Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol 7, 2012

Some Bianchi Type I Magnetized Bulk Viscous Fluid Tilted Cosmological Models

Anita Bagora(Menaria)

Department of Mathematics, Seedling Academy, Jaipur National University, Jaipur-302017, India

Rakeshwar Purohit

Department of Mathematics, College of Science,Mohanlal Sukhadia University, Udaipur-313001, India

Abstract. Expanding Bianchi type I magnetized bulk viscous models with two tilted fluid filled with disordered radiation and heat conduction are investigated. Here we assume a linear relation between shear and expansion i.e. $\frac{\sigma}{\theta}$=constant, which leads to A=BC, where A, B, C are metric potentials. The coefficient of bulk viscosity is assumed to be power function of mass density. It has been shown that tilted nature of the model is preserved due to magnetic field. The various physical and geometrical properties of the models are discussed. The nature of the models in presence and absence of magnetic field and bulk viscosity are also discussed.

Key words: Cosmology; Bianchi type I universe; Tilted models; Bulk viscosity.

1.1 *Introduction*

Many cosmologists believe that the standard cosmological models are too restrictive because of their insistence on isotropy. Several attempts have been made to study nonstandard (anisotropic) cosmological models (Narlikar [1],MacCallum [2]). It would therefore be fruitful to carry out detailed studies of gravitational fields which can be described by spacetimes of various Bianchi types. Viscosity plays an important role in explaining many physical features of the homogeneous world models. Since viscosity counteracts the cosmological collapse, a different picture of the early universe may appear due to dissipative processes caused by viscosity. Homogeneous cosmological models filled with viscous fluid have been widely studied. Murphy [3] possessed on interesting feature in that the big-bang type of singularity of infinite space time curvature does not occur to be a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. Roy and Prakash [4] have obtained some viscous fluid cosmological models of plane symmetry. It is also well known that the presence of strong magnetic field is exhibited by galaxies and interstellar space and give rise to a kind of viscous effect in the fluid flow. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in framework of general theory of relativity (Padmanabhan and Chitre [5]; Johri and Sudarshan [6]; Maartens [7]; Zimdahl [8]; Singh, Beesham and Mbokazi [9];Klimek [10];Banerjee et al. [11]; Dunn and Tupper [12]; Roy and Singh [13]; Ribeiro and Sanyal [14]; Santos et al. [15].

Homogeneous and anisotropic cosmological models have been studied widely in the framework of general relativity. These models are more restricted than the inhomogeneous models. But in spite of this, they explain a number of observed phenomena quite satisfactorily. In recent years, there has been considerable interest in investigating spatially homogeneous and anisotropic cosmological models in which matter does not move orthogonal to the hypersurface of homogeneity. Such types of models are called tilted cosmological models. The general dynamics of tilted cosmological models have been studied by King and Ellis [16] and Ellis and King [17]. The cosmological models with heat flow have been also studied by Coley and Tupper ([18], [19]), Roy and Banerjee [20]. Ellis and Baldwin [21] have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. Also, general relativity describes the state in which radiation concentrates around a star. Klein [22] worked on it and obtained an approximate solution to Einstein's field equation in spherical symmetry for a distribution of diffused radiation. Singh and Abdussattar [23] have obtained an exact static spherically symmetric solution of Einstein's field equation for disordered radiation. Roy and Singh [24] have obtained a non-static plane symmetric space time filled with disordered radiation. Teixeira, Wolk and Som [25] investigated a model filled with source free disordered distribution of electromagnetic radiation in general relativity. Bagora [26-27] obtained tilted homogeneous cosmological model with disordered radiation in different context. Motivated by these studies, in this paper, we propose to find tilted Bianchi type I cosmological models filled with disordered radiation in presence and absence of a bulk viscous fluid, heat flow and magnetic field.

1.2. *The Metric and Field Equations*

We consider the metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B and C are functions of cosmoic 't' alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction is given by Ellis [28] taken into the form

$$T_i^j = (\in + \bar{p})v_i v^j + \bar{p}\, g_i^j + q_i v^j + v_i q^j + E_i^j, \tag{2}$$

where $$\bar{p} = \in - \xi v^i_{;i} \tag{3}$$

Here $\in$, p, $\bar{p}$ and ξ are the energy density, isotropic pressure, effective pressure, bulk viscous coefficient respectively and v_i is the flow vector satisfying the relations

$$g_{ij}\, v^i v^j = 1,\ q^i q^j > 0,\ q_i v^i = 0 \tag{4}$$

where q_i is the heat conduction vector orthogonal to v_i.

