Interaction of Laser Beam with Micropolar Thermoelastic Solid

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Abstract
The present investigation deals with the deformation of micropolar generalized thermoelastic solid subjected to thermo-mechanical loading due to thermal laser pulse. Laplace transform and Fourier transform techniques are used to solve the problem. Thermo-mechanical laser interactions are taken as concentrated normal force and thermal source to describe the application of approach. The closed form expressions of normal stress, tangential stress, coupled stress and temperature are obtained in the transferred domain. Numerical inversion technique of Laplace transform and Fourier transform has been implied to obtain the resulting quantities in the physical domain after developing a computer program. The normal stress, tangential stress, coupled stress and temperature are depicted graphically to show the effect of relaxation times. Some particular cases of interest are deduced from the present investigation.
Keywords: Pulse Laser, Integral Transform, Thermoelastic, Boundary value Problem.

1. Introduction

Very rapid thermal processes (e.g., the thermal shock due to exposure to an ultra-short laser pulse) are interesting from the stand point of thermoelasticity, since they require a coupled analysis of the temperature and deformation fields. A thermal shock induces very rapid movement in the structural elements, giving the rise to very significant inertial forces, and thereby, an increase in vibration. Rapidly oscillating contraction and expansion generates temperature changes in materials susceptible to diffusion of heat by conduction [1]. This mechanism has attracted considerable attention due to the extensive use of pulsed laser technologies in material processing and non-destructive testing and characterization [2, 3]. The so-called ultra short lasers are those with pulse durations ranging from nanoseconds to femto seconds. In the case of ultra short pulsed laser heating, the high intensity energy flux and ultra short duration lead to a very large thermal gradients or ultra-high heating may exist at the boundaries. In such cases, as pointed out by many investigators, the classical Fourier model, which leads to an infinite propagation speed of the thermal energy, is no longer valid [4]. Researchers have proposed several models to describe the mechanism of heat conduction during short-pulse laser heating, such as the parabolic one-step model [5], the hyperbolic one-step model [6], and the parabolic two-step and hyperbolic two-step models [7,8]. It has been found that usually the microscopic two-step models, i.e., parabolic and hyperbolic two-step models, are useful for thin films.

Laser technology has a vital application in nondestructive materials testing and evaluation. When a solid is heated with a laser pulse, it absorbs some energy which results in an increase in localized temperature. This cause thermal expansion and generation of the ultrasonic waves in the material. First of all Scruby et al. [10] considered the point source model. He studied the heated surface by laser pulse irradiation in the thermoelastic system as a surface center of expansion (SCOE). Rose [14] later presented a more exact mathematical basis. Point source model explain main features of laser-generated ultrasound waves but this model fails to explain precursor in epicenter waves. Later introducing the thermal diffusion McDonald [11] and Spicer [12] proposed a new model known as laser-generated ultrasound model. This model reported excellent agreement between theory and experiment for metal materials. But due to the optical penetration effect this model cannot be applied to the study of laser-generated ultrasound in non-metallic material directly. The optical absorption occurs at the surface layer in metallic materials, and the heat penetration is resulted due to heat diffusion. In non-metallic materials, the laser beam can penetrate the specimen to some finite depth and induced a buried bulk- thermal source, so the features of the laser-generated ultrasound will be significantly different from that in metallic materials. Dubois [13] experimentally demonstrated that penetration depth play a very important role in the laser-ultrasound generation process.

The irradiation of the surface of a solid by pulsed laser light generates wave motion in the solid material. There are generally two mechanisms for such wave generation, depending on the energy density deposited by the laser pulse. At high energy density, a thin surface layer of the solid material melts, followed by an ablation process whereby particles fly off the surface, thus giving rise to forces that generates ultrasonic waves. At low energy density, the surface material does not melt, but it expands at a high rate and wave and wave motion is generated due to thermoelastic processes. As opposed to generation in the ablation range, laser generation of ultrasound in the thermoelastic range does not damage the surface of the material. For applications in non destructive evaluations ultrasound generates by laser irradiation in the thermoelastic regime is of interest and will be dealt with in this paper.

