Energy Bands Spectrum (g, β , γ) and Energy Ratios For Even-Even Gd (A = 150-154)

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Abstract

In the present work the interacting boson model version one (IBM-1) has been used to calculate the energy levels, energy ratios and energy bands (g, β , γ -bands) of Gadolinium Gd (A=150-154) isotopes by using the program IBSS1.for. This program calculate the eigenvalues and eigenvectors. The results were compared with the experimental data and they were found in a good agreement.

Key word : energy levels, energy ratios and energy bands (g, β , γ -bands) of Gadolinium¹⁵⁰⁻¹⁵⁴ Gd

1.Introduction

An atomic nucleus is the small, heavy, central part of atom consisting of nucleons. It has two groups of particles: protons and neutrons. Each of these groups is separately distributed over certain energy states, and they are held together by their mutual interactions which turn out to be very complicated in detail (1,2).Nuclei can be excited in different energy levels corresponding to different arrangements of nucleons in their allowed states, a nucleus in an excited state normally remains there for a very short time. Often, it decays or becomes de-excited by emitting electromagnetic radiation in the form of γ -ray transitions to states with lower energies (2)

.Many of models are currently in use in nuclear physics, some of these models are: the uniform particle model, the liquid-drop model, the shell model, the collective model and later on the interacting boson model, which has four versions. In the present work we are concerned with version one. The interacting boson model-1 (IBM-1) is an important subject that is used to study some nuclear properties of all even-mass or odd-mass nuclei. In the Interacting Boson Model of the atomic nucleus, introduced in 1974 by Arima and Iachello (1), the fundamental constituents were correlated pairs of protons and neutrons treated as bosons, able to occupy two levels, one with angular momentum restricted to zero (s boson) and the other one with angular momentum 2 (d boson) (1). The total number of bosons N depends on the number of active nucleon (or hole) pairs outside a closed shell and it can be calculated by adding the number of neutrons pairs and protons pairs of s and d bosons, namely (3),

$$N = n_s + n_d$$

where ns = number of s-bosons, nd = number of d-bosons

The IBM-1 model assumes that low-lying collective states in medium and heavy even-even nuclei away from closed shells are dominated by excitations of valance protons and valance neutrons (i.e. particles outside the major closed shells at 2, 8, 20, 28, 50, 82, 126, 184. These values are known as the magic numbers) while the closed-shell core is inert (4).

2. The interacting Boson model

Arima and Iachello (5) proposed an algebraic interacting boson model (IBM) to study the collective states of the heavy and medium mass nuclei. The IBM considers pairs of valence nucleons and treats them as bosons. The number of bosons is conventionally taken to be half the number of valence particles or holes, whichever is smaller. The simplest version of the IBM uses monopole (s) and quadrupole (d) bosons and describes nuclear structure features originating in quadrupole collectivity. An IBM Hamiltonian contains boson energy terms and boson-boson interactions with effective parameters. Electromagnetic transition operators are taken as one-body operators.

3. Theoretical basis

The most commonly used form of Interacting Boson Model-1 (IBM-1) is the Hamiltonian one, in which various boson-boson interactions are grouped so that it takes the form (6):

$$\hat{H} = \varepsilon \,\hat{n}_d + a_0(\hat{p}.\hat{p}) + a_1(\hat{I}.\hat{I}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \tag{1}$$

where $\varepsilon = \varepsilon d - \varepsilon s$ is the boson energy, a0, a 1, a 2, a 3, a 4 are the phenomenological parameters, and the other symbols are

