Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol 6, 2012

Decohering Environment And Coupled Quantum States And Internal Resonance In Coupled Spin Systems And The Conflict Between Quantum Gate Operation And Decoupling A Cormorant-Barnacle Model

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ABSTRACT

Quantum decoherence in all its locus of essence, and expression is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition. Detrimental ramifications, and pernicious implications of this dephasing leads to classical or probabilistically additive behavior. Quantum decoherence gives the appearance of wave function collapse (the reduction of the physical possibilities into a single possibility as seen by an observer. Here it is to be noted that perception is not the reality and what you see is not what you see ;what you do not see is what you do not see; what you see is what you do not see ;and what you do not see is what you see.)… Thus in all its wide ranging manifestations, it justifies the propositional subsistence and corporeal reality that justifies the framework and intuition of classical physics as an acceptable approximation: decoherence is the mechanism by which the classical limit emerges out of a perceptual field of quantum starting point and it determines the location of the quantum-classical boundary. Decoherence occurs when a system interacts with its environment in a thermodynamically irreversible way. This prevents transitive states, substantive sub states and determinate orientation different elements in the quantum superposition of the system and environment's wavefunction from interfering with each other. Decoherence has been a subject of active research since the 1980s. Decoherence can be viewed as the principal frontier of diurnal dynamics that results in loss of information from a system into the environment (often modeled as a heat bath), since every system is loosely coupled with the energetic state of its surroundings, with particularistic predicational pronouncements. . Viewed in isolation, the system's dynamics are non-unitary (although the combined system plus environment evolves in a unitary fashion). Thus, the dynamics of the system aphorism and anecdote of the system alone are irreversible. As with any coupling, entanglements are generated in its theme and potentialities between the system and environment, which have the effect of sharing quantum information with—or transferring it to—the surroundings. It is a blatant and flagrant misconception that collapse of wave function is attributable and ascribable to wave function collapse; Decoherence does not generate actual wave function collapse. It only provides an explanation for the appearance of the wavefunction collapse, as the quantum nature of the system "leaks" into the environment. So, the wave function collapse is the figment of the observer's imagination, product of puerile prognostication and resultant orientationality of his phantasmagoria. One cannot have the apodictic knowledge of reality that is; components of the wavefunction are decoupled from a coherent system, and acquire phases from their immediate surroundings. A total superposition of the global or universal wavefunction still exists (and remains coherent at the global level), but its ultimate fate remains an interpretational issue. This is a very important pharisaical provenience and plagenetious precocity with all the disembodied resemblances of the system in total form; specifically, decoherence does not attempt

to explain the measurement problem. Rather, decoherence provides an explanation for the transition of the system to a mixture of states that seem to correspond to those states observers perceive. Moreover, our observation tells us that this mixture looks like a proper quantum ensemble in a measurement situation, as we observe that measurements lead to the "realization" of precisely one state in the "ensemble".Decoherence represents a challenge for the practical realization of quantum computers, since they are expected to rely heavily on the undisturbed evolution of quantum coherences. Simply put; they require that coherent states be preserved and that decoherence is managed, in order to actually perform quantum computation. In the following we give a model for decoherence of the environment, coupled quantum states, at determinate and differential levels ,internal resonance in coupled spin systems, and the conflict in the Quantum state operations and decoupling.

KEY WORDS Coupled quantum states, Quantum mechanics

INTRODUCTION:

Quantum Coupling is an effect in quantum mechanics in which two or more quantum systems are bound such that a change in one of thequantum states in one of the systems will cause an instantaneous change in all of the bound systems. It is a state similar to quantum entanglement but whereas quantum entanglement can take place over long distances quantum coupling is restricted to quantum scales.

Trapped ion quantum computers utilize the quantum coupling effect by suspending particles representing qubits in an array of ion traps. These particles are then induced into a state of quantum coupling by using optical pumping by a laser. Information can be stored in this state by coupling two or more qubits. While the individual particles may fluctuate their values, the quantum states of the two qubits remain locked in relation to each other, via Coulomb force. Any action by one of the coupled ions instantaneously alters the other to maintain the relative value. This allows the computer to hold information despite the instability of the individual particles.

In benzene, C6H6, the charges of the individual carbon atoms exhibit coupling. There are three double bonds and three single bonds in alternating positions around the ring. The measurement of the energy in any individual bond will result in a puzzling result resembling an impossible "one and a half bond". This is because the bonds are in a quantum superposition of double and single bonds at any particular bond position. The coupling effect causes the charge at all points on the ring to be altered when a bond at any one point is altered, in order to maintain the relative charge between atoms. This results in an illusionary "bond and a half" bond between all 6 carbon atoms.

REVIEW STUDY: Nonequilibrium Dynamics of Coupled Quantum Systems (See G. Flores-Hidalgo, Rudnei O. Ramos)

The nonequilibrium dynamics of coupled quantum oscillators subject to different time dependent quenches are analyzed in the context of the Liouville's-von Neumann approach. Authors consider models of quantum oscillators in interaction that are exactly soluble in the cases of both sudden and smooth quenches. The time evolution of number densities and the final equilibration distribution for the problem of a quantum oscillator coupled to an infinity set of other oscillators (a bath) are explicitly worked out.

Recovering classical dynamics from coupled quantum systems through continuous measurement (See Shohini Ghose, Paul M. Alsing, Ivan H. Deutsch, Tanmoy Bhattacharya, Salman Habib, and Kurt Jacobs)

Continuous measurement in the quantum to classical transition has a role to play for a system with coupled internal (spin) and external (motional) degrees of freedom. Even when the measured motional degree of freedom can be treated classically, entanglement between spin and motion causes strong measurement back action on the quantum spin subsystem so that classical trajectories are not recovered in this mixed quantum-classical regime. The measurement can extract localized quantum trajectories that behave classically only when the internal action also becomes large relative to h-bar.

Coupled Quantum Dots

Coherent optical manipulation of triplet-singlet states in coupled quantum dots" Hakan E. Tureci, A. Imamoglu and J.M. Taylor, Phys. Rev. B 75: Art. No. 235313 (2007)

A system of two coupled quantum dots (sometimes called an "artificial molecule") represents the most basic example of a quantum system interacting with another quantum system. As such, these structures could be used to study fundamental quantum mechanical effects such as entanglement and decoherence. Some Physicists have recently used coupled quantum dots to demonstrate conditional dynamics, whereby the quantum state of one system controls the evolution of the state of another quantum system.

