Double Diffusive Mixed Convection in a Couple Stress Fluids with Variable Fluid Properties

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Abstract
Study of double diffusive mixed convection flow of a non-Newtonian couple stress fluid over a vertical heated plate in a sparsely packed porous medium with variable fluid properties has been investigated analytically and numerically. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformation and then solved numerically using the shooting technique that involve Runge-Kutta-Fehlberg integration scheme and Newton-Raphson corrector method to obtain the non-dimensionalised velocity, temperature and concentration distributions. The Influence of non-dimensional governing parameters on velocity, temperature and concentration profiles along with friction factor, heat and mass transfer rates were discussed and presented through graphs and tables. Comparisons of the present results made with the existed results. We have found an excellent agreement with the existed results.

1. Introduction

Double diffusive convection in fluid saturated porous medium occurs in many engineering applications exemplified by oil recovery, geothermal energy extraction, food processing, materials processing, the dispersion of chemical contaminants in various processes in the chemical industry and in the environment, and the migration of moisture in insulation and grain storage spaces. In recent years, the investigation of the flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in the books by Phillips (1991), Ingham and Pop (1998) and Nield and Bejan (1999).

The theory of couple stress fluid has been formulated by Stokes (1966) In this theory couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules and show all the important features and effects of couple stresses. Walicki and Walicka (1999) modelled synovial fluid as a couple stress fluid in human joints. Generally human joint is a dynamically loaded bearing which has auricular cartilage as the bearing and the synovial fluid as the lubricant. Sunil Sharma and Chandel (2001) discussed superposed couple stress fluids in porous medium in hydrodynamics. The instability of a plane interface between two incompressible viscous fluids of different densities was studied by Chandrasekhar (1981). Sandeep and Sugunamma (2013) discussed aligned magneticfiled and dissipative effects on a flow through vertical flat plate. Ramanamurthy et al. (2007) studied the flow of an incompressible couple stress fluid flow past a porous sphere. Srinivasacharya and Kaladhar (2011) analysed the mixed convection in a couple stress fluid with soret and dufour effects and compared there study with Kafoussias (1990) by neglecting soret and dufour parameters. Stokes assumptions helps in neglecting the non- linear inertial terms in the momentum equation and thereby making the problem more mathematically tractable. Based on this Srinivasacharya et al. (2011) analysed Soret and Dufour effects on mixed convection flow in a non-Darcy porous medium saturated with micropolar fluid and found that behaviour of Dufour and Soret parameters is self evident and the flow field is appreciably influenced by the effects. The Influence of chemical Reaction and Radiation on MHD free convection flow of Kuvshinshiki fluid over a vertical porous plate with Heat source was discussed by Mohan Krishna et al. (2013).

The above mentioned articles treat the parameters like porosity, permeability, conductivity or thermal resistance and solutal diffusivity of the medium as constants. However, porosity measurements by Shwartz and Smith (1953), Benenati and Brosilow (1962) depicts that porosity is not constant but varies from the wall to the interior. Chandrashekara et al. (1984) has incorporated the variable permeability to study the flow through a porous medium. In this study they concluded that the variation in porosity and permeability has greater influence on velocity distribution and heat transfer. Mohommadien and El-Shaer (2004) analyzed influence of variable permeability on combined free and forced convection heat transfer flow past a semi-infinite vertical plate in a saturated porous media incorporating the variation of permeability and thermal conductivity. Sandeep and Sugunamma (2014) discussed radiation effect on unsteady convective flow past an impulsively moving vertical plate in porous medium. Veerasuneela Rani et al. (2012) studied hall current effects on flow of a viscous fluid in a vertical wavy channel. Run up flow of a Rivilin-Ericksen fluid through a porous medium was discussed by Sugunamma et al. (2011).

The main objective of the present study is to investigate the effect of non-Newtonian couple stress fluid on mixed convection heat and mass transfer past a semi infinite vertical heated plate embedded in a sparsely packed porous medium incorporating the variable porosity, permeability, thermal conductivity and solutal diffusivity.

2. Mathematical Formulation

Consider the two-dimensional steady flow of a laminar, viscous, incompressible fluid past a semi-infinite vertical heated plate embedded in a sparsely packed non-Newtonian couple stress fluid saturated porous medium of variable porosity, permeability and thermal conductivity. The x-coordinate is measured along the plate from its leading edge, and y-coordinate normal to it. Let U_o be the velocity of the fluid in the upward direction and the gravitational field g is acting in the downward direction. . The surface of the plate is maintained at a uniform constant temperature T_w and concentration C_w , which are higher than the free stream values existing far away from the plate $(i.e., T_w > T_\infty, C_W > C_\infty)$. It is also assumed that the free stream velocity U_o is parallel to the vertical plate. By considering the theory of boundary layer effect for sparsely packed porous medium with high porosity $\varepsilon < 1$. The vectorial equations for the conservation of mass, momentum, energy and species concentration can be written as:

$$\nabla . \vec{q} = 0, \tag{1}$$

$$\rho_o \left(\vec{q} . \nabla \right) \vec{q} = -\nabla p + \rho \vec{g} + \bar{\mu} \nabla^2 \vec{q} - \frac{\mu \, \varepsilon}{k} \vec{q} - \mu_c \nabla^4 \vec{q}, \tag{2}$$

$$\left(\rho_o C_p\right) \left(\vec{q} . \nabla \right) T = \nabla . \left(\kappa \nabla T \right) + \Phi, \tag{3}$$

$$\left(\vec{q} . \nabla \right) C = \nabla . \left(\kappa_c \nabla C \right), \tag{4}$$

Where $\vec{q} = (u, v)$, u and v are the velocity components along the x and y directions, respectively. ρ is the density of the fluid, $\vec{g}$ is the acceleration due to gravity, p is the pressure, T is the temperature of the fluid, C is the concentration of the fluid, $\bar{\mu}$ is the effective viscosity of the fluid, μ is the fluid viscosity, μ_c is the couple stress fluid parameter, C_p is the specific heat at constant pressure, β_T is the coefficient of volume expansion, β_C is the volumetric coefficient of expansion with species concentration, κ is the thermal conductivity and κ_c is the solutal diffusivity. Equation (2) is the well-known Darcy-Brinkman equation which was first proposed by Brinkman [7] to include the boundary layer effect in the momentum equation. Φ is the viscous dissipation term.