$$\begin{split} \hat{n}_{d} &= (\hat{d}^{\dagger} \cdot \hat{\vec{d}}) & \text{the boson number operator} \\ \hat{p} &= \frac{1}{2} (\hat{\vec{d}} \cdot \hat{\vec{d}}) - \frac{1}{2} (\hat{\vec{s}} \cdot \hat{\vec{s}}) & \text{the pairing bosons operator} \\ \hat{l} &= \sqrt{10} (\hat{d}^{+} \cdot \hat{\vec{d}}) \Big[(\hat{d}^{\dagger} \cdot \hat{\vec{d}}) \Big]^{(1)} & \text{the angular momentum operator} \\ \hat{Q} &= \Big[(\hat{d}^{+} \times \hat{\vec{s}}) + (\hat{s}^{\dagger} \times \hat{\vec{d}}) \Big]^{(2)} - \frac{1}{2} \sqrt{7} \Big[(\hat{d}^{\dagger} \times \hat{\vec{d}}) \Big]^{(2)} & \text{the quadrupoleoperator} \\ \hat{T}_{3} &= \Big[(\hat{d}^{\dagger} \times \hat{\vec{d}}) \Big]^{(3)} & \text{the octupole operator} \\ \hat{T}_{4} &= \Big[(\hat{d}^{\dagger} \times \hat{\vec{d}}) \Big]^{(4)} & \text{the hexade cap de operator} \end{split}$$

The electromagnetic transition rates B(E2) values of this chain and the quadrupole moments QI are described by (3)

$$B(E_2; I+2 \to I) = \alpha_2^2 \left[\frac{I+2}{I} \right] \left[\frac{2N-1}{2} \right]$$
$$Q_I = \beta_2 \left[\left(\frac{16\pi}{70} \right)^{1/2} I \right]$$
(3)

In particular, for I = 0, or I = 2,

$$B(E_{2};2_{1}^{+}-0_{1}^{+}) = \alpha_{2}^{2}N$$

$$Q_{2_{1}^{+}} = \beta_{2} \left[\frac{32\pi}{35}\right]^{1/2}$$
(4)

The basic condition for the observation of a SU(5) symmetry in the electromagnetic transition is (5):

$$\frac{B(E_2;4_1^+ - 2_1^+)}{B(E_2;2_1^+ - 0_1^+)} = \frac{B(E_2;2_2^+ - 2_1^+)}{B(E_2;2_1^+ - 0_1^+)} = \frac{B(E_2;0_2^+ - 2_1^+)}{B(E_2;2_1^+ - 0_1^+)} = 2\left[\frac{N-1}{N}\right] < 2$$
(5)

where the necessary conditions for the observation of the SU(3) symmetry are (5)

$$\frac{B(E_2;4_1^+ \to 2_1^+)}{B(E_2;2_1^+ \to 0_1^+)} = \frac{10}{7} \frac{(N-1)(2N+5)}{N(2N+3)} < \frac{10}{7}$$
(6)

$$\frac{B(E_2; 2_2^+ \to 2_1^+)}{B(E_2; 2_1^+ \to 0_1^+)} = \frac{B(E_2; 0_2^+ \to 2_1^+)}{B(E_2; 2_1^+ \to 0_1^+)} = 0$$
(7)

The electric quadrupole transition operators can be written as (7)

(8)

$$\hat{T}^{(E_2)} = \alpha_2 \left[\hat{d}^{\dagger} \otimes \hat{\tilde{S}} + \hat{S}_{\dagger} \otimes \hat{\tilde{d}} \right]_{\mu}^2 + \beta_2 \left[\hat{d}^{\dagger} \otimes \hat{\tilde{d}} \right]$$

In the IBM-1, there are three and only three group chains of U(6) that end in O(3). They can be written as (5)

$U(6) \supset SU(5) \supset O(5) \supset O(3)$	vibrational dynamical symmetry	
$U(6) \supset SU(3) \supset O(3)$	rotational dynamical symmetry	
$U(6) \supset O(6) \supset O(5) \supset O(3)$	gamma unstable dynamical symmetry	(9)

The yarest energies increase more slowly than in rotational nucleus and more rapidly than in a harmonic vibrational nucleus. Thus, to compare between them, we may use the three limits (8).

$$E_{2}: E_{6}: E_{6}: E_{82} = \begin{cases} En_{d=1}: En_{d=2}: En_{d=3}: En_{d=4} = 1:2:3:4 & SU(5) \\ E_{\tau=1}: E_{\tau=2}: E_{\tau=3}: E_{\tau=4} = 1:2:5:7 & O(6) \\ E_{L=2}: E_{L=4}: E_{L=6}: E_{L=8} = 1:3:3:3:7:12 & SU(3) \end{cases}$$
(10)

For each state, the contributing basis states are determined by a sequential operation of the form $\Delta nd = 2$, $\Delta nB = 1$ on the first basis state. Moreover, the specific basis states that contribute are determined by the τ value; wave functions for states of the same τ have a different distribution of amplitudes for the same basis states.