"Strong electron-hole exchange in coherently coupled quantum dots"(S. Falt, M. Atature, Hakan E. Tureci, Y. Zhao, A. Badolato, A. Imamoglu, Phys. Rev. Lett. 100, 106401 (2008))

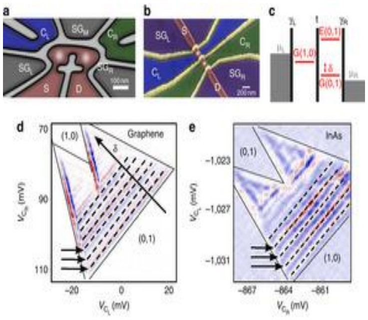
The structures HERE are grown by embedding two layers of self-assembled quantum dots, typically separated by 5-20 nm, inside a single heterostructures. Dots in the top layer tend to form directly on top of dots in the bottom layer because of the strain field produced by the latter, resulting in stacks of vertically coupled dots. Optical manipulation of the dynamics of an individual stack, is done so that the probability that one quantum dot makes a transition to an optically excited state is controlled by the presence or absence of an optical excitation in the neighboring dot. The interaction between the two dots is mediated by the tunnel coupling between optically excited states and can be optically gated by applying a laser field of the right frequency. This interaction mechanism **could form the basis** of an optically effected controlled-phase gate between two solid-state qubits.

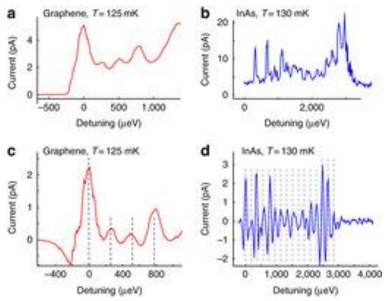
Authors also talk about the study of the conditional coupling between electrons or hole spins in separate quantum dots and to demonstrate conditional coherent quantum dynamics. In parallel, study quantum dot pairs where the exchange interaction gives rise to spin singlet and triplet ground states that are delocalized over the two dots. Such entangled states could be used to define two-level systems that are more robust against the nuclear spin decoherence and that can be manipulated using resonant optical excitations.

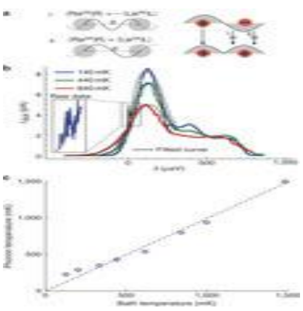
Coherent electron–phonon coupling in tailored quantum systems (See P. Roulleau, S. Baer, T. Choi, F. Molitor, J. Güttinger, T. Müller, S. Dröscher, K. Ensslin & T. IhnAffiliationsContributions)

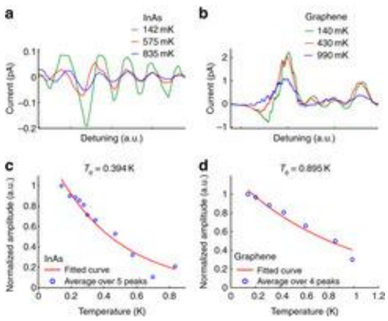
The coupling between a two-level system and its environment **leads** to decoherence. Within the context of coherent manipulation of electronic or quasiparticle states in nanostructures, it is crucial to understand the **sources of** decoherence. Here authors study the effect of electron–phonon coupling **in a** graphene and an in As nanowire double quantum dot (DQD). Reported measurements **reveal** oscillations of the DQD current periodic in energy detuning between the two levels. These periodic peaks are **more pronounced** in the nanowire than in graphene, **and disappear** when the temperature is increased. Authors` attribute the oscillations to an **interference effect** between two alternative inelastic decay paths involving acoustic phonons present in these materials. This interpretation predicts **the ramifications of** oscillations to wash out when temperature is increased, as observed experimentally. **Following images are reproduced from Nature's abstract on the same**

work.









Dissipations in coupled quantum systems (See for details Hashem Zoubi, Meir Orenstien, and Amiram Ron)

Authors investigate the dynamics of a composite quantum system, **comprised of coupled** subsystems, of which only one is significantly **interacting with** the environment. The validity of the conventional ad hoc approach—assuming that relaxation terms **can be extracted** directly from the master equation of the subsystem interacting with the reservoir—has been examined. They derived the equation of motion for the composite system's reduced density matrix—applying only the factorization approximation, but not the conventional sequence of Markoff, coarse grain, and secular approximations. The conventional ad hoc approach **is applicable** to zero-temperature reservoir, **but fails for** finite temperatures. It is further shown that at finite temperatures, the standard procedure **does not even yield a** master equation for the composite system, and its dynamics has to be studied by the equations of motion which are developed here. For demonstration we considered systems of a three-level atom, the two excited states are coupled to each other, and only one of them communicates with the ground state via a radiation reservoir.

Absolute Dynamical Limit to Cooling Weakly-Coupled Quantum Systems(See X. Wang, Sai Vinjanampathy, Frederick W. Strauch, Kurt Jacobs)

Cooling of a quantum system is limited by the size of the control forces that are available (the "speed" of control). We consider the most general cooling process, albeit restricted to the regime in which the thermodynamics of the system is preserved (weak coupling). Within this regime, authors further focus on the most useful control regime, in which a large cooling factor and good ground-state cooling can be achieved. Authors present a control protocol for cooling, and give clear structural arguments, as well as strong numerical evidence, that this protocol is globally optimal. From this the authors obtain simple expressions for the limit to cooling that is imposed by the speed of control.

'Return to equilibrium' for weakly coupled quantum systems: a simple polymer expansion (SeeW. De Roeck, A. Kupiainen)

Recently, several authors studied small quantum systems weakly coupled to free boson or fermion fields at positive temperature. All the approaches we are aware of employ complex deformations of Liouvillians or Mourre theory (the infinitesimal version of the former). Authors present an approach based on polymer expansions of statistical mechanics. Despite the fact that approach is rudimentary in its thematic and discursive form, elementary, results are slightly sharper than those contained in the literature up to now. Authors show that, whenever the small quantum system is known to admit a Markov approximation (Pauli master equation \emph{aka} Lindblad equation) in the weak coupling limit, and the Markov approximation is exponentially mixing, then the weakly coupled system approaches a unique invariant state that is perturbatively close to its Markov approximation.

