The Effects of Surface Roughness on the Squeeze Film Characteristics with Couple stress fluids in Hip joint

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Abstract

On the basis of the Stokes micro continuum theory , This paper aims to study the effects of surface roughness and couple stress on the squeeze film Characteristics in hip joint. The cartilage is modeled as biphasic poro-elastic matrix and synovial fluid is modeled as couple stress fluid . Compared to the conventional Newtonian lubricant case, the couple stress and surface roughness effects characterized by the couple stress and surface roughness parameter signify an improvement in the squeeze film Characteristics. Increasing values of the surface roughness parameter increases the load-carrying capacity and the squeeze in the squeeze film can be decreased and provides a longer time to prevent sphere –plane surface contact .The approaching time of the sphere in reducing the film thickness h*=1 to h*= 0for the couple stress fluid lubricant which is longer than surface roughness.

Keywords: Surface roughness, Couple stress fluid, Articular cartilage, Synovial fluid, Micro continuum theory, Hip joint**.**

1 Introduction

The lubrication of human synovial joints is used to reduce wear and lower friction. The hip joint is one of the most important joints in the human body. It allows us to walk, run, and jump. It bears our body's , and the hip joint is also one of our most flexible joints and allows a greater range of motion than all other joints in the body weight weigh the squeeze film phenomenon is observed in several engineering application such as gears, beatings, machin tools dampers and human joints (Nordin & Frankel 2001). The squeeze film phenomenon arises when the two lubricating surface move towards each other in normal direction so pressures are generated and in normal circums-stances surface contact does not occur until a long time has elapsed squeeze film lubrication was capable of carrying the heavy loads during the walking process even though the velocity was very low at this time (Jaw , Won & Chi 2004) . The hip joint is a spherical joint between the femoral head and acetabulum in the pelvis ,it is a synovial joint ,since it I wrapped in a capsule that contains the synovial fluid ,a biological lubricant that acts also like a shock absorber (Stokes 1966) . The hip bone is formed by three bones; ilium, ischium, and pubis. At birth, these three component bones are separated by hyaline cartilage. They join each other in a Y-shaped portion of cartilage in the acetabulum. By the end of puberty the three bones will have grown together. shown in figure(1) The synovial fluid", The inner lining of the capsule , the synovial membrane ,secretes a viscous non-Newtonian fluid called synovial fluid .It is believed to be the dialysate of blood plasma with the addition of long chain hyaluronate molecules(hyaluronate acid **T**he thin film of synovial fluid that covers the surfaces of the inner layer of the joint capsule and articular cartilage helps to keep the joint surfaces lubricated and reduces friction (Higginso & Norman 1974).The fluid nourishment for the hyaline cartilage covering the articular surfaces ,as fluid moves in and out of the cartilage as compression is applied, then released .The composition of synovial fluid also contains hyaluronate (hyaluronic acid) and glycoprotein called lubricin .The hyaluronic acid component of synovial fluid is responsible for the viscisity of the fluid and is essential for joint lubrication . hyaluronate reduces the friction between the synovial folds of the capsule and the articular surface .Lubricin is the component of synovial fluid thought to be responsible for cartilage on cartilage lubrication (Mrtin 1953) .Changes in the concentration of hyaluronate or lubricin in the synovial fluid will affect the overall lubrication and the amount of frication that is present. Many experiments have confirmed that articular coefficient of friction in synovial joints are far lower than those created with manufactured lubricants. The lower the coefficient of friction ,the lower the resistance to movement (Mrtin 1953) .Normal synovial fluid appears as a clear, pale yellow fluid present in small amounts at all synovial joints. The synovial fluid ,like all viscous substances ,resists shear loads .The viscosity of the fluid varies inversely with the joint velocity or rate of shear; that is ,it becomes less viscous at high rates. Thus, synovial fluid is referred to as thixotropic .When the bony components of a joint are moving