Here E_i^j is the electromagnetic field given by Lichnerowicz [29] as

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2} g_i^j\right) - h_i h^j\right], \tag{5}$$

where $\bar{\mu}$ is magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijk\ell} F^{k\ell} v^j. \tag{6}$$

$F_{k\ell}$ is the electromagnetic field tensor and $\in_{ijk\ell}$ the Levi-Civita tensor density.

From (6) we find that $h_1 \neq 0$, $h_2 = 0$, $h_3 = 0$, $h_4 \neq 0$. This leads to $F_{12} = 0 = F_{13}$ by virtues of (6). We also find that $F_{14} = 0 = F_{24}$ due to the assumption of infinite conductivity of the fluid. We take the incident magnetic field to be in the direction of x-axis so that the only non-vanishing component of F_{ij} is F_{23}.

The first set of Maxwell's equation $F_{ij;k} + F_{jk;i} + F_{ki;j} = 0$,leads to F_{23} = constant H (say).

From the equation (6) and (5), we have

$$E_1^1 = \frac{-H^2}{2\bar{\mu}B^2C^2} = -E_2^2 = -E_3^3 = E_4^4. \tag{7}$$

The fluid flow vector v^i has the components $\left(\frac{\sinh\lambda}{A},\ 0,\ 0,\ \cosh\lambda\right)$ satisfying (4) and λ is the tilt angle.

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j, \qquad (c = G = 1) \tag{8}$$

The field equation (8) for the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -8\pi\left[(\in + \bar{p})\sinh^2\lambda + \bar{p} + \frac{2q_1\sinh\lambda}{A} - \frac{H^2}{2\bar{\mu}(BC)^2}\right], \tag{9}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=-8\pi\left[\bar{p}+\frac{H^2}{2\bar{\mu}(BC)^2}\right], \tag{10}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=-8\pi\left[\bar{p}+\frac{H^2}{2\bar{\mu}(BC)^2}\right], \tag{11}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}=-8\pi\left[-(\in+\bar{p})\cosh^2\lambda+\bar{p}-2q_1\frac{\sinh\lambda}{A}-\frac{H^2}{2\bar{\mu}(BC)^2}\right], \tag{12}$$

$$(\in+\bar{p})A\sinh\lambda\cosh\lambda+q_1\cosh\lambda+q_1\frac{\sinh^2\lambda}{\cosh\lambda}=0, \tag{13}$$

where the suffix '4' stands for ordinary differentiation with respect to the cosmic time 't' alone.

1.3. *Solutions of Field Equations*

Equations from (9) to (13) with (3) are six independent equations in eight unknowns A, B, C,$\in$, p, ξ, q and λ. For the complete determinacy of the system we need three extra conditions.

(i) We assume that the model is filled with disordered radiation which leads to

$$\in=3p. \tag{14}$$

(ii) A relation between metric potential as

$$A=BC. \tag{15}$$

(iii) The coefficient of bulk viscosity (ξ) is inversely proportional to expansion (θ) i.e.

$$\xi\propto\frac{1}{\theta} . \tag{16}$$

The motive behind assuming the conditions $A = BC$ is explained as follows: Referring to Throne [30].the observations of velocity-redshift relation for extra-galactic sources suggest that the Hubble expansion of the universe is isotropic to within 30% [31-32].More precisely, the redshift studies place the limit $\sigma/H\le 0.30$ where σ is the shear and H the Hubble constant. Collins et al.[33] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hypersurface satisfies the condition $\sigma/\theta=$constant. The condition σ_1^1/θ=constant leads to A = $(BC)^n$ when the tilt angle $\lambda=0$.Here σ_1^1 is the eigenvalue of shear tensor σ_i^j.Also, the condition $\xi\propto\frac{1}{\theta}$ is due to the peculiar characteristic of the bulk viscosity. It acts like a negative energy field in an expanding universe (Johri and Sudarshan [34]).