Decoherence-protected quantum gates for a hybrid solid-state spin register (T. van der SAR, Z. H. Wang, M. S. Blok, H. Bernien, T. H. Taminiau, D.M. Toyli, D. A. Lidar, D. D. Awschalom, R. Hanson, V. V. Dobrovitski)

Protecting the dynamics of coupled quantum systems from decoherence by the environment is a key challenge for solid-state quantum information processing. An idle qubit can be efficiently insulated from the outside world via dynamical decoupling, as has recently been demonstrated for individual solid-state qubits. However, protection of qubit coherence during a multi-qubit gate poses a non-trivial problem: in general the decoupling disrupts the inter-qubit dynamics, and hence conflicts with gate operation. This problem is particularly CARDINAL, salient for hybrid systems, wherein different types of qubits evolve and decohere at vastly different rates. Here Authors present the integration of dynamical decoupling into quantum gates for a paradigmatic hybrid system, the electron-nuclear spin register. Design harnesses the internal resonance in the coupled-spin system to resolve the conflict between gate operation and decoupling. Authors experimentally demonstrate these gates on a two-qubit register in diamond operating at room temperature. Quantum tomography reveals that the qubits involved in the gate operation are protected as accurately as idle qubits. Illustration and exemplary notification is made about the proposed design by Grover's quantum search algorithm, achieving fidelities above 90% even though the execution time exceeds the electron spin dephasing time by two orders of magnitude. Results directly enable decoherence-protected interface gates between different types of promising solid-state qubits. Ultimately, quantum gates with integrated decoupling may enable reaching the accuracy threshold for fault-tolerant quantum information processing with solid-state devices.

Stark tuning spin qubits in diamond for quantum optical networks (Victor Acosta , Charles Santori , Andrei Faraon , Zhihong Huang , Kai-Mei Fu , Alastair Stacey , David Simpson , Timothy Karle , Brant Gibson , Liam McGuiness, Kumaravelu Ganesan , Snjezana Tomljenovic-hanic , Andrew Greentree , Steven Prawer , Raymond Beausoleil)

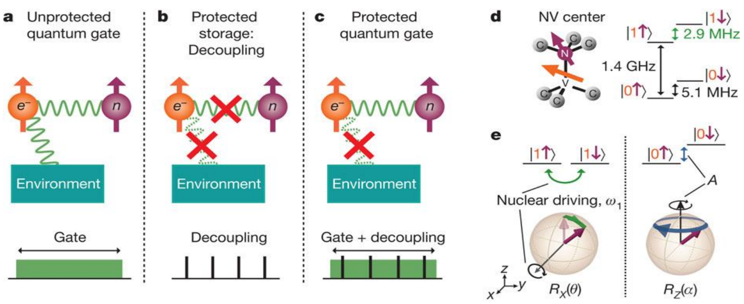
Integrated diamond networks based on cavity-coupled spin impurities offer a promising platform for scalable quantum computing. A key ingredient for this technology involves heralding entanglement by interfering indistinguishable photons emitted by pairs of identical spin qubits. Here the demonstration of the authors bear ample testimony and infallible observatory to the required control over the internal level structure of nitrogen-vacancy (NV) centers located within 100 nm of the diamond surface using the DC Stark effect. By varying the voltages applied to lithographically-defined metal electrodes, we tune the zero-phonon emission wavelength of a single NV center over a range of $\sim $0.5 nm. Using high-resolution emission spectroscopy direct observation is done of electrical tuning of the relative strengths of spin-altering lambda transitions to arbitrary values. Under resonant excitation, dynamic feedback is applied to stabilize the optical transition against spectral diffusion. Progress on application of gated control to single NV centers **coupled to** single-crystal diamond photonic crystal cavities and other nanophotonics structures are presented.

NUCLEAR SPUN RELAXATION STUDIES IN MULTIPLE SPIN SYSTEMS(B. D. NAGESWARA RAO)

Nuclear spin relaxation in multiple spin systems in diamagnetic liquids, studied by the techniques of high-resolution nuclear magnetic double resonance and T1-measurements, is discussed. The principle of the double resonance method **along with** features of strong and weak irradiation spectra and in homogeneity effects are given. The information obtainable on relaxation mechanisms is presented, including a discussion of the isotropic random field model, its applicability and **limitations i**n relation to intermolecular dipolar interactions. Scalar coupling with quadrupole nuclei and symmetry features of relaxation effects are also considered. T1-measurements are discussed with emphasis on cross-relaxation effects, multiple exponential relaxation decays and their analysis. It is pointed out that even for systems dominated by a single exponential decay mode the dipolar relaxation rate is not usually a linear superposition of the intermolecular and intermolecular contributions.

It has been generally recognized that the study of nuclear spin relaxation is a simple but powerful tool for probing the micro dynamical behaviour in liquids since the relaxation parameters are often direct measures of various types of correlation times for fluctuations in molecular orientation, angular velocity, position and so on16. In practice, for liquids containing several spins per molecule the determination and analysis of the relaxation parameters are usually complicated, owing to, among other causes, the multiplicity and line width variations in the resonance spectra that arise from the chemical environments of the spins7, and the cross-relaxation effects and internal motions that arise from their geometrical arrangement91t.

Quantum gate operation in the presence of decoherence.FromDecoherence-protected quantum gates for a hybrid solid-state spins register (T. van der Sar, Z. H. Wang, M. S. Blok,H. Bernien, T. H. aminiau, D. M. Toyli, D. A. Lidar, D. D. Awschalom,R. Hanson & V. V. Dobrovitski)



