

Analytical Solution of Dufour and Soret Effects on Hydromagnetic Flow Past a Vertical Plate Embedded in a Porous Medium

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Abstract
This study investigated among others, the effect of thermal diffusion and diffusion thermo on heat and mass transfer over a vertical porous surface with convective heat transfer. A similarity analysis was used to transform the system of partial differential equations describing the problem into ordinary differential equations. The reduced system was solved using the Forth-order Runge - Kutta algorithm alongside the Newton Raphson shooting method. The results are presented graphically and in tabular form for various controlling parameters.
Keywords: Permeability; Soret and Dufour; Brinkmann number; heat and mass transfer; vertical plate, Buoyancy.

1.0 Introduction

The need to alter flow kinematics to enhancing heat and mass transfer in engineering practice and industry has been one of the top issues for some time now. In view of this, many researchers (Bestman and Adiepong, 1988, Chamkha and Quadri, 2001, Hayat, T., Hameed et al., 2004, Ishak et al., 2006, Ibrahim and Makinde, 2011, Gangadhar, 2012, Kazi et al., 2013, Arthur and Seini, 2014a, Arthur and Seini, 2014b, Arthur et al., 2014, Christian et al., 2014, Imoro et al., 2014) have done studies in this area in their quest to help this situation. When heat and mass transfer are occurring simultaneously the relationship between the energy and mass fluxes and the driving potentials become more intricate in nature. In this case, rigorous studies have revealed that, the generation of energy flux is not dependent on only the temperature gradient but also the concentration gradient. This is same for the generation of the mass flux as well. In an enlightened form, when heat transfer occurring in a fluid is as a result of the concentration gradient, the effect is called diffusion thermo. This is also called the Dufour effect. Equivalently, when mass transfer occurring in a fluid is as a result of the temperature gradient, the effect is called thermal-diffusion. This is also called Soret effect. Many of the research works on heat and mass transfer have neglected the effects of Dufour and Soret on the heat and mass transfer under the assumption that they are negligible in magnitude than that prescribed by the Fourier's and Fick's laws. Recent advancement in heat and mass transfer shows that Dufour effect is important in transport problems while Soret effect is influential in mass transfer phenomenon. The Soret effect, for instance, has been utilized for isotope separation and in a mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (H_2, air). According to Eckert and Drake (1972), Dufour effect is of considerable magnitude and should not be neglected. There have been many investigations in hydrodynamics over the years focusing on this problem. Kafoussias and Williams (1995) presented thermal-diffusion and diffusion-thermo effects on mixed free-convective and mass transfer boundary layer flow with temperature dependent viscosity. Joly et al. (2000) reported on Soret-driven thermosolutal convection in a vertical enclosure while Postelnicu (2004) studied the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Similarly, Alam and Rahman (2006) examined Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction while Alam et al., (2006) studied Dufour and Soret effects on steady free convection and mass transfer flow past a semi-infinite vertical porous plate in a porous medium. Gaikwad et al., (2007) presented an analytical study of linear and non-linear double diffusive convection with Soret and Dufour effects in couple stress fluid. Osalusi et al., (2008) studied thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer of a steady MHD convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating.

Meanwhile, Beg et al., (2009) did a numerical study of free convection Magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects while Nithyadevi and Yang (2009) reported on double diffusive natural convection in a partially heated enclosure with Soret and Dufour effects. Mahdy (2009) also presented MHD non-Darcian free convection from a vertical wavy surface embedded in porous media in the presence of Soret and Dufour effect while Nithyadevi and Yang (2009) studied double diffusive natural convection in a partially heated enclosure with Soret and Dufour effects. Heat and mass transfer for Soret and Dufour effects on Hiemenz flow through a porous medium on stretching surfaces was investigated by Tsai and Huang (2009) and Afify (2009) presented similarity solutions for MHD thermal-diffusion and diffusion-thermo on free convective heat and mass transfer over a stretching surface in relation to suction or injection.