The governing basic equations (1) – (4) for steady two-dimensional flow can be written in the form

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{5}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_o} \frac{\partial p}{\partial x} + g \, \beta_T \left(T - T_\infty\right) - g \, \beta_C \left(C - C_\infty\right) + \frac{\bar{\mu}}{\rho_o} \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{\rho_o} \frac{\varepsilon \, (y)}{k \, (y)} u - \frac{\mu_c}{\rho_o} \frac{\partial^4 u}{\partial y^4}, \tag{6}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha(y) \frac{\partial T}{\partial y} \right) + \frac{\bar{\mu}}{\rho_o C_p} \left(\frac{\partial u}{\partial y} \right)^2, \tag{7}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(\gamma(y) \frac{\partial C}{\partial y} \right) \tag{8}$$

where $\Phi = \dfrac{\overline{\mu}}{\rho_o C_p}\left(\dfrac{\partial u}{\partial y}\right)^2$, T_∞ is the ambient temperature, C_∞ is the ambient concentration, $k(y)$ the variable permeability of the porous medium is, $\varepsilon(y)$ is the variable porosity of the saturated porous medium, $\alpha(y)$ is the variable effective thermal diffusivity of the medium and $\gamma(y)$ is the effective solutal diffusivity. For simplicity we consider $\rho = \rho_o$.

To determine the flow field the above governing equations need to be solved subject to the boundary conditions. The different types of rigid surfaces boundary conditions have been stated to describe flow characteristics at the boundary, near the plate and far away from the plate embedded in a sparsely packed porous medium.

The following are the boundary conditions on velocity, temperature and concentration fields:

$$u = 0, \quad v = 0, \quad v_x = u_y, \quad T = T_W, \quad C = C_W \quad \text{at} \quad y = 0,$$

$$u = U_o, \quad v_x = u_y, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \to \infty. \tag{9}$$

Since the flow field is uniform at a sufficiently large distance from the porous surface, in the free stream $u = U_0$, where U_0 is the free stream velocity. The expression for free stream velocity is obtained from equation (2) and is given by,

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{\mu \varepsilon(y)}{\rho k(y)} U_0, \tag{10}$$

Eliminating $\dfrac{\partial p}{\partial x}$ in equation (6) by using equation (10), we finally obtain the momentum equation as,

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \vec{g}\,\beta(T - T_\infty) - \vec{g}\,\beta^*(C - C_\infty)$$
$$+ \frac{\overline{\mu}}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\mu_c}{\rho}\frac{\partial^4 u}{\partial y^4} + \frac{\mu}{\rho}\frac{\varepsilon(y)}{k(y)}(U_o - u) \tag{11}$$

Equations (5), (7), (8) and (11) are the governing equations: conservation of mass, energy, species concentration and momentum equations respectively. They are highly nonlinear partial differential equations, in order to solve them the following dimensionless variables f, θ and ϕ and as well as the similarity variable η are introduced (see, Mohammad in and El-shaer [15])

$$\eta = \left(\frac{y}{x}\right)\left(\frac{U_o x}{v}\right)^{1/2}, \qquad \psi = \sqrt{v U_o x}\, f(\eta), \qquad \theta = \frac{T - T_\infty}{T_W - T_\infty}, \qquad \phi = \frac{C - C_\infty}{C_W - C_\infty}, \tag{12}$$

The stream function $\psi(x, y)$ is defined by $u = \dfrac{\partial \psi}{\partial y}$, $v = -\dfrac{\partial \psi}{\partial x}$, such that the continuity equation (5) is satisfied automatically and the velocity components are given by

$$u = U_o f'(\eta), \qquad v = -\frac{1}{2}\sqrt{\frac{v U_o}{x}}\left(f(\eta) - \eta\, f'(\eta)\right), \tag{13}$$

where, a prime represents differentiation with respect to η ,

Following Chandrashekhara and Namboodiri [16, 17], the variable permeability $k(\eta)$, the variable porosity $\varepsilon(\eta)$, variable effective thermal conductivity $\alpha(\eta)$ and $\gamma(\eta)$ is the variable effective solutal diffusivity are given by

$$k(\eta) = k_o\left(1 + d e^{-\eta}\right), \tag{14}$$

$$\varepsilon(\eta) = \varepsilon_o\left(1 + d^* e^{-\eta}\right), \tag{15}$$

$$\alpha(\eta) = \alpha_o[\varepsilon_o\left(1 + d^* e^{-\eta}\right) + \sigma^*\{1 - \varepsilon_o\left(1 + d^* e^{-\eta}\right)\}], \tag{16}$$

$$\gamma(\eta) = \gamma_o[\varepsilon_o\left(1 + d^* e^{-\eta}\right) + \gamma^*\{1 - \varepsilon_o\left(1 + d^* e^{-\eta}\right)\}], \tag{17}$$

where k_o, ε_o, α_o and γ_o are the permeability, porosity, thermal conductivity and solutal diffusivity at the edge of the boundary layer respectively, σ^* is the ratio of the thermal conductivity of solid to the conductivity of the fluid, γ^* is the ratio of the thermal diffusivity of solid to the diffusivity of the fluid, d and d^* are treated as constants having values 3.0 and 1.5 respectively for variable permeability and $d = d^* = 0$ for uniform permeability.
Substituting (12) and (13) in Equations (11), (7) and (8) and using (14) to (17), we get the following transformed equations

$$f''' + \frac{1}{2} ff'' + \frac{Gr}{\mathrm{Re}^2}(\theta - N\phi) + \frac{\alpha^*\left(1 + d^* e^{-\eta}\right)}{\sigma \mathrm{Re}\left(1 + de^{-\eta}\right)}(1 - f') - C_\alpha \, \mathrm{f}^{(\mathrm{v})} = 0 \tag{18}$$