Equations (9) and (12) with (3) and (14) lead to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}+\frac{A_4C_4}{AC}+\frac{A_4B_4}{AB}=8\pi(2p+\xi\theta)+\frac{K}{B^2C^2} , \tag{17}$$

where $$\frac{8\pi H^2}{\bar{\mu}}=K. \tag{18}$$

Equations (10) and (11) with (3), lead to

$$\frac{2A_{44}}{A}+\frac{C_{44}}{C}+\frac{B_{44}}{B}+\frac{A_4C_4}{AC}+\frac{A_4B_4}{AB}=-16\pi(p-\xi\theta)-\frac{K}{B^2C^2} . \tag{19}$$

Again equations (10) and (11) lead to

$$\frac{\nu_4}{\nu}=\frac{a}{\mu^2} \ , \tag{20}$$

where 'a' is constant of integration and $BC=\mu$, $\frac{B}{C}=\nu$. (21)

Equations (17) and (19) lead to

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}+\frac{A_4B_4}{AB} = 12\xi\theta \ . \tag{22}$$

By condition (16) we can assume that

$$12\xi = \gamma \ . \tag{23}$$

Equation (22) with use of (21) and (23), leads to

$$\frac{2\mu_{44}}{\mu}+\frac{3}{4}\left(\frac{\mu_4}{\mu}\right)^2+\frac{1}{4}\left(\frac{\nu_4}{\nu}\right)^2=\gamma \ , \tag{24}$$

where $A=\mu$.

From equations (20) and (24), we have

$$f^2=\frac{11a^2+55b\mu^{5/4}+20\gamma\mu^4}{55\mu^2} \ , \tag{25}$$

where $\mu_4 = f(\mu)$.

From (20), we have

$$\log\nu = a\sqrt{55}\int\frac{d\mu}{\mu\sqrt{11a^2+55b\mu^{5/4}+20\gamma\mu^4}} \ . \tag{26}$$

By introducing the following transformations

$$\mu = T,\ x=X,\ y=Y,\ z=Z.$$

The metric (1) becomes

$$ds^2=\left(\frac{-55T^2}{11a^2+55bT^{5/4}+20\gamma T^4}\right)dT^2+T^2dx^2+T\nu dy^2+\frac{T}{\nu}dz^2 \ . \tag{27}$$

where ν is determined by (26) when $\mu = T$.

1.4. *Some physical and geometrical features*

The pressure and matter density for the model (27) are given by

$$8\pi(p-\xi\theta)=\frac{1}{176T^{11/4}}\left(55b-112\gamma T^{11/4}-88KT^{3/4}\right), \tag{28}$$

$$8\pi(\in-3\xi\theta)=\frac{3}{176T^{11/4}}\left(55b-112\gamma T^{11/4}-88KT^{3/4}\right). \tag{29}$$

The tilt angle λ is given by

$$\cosh\lambda = \sqrt{\frac{275b - 220\gamma T^{11/4} - 176KT^{3/4}}{2(165b + 4\gamma T^{11/4})}} . \tag{30}$$

The scalar of expansion θ is given by

$$\theta = \frac{22\psi_1}{T^2}\sqrt{\frac{11a^2 + 55bT^{5/4} + 20\gamma T^4}{110(165b + 4\gamma T^{11/4})^3(275b - 220\gamma T^{11/4} - 176KT^{3/4})}} . \tag{31}$$

If we put $\xi = 0$ in equations (28) – (31), we get the following solutions

$$p = \frac{1}{128\pi T^{11/4}}(5b - 8KT^{3/4}) ,$$

$$\in = \frac{3}{128\pi T^{11/4}}(5b - 8KT^{3/4}) ,$$

$$\cosh\lambda = \sqrt{\frac{25b - 16KT^{3/4}}{30b}} ,$$

$$\theta = \frac{2(25b - 19KT^{3/4})}{5T^2}\sqrt{\frac{a^2 + 5bT^{3/4}}{6b(25b - 16KT^{3/4})}} .$$