Moreover, Prasad et al. (2011) investigated into thermo-diffusion and diffusion-thermo effects on MHD free convection flow past a vertical porous plate embedded in a non-Darcian porous medium. Srinivasacharya and Reddy (2011) examined Soret and Dufour effects on mixed convection in a non-Darcy porous medium saturated with micropolar fluid. The hydromagnetic mixed convection flow with Soret and Dufour effects past a vertical plate embedded in a porous medium was investigated by Makinde (2011). Olanrewaju and Makinde (2011) then analysed the effect of thermal-diffusion and diffusion-thermo on chemically reacting MHD boundary layer flow of heat and mass transfer past a moving vertical plate with suction/injection.

Subhakar and Gangadhar (2012) investigated Soret and Dufour effects on MHD free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source/sink. Sivaraman et al., (2012) presented Soret and Dufour effects on MHD free convective heat and mass transfer with thermophoresis and chemical reaction over a porous stretching surface: group theory transformation. Sarada and Shankar (2013) examined the effects of Soret and Dufour on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Nalinakshi et al., (2013), Soret and Dufour effects on mixed convection heat and mass transfer with variable fluid properties. Srinivasacharya and Upendar (2013) studied Soret and Dufour effects on MHD mixed convection heat and mass transfer in a micropolar fluid. El-Kabeir et al. (2013) studied Soret and Dufour effects on heat and mass transfer from a continuously moving plate embedded in porous media with temperature dependent viscosity and thermal conductivity. Olanrewaju et al. (2013) examined Dufour and Soret effects on convection heat and mass transfer in an electrical conducting power law flow over a heated porous plate. Seini and Makinde (2013) studied hydromagnetic flow with Dufour and Soret effects past a vertical plate embedded in porous media.

More recently, Gadipally and Gundagani (2014) did analysis of Soret and Dufour Effects on Unsteady MHD Convective Flow past a Semi-Infinite Vertical Porous Plate via Finite Difference Method. Animasaun and Oyem (2014) reported on the effect of Variable Viscosity, Dufour, Soret and thermal conductivity on free convective heat and mass transfer of non-Darcian flow past porous flat surface.

In this paper, we seek to study the combined effects of radiation and convective boundary conditions on the hydromagnetic mixed convection heat and mass transfer past a vertical porous plate with Dufour and Soret effects is investigated.

MATHEMATICAL MODEL

Consider a steady incompressible hydromagnetic fluid flow over a vertical porous stretching surface at y= 0, in the presence of a transverse magnetic field. Let the x-axis be taken along the direction of plate and y-axis normal to it. The fluid occupies the half space y> 0. The flow is subjected to a constant applied magnetic field B_0 in the y direction. The magnetic Reynolds number is considered to be very small so that the induced magnetic field is negligible in comparison to the applied magnetic field. The tangential velocity $u_w = U_0$, due to the stretching surface is assumed to vary proportional to the distance x so that $u_w = U_0 = ax$, where a is a constant. Following convection flow with heat and mass transfer over a vertical plate in a stream of cold fluid at temperature T , we assume the left surface of the plate is heated by convection from a hot fluid at temperature T_w which provides a heat transfer coefficient h_w. The cold fluid at the right side of the plate is assumed to be Newtonian, and its property variations due to temperature and chemical species concentration are limited to fluid density. It is also assumed that it is a viscous dissipative fluid. The density variation and the effects of buoyancy are taken into account in the momentum equation.

If u, v, T and C are the fluid x-component of velocity, y component of velocity, temperature and concentration respectively, then under the Boussinesq and boundary-layer approximations, the continuity, momentum, energy and mass transfer (concentration) equations for the problem under consideration can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty)-\left(\frac{\upsilon}{\tilde{K}}+\frac{\sigma B_\circ^2}{\rho}\right)u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m\kappa_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{\alpha}{\kappa}\frac{\partial q_r}{\partial y}+\frac{\sigma B_\circ^2}{\rho c_p}u^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m\kappa_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Subject to the following boundary conditions:

$$u(x,0)=U_0,\ v(x,0)=V(x),\ -\kappa\frac{\partial}{\partial y}T(x,0)=h_w[T_w-T(x,0)],\ C(x,0)=C_w,$$
$$u(x,\infty)=0,\ T(x,\infty)=T_\infty,\ C(x,\infty)=C_\infty, \tag{5}$$