A–c, Challenge of high-fidelity quantum gates for qubits (orange, electron spin; purple, nuclear spin) **coupled to a** decohering environment. a, without decoherence protection, the fidelity of two-qubit gates is limited by interactions with the environment. b, Dynamical **decoupling efficiently preserves** the qubit coherence (protected storage) **by turning off the** interaction between the qubit and its environment. However, this generally **also decouples the** qubit from other qubits and prevents two-qubit gate operations. In the eventuality of the fact, the decoupling and the gate are separated in time; the unprotected gate is **still susceptible to** decoherence-induced errors. The goal is to perform dynamical decoupling during the gate operation, thus ensuring that the gates **are protected** against decoherence. The gate operation should therefore **be compatible** with decoupling. The dephasing rate of the nuclear spin is negligible in our experiments. However, nuclear spin protection **can easily** be incorporated using another layer of decoupling. d, The two-qubit system used in this work: a nitrogen–vacancy (NV) colour centre in diamond carries an electron spin S = 1 (orange) coupled to a 14N nuclear spin I = 1 (purple). The states of the electronic qubit, |0〉 and |1〉, are split by 1.4 GHz in an external field B0 = 510 G. The states |0↑〉 and |0↓〉 are split by 5.1 MHz owing to nuclear quadrupole and Zeeman interactions. The hyperfine coupling yields an additional splitting, such that the levels |1↑〉 and |1↓〉 are separated by 2.9 MHz. The Rabi driving is applied in resonance with this transition. e, Dynamics of the electron–nuclear spin system in the limit $\omega_1 \ll A$, visualized in a coordinate frame that rotates with frequency 1.4 GHz in the electron spin subspace and frequency 2.9 MHz in the nuclear spin subspace. In this frame, the states |1↑〉 and |1↓〉 have the same energy. The Rabi driving field, which is directed along the x axis, coherently rotates the nuclear spin if the electronic qubit is in |1〉 (the resulting rotation around the x axis by angle θ is denotedRX (θ)). However, the Rabi driving is negligible for the states |0↑〉 and |0↓〉, which differ in energy by A = 2π × 2.16 MHz. The phase accumulation between |0↑〉 and |0↓〉corresponds to a coherent rotation of the nuclear spin around the z axis with frequency A (denoted RZ (α), where α is the rotation angle).

Decoherence-protected quantum gates for a hybrid solid-state spin register (T. van der Sar, Z. H. Wang M. S. Blok, H. Bernien, T. H. Taminiau, D. M. Toyli, D. A. Lidar, D. D. Awschalom, R. Hanson& V. V. Dobrovitski)

Protection of qubit coherence during a multi-qubit gate is a non-trivial problem. Decoupling disrupts the interqubit dynamics and hence conflicts with gate operation. This problem is particularly salient for hybrid systems, in which different types of qubit evolve and decohere at very different rates. Authors make a through presentation of the integration of dynamical decoupling into quantum gates for a standard hybrid system, the electron–nuclear spin register. Design harnesses the internal resonance in the coupled-spin system to resolve the conflict between gate operation and decoupling. Authors experimentally demonstrate these gates using a two-qubit register in diamond operating at room temperature.

ESSENTIAL PREDICATIONS AND INTERFACIAL INTERFERENCE OF MATTER AND ANTIMATTER:A NECESSARY PRELUDE FOR DECOUPLING AND DECOHERENCE IN ENVIRONMENT:

Physicists at the Stanford Linear Accelerator Center (SLAC) in California and the High Energy Accelerator Research Organization (KEK) in Japan are colliding particles and anti-particles at high energies to study minute differences between the ways matter and antimatter interact. Their goal is to contribute to our understanding of the workings of the universe at its largest and smallest scales, from revealing the origin of matter shortly after the Big Bang, to uncovering the secrets of elementary particles and their interactions.

After decades of particle physics experiments, we now know that every type of particle has a corresponding antimatter particle, called an anti-particle. A particle and its anti-particle are identical in almost every way - they have the same mass, for example - but they have opposite charges. The existence of the positron, the positively charged anti-particle of the negative electron, was first hypothesized by Dirac in 1928. Its existence was experimentally proven in 1933 by Anderson, who received the 1936 Nobel Prize for this achievement. Since then, physicists have discovered the anti-particles of all the known elementary particles, and have even been able to combine positrons with antiprotons to make antihydrogen "antiatoms".

Matter and antimatter are created together.

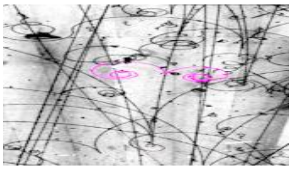


Fig 1. Particles and antiparticles (such as the pair highlighted in pink) are **created in** pairs from the energy released by the collision of fast-moving particles with atoms in a bubble chamber. Since particles and antiparticles have opposite electrical charges, they curl in opposite directions in the magnetic field applied to the chamber.

From the physicists' point of view, what is strange about antimatter is that we don't see more of it. When we **collide** high-energy particles in accelerators, their energy is **converted into** equal amounts of matter and antimatter particles (according to Einstein's famous formula, the energy (E) it takes to create matter and antimatter of total mass (m) is E=mc^2).

For example, you can see matter and antimatter particles **created in** the bubble chamber photo on the left. The photo shows many bubble tracks **generated by** charged particles **passing through** superheated liquid. Due to the magnetic field applied to the chamber, positive particles **curl to the right** and **negative particles curl** to the left. The two curled tracks highlighted in pink show an electron-positron pair **created by** the collision of a gamma ray photon (a highly energetic particle of light) with an atom in the chamber, in a process called pair production.

Where did all the antimatter go?

Since we see matter and antimatter created in equal amounts in particle experiments, we expect that shortly after the Big Bang, when the universe was extremely dense and hot, equal amounts of matter and antimatter were created from the available energy. The obvious question is, therefore, where did the antimatter go?.

One survivor for every billion.

Based on numerous astronomical observations and the results of particle physics and nuclear physics experiments, **we deduce** that all the matter in the universe today is only about a billionth of the amount of matter that **existed during** the very early universe. As the universe expanded and cooled, almost every matter particle **collided with** an antimatter particle, and the two turned into two photons - gamma ray particles - in a process called annihilation, the opposite of pair production. But roughly a billionth of the matter particles survived, and it is those particles that now make the galaxies, stars, planets, and all living things on Earth, including our own bodies.

The universe and the particles.

The survival of a small fraction of the matter particles indicates that, unlike what we wrote above, matter and antimatter are not exactly identical. There is a small difference between the ways they interact. This difference between matter and antimatter was first observed in particle accelerators in 1964 in an experiment for which Cronin and Fitch were awarded the 1980 Nobel Prize, and its connection to the existence of matter in the universe was realized in 1967 by Sakharov.

Physicists call this difference CP violation. Jargon, but it just means that if you are conducting a particular experiment on particles, from which **you deduce** a certain theory of the laws of physics, then conducting the same experiment on anti-particles would lead you **to deduce** different laws. The only way to end up with a consistent set of physical laws is to incorporate the matter-antimatter difference into your theory. Because this difference is small, conducting any old experiment would not reveal it. For example, if your experiment involves gravity, you would find that apples are attracted by massive bodies like the earth, and that anti-apples are also attracted by massive bodies. So gravity **affects** matter and antimatter identically, and this experiment would **not reveal CP violation**. A much more sophisticated experiment is required.