$$\theta'' = -\frac{\left(\frac{1}{2}\right)\Pr \theta' f + \Pr E f''^2 + \varepsilon_o d^* e^{-\eta}\left(\sigma^* - 1\right)\theta'}{\varepsilon_o + \sigma^*(1 - \varepsilon_o) + \varepsilon_o d^* e^{-\eta}\left(1 - \sigma^*\right)} \tag{19}$$

$$\phi'' = -\frac{\left(\frac{1}{2}\right)Sc\, \phi' f + \varepsilon_o d^* e^{-\eta}\left(\gamma^* - 1\right)\phi'}{\varepsilon_o + \gamma^*(1 - \varepsilon_o) + \varepsilon_o d^* e^{-\eta}\left(1 - \gamma^*\right)}, \tag{20}$$

where, $\Pr = \bar{\mu}/\rho\alpha_o$ is the Prandtl number, $Sc = \bar{\mu}/\rho\gamma_o$ is the Schmidt number, $\alpha^* = \mu/\bar{\mu}$ is the ratio of viscosities, $N = \dfrac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$ is the Buoyancy ratio, $E = U_o^2/C_p(T_w - T_\infty)$ is the Eckert number, $\sigma = k_o/x^2\varepsilon_o$ is the local permeability parameter, $\mathrm{Re} = U_o x/\nu$ is the local Reynolds number and $Gr_T = g\beta_T(T_w - T_\infty)x^3/\nu^2$ is the thermal Grashof number, $Gr_c = g\beta_C(C_w - C_\infty)x^3/\nu^2$ is the solutal Grashof number, $C_\alpha = \dfrac{\mu_c}{\mu \, \mathrm{x}^2}\mathrm{Re}$ is the couple stress parameter and $Ri = Gr/\mathrm{Re}^2$ is the Richardson number which is the mixed convection parameter. Here $Gr_T = Gr_C$.

The transformed boundary conditions are:

$$f = 0, \quad f' = 0, \quad f'' = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0,$$
$$f' = 1, \quad f'' = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty. \tag{21}$$

Once the velocity, temperature and concentration distributions are known, for many practical applications, it is important to find an expression for the skin friction, the rate of heat transfer and the rate of mass transfer. The Nusselt number is defined as the ratio of the vertical heat flux to the conductive vertical heat flux. In the steady state the vertical heat flux is independent of the vertical coordinate. Hence, they can be calculated respectively by using

$$\tau = -f''(0) / \sqrt{\mathrm{Re}}, \quad Nu = -\sqrt{\mathrm{Re}}\;\theta'(0) \quad \text{and} \quad Sh = -\sqrt{\mathrm{Re}}\;\phi'(0), \tag{22}$$

Where τ is the skin friction, Nu is the Nusselt number and Sh is the Sherwood number.

3. Results and Discussion

The system of first-order differential equations (18) - (20) with respect to the boundary conditions (21) are solved numerically using shooting technique with Runge-Kutta Fehlberg and Newton-Raphsonmethod. In order to know the accuracy of the method used, computed values of $f'''(0)$, $\theta'(0)$ and $\phi'(0)$ are obtained for buoyancy ratio $N = 0$ and compared with those obtained by Mohammadin and El-Shaer (2004) with only the heat transfer, for the variable permeability $(d = 3.0,\ d^* = 1.5)$ case and good agreement has been obtained with their results. The values tabulated in Table 1 are for $\varepsilon_o = 0.4, Ec = 0.1, \Pr = 0.71, \mathrm{Sc} = 0.22$ with selected values of Gr/Re^2, σ^* and $\alpha^*/\sigma\mathrm{Re}$. As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters namely, the Buoyancy ratio N,

$Gr\ /\ \mathrm{Re}^2$, $\alpha^* / \sigma \mathrm{Re}$, and σ^* with the fixed Prandtl number $\Pr$, Eckert number Ec, Schmidt number Sc, porosity of the saturated porous medium at the edge of the boundary layer ε_0 in both UP and VP cases.

The effects of couple stress parameter C_α for variable permeability case on the dimensionless velocity, temperature and concentration profiles are presented for fixed values of Buoyancy ratio, mixed convection parameter, Prandtl number and Eckert number in Figs. (1) – (3). As C_α increases, it can be observed from the Fig. 1 the maximum velocity decreases in amplitude and location of the maximum velocity moves far away from the plate. This is due to the fact that the rotational field of the velocity generated in couple stress fluid. It is clear from Fig. 2 that the temperature increases with the increase of couple stress fluid parameter C_α. It can be seen from Fig. 3 that the concentration of the fluid increases with the increase of couple stress fluid parameter C_α. The temperature and concentration in case of couple stress fluid is more than that of Newtonian fluid case.

Variations of Buoyancy ratio N for variable permeability case, fixing the couple stress parameter $C_\alpha = 1.0$, the velocity, temperature and concentration profiles are depicted in Figs. (4) – (6). Buoyancy ratio N, which defines as ratio of solutal and thermal buoyancy effects increases the velocity boundary layer as it increases. This is due to the fact that species buoyancy has an accelerating effect on the flow field. With $N \to 10$ the velocity boundary layer is little high when compared to the other layers. From Fig. 4, it can be observed that increase in buoyancy ratio N results a decrease of the thermal boundary layer thickness and in general little high average temperature within the boundary layer. From Fig. 5, it can be seen that increase in buoyancy ratio N results a decrease of species boundary layer thickness and uniformity in the decrease of boundary layer.