where υ is the kinematic viscosity, T_m is the mean fluid temperature, T_∞ is the free stream temperature, C_w is the species concentration at the plate surface, C_∞ is the free stream concentration, q_r is the radiation heat flux, $V(x)$ is the suction velocity at the plate surface, α is the thermal diffusivity, D_m is the mass diffusivity, β_T is the thermal expansion coefficient, β_C is the solutal expansion coefficient, ρ is the fluid density, g is gravitational acceleration, σ is the electrical conductivity, c_s is the concentration susceptibility, c_p is the specific heat capacity at constant pressure, κ_T is the thermal diffusion ratio, κ is the thermal conductivity, and $\check{K}$ is the permeability of the porous medium.

The velocity components u and v can be expressed in terms of the stream function, $\psi(x, y)$ such that

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{6}$$

We seek for the stream function and the similarity variable respectively such that

$$\psi=(2\upsilon U_0 x)^{1/2} f(\eta),\quad \eta=y\sqrt{\left(\frac{U_0}{2\upsilon x}\right)} \tag{7}$$

It may be verified that the continuity equation in (1), is identically satisfied.

Using the Rosseland approximation for radiation, Ibrahim and Makinde (2011) simplified the heat flux as

$$q_r=-\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y} \tag{8}$$

where σ and K' are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the temperature differences within the flow such as the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, we get;

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{9}$$

Also, the following non-dimensional temperature and concentration can be introduced as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{10}$$

Considering (6)-(10), we can transform (1)-(5) into the following ordinary nonlinear system of differential equations:

$$f'''+ff''+Gt_x\theta+Gc_x\phi-(M_x+K)f'=0, \tag{11}$$

$$\left(1+\frac{4}{3}Ra\right)\theta''+\Pr f\theta'+Brf''^2+\Pr Du\phi''+M_x Brf'^2=0, \tag{12}$$

$$\phi''+Scf\phi'-ScSr\theta''=0. \tag{13}$$

The transformed boundary conditions are;

$$f'(0)=1,\ f(0)=fw,\ \theta'(0)=Bi_x(\theta(0)-1),\ \phi(0)=1,$$
$$f'(\infty)=0,\ \theta(\infty)=0,\ \phi(\infty)=0. \tag{14}$$

The prime symbol denotes differentiation with respect to the similarity variable (η), whereas $M_x=2\sigma B_0^2 x/\rho U_0$ is the local magnetic parameter, $fw=-V(x)\big/(2U_0\upsilon/x)^{\frac{1}{2}}$ is the suction parameter, $\Pr=\upsilon/\alpha$ is the Prandtl number, $Br=\mu U_0^2/\kappa(T_w-T_\infty)$ is the Brinkmann number,

$Bi_x = -h_w(2\upsilon x/U_0)^{\frac{1}{2}}/\kappa$ is the local Biot number, $Ra = 4\sigma^* T_\infty^3/3\kappa K'$ is the radiation parameter, $Gt_x = 2g\beta_T(T_w - T_\infty)x/U_0^2$ is the local thermal Grashof number, $Gc_x = 2g\beta_C(C_w - C_\infty)x/U_0^2$ is the local solutal Grashof number, $Du = \dfrac{D_m\kappa_T(C_w - C_\infty)}{\upsilon c_s c_p(T_w - T_\infty)}$ is the Dufour number, $Sr = \dfrac{D_m\kappa_T(T_w - T_\infty)}{\upsilon T_m(C_w - C_\infty)}$ is the Soret number, $K = 2\upsilon x/\tilde{K}U_0$ is the permeability parameter and $Sc = \upsilon/D_m$ is the Schmidt number

NUMERICAL SOLUTION

The numerical technique chosen for the solution of the coupled nonlinear ordinary differential equations (11)-(13) together with the associated transformed boundary conditions (14) is the standard Newton-Raphson shooting method alongside the fourth-order Runge-Kutta integration algorithm. From the process of numerical computation, the plate surface temperature, the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $-f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ are also sorted out and their numerical values presented in a tabular form.