Fig.(1)show that bones in human hip joint

2 Analysis

On the basis of the Stockes (Lin, Liao& Hung2004) micro-continuum theory the continuity and momentum equations of the flow filed with couples stresses are :

$$\nabla\vec{V} = 0 \qquad (1)$$

$$\rho\frac{D\vec{V}}{Dt} = -\nabla p + \mu\nabla^2 - \eta\nabla^4 \qquad (2)$$

Where $\vec{V}$ the fluid velocity vector ρ is the density, p is the pressure , μ is the viscosity and η is the material Constant responsible for the couple-stress fluid property. In this theoretical study, the lubricant in the system is blended with long chain polymers and can be considered as a Stokes couple stress fluid. In the mean while, the fluid film is assumed to be thin, and the body force and body couples are assumed to absent (Bujurke & Jayaraman1982). Then, the governing equations of the lubricant system in Cartesian co-ordinates reduce to where u and v are the velocity components in the x and y directions,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

$$\mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial x} - R_a \qquad (4)$$

The ratio ($\frac{\eta}{\mu}$) is a dimensional square length and hence characterizes the chain length (Albert. E and Ali.

A. ,(2013).

$$\iota = \sqrt{\frac{\eta}{\mu}} \tag{5}$$

The flow of couple stress fluid in a porous matrix is governed by the modified Darcy law, which accounts for the polar effects

$$q^{\rightarrow} = \frac{-\emptyset}{\mu(1-\beta)} \nabla p \tag{6}$$

where q is the flux (discharge per unit area, with units of length per time ,m/s) ∇P is the pressure gradient vector (Pa/m)

$$as\ q^{\rightarrow} = (u, v)\ and\ \beta = \frac{l^2}{\emptyset} \tag{7}$$

The parameter (β) represents the ratio of the microstructure size to the pore size(Nordin & Frankel 2001) The (*p*) is the pressure in the porous region. Owing to continuity, the Laplace equation is satisfied:-

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \tag{8}$$

Integrating equation (8) with respect to (y) over the porous Layer thickness ($H_\circ$), and using the boundary conditions of solid bearing.

$$\frac{\partial p}{\partial y} = -\int_{-H_\circ}^{0} \left(\frac{\partial^2 p}{\partial x^2}\right) dy \tag{9}$$

Assuming that the porous layer thickness pressure ($H_\circ$)is very small and using the continuity condition $p = p^*$ at the interface y =0 of porous matrix and fluid film, equation (9) reduces to:-

$$v = \frac{\emptyset H_\circ}{\mu(1-\beta)} \left(\frac{\partial^2 p}{\partial x^2}\right) \tag{10}$$

The parameter is a characteristic measure of the polarity of the fluid which is zero in the case of non polar fluid

$$-v_\theta - \frac{\emptyset H_\circ}{\mu(1-\beta)} \left(\frac{\partial^2 P}{\partial x^2}\right) = -\frac{\partial}{\partial x} \int_0^h u\ dy \tag{11}$$

boundary condition for u(x,y) are

$$u|_{y=0} = 0 \qquad \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = 0$$

$$u|_{y=h} = 0 \qquad \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = 0 \tag{12}$$

The solution of Eq. (4) in addition to boundary condition (12)and applying since and cosine hyperbolic expressions to get final form of velocity in Cartesian coordinates :-

$$P^*(\theta) = \frac{3(\pi^2 - 4\theta^2)}{2((-1+\varepsilon)^3 - 12(-1+\varepsilon)l^2 - \left(\frac{12\emptyset H}{1-\beta}\right) - 24l^3 \tanh\left(\frac{1-\varepsilon}{2l}\right)(1-R_a)} \quad (13)$$

Now integrate the continuity Eq.(4)with respect to y with the boundary conditions of u(x,y).Then the modified Reynolds equation governing the film pressure is derived as

$$u(x,y) = \frac{1}{2\mu(1-R_a)}\frac{\partial p}{\partial x}\left\{y(y-h) + 2l^2\left[1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\} \quad (14)$$

Where function $f(h,l)$ is given as:-

$$f(h,l) = h^3 - 12l^2 h + 24l^3 \tanh\left(\frac{h}{2l}\right) \quad (15)$$

Introduce the dimensionless variables and parameters:

$$p^* = \frac{pC^2}{\mu R^2(\frac{d\varepsilon}{dt})}, h^* = \frac{h}{C} = 1 - \varepsilon\cos(\theta), \quad \theta = \frac{X}{R}$$
$$l^* = \frac{l}{C}, \emptyset^* = \frac{\emptyset}{C^2}, H^* = \frac{H}{C},$$
$$R^* = \frac{R}{C}, \Psi = \frac{KH}{C^3}, \varepsilon = \frac{e}{C} \quad (16)$$