The dimensionless parameter Gr/Re^2 is used to represent the free, forced and combined convection regimes. The case $Gr/\mathrm{Re}^2 << 1$ corresponds to pure forced convection, $Gr/\mathrm{Re}^2 = 1$ corresponds to combined free-forced convection and $Gr/\mathrm{Re}^2 >> 1$ corresponds to pure free convection. As the thermal Grashof number Gr_T is a measure of the Buoyancy forces due to temperature differences to the viscous forces and the solutalGrashof number Gr_C is a measure of the buoyancy forces due to concentration differences to the viscous forces, the dimensionless parameter Gr_T / Re^2 has the same meaning as the parameter Gr_C / Re^2. Hence Gr/Re^2 variations are considered in the Figs. (7) – (9). It is observed in Fig. 7, that increase in the magnitude of Gr/Re^2 increases the velocity distribution for both UP and VP cases considered which is very significant for higher value in the boundary layer. This is due to the fact that in the process of cooling the heated plate, the free convection currents are carried away from the plate to the free stream and as the free stream is in the upward direction, the free convection currents induce the mean velocity to increase. The increase in buoyancy effects and hence increases in the value of Gr/Re^2 results in causing more induced flow along the plate in the vertical direction which is reflected by the increase in the fluid velocity. It is clearly seen in the velocity profiles that, effect of variable permeability (VP) is more prominent compared to uniform permeability (UP) case. As Gr/Re^2 increases, it can be observed that from Figs. 8 and 9 that the temperature and concentration profile decreases. This is due to the fact that the free convection currents are carried away from the plate to the free stream and as the free stream is in the downward direction, the free convection currents induces the energy and species concentration to decrease.

Figs. (10) – (12) illustrates the velocity, temperature and concentration profiles for fixed $C_\alpha = 1.0$ by varying $\alpha^* / \sigma \mathrm{Re}$ for both uniform permeability and variable permeability cases. $\alpha^* / \sigma \mathrm{Re}$ is defined to be ratio of viscosities to the permeability parameter with the Reynolds number. Increase in $\alpha^* / \sigma \mathrm{Re}$ shows increase in the velocity boundary layer observed from Fig. 10 for uniform and variable permeability cases. This is due to high viscous force leading to low Reynolds number, thereby increase in $\alpha^* / \sigma \mathrm{Re}$ with high viscosities inducing the average velocity to increase in the process of cooling of heated plate. It is also observed that both uniform and variable permeability cases are of equal prominence. Figs. 11 and 12 depicts that temperature and concentration profiles decreases, as $\alpha^* / \sigma \mathrm{Re}$ increases for both uniform and variable permeability cases due to high viscous forces. The temperature profiles shows slight prominence in variable permeability (VP) case when compared to uniform permeability (VP) case, whereas the concentration profiles shows that the UP and VP cases are almost same.

Figure 13 depicts the temperature profiles with variation of σ^* for fixed value of $C_\alpha = 1.0$. σ^* defines to be the ratio of thermal conductivity of the solid to the fluid. Increase in σ^* leads to decrease in temperature profiles. For all the increasing values of σ^* , the variable permeability is less prominent compared to uniform permeability case. Figure 14 depicts the concentration profiles with variation of γ^* for fixed value of $C_\alpha = 1.0$. γ^* defines to be the ratio of solutal conductivity of the solid to the fluid. Increase in γ^* leads to decrease in concentration profiles.

The local Nusselt number with Gr/Re^2 for various values of Prandtl number Pr are shown in Fig. 15. Increase in Gr/Re^2 results in linear increase of heat transfer for different values of Prandtl number ranging from 0.71 to 7.0 for both uniform permeability and variable permeability cases. It is observed that variable permeability case results in faster heat transfer which is more dominant when compared to uniform permeability case. The local Sherwood number with Gr/Re^2 for various values of Schmidt number Sc are shown in Fig. 16. Schmidt number for mass transfer is an analogous number to Nusselt number which is for heat transfer. It is clearly seen here also that, increase in Gr/Re^2 results in linear increase of mass transfer for different values of Schmidt number ranging from 0.22 to 0.60 for both uniform permeability and variable permeability cases.

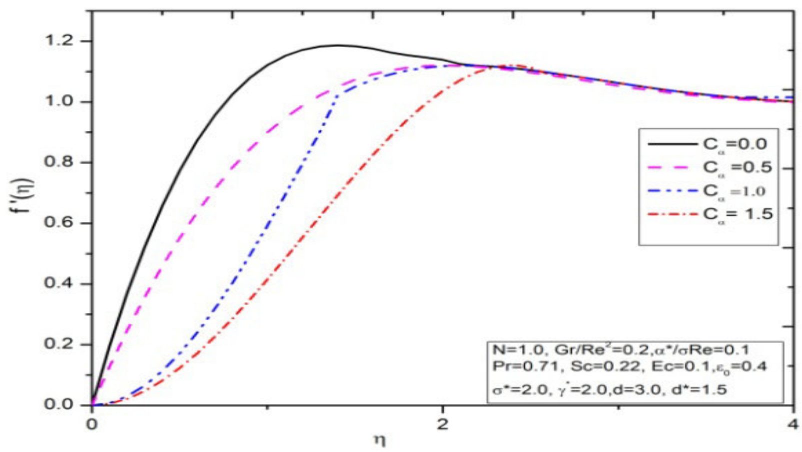

Fig. 1 Velocity profiles for various values of Couplestress parameter C_α for VP case

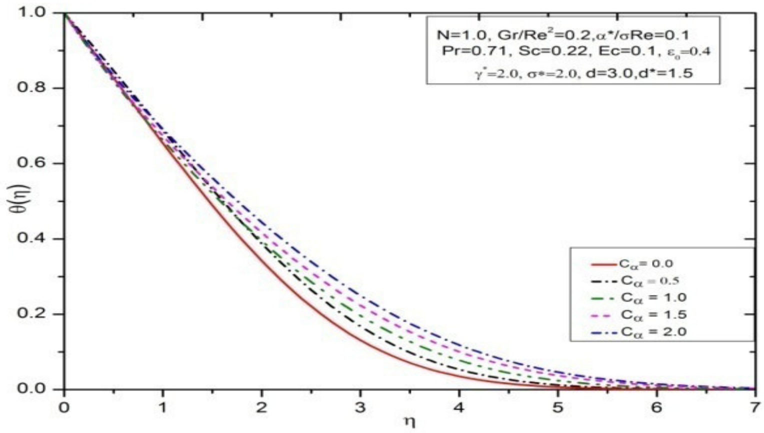

Fig. 2 Temperature distributions for various values of couple stress parameter C_α for VP case