4. Transitional Regions in IBM-1

4.1.1 - SU(3)-SU(5) transitional dynamical symmetry

This transitional region includes the two groups, SU(3) and SU(5). The SU(3) has to be broken with ε nd term. The general form of Hamiltonian operator of this region can be written as (9);

$$\hat{H} = \varepsilon \,\hat{n}_d + a_1 \hat{I} \cdot \hat{I} + a_2 \hat{Q} \cdot \hat{Q} \tag{11}$$

The solution of Eq.[10] depends on the ratio of $\epsilon/a2$: when the ratio $\epsilon/a2$ is large the eigen-functions of \hat{H} are those appropriate to the limiting SU(5). Also the B(E2) values are affected by the ratio $\epsilon/a2$. The B(E2) ratios (Branching Ratios) R are given by (9);

$$R = \frac{B(E_2; 2_2^+ \to 0_1^+)}{B(E_2; 2_2^+ \to 2_1^+)}$$
(12)

and

R = 0 in SU(5) region

R = 7/10 in SU(3) region-

4.1.2 - SU(3)-O(6) transitional dynamical symmetry

The breaking of SU(3) symmetry in the direction of O(6) symmetry can be treated in this transitional region by adding the term $\hat{P}^{\dagger}.\hat{P}$, so that the Hamiltonian form can be written as (9,10)

$$\hat{H} = a_0 \hat{P}^{\dagger} \cdot \hat{P} + a_1 \hat{I} \cdot \hat{I} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3$$
⁽¹³⁾

The solutions of Eq.[12] depend on the ratio ao/a2:: when it is large, the eigenfunctions of the Hamiltonian \hat{H} are appropriate to O(6) symmetry, but if it is small, the eigenfunctions are appropriate to SU(3) symmetry. In this region the change in the electromagnetic rates can be seen from the branching ratios R which takes values

R = 7/10 in SU(3) symmetry

R=0 in O(6) symmetry

4.1.3 - O(6)- SU(5) transitional dynamical symmetry

The form of Hamiltonian in this region can be written as (11,12)

$$\hat{H} = \varepsilon \,\hat{n}_d + a_0 \hat{P}^{\dagger} \cdot \hat{P} + a_1 \hat{I} \cdot \hat{I}_3 \tag{14}$$

The solution of Eq.[13] depends on the ratio $\varepsilon/a0$ -

The B(E2) values show a smooth transition towards typical O(6) vales, the branching ratios take a constant value (11);

R = 0 in SU(5) symmetry

R = 0 in O(6) symmetry

Also, when the ratio $\varepsilon/a0$ - is large, the eigenfunctions of \hat{H} are those appropriate to the limiting SU(5), while when it is small, the eigenfunctions are appropriate to symmetry O(6).

5. Calculations and results

5.1 - Energy Levels and Energy Bands

The behavior of the structure of each nucleus considered in this work is deduced by studying the dynamical symmetry of deformed Gd (A = 150-154) and the energy spectrum according to the sequences of energy bands g, β , γ .

The parameters of Eq.[1] fitted to the experimental data are used to calculate the eigenvalues and eigenvectors of Gd (A = 150-160) isotopes, which are tabulated in Table 1.

To carry out the calculation of ^{150 152 154}Gd using interacting boson model-1(IBM-1), which does not distinguish between the neutron- and proton- boson, we must evaluate the total number of bosons N and the dynamical symmetry.