Basic Equations:
The equations of motion are:-

$$\sigma_{ij,j} = \rho\left[\frac{\partial^2 u_i}{\partial t^2}\right] \tag{1.1}$$

$$\epsilon_{ijk}\sigma_{ji} + m_{ji,j} = j\rho\left[\frac{\partial^2 \phi_i}{\partial t^2}\right], \tag{1.2}$$

The constitutive laws are:

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \epsilon_{ijk}\phi_k\right) - \gamma_1\left(1 + \nu\frac{\partial}{\partial t}\right)\delta_{ij}T, \tag{1.3}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}\,, \tag{1.4}$$

The heat conduction equation is:

$$K^*\nabla^2 T = \rho c_E\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \left(1 + \eta_0\tau_0\frac{\partial}{\partial t}\right)\left(T_0\gamma_1\frac{\partial u_{i,i}}{\partial t} - \rho Q\right), \tag{1.5}$$

Here,

$$Q = I_0 f(t) g(x) h(z) \tag{1.6}$$

$$f(t) = \frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)}$$

$$g(x) = \frac{1}{2\pi r^2}e^{-\left(\frac{x^2}{r^2}\right)}$$

$$h(z) = \gamma^* e^{-\gamma^* z}$$

Putting these values of $f(t), g(x), h(z)$ in equation (6), we have:

$$Q = \frac{I_0\gamma^*}{2\pi r^2 t_0^2}te^{-\left(\frac{t}{t_0}\right)}e^{-\left(\frac{x^2}{r^2}\right)}e^{-\gamma^* z}$$

Here λ, μ are Lame's constants, K is the thermal conductivity, ρ is density, $\boldsymbol{j}$ is current density vector, C_E is the specific heat at constant strain, e is dilatation, t is time, m_{ij} is couple stress tensor, τ_0, υ are relaxation times, T_0 is the reference temperature, T is the temperature, ε_{ijk} is the alternate tensor, σ_{ij} is the stress tensor, t_0 is the pulse rise time, u_i are the components of displacement vector, $\boldsymbol{\phi}$ is the microrotation vector, δ_{ij} is Kroneker's delta function, I_0 is the energy absorbed and k, α, β, γ are the micropolar constants.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot ("˙") denotes the derivative with respect to time respectively.

2. Formulation of the problem:
We consider a micropolar generalized thermoelastic solid with rectangular Cartesian coordinate system $OX_1X_2X_3$ having origin on x_3-axis with x_3-axis pointing vertically downward the medium. A normal force/thermal source is assumed to acting on the origin of the rectangular Cartesian co-ordinate system.

If we restrict our problem for plane strain parallel to x_1x_3-plane with

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \tag{2.1}$$

Then the field equations in micropolar generalized thermoelastic solid in the absence of body forces and body couples the equations of motion can be written as:

$$(\lambda + \mu)\frac{\partial e}{\partial x_1} + (\mu + K)\nabla^2 u_1 - K\frac{\partial \phi_2}{\partial x_3} - \gamma_1\left(1 + \nu\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{2.2}$$

$$(\lambda + \mu)\frac{\partial e}{\partial x_3} + (\mu + K)\nabla^2 u_3 + K\frac{\partial \phi_2}{\partial x_1} - \gamma_1\left(1 + \nu\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{2.3}$$

$$K\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) + \gamma\nabla^2\phi_2 - 2K\phi_2 = j\rho\ddot{\phi}_2\,, \tag{2.4}$$

$$K^*\nabla^2 T = \rho c_E\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \left(1 + \eta_0\tau_0\frac{\partial}{\partial t}\right)\left(T_0\gamma_1\frac{\partial e}{\partial t} - \rho Q\right), \tag{2.5}$$

For further consideration it is convenient to introduce in equations (1.1)-(1.5) the dimensionless quantities defined as:

$$(x_1', x_3') = \frac{\eta_0}{c_0}(x_1, x_3), (u_1', u_3') = \frac{\rho\eta_0 c_0}{\gamma_1 T_0}(u_1, u_3), (t', \tau_0', \nu') = \eta_0(t, \tau_0, \nu), T' = \frac{T}{T_0}, \sigma_{ij}' = \frac{\sigma_{ij}}{\gamma_1 T_0},$$
$$\phi_2' = \frac{\rho c_0^2}{\gamma_1 T_0}\phi_2, m_{ij}' = \frac{\eta_0}{c_0\gamma_1 T_0}m_{ij}, Q' = \frac{\gamma_1^2}{\rho c_0^2}Q, \tag{2.6}$$