If we put K = 0, then above mentioned quantities lead to

$$p = \frac{5b}{128\pi T^{11/4}} ,$$

$$\in = \frac{15b}{128\pi T^{11/4}} ,$$

$$\cosh\lambda = \sqrt{\frac{5}{6}} ,$$

$$\theta = \frac{2}{T^2}\sqrt{\frac{a^2 + 5bT^{3/4}}{6}} .$$

Thus given $\xi(t)$ we can solve the system for the physical quantities. Therefore, to apply the third conditions, let us assume the following adhoc law (Maartens [7], Zimdahl [8]).

$$\xi(t) = \xi_0 \in^m . \tag{32}$$

where ξ_0 and m are real constants. If m = 1, equation (32) may corresponds to a radiative fluid (Weinberg [35]), whereas m = 3/2 may correspond to a string dominated universe. However, more realistic models (Santos [15]) are based on lying the regime $0 \le m \le \frac{1}{2}$.

Model-I $\xi = \xi_0$.

When m = 0, equation (32) reduces to $\xi = \xi_0$ and hence equations (28) and (29) with the use of equation (31) lead to

$$p = \xi_0\theta + \frac{1}{1408\pi T^{11/4}}(55b - 112\gamma T^{11/4} - 88KT^{3/4}) , \tag{33}$$

$$\in = 3\xi_0\theta + \frac{3}{1408\pi T^{11/4}}(55b - 112\gamma T^{11/4} - 88KT^{3/4}) . \tag{34}$$

Model-II $\xi = \xi_0 \in$.

When m = 1, equation (32) reduces $\xi = \xi_0 \in$ and hence equations (28) and (29) with the use of equation (31) lead to

$$p = \frac{(55b - 112\gamma T^{11/4} - 88KT^{3/4})}{1408\pi T^{11/4}(1 - 3\xi_0\theta)}, \tag{35}$$

$$\in = \frac{3(55b - 112\gamma T^{11/4} - 88KT^{3/4})}{1408\pi T^{11/4}(1 - 3\xi_0\theta)}. \tag{36}$$

The non-vanishing components of shear tensor, rotation tensor, flow vector v^i and heat conduction q^i are given by

$$v^4 = \sqrt{\frac{275b - 220\gamma T^{11/4} - 176KT^{3/4}}{2(165b + 4\gamma T^{11/4})}}, \tag{37}$$

$$q^4 = \frac{(55b + 228\gamma T^{11/4} + 176KT^{3/4})}{1408\pi T^{11/4}}\sqrt{\frac{(275b - 220\gamma T^{11/4} - 176KT^{3/4})}{2(165b + 4\gamma T^{11/4})}}. \tag{38}$$

$$\sigma_{11} = \frac{11\psi_2}{6}\sqrt{\frac{(11a^2 + 55bT^{5/4} + 20\gamma T^4)(275b - 220\gamma T^{11/4} - 176KT^{3/4})}{110(165b + 4\gamma T^{11/4})^5}}, \tag{39}$$

$$\sigma_{22} = \frac{11v\left[-\psi_2\sqrt{11a^2 + 55bT^{5/4} + 20\gamma T^4} + 3a\psi_3 T\sqrt{55}\right]}{6T^2\sqrt{110(165b + 4\gamma T^{11/4})^3(275b - 220\gamma T^{11/4} - 176KT^{3/4})}}, \tag{40}$$

$$\sigma_{33} = \frac{-11\left[\psi_2\sqrt{11a^2 + 55bT^{5/4} + 20\gamma T^4} + 3a\psi_3 T\sqrt{55}\right]}{6vT^2\sqrt{110(165b + 4\gamma T^{11/4})^3(275b - 220\gamma T^{11/4} - 176KT^{3/4})}}, \tag{41}$$

$$\sigma_{44} = \frac{-11(55b + 228\gamma T^{11/4} + 176KT^{3/4})\psi_2}{6}\sqrt{\frac{(11a^2 + 55bT^{5/4} + 20\gamma T^4)}{110(165b + 4\gamma T^{11/4})^5(275b - 220\gamma T^{11/4} - 176KT^{3/4})}}, \tag{42}$$

$$\omega_{14} = \frac{-(2\psi_4 - \psi_5\psi_6)}{8T}\sqrt{\frac{(11a^2 + 55bT^{5/4} + 20\gamma T^4)}{110(165b + 4\gamma T^{11/4})^5(-55b - 228\gamma T^{11/4} - 176KT^{3/4})}}. \tag{43}$$