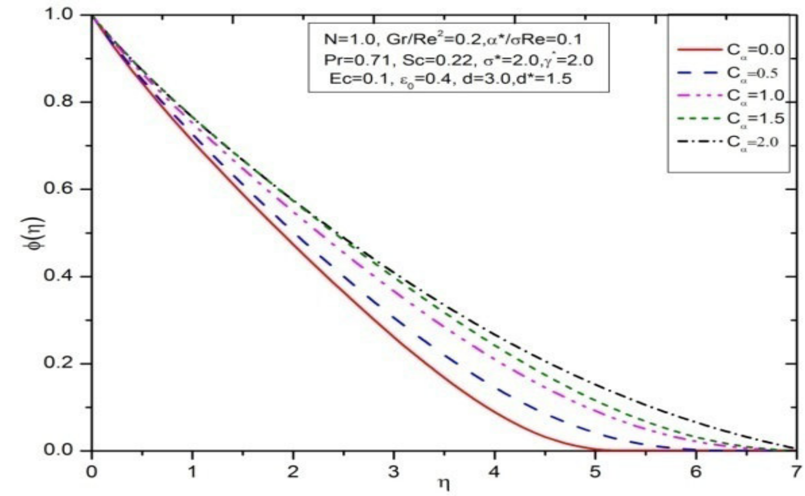

Fig. 3 Concentration Variations for different values of Couple stress parameter C_α for VP case

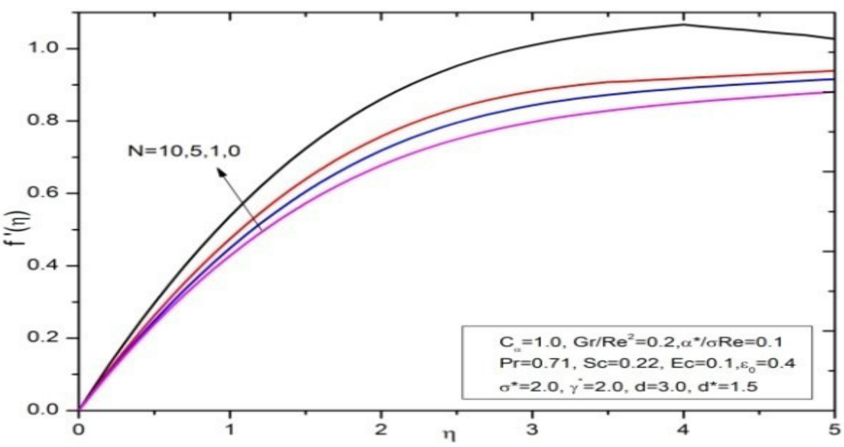

Fig. 4 Velocity profiles for different values of buoyancy ratio N for VP case

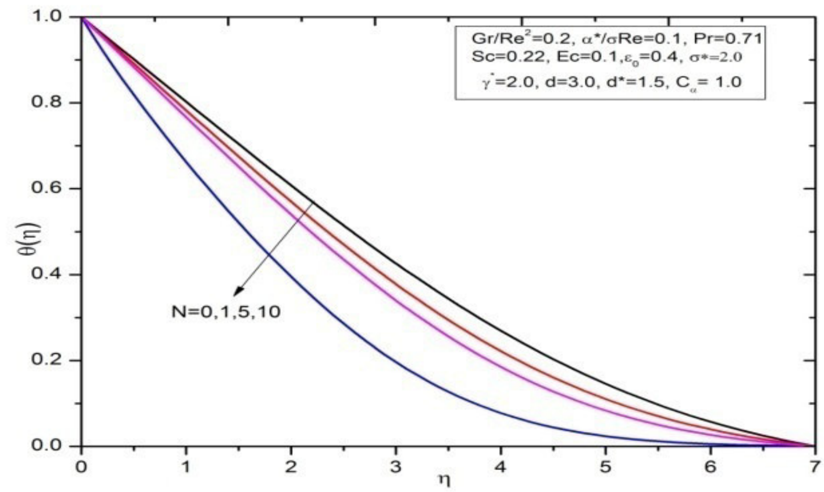

Fig. 5 Temperature profiles for different values of buoyancy ratio N for VP case

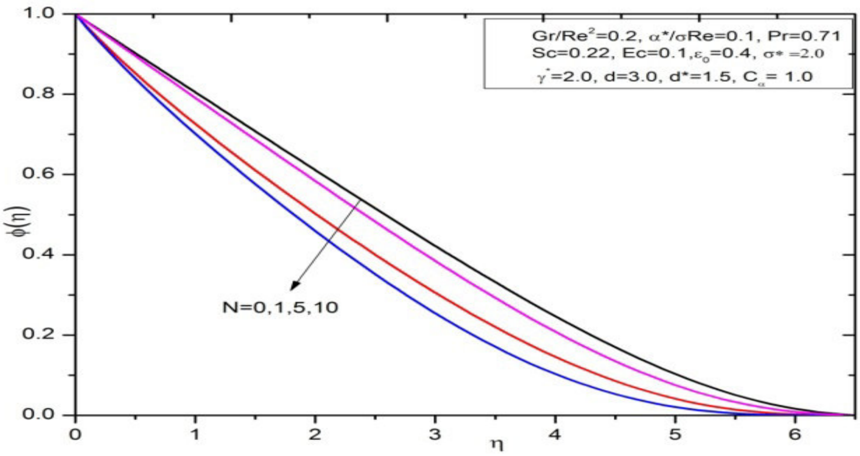

Fig. 6 Concentration profiles for different values of buoyancy ratio N for VP case

