subspace method, GMRES (Saad & Schultz 1986). The GMRES algorithm aims at projecting the coefficient matrix A onto the Krylov subspace and henceforth solves a least square system of lower dimension, which is less expensive. The solutions are assumed to converge when the following convergence criterion is satisfied for every dependent variable at every point in the solution domain:

$$|(\Psi_{new} - \Psi_{old})/\Psi_{old}| < 10^{-6},$$

where Ψ represents a dependent variable u, v, p and T.

3. Experiments

The simulation is performed for Gr = 167.8 with different angle of inclination θ, which is varied from 0 to 90^0. Results are obtained for the average Nusselt number (Nu) with respect to θ as shown in Fig. 2. The minimum average Nusselt number Nu_{min} is 0.7426 occurring at θ = 53 .6^0 and the maximum Nu_{max} is θ = 38.8^0 with value 1.1144.

Figs. 3 to 6 display the velocity vector plots and isotherms for different angle of inclination, calculated on a 21×21 grid. It can be observed that near the hot wall the temperature of the fluid becomes positive, causing a pronounced movement of the fluid against the gravity direction. In a similar manner, the fluid gets colder near the opposite wall. A negative buoyancy effect is produced, causing the fluid to move in the direction of gravity. Hence, an overall anticlockwise movement of fluid is produced.

The isotherm patterns are uniformly distributed, which indicate that the heat transfer mechanism is mainly driven by conduction and the temperature field is only marginally distorted by the convective flows. The effect of cavity inclination is clearly visible on both the flow patterns and isotherms.

4. Conclusions

The natural convection in a two-dimensional rectangular enclosure has been analyzed numerically using the finite difference method. The top and bottom walls of the cavity were considered adiabatic and the two vertical walls were maintained at constant low and hot temperatures. The resulting discretized formulation of the governing equations was solved using the GMRES algorithm. The results obtained demonstrate promising avenues for further research.

References

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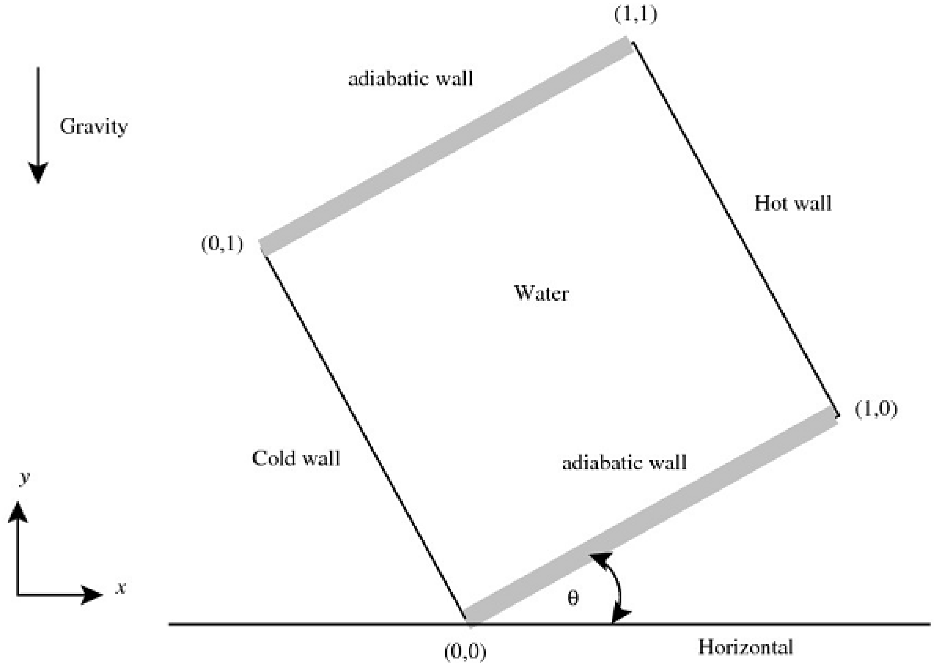