Introduce the dimensionless Reynolds Eq.is expressed in a non- dimensionless forms as

$$\frac{\partial}{\partial\theta}\left\{\left[f(h^*,l^*) + \frac{12H^*\emptyset^*}{(1-\beta)}\right]\frac{\partial p^*}{\partial\theta(1-R_a)} = -12\cos(\theta)\right. \quad (17)$$

boundary condition for the film pressure are:

$$p^* = 0 \quad \text{at} \quad \theta = \pm\frac{\pi}{2}$$
$$\frac{dp^*}{d\theta} = 0 \quad \text{at} \quad \theta = 0 \quad (18)$$

Integrating the Reynolds equation with respect to θ with the above conditions, the squeeze film pressure is obtained.

$$P^*(\theta) = \frac{3(\pi^2 - 4\theta^2)}{2((-1+\varepsilon)^3 - 12(-1+\varepsilon)l^2 - \left(\frac{12\emptyset H}{1-\beta}\right) - 24l^3 \tanh\left(\frac{1-\varepsilon}{2l}\right)(1-R_a)} \quad (19)$$

3 Squeeze film characteristics

By integrating the film pressure action upon sphere we can computer load –carrying capacity.

Squeezing

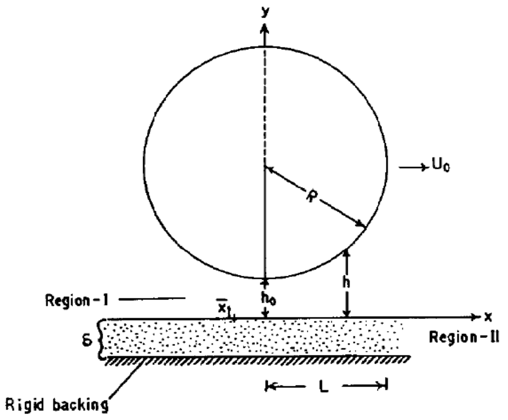

Fig.(2) Squeeze film geometry between a sphere and plan surface.

$$W = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} P(\theta).R\,.\cos(\theta)\,d\theta \qquad (20)$$

Introducing the dimensionless quantity:

$$W^* = \frac{WC^2}{\mu R^3(\frac{d\varepsilon}{dt})} \qquad (21)$$

the dimensionless load –carrying capacity. is given by

$$W^* = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} P^*(\theta).\cos(\theta)\,d\theta \qquad (22)$$

Although the value of the dimensionless load –carrying capacity in Eq. (22) cannot to be calculated by direct integration, they could be numerically evaluated by the , method of power series. then Eq.(22) becomes :-

$$W^* = \frac{\pi^2(1280 - 32\pi^2 + \pi^4)}{1280((-1+\varepsilon)^3 - 12(-1+\varepsilon)l^2 - \left(\frac{120\text{ØH}}{1-\beta}\right) - 24l^3\tanh\left(\frac{1-\varepsilon}{2l}\right)(1-R_a)} \qquad (23)$$

Now dimensionless response time be :

$$t^* = \int_{-h^*}^{1} W^*\,d\varepsilon \qquad (24)$$

4 .Results and Discussion

Using the Stokes micro continuum theory, the influence of couple stresses and surface roughness on the squeeze film characteristics between a sphere and a flat plate is theoretically examined .To take into account the couple stresses effects due to lubricant blended with various additives, the modified Reynolds equation is derived by the Stockes constitutive equation (Nordin & Frankel 2001).The film pressure. I solved and ,then ,applied to evaluate the load –carrying capacity and the evaluate the load –carrying capacity and the time approach .From the Stokes theory , the new material constant η is responsible for the property of couple stresses since the dimension of l is of length ,so addition new material constant R_a to Stokes Eq. where R_a is responsible for the property of surface roughness of articular cartilage ,it may be identified as the characteristic length of additives in a Newtonian lubrication . Therefore ,the effects of couple stresses and surface roughness are characterized by the couple stresses and surface roughness parameter. in the present analysis the result are presented for the parameters following in table (1)

table (1) the numerical values of the parameters involved([Ruggiero. A, Gomez. E.& Damato . R, 2013), Bernaed .H. & Steren. S 1994)