Sophisticated experiment

The new generation of experiments at SLAC and KEK, called BaBar and Belle, offer new tools with which to probe the nature of CP violation, hopefully shedding light on what happened a tiny fraction of a second after the Big Bang, and expanding our understanding of elementary particles and their interactions. These experiments work as follows: an accelerator accelerates electrons and positrons to high energies. They are **then "stored"** in bunches of about a hundred billion particles each, running around in a circular accelerator called a storage ring at about 0.99997 of the speed of light. Electrons are made to go one way, and positrons go the other way, so that the bunches cross through each other every time they go around the ring.

Making quarks

On some bunch crossings, a positron and an electron come close enough to collide, and the high energy that they have been given by the accelerator **turns into a** new particle & anti-particle pair: a B meson and its anti-particle, called a B-bar meson (mesons are particles composed of a quarkand an anti-quark). These mesons undergo radioactive decay within about a picosecond (a trillionth of a second). Because they are quite heavy - their mass is about five times that of the proton - they can **decay** in numerous ways into different combinations of lighter particles.

Physicists have built a living-room size detector (see pictures) around the collision point in order to detect the lighter particles which **are produced** in the decay of the two mesons. These detectors allow them to identify the types of particles produced, measure their momenta and energies, and trace them to their points of origin to within less than a 10th of a millimeter.

The huge amounts of data collected by the detectors is stored in large databases and analyzed by computer "farms" with many hundreds of computers. Together, BaBar and Belle have produced almost 300 million B mason & B-bar meson pairs, and physicists around the world are hard at work analyzing the mountains of data and publishing their results. 300 million is a large number, but when it comes to some CP violation measurements, it can be barely enough.

Measuring CP violation

Physicists detect differences between matter and antimatter and **determine the** strength of CP violation by measuring the ways the B and the B-bar **decay**. For example, decays into particular sets of particles exhibit a peculiar time structure which is different for B and B-**bar decays**.

To expose this behavior, the physicists conduct the following analysis:

First, they select "events" in which they see one of the heavy mesons undergoing the desired decay. This is done by looking at all particle signatures in the detector and determining which combinations of particles may have been produced in the decay of interest, given the constraints imposed by Einstein's theory of relativity.

Next, they analyze the decay products of the other meson **to determine** whether it was a B or a B-bar. This process is called "tagging", and it **makes use of** the fact that B-bar meson decay tends **to produce a** certain particle, such as an electron, whereas the decay of a B **usually produces** the corresponding anti-particle, such as a positron.

Third, by measuring the points of origin of the decay products of the two mesons, they can find the distance between them, which is typically about a quarter of a millimeter. They divide this distance by the velocity with which the mesons move, to obtain the difference between their decay times, known as dt, which is typically about a picosecond.

Finally, they plot the number of events observed in different ranges of dt.

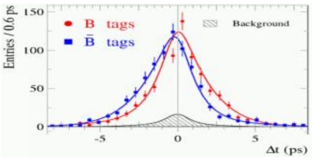


Fig 2. The difference between the red and the blue lines **shows the** difference in how a particle and its antiparticle behave. This is CP violation, and **indicates** that matter and anti-matter are not exactly opposites.

Observation template:

A plot appears to the left, with events in which the other Meson was "tagged" as a B shown in red, and those in which it was "tagged" as a B-bar shown in blue. You can see that the plots are not the same: events with a B tag tend to have a larger dt than events with a B-bar tag. This subtle difference is exactly what we are looking for: this is CP violation, observed for the first time in almost four decades!

Using their results, BaBar and Belle have measured with high precision a parameter called sin(2 beta), which describes part of the mechanism thought to be responsible for CP violation. According to our understanding of particle physics, if sin (2 beta) had been equal to zero, there would have been no CP violation, and matter and antimatter would have been identical. Recalling that the difference between matter and antimatter is necessary for the existence of matter in the universe today, a zero value for sin (2 beta) would have meant that the universe would have been a totally different place, with no stars or planets, not even people to ponder the mysteries of the universe and the underlying laws of physics.

Particle physicists are motivated to study CP violation both because it's an interesting phenomenon in its own right and because it is intimately **related to the** universe as a whole and to our very existence within it.

What next?

Having measured sin(2 beta), BaBar and Belle are now collecting more data about B and B-bar decays and measuring more CP violation parameters, to improve our understanding of the difference between matter and antimatter.

More data is coming from other experiments as well. Physicists at the CDFand D0 experiments in Fermilab are also studying the decays of B mesons **produced in** collisions of protons with anti-protons. Additional experiments using the Large Hadron Collider at CERN, which **will produce** B mesons copiously by colliding protons with protons at even higher energies, are scheduled to begin operation in a few years.

There are many open questions that these experiments seek to address. Some of the most intriguing questions are prompted by the fact that the matter-antimatter difference we see in the laboratory appears too small to be solely responsible for all the matter in the universe today. This suggests that there may be additional differences between matter and antimatter, additional sources of CP violation that we have not been able to detect yet, but which could have played an important role during the very early universe, when most matter and antimatter annihilated and a small fraction of the matter survived.

Physicists are searching for these unknown CP violation effects. We never know what exactly this quest will yield, but as has always been the case in the history of particle physics, we expect to learn a great deal about nature in the process. (For more details Please see Abner Soffer of Colorado State University)

NOTATION :

Coupled Quantum Systems And Decohering Environment:

G_{13} : Category One Of Coupled Quantum System

G_{14} : Category Two Of Coupled Quantum System

G_{15} : Category Three Of Coupled Quantum System

T_{13} : Category One Of Decohering Environment

T_{14} : Category Two Of Decohering Environment

T_{15} :Category Three Of Decohering Environment

Conflict Between Quantum Gate Operation And Decoupling And Internal Resonance In Coupled Spin Systems: Module Numbered Two:

G_{16} : Category One Of Conflict Between Quantum Gate Operation And Decoupling

G_{17} : Category Two Of Conflict Between Quantum Gate Operation And Decoupling

G_{18} : Category Three Conflict Between Quantum Gate Operation And Decoupling

T_{16} : Category One Of Internal Resonance In Coupled Spin Systems

T_{17} : Category Two Of Internal Resonance In Coupled Spin Systems

T_{18} : Category Three Of Internal Resonance In Coupled Spin Systems

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$

$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$: are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$