Table 1 Comparison of results for different values of increasing values of *Du* and decreasing values for Pr =0.71, Gt_x= 10, Gc_x = 4, Sc = 0.22, $M_x = Ra = Br$ =0, K =0.3, fw= 0.5, Bi_x=10^7

		Alam et al. (2006)			Seini and Makinde (2014)			Present Study		
Du	*Sr*	***-f ''(0)***	*-θ'(0)*	*-φ'(0)*	***-f ''(0)***	*-θ'(0)*	*-φ'(0)*	***f ''(0)***	*-θ'(0)*	*-φ'(0)*
0.003	2.0	**6.2285**	1.1565	0.1531	**6.22933**	1.15640	0.153205	**6.22933**	1.15640	0.15321
0.037	1.6	**6.1491**	1.1501	0.2283	**6.15001**	1.15001	0.228403	**6.15001**	1.15000	0.22840
0.050	1.2	**6.0720**	1.1428	0.3033	**6.07293**	1.14265	0.303335	**6.07293**	1.14265	0.30333
0.075	0.8	**6.0006**	1.1333	0.3781	**6.00160**	1.13316	0.378167	**6.00160**	1.13316	0.37817
0.150	0.4	**5.9553**	1.1157	0.4540	**5.95636**	1.11555	0.454027	**5.95636**	1.11555	0.45403

Table 1 shows the comparison of the works of Alam et al. (2006) and Seini and Makinde (2014) with the present study for Dufour and Soret numbers and it is clear from the table that our present study is consistent with their works.

The results of varying parameter values on the local skin friction coefficient, the local Nusselt number and the local Sherwood number, are shown in Table 2. It is observed that the skin friction increases with increasing values of M_x, fw, Pr, Sc and K; and decreases with increasing values of Br, Bi_x, Ra, Gt_x, Gc_x, Du and Sr. This means that the combined effect of magnetic field, suction parameter, Prandtl number, Schmidt number, porous medium permeability is to increase the local skin friction; and the combined effect of the viscous dissipation, convective heat transfer, radiation, buoyancy forces, species concentration and thermal energy gradients is to decrease the local skin friction at the surface of the plate. Similarly, the rate of heat transfer at the plate surface increases with increase in parameter values of fw, Pr, Bi_x, Gt_x, Gc_x and Sr; and reduces with increasing values of M_x, Br, Ra, Du, K and Sc. Moreover, it is observed that the rate of mass transfer increases with increasing values of fw, Br, Ra, Gt_x, Gc_x, Du, K, and Sc; and decreases with increasing values of M_x, Pr, Bi_x, and Sr.

Table 2 Numerical results of skin friction coefficient, Nusselt number and the Sherwood number

M_x	Pr	Sc	Gt_x	Gc_x	Br	Bi_x	Ra	K	Du	Sr	fw	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1.0 **3.0**	0.71	0.22	0.1	0.1	0.1	0.1	0.1	0.1	0.03	2	0.1	1.15078 1.82578	0.06631 0.04680	0.19592 0.17963
	4.00 **7.10**											0.68787 0.68993	0.09205 0.09436	0.21011 0.20768
		0.78 **2.62**										0.71051 0.73647	0.07598 0.07351	0.51000 1.16525
			0.5 **1.5**									0.60140 0.44943	0.07864 0.07989	0.22889 0.24047
				0.5 **1.5**								0.25946 -0.57150	0.08182 0.08263	0.26915 0.32875
					0.5 **1.5**							0.64642 0.59359	0.05757 0.01233	0.24243 0.28523
						0.5 **1.5**						0.64009 0.61707	0.23834 0.36451	0.17101 0.12999
							0.5 **1.5**					0.66479 0.65365	0.07467 0.06862	0.22642 0.23335
								0.5 **1.5**				0.91340 1.35660	0.07513 0.06881	0.02054 0.18172
									0.05 **0.15**			0.67005 0.66796	0.07768 0.07643	0.22346 0.22485
										1.0 **1.5**		0.67299 0.67172	0.07783 0.07788	0.23488 0.22904
											0.5 **1.5**	0.91932 1.69726	0.08115 0.08561	0.26551 0.40603