Parameters	Numerical values	Units
Surface roughness	1-3	µm
Permeability of the cartilage Matrix (Φ)	10^{-18}	m^2
Dimensionless couple stress Length (l^*)	0.1-0.5	--------
Eccentricity ratio parameter (ε)	0.1-0.5	---------
Thickness layer of cartilage (H)	3-4	µm
Ratio of microstructure size to Poro size (β)	0.05	---------
Radius of curvature	0.1-1	m
Viscosity of synovial fluid	10^{-2} -10^{-3}	pa.s

4.1. Squeeze film pressure

Fig.3 shows the dimensionless pressure (p^*)generated by squeeze film action with angle (θ°) for different values of couple stress length parameter (l^*),the solid line describes the Newtonian lubrication case. The dashed lines represent the results of the sphere –plate system lubricant with couple stress fluid for l^*=0.3 to l^*=0.1 the influence of couple stress is visibly apparent. The couple stress effects result in higher film pressure , especially in the center of the region of angle.

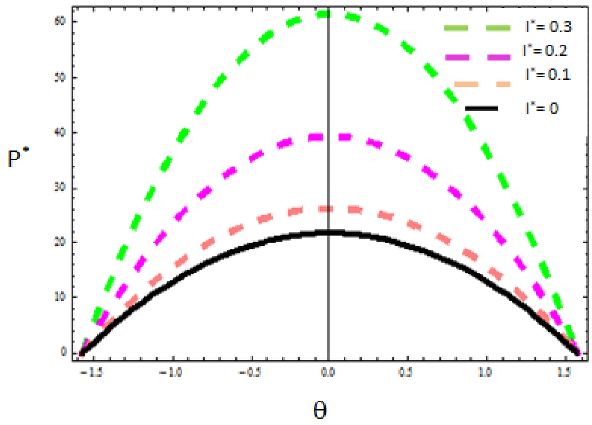

Fig.3 .Dimensionless pressure $p^*$with angle θ for different l^*

Figure (4) depicts the dimensionless pressure (p^*) generated by squeeze film action with angle (θ°) for different values of surface roughness of articular cartilage. Compared with couple stresses find surface roughness much effects an increase in the value of P*, As is clear to us of Fig.(4) that the value of pressure increasing with decreasing value of roughness while ,in couple stress we find that the value of the pressure increasing with the increasing in the value of couple stresses.

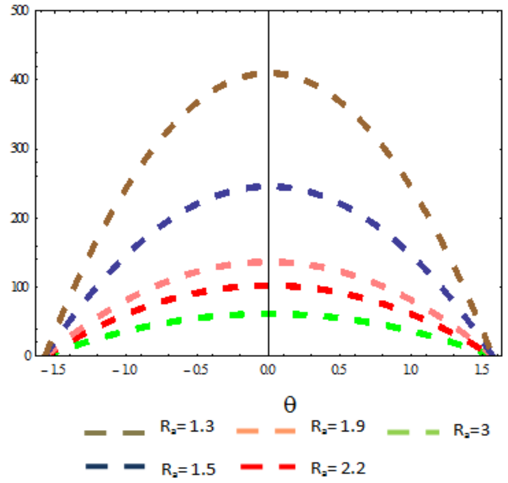

Fig.4.Dimensionless pressure p^* with angle θ for different R_a

Figure(5) describes the dimensionless pressure (p^*) generated by squeeze film action with angle (θ°) for different values of eccentricity ratio (ε) .As shown ,the effects of eccentricity ratio an increase in the pressure .Moreover ,a larger increment of the pressure is obtained with decreasing value of R_a .