Where

$$\eta_0 = \frac{\rho c_E c_0^2}{K^*}, \gamma_1 = (3\lambda + 2\mu + K)\alpha_t \ \& \ c_0^2 = \frac{(\lambda + 2\mu + K)}{\rho}$$

Using these non-dimensional parameters, dropping suffices and introducing the potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} + \frac{\partial\psi}{\partial x_3} \text{ and } u_3 = \frac{\partial\phi}{\partial x_3} - \frac{\partial\psi}{\partial x_1}, \tag{2.7}$$

With the aid of equation (2.1), (2.6) and (2.7), the equations (1.1)-(1.5) reduce to:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi + \left(1 + \nu\frac{\partial}{\partial t}\right)T = 0, \tag{2.8}$$
$$\left(\nabla^2 - a_3\frac{\partial^2}{\partial t^2}\right)\psi - a_4\phi_2 = 0, \tag{2.9}$$
$$\left(\nabla^2 - 2a_1 + a_2\frac{\partial^2}{\partial t^2}\right)\phi_2 + a_1\nabla^2\psi = 0, \tag{2.10}$$
$$\nabla^2 T - \left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} - a_5\left(\frac{\partial}{\partial t} + \eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi = -Q_0\left[t + \eta_0\tau_0 - \frac{\eta_0\tau_0}{t_0}t\right]e^{-\left(\frac{t}{t_0}\right)}e^{-\left(\frac{x^2}{r^2}\right)}e^{-\gamma^* z}, \tag{2.11}$$

3. Solution of the problem:

Applying the Laplace transform on equations (2.7)-(2.10) as defined by:

$$\bar{f}(s, x_1, x_3) = \int_0^\infty f(t, x_1, x_3)e^{-st}dt, \tag{3.1}$$

And then applying the Fourier transform on the resulting equations defined by:

$$\hat{f}(x_3, \xi, s) = \int_{-\infty}^{\infty}\bar{f}(s, x_1, x_3)e^{i\xi x_1}dx_1, \tag{3.2}$$

The equations (2.7)-(2.10) reduce to the following equations:

$$\left[s^2 - \left(\frac{d^2}{dx_3^2} - \xi^2\right)\right]\hat{\phi} + (1 + \nu s)\hat{T} = 0, \tag{3.3}$$
$$\left[\frac{d^2}{dx_3^2} - \xi^2 - s - \tau_0 s^2\right]\hat{T} - a_5(s + \eta_0\tau_0 s^2)\left(\frac{d^2}{dx_3^2} - \xi^2\right)\hat{\phi} = -\frac{Q_0\sqrt{\pi}re^{-\gamma^* z}}{2}\left[\frac{\eta_0\tau_0 t_0}{1 + t_0 s} + \frac{t_0^2 - \eta_0\tau_0 t_0}{(1 + t_0 s)^2}\right]e^{-\frac{\xi^2 r^2}{4}}, \tag{3.4}$$
$$\left[\frac{d^2}{dx_3^2} - \xi^2 - a_3 s^2\right]\hat{\psi} - a_4\widehat{\phi_2} = 0, \tag{3.5}$$
$$\left[\frac{d^2}{dx_3^2} - \xi^2 - 2a_1 + a_2 s^2\right]\widehat{\phi_2} + a_1\left[\frac{d^2}{dx_3^2} - \xi^2\right]\hat{\psi} = 0, \tag{3.6}$$