$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$ are Dissipation coefficients

Governing Equations of The System Decohering Environment And Coupled Quantum Systems:
The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right] G_{13}$$ 1

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right] G_{14}$$ 2

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right] G_{15}$$ 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right] T_{13}$$ 4

$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t) \right] T_{14}$ 5

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t) \right] T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor 7

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor 8

Governing Equations of the System Conflict Between Quantum Gate Operations And Decoupling And Internal Resonance In Coupled Spin Systems:

The differential system of this model is now

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t) \right] G_{16}$ 9

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t) \right] G_{17}$ 10

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t) \right] G_{18}$ 11

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right) \right] T_{16}$ 12

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right) \right] T_{17}$ 13

$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right) \right] T_{18}$ 14

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor 15

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor 16

Coupled Quantum Systems And Decohering Environment And The Conflict Between Quantum Gate Operation And Decoupling And Internal Resonance In Coupled Spin Systems - The Final Governing Equations

$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2)}(T_{17},t)} \right] G_{13}$ *17*

$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2)}(T_{17},t)} \right] G_{14}$ *18*

$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2)}(T_{17},t)} \right] G_{15}$ *19*

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 *20*

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3 *21*

$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{+(b''_{16})^{(2,2)}(G_{19},t)} \right] T_{13}$ *22*

$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{+(b''_{17})^{(2,2)}(G_{19},t)} \right] T_{14}$ *23*

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{+(b''_{18})^{(2,2)}(G_{19},t)} \right] T_{15}$ *24*

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 *25*

$\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1)}(T_{14},t)}\right]G_{16}$ 26

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1)}(T_{14},t)}\right]G_{17}$ 27

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1)}(T_{14},t)}\right]G_{18}$ 28

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 29

$\boxed{+(a''_{13})^{(1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1)}(T_{14},t)}$ are second detrition coefficients for category 1, 2 and 3

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1)}(G,t)}\right]T_{16}$ 30

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1)}(G,t)}\right]T_{17}$ 31

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1)}(G,t)}\right]T_{18}$ 32

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 33
$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$ 34

$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

<u>**Definition of**</u> $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)}$ 35

$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = \ (r_i)^{(1)}$

<u>**Definition of**</u> $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 36

$|(a''_i)^{(1)}(T'_{14},t) - (a''_i)^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T'_{14}|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b''_i)^{(1)}(G',t) - (b''_i)^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14},t)$ and $(a''_i)^{(1)}(T_{14},t)$. (T'_{14},t) and (T_{14},t) are points belonging to the interval

$\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)} \right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 37

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 38

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

(F) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$ 39

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded. 40

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 41$$

$$(b_i'')^{(2)}(G, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 42$$

(H) $\lim_{T_2 \to \infty} (a_i'')^{(2)} (T_{17}, t) = (p_i)^{(2)}$ 43

$\lim_{G \to \infty} (b_i'')^{(2)} \left((G_{19}), t \right) = (r_i)^{(2)}$ 44

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$:

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \le (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 45$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\left((G_{19}), t\right)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 46$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)} \right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the SECOND augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 47

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 48

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1$ 49

Theorem 1: if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions 50

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \ (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions 51

Definition of $G_i(0), T_i(0)$

$G_i(t) \leq \ (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq \ (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$, $T_i(0) = T_i^0 > 077$

PROOF: 52

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0\ , T_i(0) = T_i^0\ , G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)},$ 53

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 54

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 55

By 56

$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)}\left(T_{14}(s_{(13)}), s_{(13)}\right)\right) G_{13}(s_{(13)})\right] ds_{(13)}$

$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)}\left(T_{14}(s_{(13)}), s_{(13)}\right)\right) G_{14}(s_{(13)})\right] ds_{(13)}$ 57

$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)}\left(T_{14}(s_{(13)}), s_{(13)}\right)\right) G_{15}(s_{(13)})\right] ds_{(13)}$ 58

$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{13}(s_{(13)})\right] ds_{(13)}$ 59

$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$ 60

$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}')^{(1)} - (b_{15}'')^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{15}(s_{(13)})\right] ds_{(13)}$ 61

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$

which satisfy

$G_i(0) = G_i^0 \,,\, T_i(0) = T_i^0 \,,\, G_i^0 \leq (\,\hat{P}_{16}\,)^{(2)} \,, T_i^0 \leq (\,\hat{Q}_{16}\,)^{(2)},$ 62

$0 \leq G_i(t) - G_i^0 \leq (\,\hat{P}_{16}\,)^{(2)} e^{(\,\hat{M}_{16}\,)^{(2)} t}$ 63

$0 \leq T_i(t) - T_i^0 \leq (\,\hat{Q}_{16}\,)^{(2)} e^{(\,\hat{M}_{16}\,)^{(2)} t}$ 64

By 65

$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$

$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$ 66

$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$ 67

$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$ 68

$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$ 69

$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$ 70

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying CONCATENATED EQUATIONS into itself .Indeed it is obvious that 71

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\,\hat{P}_{13}\,)^{(1)} e^{(\,\hat{M}_{13}\,)^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\,\hat{P}_{13}\,)^{(1)}}{(\,\hat{M}_{13}\,)^{(1)}} \left(e^{(\,\hat{M}_{13}\,)^{(1)} t} - 1\right)$$

From which it follows that 72

$$(G_{13}(t) - G_{13}^0) e^{-(\,\hat{M}_{13}\,)^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\,\hat{M}_{13}\,)^{(1)}} \left[\left((\,\hat{P}_{13}\,)^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\,\hat{P}_{13}\,)^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\,\hat{P}_{13}\,)^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\,\hat{P}_{16}\,)^{(6)} e^{(\,\hat{M}_{16}\,)^{(2)} s_{(16)}}\right)\right] ds_{(16)} =$$ 73

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\,\hat{P}_{16}\,)^{(2)}}{(\,\hat{M}_{16}\,)^{(2)}} \left(e^{(\,\hat{M}_{16}\,)^{(2)} t} - 1\right)$$

From which it follows that 74

$$(G_{16}(t) - G_{16}^0) e^{-(\,\hat{M}_{16}\,)^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\,\hat{M}_{16}\,)^{(2)}} \left[\left((\,\hat{P}_{16}\,)^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\,\hat{P}_{16}\,)^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\,\hat{P}_{16}\,)^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\,\hat{M}_{13}\,)^{(1)}}, \frac{(b_i)^{(1)}}{(\,\hat{M}_{13}\,)^{(1)}} < 1$ and to choose 75