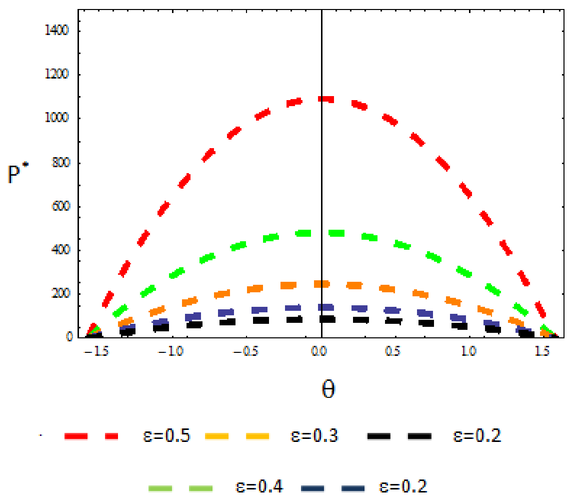

Fig.5.Dimensionless pressure p^* with angle θ for different ε

Figure (6) depicts the dimensionless pressure (p^*) generated by squeeze film action with angle (θ°) for different of permeability . The porosity of the main charge that controls the size of the permeability. With decreasing porosity value, we find that there is a clear increase of the value of the pressure off the articular cartilage. It is worth mentioning that the surface roughness of the important role where we find that with decreasing surface roughness There is a significant increase in pressure.

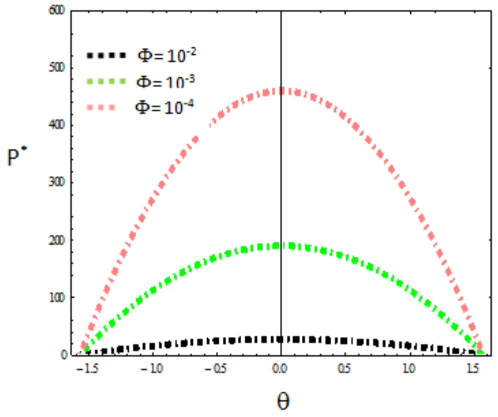

Fig. 6.Dimensionless pressure $p^*$with angle θ for different permeability Φ.

4.2 .Load carry capacity

The variation dimensionless load carry capacity W^* with eccentricity ratio (ε) for different couple stress length parameters (l^*) .Since the couple stress effects results in a higher film pressure ,the integrated load carry capacity is similarly affected. ,the solid line describes the Newtonian lubrication case. The dashed lines represent the results of the sphere –plate system lubricant with couple stress fluid for l^*=0.3 to l^*=0.1.

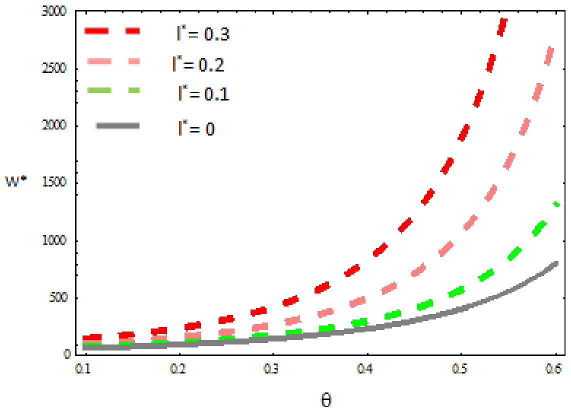

Fig 7.Dimensionless load $W^*$with eccentricity ratio ε for different couple stress length l^*

Fig.8 depicts the dimensionless load –carry capacity W* with eccentricity ratio ε for different values of surface roughness of articular cartilage. It is observed that W* increases (decreases) for increasing (decreasing values of surface roughness and This explains the Infection with diseases special of the joints Where with each decrease of surface roughness and load carry capacity increases endurance of rates wear and friction between the bones

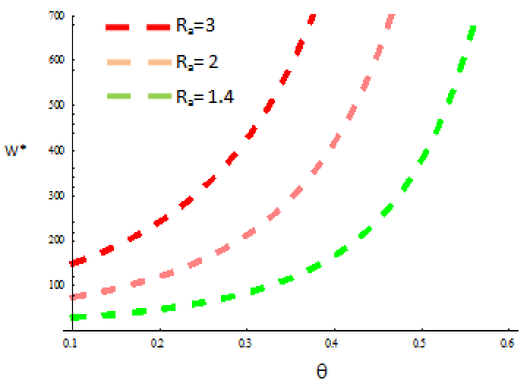

Fig8.Dimensionless load $W^{*}$with eccentricity ratio ε for different surface roughness R_a

Figure 9 depicts the dimensionless load carry capacity W^{*} with permeability Φ for different values of eccentricity ratio ε,at ε = 0.5 we obtain increase in value load carry capacity while ε = 0.3 occur decrees in value load carry capacity. This is evident in the following figure Also, the apparent increase in value load carry capacity commensurate with the increase in the value of roughness.