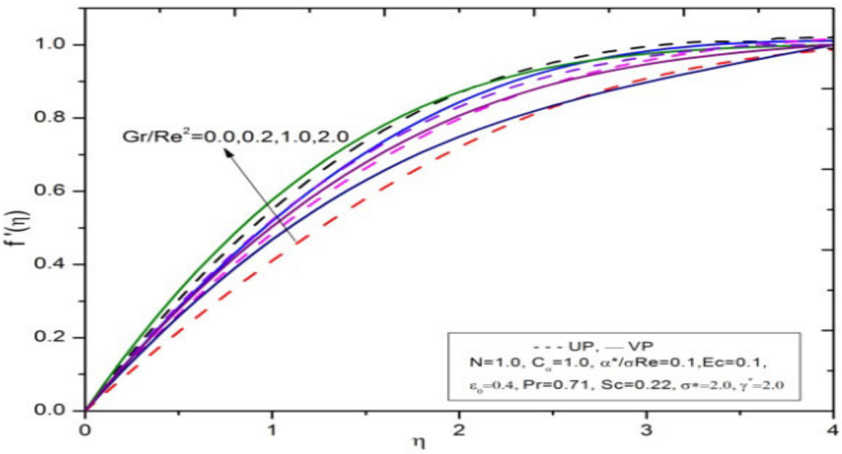

Fig. 7 Velocity profiles for various values of Gr/Re^2 for UP and VP cases

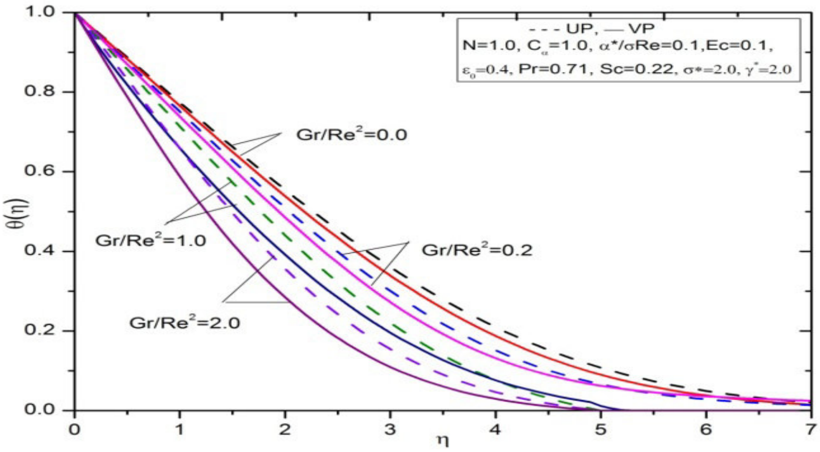

Fig. 8 Temperature distributions for various values of Gr/Re^2 for UP and VP cases

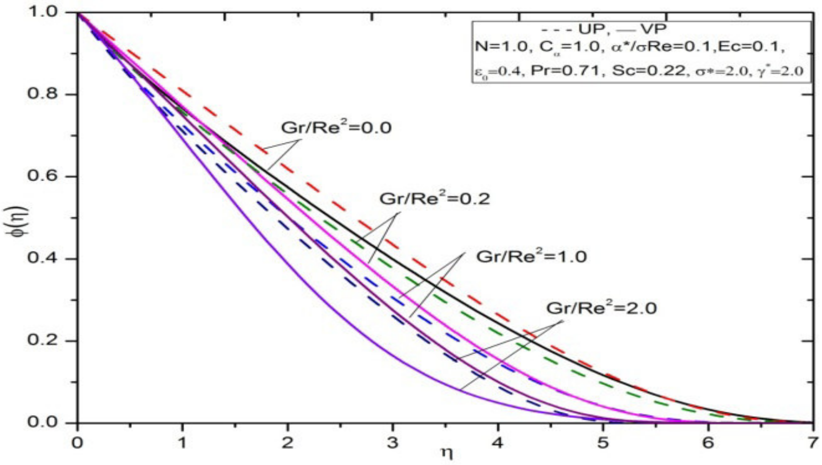

Fig. 9 Concentration profiles for various values of Gr / Re^2 for UP and VP cases

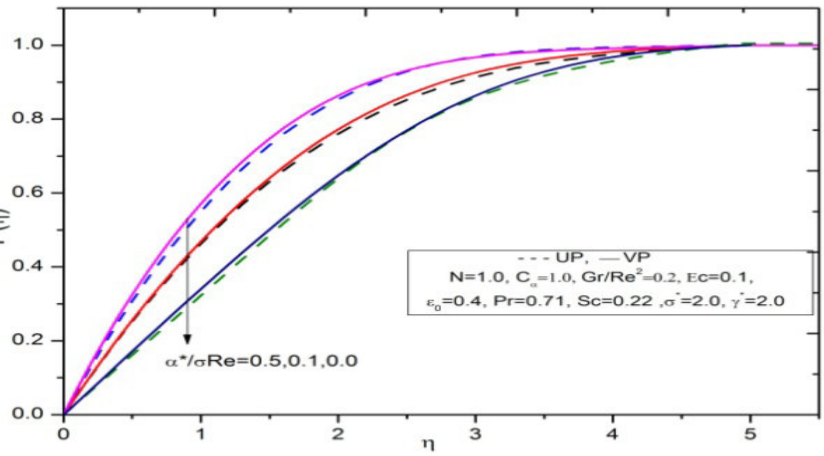

Fig. 10 Velocity profiles for various values of $\alpha^* / \sigma\,\mathrm{Re}$ for UP and VP cases

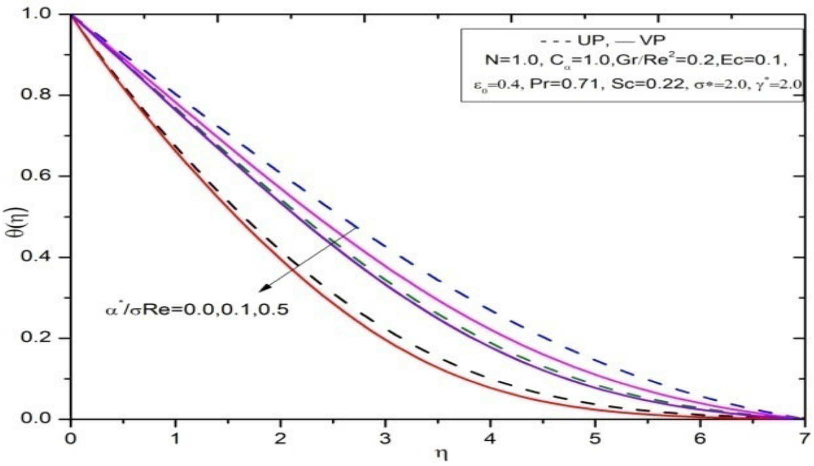

Fig. 11 Temperature profiles for various values of $\alpha^* / \sigma\,\mathrm{Re}$ for UP and VP cases