Here,

$a_1 = \frac{Kc_0^2}{\gamma\eta_0^2}, a_2 = \frac{j\rho c_0^2}{\gamma}, a_3 = \frac{\rho c_0^2}{\mu + K}, a_4 = \frac{K}{\mu + K}, a_5 = \frac{T_0\gamma_1^2}{\rho K^*\eta_0^2}, a_6 = \xi^2 + s + \tau_0 s^2, a_7 = \xi^2 - s^2, a_8 = a_5(s + \eta_0\tau_0 s^2), a_9 = -\frac{Q_0\sqrt{\pi}r}{2}\left[\frac{\eta_0\tau_0 t_0}{1 + t_0 s} + \frac{t_0^2 - \eta_0\tau_0 t_0}{(1 + t_0 s)^2}\right]e^{-\frac{\xi^2 r^2}{4}}, a_{10} = (1 + \nu s)a_9, a_{11} = \xi^2 + a_3 s^2, a_{12} = \xi^2 + 2a_1 - a_2 s^2$

Eliminating $\hat{T}$ from the equations (3.3)-(3.4), we obtain:

$$(D^4 + A_1 D^2 + A_2)\hat{\phi} = a_{10}e^{-\gamma^* z}, \tag{3.7}$$

Here,

$$D = \frac{d}{dx_3}$$
$$A_1 = -(a_6 + a_7 + a_8)$$
$$A_2 = (a_6 a_7 + \xi^2 a_8)$$

Eliminating $\widehat{\phi_2}$ from the equations (3.5)-(3.6), we obtain:

$$(A_3 D^4 + A_4 D^2 + A_5)\hat{\psi} = 0, \tag{3.8}$$

Here,

$$A_3 = a_1, A_4 = -a_1(\xi^2 + a_{11}) + a_4, A_5 = a_1 a_{11}\xi^2 - a_4 a_{12}$$

Also,

$$\hat{T} = \left[\frac{1}{(1 + \nu s)}[D^2 - \zeta^2 - s^2]\right]\hat{\phi}, \tag{3.9}$$

And $\widehat{\phi_2} = \frac{1}{a_4}[D^2 - \zeta^2 - a_3 s^2]\hat{\psi}$ (3.10)

The general solution of the above equations (3.7) and (3.8) satisfying the radiation conditions that $(\hat{\phi}, \hat{T}, \widehat{\phi_2}, \hat{\psi}) \to 0$ as $x_3 \to \infty$ are given as following:

$$\hat{\phi} = B_{11}e^{-m_1 x_3} + B_{12}e^{-m_2 x_3} + L_1 e^{-\gamma^* x_3} \tag{3.11}$$
$$\hat{T} = B_{21}e^{-m_1 x_3} + B_{22}e^{-m_2 x_3} + L_2 e^{-\gamma^* x_3} \tag{3.12}$$

$\hat{\psi} = B_{31}e^{-m_1x_3} + B_{32}e^{-m_2x_3}$ (3.13)
$\widehat{\phi_2} = B_{41}e^{-m_1x_3} + B_{42}e^{-m_2x_3}$ (3.14)

Here m_i^2 $(i = 1,2)$ are the roots of the characteristic equation of equation (3.7) and $m_i^2 (i = 3,4)$ are the roots of the characteristic equation of equation (3.8).

And $L_1 = \frac{K}{f(-\gamma^*)}$, $L_2 = \frac{L_1}{(1+\upsilon s)}[\gamma^{*2} - \zeta^2 - s^2]$,

$B_{21} = \left[\frac{m_1^2}{1+\upsilon s} - \frac{\zeta^2+s^2}{1+\upsilon s}\right]B_{11}$, $B_{22} = \left[\frac{m_2^2}{1+\upsilon s} - \frac{\zeta^2+s^2}{1+\upsilon s}\right]B_{12}$,

$B_{41} = \frac{1}{a_4}[m_3^2 - \zeta^2 - a_3s^2]B_{31}$, $B_{22} = \frac{1}{a_4}[m_4^2 - \zeta^2 - a_3s^2]B_{32}$,

$B_{11}, B_{12}, B_{31}, B_{31}$, are arbitrary constants.

4. Boundary Conditions:

The boundary conditions at the surface $z = 0$, are:

Case 1: for the thermal source:

$t_{33} = 0, t_{31} = 0$
$m_{32} = 0$,
$T = F_1\delta(x_1)\delta(t)$, (4.1)

Case 2: for the normal force:

$t_{33} = -F_2\delta(x_1)\delta(t), t_{31} = 0$
$m_{32} = 0$,
$T = 0$, (4.2)

Here F_1 is the temperature gradient source and F_1 is the magnitude of the applied force.