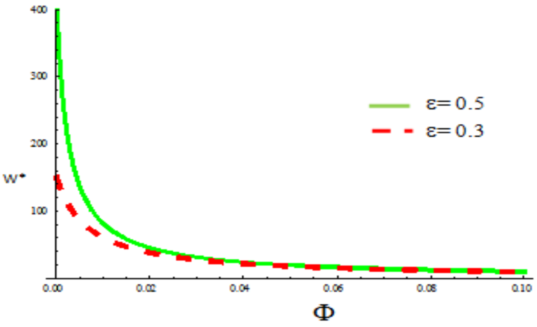

Fig 9.Dimensionless load $W^{*}$with permeability for different eccentricity ratio ε

4.3 Time approach

Figure 10 present the dimensionless time of approach t* with dimensionless film thickness h* for different values of couple stress parameter l*.it is seen that the presence of couple stress provides an increase in the time of approach .these phenomena can be realized that since the couple stress a effects yield higher load carry capacity , a higher h* would be attained for the same time to be taken as compared to the Newtonian - lubricant case.

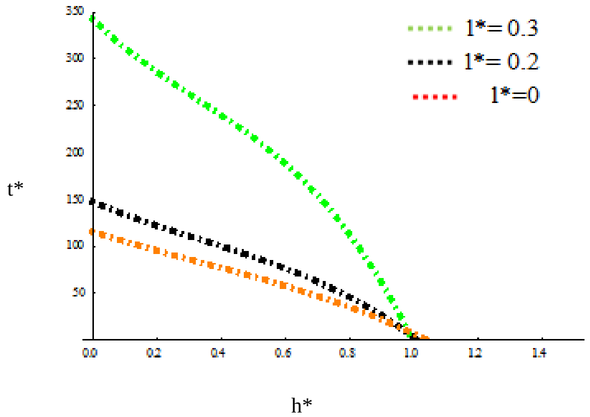

Fig10. dimensionless time of approach t with dimensionless film thickness h* for different values of couple stress parameter l**

Figure 11 present the dimensionless time of approach t* with dimensionless film thickness h* for different values of surface roughness R_a . It observed that increase surface roughness of articular cartilage provides an increase in the time of approach thus increase in film thickness. In other words increase surface roughness provides more times to avoid sphere –plate contact . This explains with age and decreases surface roughness increases the likelihood of communication between the plane and the ball resulting in a lot of problems during perform daily activities

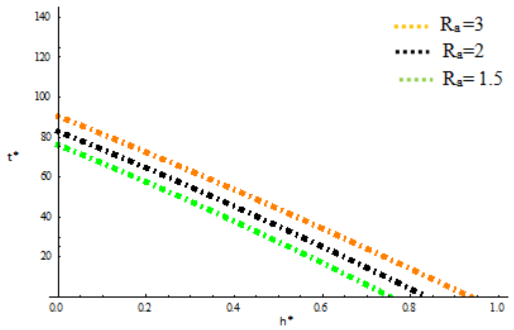

Fig11. dimensionless time of approach t with dimensionless film thickness h* for different values of couple stress parameter l**

5.Conclusions

The effects of couple stresses on the squeeze film between a sphere and a flat plate are presented on the basis of Stockes micro continuum theory . We addition surface roughness parameter to Stockes equation . The modified Reynolds equation, governing the squeeze film pressure is derived using the Stockes constitutive equation ,form a Newtonian- lubricant blended with various additives , the presence of couple stresses and surface roughness characterized by the couple stress and surface roughness parameter have a significant effect upon the squeeze film pressure .Comparing between couple stress and surface roughness we find decreasing the value of surface roughness(increasing couple stress lead to increment squeeze film pressure the effects of the couple stress and surface roughness provide an increase in the load carry capacity and thereby provides a longer time to prevent the sphere plane surface contact .The approaching time of the sphere in reducing the film thickness $h^{*}=1$ to $h^{*}=0$ for the couple stress fluid lubricant $l^{*}=0.3$ is about 340 which is longer than the approaching time 110 for the a Newtonian lubricant . Camper with surface roughness we find couple stress have approaching time.

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