Figure 1. Geometry of the problem in the domain $\Omega(x, y)$

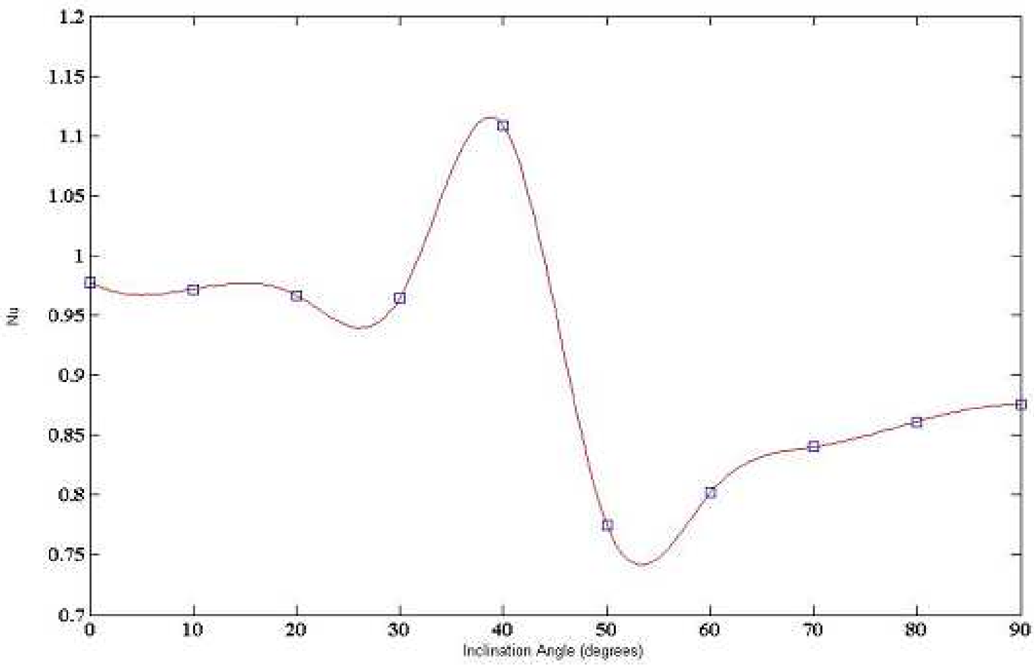
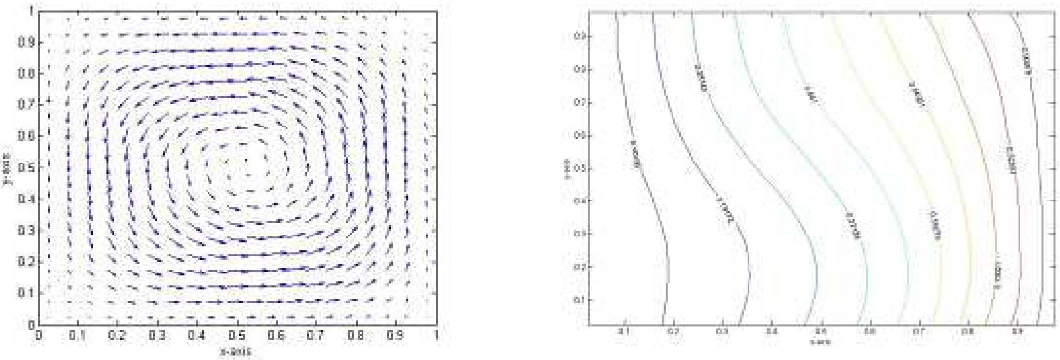
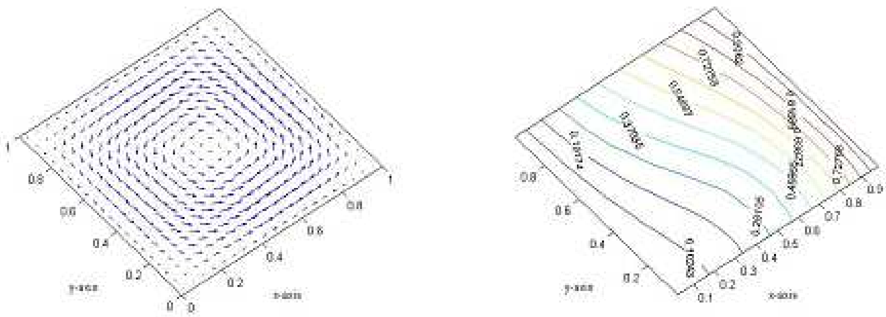


Figure 2. The average Nusselt number versus tilt angle for *Gr* = 167.8



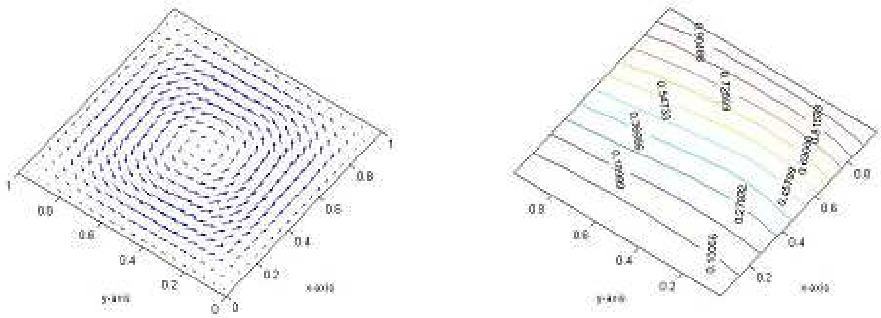
(a) Flow field (b) Isotherms

Figure 3. Velocity vectors and isotherms for $\theta = 0^0$



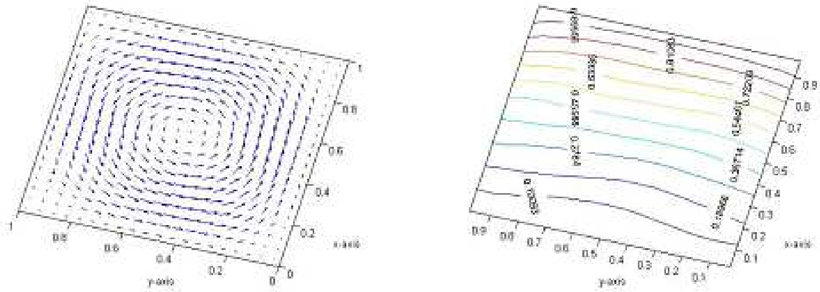
(a) Flow field (b) Isotherms

Figure 4. Velocity vectors and isotherms for $\theta = 35^0$



(a) Flow field (b) Isotherms

Figure 5. Velocity vectors and isotherms for $\theta = 55^0$



(a) Flow field (b) Isotherms

Figure 6. Velocity vectors and isotherms for $\theta = 75^0$