Isotope	N _x	Ν	Ν	Esp	\hat{P} . \hat{P}	$\hat{I}.\hat{I}$	$\hat{Q}.\hat{Q}$	$\hat{T}_3.\hat{T}_3$	$\hat{T}_{A}.\hat{T}_{A}$	χ
		У	Tot	Mev	MeV	MeV	MeV	MeV	MeV	
$^{150}_{64}Gd_{86}$	7	2	9	0.5500	0.0100	-0.0055	-0.0001	-0.005	-0.045	-1.0220
$^{152}_{64}Gd_{88}$	7	3	10	0.3250	0.0100	-0.0090	-0.0142	0.1691	0.0312	-1.2400
$^{154}_{64}Gd_{90}$	7	4	11	0.0000	0.0000	0.0195	-0.0089	0.0019	0.0000	1.6500

Table 1. The parameters of Hamiltonian function operator for Gd (A = 150-154) isotopes.

In this work we have studied the energy levels of even-even Gd (A =150 -154) isotope transitions with change in number of neutrons observed when moving from the lighter to heavier isotopes, i.e. SU(5) - SU(3) transitional regions (Table 2). The N = 90 isotopes ¹⁵⁴Gd were seen to provide a good example to transition from spherical to axially deformed (13,14).

		Energ	y level			Energ	y level			Energ	y level
Isotope	\mathbf{I}^{n}	(Me	eV)	Isotope	I ⁿ	(M	eV)	Isotope	\mathbf{I}^{n}	(M	eV)
		(15)	IBM-1			$Exp^{(15)}$	IBM-1			$\operatorname{Exp}^{(15)}$	IBM-1
		Exp ⁽¹⁵⁾	(pw)				(pw)				(pw)
	0_1^+	0.000	0.000		0_1^+	0.000	0.000		0_1^+	0.000	0.000
	2_1^+	0.6360	0.609		2_1^+	0.3442	0.342		2_1^+	0.1230	0.1420
	4_1^+	1.2884	1.296		41 ⁺	07553	0.790	$^{154}_{90}Gd_{64}$	4_1^+	0.371	0.463
$^{150}_{64}Gd_{86}$	6_1^+	2.116	2.061	$^{152}_{64}Gd_{88}$	6_1^+	1.2272	1.328		6_1^+	0.7177	0.973
	8_1^+	2.554	2.061		81+	1.746	1.943		8_1^+	1.1445	1.667
	0_2^+	1.2072	1.200		0_2^+	0.615	0.725		0_2^+	0.6800	0.701
	2_2^+	1.5165	1.358		2_2^+	0.9305	1.134		2_2^+	0.8155	0.842
	4_2^+	1.7001	2.145		4_2^+	1.2822	1.718		42+	1.0475	1.172
	6_2^+		3.007		6_2^+	1.668	2.392		6_2^+	1.3659	1.69
	8_2^+		3.947		8_2^+	2.3004	3.151		8_2^+	1.7567	2.395
	2_3^+	1.4305	1.956		0_3^+	1.0478	1.571	(9)	2_3^+	0.9962	0.923
	33+	1.9880	2.200	(9)	23+	1.3183	2.107	Õ	33+	1.1278	1.064
)(6	43+	2.0800	2.789	0	4 ₃ ⁺	1.6923	2.880		43+	1.2637	1.253
	5 ₃ ⁺		3.090	(3)	63 ⁺		3.660](3	5 ₃ ⁺	1.4323	1.488
(2)	63 ⁺		3.702	SU	83+		4.600	SI	63 ⁺	1.6066	1.771
SUC	7 ₃ ⁺		4.000	5)-	2_4^+	1.109	1.445	$\frac{1}{10}$	7 ₃ ⁺	1.8103	2.101
•1	83+		4.694	n(31+	1.4339	2.173)(E	83+		2.477
				01	4_4^+	1.550	2.185	SL	03+	1.2951	1.2951
					5_1^+	1.861	3.032		2_4^+	1.4183	1.431
					6_4^+	1.997	3.1		4_4^+	1.6923	1.759
					7_1^+	2.394	3.798		6_4^+		2.273
					8_4^+		3.963		8_4^+		2.97
					2_5^+	1.605	2.507		2_5^+	1.5431	1.431
					3_2^+	1.839	3.314		3_2^+	1.668	1.62
					4 ₅ ⁺		3.253		4_{5}^{+}	1.7902	1.809
					5 ₂ ⁺		4.281		5_4^+		2.043
					6_{5}^{+}		4.5342		6_{5}^{+}		2.372
					7_2^+		5.342		7_2^+		2.652
					8_{5}^{+}		5.614		8_{5}^{+}		3

Table 2 .Theoretical energy levels and energy transitions compared with experimental datafor the chosen even – even isotopes by using IBSS1.For program.(15).