Using these boundary conditions and using (1.3)-(1.4), (3.1)-(3.2) and solving the linear equations formed, we get:

$\widehat{t_{33}} = \sum_{i=1}^4 G_{1i}e^{-m_ix_3} + M_1e^{-\gamma^*x_3}, i = 1,2, \dots ,4$ (4.3)
$\widehat{t_{31}} = \sum_{i=1}^4 G_{2i}e^{-m_ix_3} + M_2e^{-\gamma^*x_3}, i = 1,2, \dots ,4$ (4.4)
$\widehat{m_{32}} = \sum_{i=1}^4 G_{3i}e^{-m_ix_3} + M_3e^{-\gamma^*x_3}, i = 1,2, \dots ,4$ (4.5)
$\hat{T} = \sum_{i=1}^4 G_{7i}e^{-m_ix_3} + M_4e^{-\gamma^*x_3}$ $i = 1,2, \dots ,4$ (4.6)

Particular cases

(i) If we take $\tau_1 = \tau^1 = 0$, $\varepsilon = 1$, $\gamma_1 = \tau_0$, in Eqs. (4.2)- (4.9), we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.

(ii) If we take $\varepsilon = 0$, $\gamma_1 = \tau^0$ in Eqs. (4.2)- (4.9), the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.

(iii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = \gamma_1 = 0$ in Eqs. (4.2) - (4.9), yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

5. Numerical Results and Discussions: The analysis is conducted for a micropolar material. The values of physical constants are: $\lambda = 9.4 \times 10^{10}\ Nm^{-2}$, $\mu = 4.0 \times 10^{10}\ Nm^{-2}$, $K = 1.0 \times 10^{16}\ Nm^{-2}$, $\rho = 1.74 \times 10^3\ Kgm^{-3}$, $j = 0.2 \times 10^{-19}m^2$, $\gamma = 0.779 \times 10^{-9}\ N$

Thermal and diffusion parameters are given by

$c^* = 1.04 \times 10^3\ JKg^{-1}K^{-1}$, $K^* = 1.7 \times 10^6\ Jm^{-1}s^{-1}K^{-1}$, $\alpha_{t1} = 2.33 \times 10^{-5}K^{-1}$,
$\alpha_{t2} = 2.48 \times 10^{10}K^{-1}$, $T_0 = 0.298 \times 10^3\ K$, $\tau_0 = 0.02$, $\tau_1 = 0.01$,
$\alpha_{c1} = 2.65 \times 10^{-4}m^3Kg^{-1}$, $\alpha_{c2} = 2.83 \times 10^{-4}m^3Kg^{-1}$, $a = 2.9 \times 10^4m^2s^{-2}K^{-1}$,
$b = 32 \times 10^5\ Kg^{-1}m^5s^{-2}$, $\tau^1 = 0.04$, $\tau^0 = 0.03$, $D = 0.85 \times 10^{-8}\ Kgm^{-3}s$

And, the microstretch parameters are taken as:

$j_0 = 0.19 \times 10^{-19}m^2$, $\alpha_0 = 0.779 \times 10^{-9}\ N$, $b_0 = 0.5 \times 10^{-9}\ N$, $\lambda_0 = 0.5 \times 10^{10}\ Nm^{-2}$
$\lambda_1 = 0.5 \times 10^{10}\ Nm^{-2}$

The trends for the normal stress t_{33} , tangential couple stress m_{32} , tangential stress t_{31} and microstress λ_3 on the surface of plane $x_3 = 1$ due to applied concentrated and uniformly distributed normal sources are shown in Figs. 1-3. The comparison of three theories of generalized thermoelasticity, namely, Lord-Shulman (LS), and Coupled theory (CT) has been shown in graphs. The trends are shown for two different temperatures i.e. one by black lines and other by red lines. The dotted curves represent the trend of various stresses for CT theory of thermoelasticity and the solid curves represent the trend of stresses for Lord- Shulman ‘ s theory of thermoelasticity.