Here

$$\psi_1 = 4125b^2 - 5537.5b\gamma T^{11/4} - 3135bKT^{3/4} - 80\gamma^2 T^{11/2} - 32K\gamma T^{7/2},$$

$$\psi_2 = 4125b^2 - 12550b\gamma T^{11/4} - 4620bKT^{3/4} - 80\gamma^2 T^{11/2} + 64K\gamma T^{7/2},$$

$$\psi_3 = 4125b^2 - 3200b\gamma T^{11/4} - 2640bKT^{3/4} - 80\gamma^2 T^{11/2} - 64K\gamma T^{7/2},$$

$$\psi_4 = 4(165b + 4\gamma T^{11/4})^2(55b + 228\gamma T^{11/4} + 176KT^{3/4}),$$

$$\psi_5 = 1375b + 260\gamma T^{11/4} + 176KT^{3/4},$$

$$\psi_6 = 11[9350b\gamma T^{11/4} + 1980bKT^{3/4} - 128K\gamma T^{7/2}].$$

The rates of expansion (H_i) in the direction of x, y and z axes are given by

$$H_1 = \frac{1}{T^2}\sqrt{\frac{11a^2 + 55bT^{5/4} + 20\gamma T^4}{55}}, \tag{44}$$

$$H_2 = \frac{1}{2T^2}\left[\sqrt{\frac{11a^2 + 55bT^{5/4} + 20\gamma T^4}{55}} + a\right], \tag{45}$$

$$H_3 = \frac{1}{2T^2}\left[\sqrt{\frac{11a^2 + 55bT^{5/4} + 20\gamma T^4}{55}} - a\right]. \tag{46}$$

1.5. *Conclusion*

The model has point type singularity at T = 0.The model starts expanding with a big-bang at T=0 and the expansion in the model stops at $T=\infty$.The Hubble parameters $H_i \to \infty$ at T=0.The magnetic field and bulk viscosity posses the expansion in the model. At the initial stage their energy density $\in \to \infty$ but for $T\to\infty, \in \to 0$ i.e. $\in$ is decreasing function of time and metric is asymptotically empty. In the absence of magnetic field and bulk viscosity the model reduces to non-tilted model otherwise model is tilted model. In general, model is expanding, shearing and rotating. Since $\underset{T\to\infty}{\mathrm{Lt}} \frac{\sigma}{\theta} \neq 0$ then the model does not approach isotropy for large values of T.

References

[1] Narlikar, J.V. (1983). An *introduction to cosmology*. London: Cambridge University Press, 231-249.

[2] MacCallum, M.A.H. (1979). *General relativity on Einstein centenary survey,*London:Cambridge University Press,533-580.

[3] Murphy, G.L. (1973). Big-bang model without singularities. *Phys. Rev. D8,* 4231-4233.

[4] Roy, S.R., & Prakash, S. (1976).Some viscous fluid cosmological models of plane symmetry. J.*Phys.*A 9, 261-267.

[5] Padmanabhan, T., & Chitre, S.M. (1987).Viscous universes. *Phys. Lett. A.* 120**,** 433.

[6] Johri, V.B., & Sudarshan, R. (1988). Friedmann universes with bulk viscosity. Phys Lett. A132, 316-320.

[7] Maartens, R. (1995).Dissipative cosmology.Class. Quantum Gravit. 12, 1455.

[8] Zimdahl, W. (1996). Bulk viscous cosmology. Phys. Rev. D 53,5483-5493.

[9] Singh, T., Beesham, A., & Mbokazi, W.S. (1998). Bulk viscous cosmological models with variable G and Λ. Gen. Relativ. Gravit.30 (4), 537-581.

[10] Klimek, Z. (1974). Some thermodyanamic features of cosmological models with viscosity.Acta *Cosmologica-Z.* 2, 49-52.

[11] Banerjee, A., Riberiro, M.B.,& Santos, N.O. (1987). Anisotropic viscous fluid cosmological model. *Astrophys. Space Sci.* 136,331-336.