$(\,\hat{P}_{13}\,)^{(1)}$ and $(\,\hat{Q}_{13}\,)^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)}$$ 76

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)}$$ 77

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 78

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote 79

Definition of $\tilde{G},\tilde{T}$: $(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{13})^{(1)}|G_{14}^{(1)}-G_{14}^{(2)}|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}|G_{13}^{(1)}-G_{13}^{(2)}|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}(T_{14}^{(1)},s_{(13)})|G_{13}^{(1)}-G_{13}^{(2)}|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}(T_{14}^{(1)},s_{(13)})-(a''_{13})^{(1)}(T_{14}^{(2)},s_{(13)})|\;e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$|G^{(1)}-G^{(2)}|e^{-(\hat{M}_{13})^{(1)}t}\leq$$ 80

$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left((G^{(1)},T^{(1)};\,G^{(2)},T^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$. 81

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i=13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 82

From 19 to 24 it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0e^{(-(b'_i)^{(1)}t)}>0\quad\text{for } t>0$$

Definition of $\left((\hat{M}_{13})^{(1)}\right)_1,\left((\hat{M}_{13})^{(1)}\right)_2$ *and* $\left((\hat{M}_{13})^{(1)}\right)_3$: 83

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)}G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 84

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(1)}(G(t),t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 85

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b''_i)^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 86

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.

The same property holds for T_{15} if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions OF THE GLOBAL SYSTEM

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 87

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 88$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 89$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 90

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 91

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 92

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$

$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$

$G_{16}^{(2)}|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$ 93

$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 94

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 95

From 19 to 24 it results

$G_i(t)\geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(2)}-(a''_i)^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$

$T_i(t)\geq T_i^0 e^{(-(b'_i)^{(2)}t)}>0$ for $t>0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1,\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 96

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\left((\widehat{M}_{16})^{(2)}\right)_1-(a'_{17})^{(2)}G_{17}$ and by integrating

$G_{17}\leq\left((\widehat{M}_{16})^{(2)}\right)_2=G_{17}^0+2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a'_{17})^{(2)}$

In the same way , one can obtain

$G_{18}\leq\left((\widehat{M}_{16})^{(2)}\right)_3=G_{18}^0+2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 97

Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(2)}((G_{19})(t),t))=(b'_{17})^{(2)}$ then $T_{17}\to\infty$. 98

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t>t_2$

$(b_{17})^{(2)}-(b''_i)^{(2)}((G_{19})(t),t)<\varepsilon_2, T_{16}(t)>(m)^{(2)}$

Then $\frac{dT_{17}}{dt}\geq(a_{17})^{(2)}(m)^{(2)}-\varepsilon_2 T_{17}$ which leads to 99

$T_{17}\geq\left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1-e^{-\varepsilon_2 t})+T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t}=\frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 100

The same property holds for T_{18} if $\lim_{t\to\infty}(b''_{18})^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions OF THE GLOBAL SYSTEM: 101

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 102

(b) By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 103

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 104

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)},\ if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 105

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)},\ if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined respectively

Then the solution of GLOBAL CONCATENATED EQUATIONS satisfies the inequalities 106

$G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$

$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq\right.$ 107

$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}}$$ 108

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}$$ 109

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b_{15}')^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b_{15}')^{(1)}t}\right] + T_{15}^0 e^{-(b_{15}')^{(1)}t} \leq T_{15}(t) \leq$$ 110

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 111

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a_{13}')^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b_{13}')^{(1)}$

$(R_2)^{(1)} = (b_{15}')^{(1)} - (r_{15})^{(1)}$

Behavior of the solutions of GLOBAL EQUATIONS 112

If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 113

(d) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a_{16}')^{(2)} + (a_{17}')^{(2)} - (a_{16}'')^{(2)}(\mathrm{T}_{17}, t) + (a_{17}'')^{(2)}(\mathrm{T}_{17}, t) \leq -(\sigma_1)^{(2)}$$ 114

$$-(\tau_2)^{(2)} \leq -(b_{16}')^{(2)} + (b_{17}')^{(2)} - (b_{16}'')^{(2)}\big((G_{19}), t\big) - (b_{17}'')^{(2)}\big((G_{19}), t\big) \leq -(\tau_1)^{(2)}$$ 115

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 116

By $(\nu_1)^{(2)} > 0, (\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots

(e) of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 117

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 118

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 119

By $(\bar{\nu}_1)^{(2)} > 0, (\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the 120

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 121

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 122

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 123

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 124

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)},$ ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 125

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)},$ ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 126

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},$ ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 127

and analogously 128

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},$ ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)},$ ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$, and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 129

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 130

$G_{16}^0 e^{((S_1)^{(2)}-(p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)}-(p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 131

$\left(\frac{(a_{18})^{(2)}G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)}-(p_{16})^{(2)}-(S_2)^{(2)})}\left[e^{((S_1)^{(2)}-(p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq\right.$ 132

$\left.\frac{(a_{18})^{(2)}G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)}-(a'_{18})^{(2)})}[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)}+(r_{16})^{(2)})t}}$ 133

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)}+(r_{16})^{(2)})t}$ 134

$\frac{(b_{18})^{(2)}T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)}-(b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$ 135

$\frac{(a_{18})^{(2)}T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 136

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 137

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

PROOF : From GLOBAL EQUATIONS we obtain 138

$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}$, $\boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 139

$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}$, $\boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 140

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 141

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(1)}(t)$:-

$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}$, $\boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$

In a completely analogous way, we obtain

<u>Definition of</u> $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in CONCATENATED SYSTEM OF EQUATIONS we get easily the result stated in the theorem.

<u>Particular case :</u>

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

<u>PROOF</u> : From GLOBAL EQUATIONS we obtain (PLEASE REFER PART ONE OF THE PAPER) 142

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right) - (a''_{17})^{(2)}(T_{17},t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

<u>Definition of</u> $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 143

It follows 144

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 145

<u>Definition of</u> $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 146

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 147

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 148

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(2)}(t)$:- 149

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 150

<u>Definition of</u> $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in GLOBAL SOLUTIONS we get easily the result stated in the theorem.