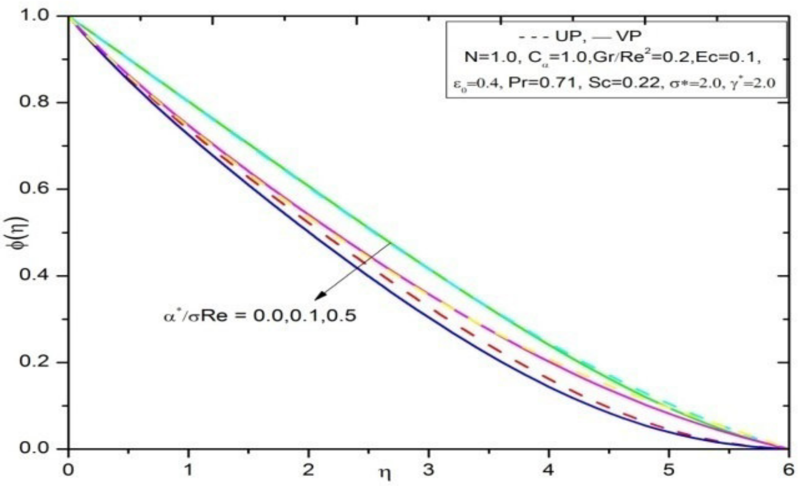

Fig. 12 Concentration profiles for various values of $\alpha^* / \sigma \, \mathrm{Re}$ for UP and VP cases