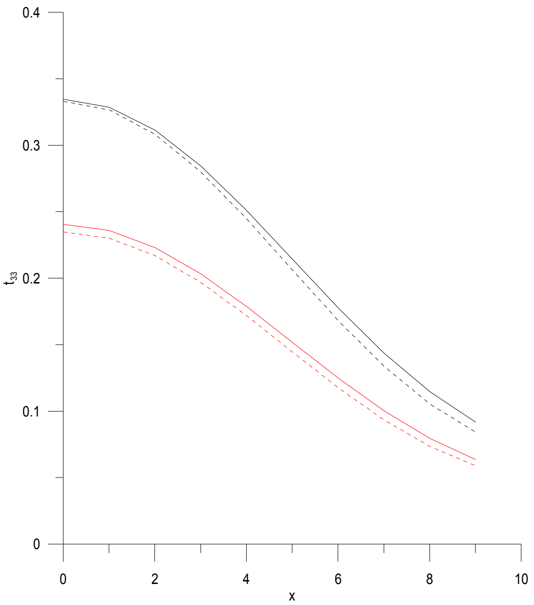

Fig. 1, variation of normal stress w. r. t. distance

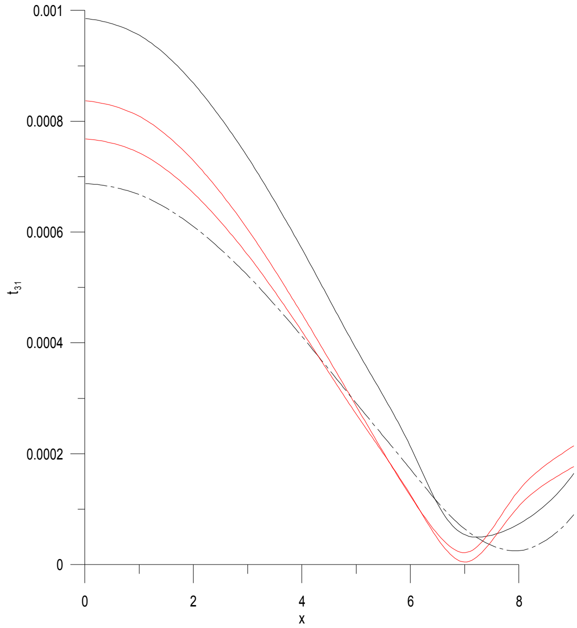

Fig. 2, variation of tangential stress w. r. t. distance

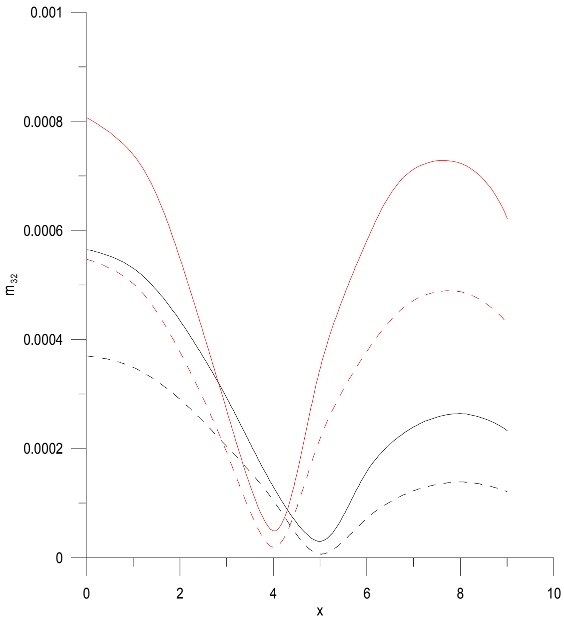

Fig. 3, variation of coupled stress w. r. t. distance

6. Conclusions

(i) The Laplace and Fourier transforms are used to derive the components of normal stress, shear stress, couple stress, microstress, temperature distribution and the mass concentration.

(ii) Values of displacement components, stress components are close to each other due to LS and CT theories.

(iii) Behavior of variation of stress components is shown in figures.

(iv) The stress components show a similar trend far from the source.

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