<u>Particular case :</u> 151

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

We can prove the following 152

<u>Theorem 3</u>: If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined are satisfied , then the system

If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions

$(a_{16}')^{(2)}(a_{17}')^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 153

$(a_{16}')^{(2)}(a_{17}')^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a_{17}')^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$

$(b_{16}')^{(2)}(b_{17}')^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$,

$(b_{16}')^{(2)}(b_{17}')^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16}')^{(2)}(r_{17})^{(2)} - (b_{17}')^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14})\right]G_{13} = 0$ 154

$(a_{14})^{(1)}G_{13} - \left[(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14})\right]G_{14} = 0$ 155

$(a_{15})^{(1)}G_{14} - \left[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14})\right]G_{15} = 0$ 156

$(b_{13})^{(1)}T_{14} - [(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G)\ ]T_{13} = 0$ 157

$(b_{14})^{(1)}T_{13} - [(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G)\ ]T_{14} = 0$ 158

$(b_{15})^{(1)}T_{14} - [(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G)\ ]T_{15} = 0$ 159

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17})\right]G_{16} = 0$ 160

$(a_{17})^{(2)}G_{16} - \left[(a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17})\right]G_{17} = 0$ 161

$(a_{18})^{(2)}G_{17} - \left[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17})\right]G_{18} = 0$ 162

$(b_{16})^{(2)}T_{17} - [(b_{16}')^{(2)} - (b_{16}'')^{(2)}(G_{19})\ ]T_{16} = 0$ 163

$(b_{17})^{(2)}T_{16} - [(b_{17}')^{(2)} - (b_{17}'')^{(2)}(G_{19})\ ]T_{17} = 0$ 164

$(b_{18})^{(2)}T_{17} - [(b_{18}')^{(2)} - (b_{18}'')^{(2)}(G_{19})\ ]T_{18} = 0$ 165

has a unique positive solution , which is an equilibrium solution for the GLOBAL SYSTEM

Proof: 166

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a_{13}')^{(1)}(a_{14}')^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13}')^{(1)}(a_{14}'')^{(1)}(T_{14}) + (a_{14}')^{(1)}(a_{13}'')^{(1)}(T_{14}) +$
$(a_{13}'')^{(1)}(T_{14})(a_{14}'')^{(1)}(T_{14}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if 167

$\mathrm{F}(T_{19}) = (a_{16}')^{(2)}(a_{17}')^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16}')^{(2)}(a_{17}'')^{(2)}(T_{17}) + (a_{17}')^{(2)}(a_{16}'')^{(2)}(T_{17}) +$
$(a_{16}'')^{(2)}(T_{17})(a_{17}'')^{(2)}(T_{17}) = 0$

Definition and uniqueness of T_{14}^* :- 168

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a_{13}')^{(1)}+(a_{13}'')^{(1)}(T_{14}^*)]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a_{15}')^{(1)}+(a_{15}'')^{(1)}(T_{14}^*)]}$

Definition and uniqueness of T_{17}^{*} :- 169

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^{*} for which $f(T_{17}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a_{16}')^{(2)}+(a_{16}'')^{(2)}(T_{17}^{*})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a_{18}')^{(2)}+(a_{18}'')^{(2)}(T_{17}^{*})]}$$ 170

(c) By the same argument, the equations(CONCATENATED SET OF THE GLOBAL SYSTEM) 171 admit solutions G_{13}, G_{14} if

$$\varphi(G) = (b_{13}')^{(1)}(b_{14}')^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$
$$\left[(b_{13}')^{(1)}(b_{14}'')^{(1)}(G) + (b_{14}')^{(1)}(b_{13}'')^{(1)}(G)\right]+(b_{13}'')^{(1)}(G)(b_{14}'')^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

(d) By the same argument, the equations (SOLUTIONAL EQUATIONS OF THE GLOBAL EQUATIONS) admit solutions G_{16}, G_{17} if 172

$$\varphi(G_{19}) = (b_{16}')^{(2)}(b_{17}')^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$
$$\left[(b_{16}')^{(2)}(b_{17}'')^{(2)}(G_{19}) + (b_{17}')^{(2)}(b_{16}'')^{(2)}(G_{19})\right]+(b_{16}'')^{(2)}(G_{19})(b_{17}'')^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is 173 a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$

Finally we obtain the unique solution 174

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a_{13}')^{(1)}+(a_{13}'')^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a_{15}')^{(1)}+(a_{15}'')^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b_{13}')^{(1)}-(b_{13}'')^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b_{15}')^{(1)}-(b_{15}'')^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution THE GLOBAL SYSTEM

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 175

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a_{16}')^{(2)}+(a_{16}'')^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a_{18}')^{(2)}+(a_{18}'')^{(2)}(T_{17}^{*})]}$$ 176

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b_{16}')^{(2)}-(b_{16}'')^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b_{18}')^{(2)}-(b_{18}'')^{(2)}((G_{19})^{*})]}$$ 177

Obviously, these values represent an equilibrium solution of THE GLOBAL SYSTEM

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof:_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(\,G^*\,) = s_{ij}$

Then taking into account equations GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14}$ 179

$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14}$ 180

$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14}$ 181

$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right)$ 182

$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right)$ 183

$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right)$ 184

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\,\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$ 185

$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}(\,(G_{19})^*\,) = s_{ij}$ 186

taking into account equations (SOLUTIONAL EQUATIONS TO THE GLOBAL EQUATIONS) and neglecting the terms of power 2, we obtain 187

$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17}$ 188

$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17}$ 189

$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17}$ 190

$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right)$ 192

$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right)$ 193

$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right)$ 194

The characteristic equation of this system is 195

$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$

$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$

$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$

$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)$

$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left(\,(a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right) (\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right) (\lambda)^{(1)}\right) (q_{15})^{(1)} G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right) \left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

+

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T^*_{17} + (b_{17})^{(2)}s_{(16),(17)}T^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}G^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T^*_{17} + (b_{17})^{(2)}s_{(16),(16)}T^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right) (\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right) (q_{18})^{(2)} G_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right) \left((a_{18})^{(2)}(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}T^*_{17} + (b_{17})^{(2)}s_{(16),(18)}T^*_{16}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Article Abstracts, NASA PAGES FOR FIGURES, STANFORD ENCYCLOPEDIA, NATURE review articles, Face Book appraisals for the common man, ask a physicist column, Journal Reviews, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, We regret with great deal of compunction, contrition, and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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