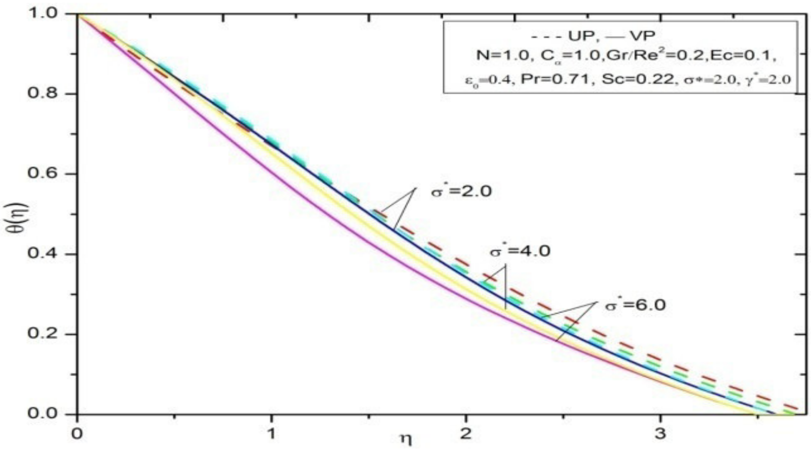

Fig. 13 Temperature profiles for various values of σ^* for UP and VP cases

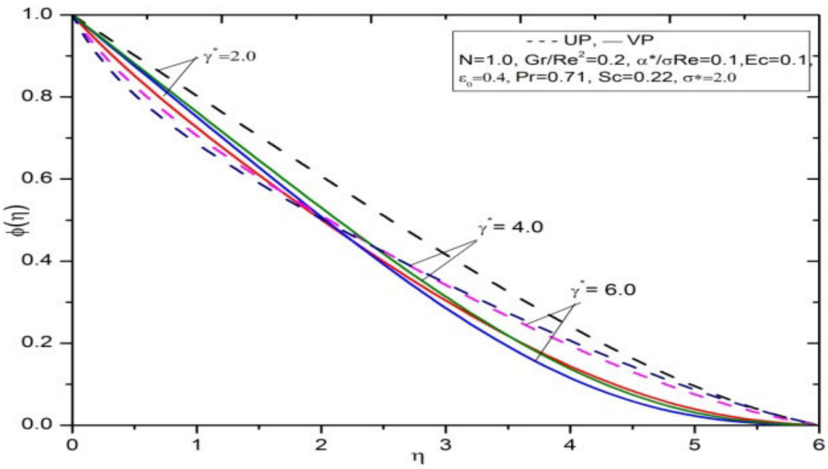

Fig. 14 Concentration profiles for various values of γ^* for UP and VP cases

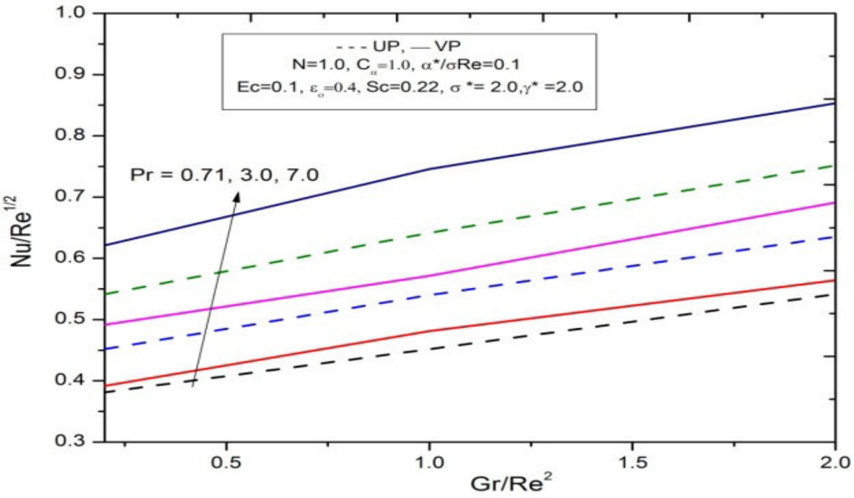

Fig. 15 Variations of local Nusselt number with Gr/Re^2 for various values of Pr

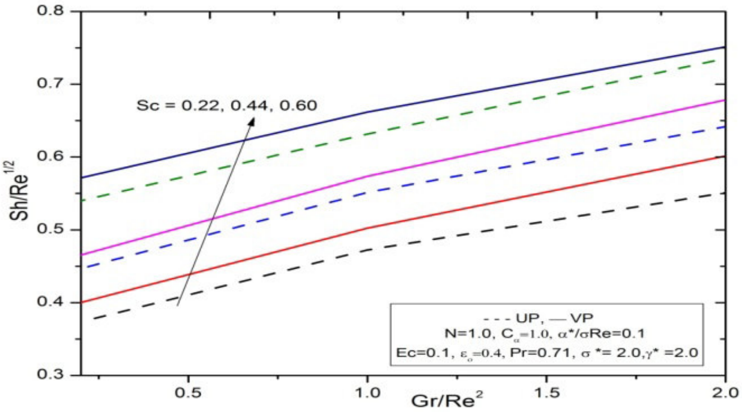

Fig. 16 Variations of local Sherwood number with Gr/Re^2 for various values of Sc

Table 1 Results for $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for Uniform Permeability (UP) and Variable Permeability (VP) cases

C_α	N	σ^* γ^*	Gr/Re^2	$\alpha^*/\sigma\mathrm{Re}$	Uniform Permeability (UP)			Variable Permeability (VP)		
					$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1.0	1.0	2.0	0.2	0.0	0.453210	0.351230	0.341110	0.500345	0.361780	0.350020
				0.1	0.356784	0.281765	0.271245	0.371650	0.321340	0.311450
				0.2	0.486750	0.381560	0.361450	0.500323	0.425678	0.400345
				0.5	0.881340	0.451650	0.441030	0.900544	0.481954	0.461345
			0.0	0.1	0.443210	0.381340	0.373450	0.463450	0.402563	0.391786
			0.2		0.356784	0.251678	0.281765	0.371650	0.321340	0.311450
			1.0		0.754320	0.481230	0.472310	0.773567	0.502345	0.489876
			2.0		0.897650	0.564320	0.551120	0.924567	0.594322	0.581567
		4.0	0.2	0.1	0.551430	0.381450	0.371110	0.554780	0.435670	0.430020
		6.0			0.651780	0.423130	0.411110	0.661760	0.451340	0.440230
	0.0	2	0.2	0.1	0.356430	0.287650	0.271560	0.451320	0.321560	0.310050
	1.0				0.356784	0.251678	0.243765	0.881650	0.421560	0.420030
	5.0				0.987650	0.657867	0.641230	0.987060	0.564530	0.551003
	10.0				0.998760	0.682430	0.671220	0.999340	0.657890	0.641560
0.0	1.0	2.0	0.2	0.1	0.674535	0.567430	0.572340	0.773567	0.591678	0.581678
0.5					0.656784	0.351423	0.341110	0.683220	0.356750	0.350120
1.0					0.356784	0.281765	0.271245	0.371650	0.321340	0.311450
1.5					0.451320	0.371540	0.370020	0.519430	0.426510	0.420110

5. Conclusions

In this chapter, the problem of steady, laminar, viscous, incompressible two-dimensional mixed convection flow due to vertical heated plate in a non-Newtonian couple stress fluid embedded in a sparsely packed porous medium with variable fluid properties such as variable porosity, permeability, thermal conductivity and solutal diffusivity is investigated. The boundary layer flow in the porous medium is governed by Lapwood- Brinkman extended Darcy model. Using the similarity variables, the governing equations are transformed into a set of highly coupled non linear ordinary differential equations. These equations are then solved numerically by shooting technique using Runge-KuttaFehlberg method. Since no experimental results of the corresponding studies are available, comparison between the obtained results with the existing results is numerically simulated. The computed results are presented to illustrate the details of flow and heat and mass transfer characteristics and also their dependence on the physical parameters, the following conclusions are drawn:

(i) Increase in couple stress parameter C_α, decreases the maximum velocity in amplitude and the location of the maximum velocity moves far away from the wall due to the rotational field of the velocity generated in couple stress fluid for variable permeability case. The reciprocal of the couple stress parameter is multiplied to the momentum equation. Temperature and concentration of the fluid increases with the increase of couple stress fluid parameter C_α.

(ii) Increase in the buoyancy ratio N leads to increase the dimensionless velocity due to more solutal buoyancy force. Increase in N leads to decrease in both temperature and concentration layers for variable permeability case. For higher value of N=10, the velocity overshoot is more significant when compared to the lower N = 0.0.

(iii) The mixed convection parameter Gr/Re^2 variations, shows increase in the parameter results in increasing the velocity distributions. When the parameter $Gr/\mathrm{Re}^2 = 2.0$ also the velocity boundary layer proportionally increases both in uniform permeability and variable permeability cases. In the absence of couple stress parameter, i.e., for a Newtonian fluid when the parameter $Gr/\mathrm{Re}^2 = 2.0$ the velocity overshoot is more significant. The temperature and concentration decreases with the increase in the mixed convection parameter.

(iv) Increase in the parameter $\alpha^*/\sigma\,\mathrm{Re}$ results in increasing the velocity distributions and increase in the parameter results in decreasing the temperature and concentration profiles for both uniform and variable permeability cases. The concentration profiles are almost overlapping with uniform and variable permeability for different values of the parameter.

(v) The temperature variations with the varying parameter σ^* results in decrease of the boundary layer with the increase in the parameter. The concentration variations with the varying parameter γ^* results in decrease in the boundary layer thickness.

(vi) The local Nusselt number and local Sherwood number with variations of the parameter Gr/Re^2 as the Prandtl number and Schmidt number varies respectively results in the increasing behaviour for higher values of Prandtl and Schmidt number respectively.

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