Figures 1 to 3 show the energy levels and the energy bands arrangement compared with identical bands for the three limits SU(5) - SU(3) and O(6).



Fig1. Comparison between calculated IMB (PW); and experimental energy bands states (g, β , γ -bands) in ₅₄ Gd_{56}^{150} isotope of the dynamical symmetry SU(5)-O(6).



Fig2. Comparison between calculated IMB (PW); and experimental energy bands states $(g,\beta,\gamma$ -bands) in isotope ${}_{s4}Gd_{ss}^{152}$ of the dynamical symmetry SU(5)-SU(3)-O(6).



Fig3. Comparison between calculated IMB (PW); and experimental energy bands states (g,β,γ -bands) in isotope ${}_{44}Gd_{20}^{154}$ of the dynamical symmetry SU(5)-SU(3)-O(6)

5.2 - Energy Ratios

Figures A, B and C show that the deformed transitions are caused by the distorting influence of particles (holes) outside a closed shell near closed cells.





Fig. A,B,C ;The relation between the energy ratios as a function of number of neutron N for the even-even Gd (A=150-154) isotopes.

The Gd isotopes with mass numbers A = 150 - 154 are laying in region of 150 < A < 190and A > 230. In this region the nuclei well over come of γ – soft and then to vibrational nuclei, i.e. the nuclei in this region take the form of SU(5) – O(6) – SU(3). The nuclei in the region N \ge 90 are deformed with the highest energy ratios;

$E(4_{1}^{+})$		$E(6_{1}^{+})$,	$E(8_{1}^{+})$
$E(2_{1}^{+})$,	$E(2_{1}^{+})$		$\overline{E(2_{1}^{+})}$

The light rare-earth region is well known for its transitional nature, beautifully illustrated by the differences exhibited between the N =88 and N= 90 isotones(any of two or more species of atoms or nuclei that have the same number of neutrons). where the addition of just a few neutrons accounts for a dramatic shift from a vibrational to a rotational character (16, 17).

Gadolinium isotopes are at the center of this transitional region). With respect to the condition of energy ratios of the corresponding limits shown in Table 3, it has been found from the calculations of energy levels from IBM-1, that ratios $E(4_1^+)/E(2_1^+) = 2.0194$, $E(6_1^+)/E(2_1^+) = 3.3166$ and $E(8_1^+)/E(2_1^+) = 4.0$, therefore the nucleus ¹⁵⁰Gd is considered as a (deformed) vibrational nucleus and the dynamical symmetry is SU(5) limit in the interacting boson model.

Table 3The energy ratios of corresponding limits (18).

Limit	$E(4_1^+)/E(2_1^+)$	$E(6_1^+)/E(2_1^+)$	$E(8_1^+)/E(2_1^+)$
SU(5)	2.3	3	4
O(6)	2,5	4.5	7
SU(3)	3.33	7	12

Table	4
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Theoretical energy ratios compared with the experimental data for chosen even-even isotopes

	$E(4_1^+)$	$/E(2_1^+)$	$E(6_1^+)$	$/E(2_1^+)$	$E(8_1^+)$	$/E(2_1^+)$
Isotope		IBM-1		IBM-1		IBM-1
	Exp.	(pw)	Exp.	(pw)	Exp.	(pw)
$^{150}_{86}Gd_{64}$	2.0194	2.1289	3.3166	3.3842	4.0031	4.7717
$^{152}_{88}Gd_{64}$	2.1943	2.3099	3.5654	3.8830	5.0746	5.6812
$^{154}_{90}Gd_{64}$	3.0162	3.2605	5.8340	6.8521	9.3048	11.7394

Conclusion

The Interacting Boson Model version one (IBM-1) gives a good valuess for the energy levels as compare with the experimental values. The dynamical symmetries of Gd (A=150-156) show mixture of SU(5)-O(6) and SU(5)-SU(3)-O(6) dynamical symmetry.

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