[12] Dunn, K.A., & Tupper, B.O.J. (1978).Tilting and viscous models in a class of Bianchi type VI0 cosmologies. *Astrophys. J.*222**,** 405-411.

[13] Roy, S.R., & Singh, J.P. (1983). Some Bianchi type VI0 cosmological models with free gravitational field of the magnetic type. *Ada Physica Austrica* 55, 57-66.

[14] Ribeiro, M.B., & Sanyal, A.K. (1987). Bianchi VI0 viscous fluid cosmology with magnetic fluid. *J. Math. Phys.* 28, 657-660.

[15] Santos, N.O., Dias, R.S., & Banerjee, A. (1985). Isotropic homogeneous universe with viscous fluid, *J. Math. Phys.*28, 878-881.

[16] King, A.R., & Ellis, G.F.R.(1973). Tilted homogeneous cosmological models. Comm. Math. Phys. 31,209.

[17] Ellis, G.F.R., & King, A.R. (1974). Was the big bang a whimper?. Comm. Math. Phys. 38(2), 119-156.

[18] Coley, A.A., & Tupper, B.O.J. (1983).An exact viscous fluid FRW cosmology. Phys. Lett. A95, 357-360.

[19] Coley, A.A., & Tupper, B.O.J. (1984).Imperfect fluid cosmologies with thermodynamics: some exact solutions.Astrophys.J. 280, 26.

[20] Roy, S.R., & Banerjee, S.K. (1988). Some cylindrically-symmetric cosmological models in general relativity representing viscous fluid and perfect fluid with heat conduction.Astrophys. Space Sci.150 (2), 213-222.

[21] Ellis, G.F.R., & Baldwin, J.E. (1984). On the expected anisotropy of radio source content. Mon. Not. Roy. Astro. Soc.206, 377-381.

[22] Klein, O. (1948). On a case of radiation equilibrium in general relativity theory and its bearing on the early stage of stellar evolution. Ark . Mat. Astron. Frys 34A (19), 11-15.

[23] Singh, K.P., & Abdussattar.(1973).Perfect fluid surrounded by a frozen photon field. Ind. J. Pure Appl. Math. 4(4), 468-472.

[24] Roy, S.R., & Singh, P.N. (1977). A plane symmetric universe filled with disordered radiation. J. Phys. A Math. and Gen.10(1),49-54.

[25] Teixeira. Da.F., I.Wolk, A.F., & Som, M.M.(1977). Exact relativistic cylindrical solution of disordered radiation. Nuovo Cimento Sezione B 41,387-396.

[26] Bagora, A., Rathore, G.S., & Gandhi, S. (2009). A tilted homogeneous cosmological model with disordered radiation in the presence of magnetic field. Rom.J.Phys.54 (7-8), 777-778.

[27] Bagora, A. (2010). A tilted homogeneous cosmological model with disordered radiation and heat conduction in presence of magnetic field. EJTP 14, 373-382.

[28] Ellis, G.F.R. (1971). General relativity and cosmology. R.K. Sachs (Eds.) (p.117). New York: Academic Press.

[29] Lichnerowicz, A. (1967).Relativistic Hydrodynamics and Magneto Hydrodynamics. New York: W.A. Benjamin Inc, p.13.

[30] Thorne, K.S. (1967). Primordial element formation, primordial magnetic fields and the isotropy of the universe.Astrophys. Space Sci.148, 51.

[31] Kantowski, R.,& Sachs, R.K. (1966). Some spatially homogeneous anisotropic relativistic cosmological models. J. Math.Phys.7, 433.

[32] Kristian, J., & Sachs, R.K. (1966). Observations in cosmology.Astrophys.J. 143, 379-399.

[33] Collins, C.B., Glass, E.N., & Wilkinson, D.A. (1980). Exact spatially homogeneous cosmologies.*Gen. Relativ.Gravit.* 12(10), 805–823.

[34] Johri, V.B., Sudarshan, R. (1988).Proc.Int.Conf. on Mathematical Modelling in Science and Technology,World Scientific,Singapore.

[35] Weinberg, S. (1972). Gravitation and Cosmology. New York: Wiley, p.57.