RESULTS AND DISCUSSION

Effects of Parameter Variation on Velocity Profiles

The effects of parameter variation on the velocity boundary layer are shown in Figures 1-11. Normally, the fluid velocity is minimal at the plate surface and increases to the free stream value satisfying the far field boundary condition.The effect of the local Magnetic Parameter, M_x on the velocity is shown in Figure 1. From this, it is observed that the velocity decreases with increasing the magnetic field intensity. This is due to the fact that the applied magnetic field normal to the flow direction induces the drag in terms of a Lorentz force which provides resistance to flow. For various values of suction parameter (*fw*) and permeability parameter (*K*), the profiles of the velocity across the boundary layer are shown in Figures 2 and 3, respectively. The velocity decreases for increasing values of *fw* and *K*. This can be attributed to the fact that suction and permeability are agents which cause resistance to the fluid flow hence retarding the fluid velocity. Figures 4 and 5 respectively depict the effects of local thermal Grashof number (Gt_x) and local solutal Grashof number (Gc_x) on the dimensionless velocity. It is observed that increasing values of both Gt_x and Gc_x increases the velocity profiles. In application increasing Gt_x and Gc_x means increase in buoyancy which enhances convection and leads to enhancement in velocity. This fact is also adequate to explain the observed increase in the velocity profile as a result of increasing the, Brinkmann number (*Br*) and Biot number (Bi_x)and radiation parameter (*Ra*) asin Figures 6, 7 and 8 respectively. We can note here that, increasing buoyancy forces will lead to a better flow kinematics.

Figures 9 and 10 represent the variation of different values of Dufour Number (*Du*) and Soret (*Sr*) respectively on the dimensionless velocity distributions. It is observed that increasing the *Du* and *Sr* lead to an increase in the fluid flow, which causes the momentum boundary layer thickness generally, to increases away from the plate satisfying the boundary conditions. Moreover it is observed from Figure 11 that increasing values of the Schmidt number (*Sc*) tends to decrease the fluid velocity at the surface due to dominance of momentum diffusion over mass diffusion of the fluid.

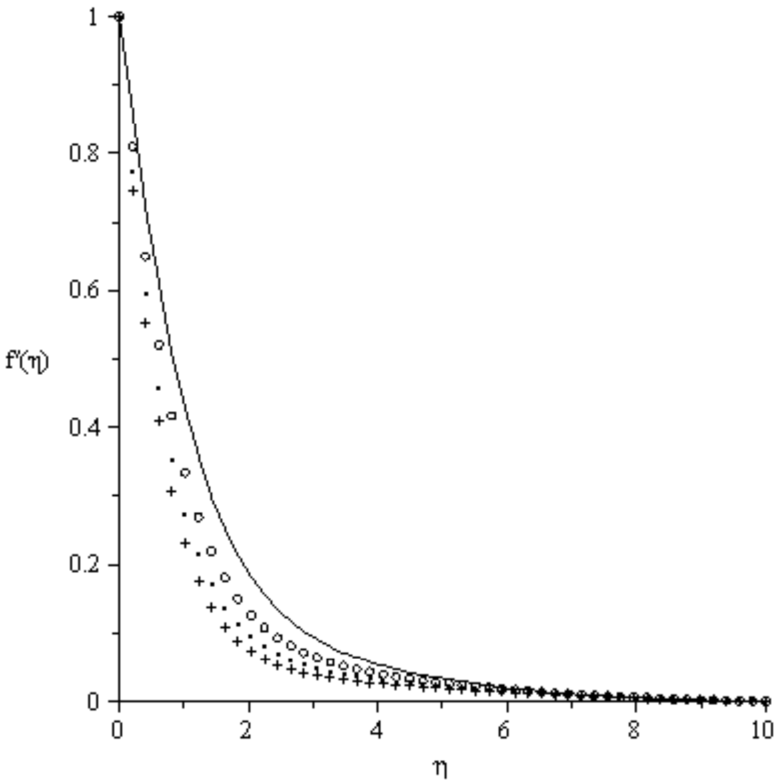

Figure 1: Velocity profiles for varying values of magnetic field parameter

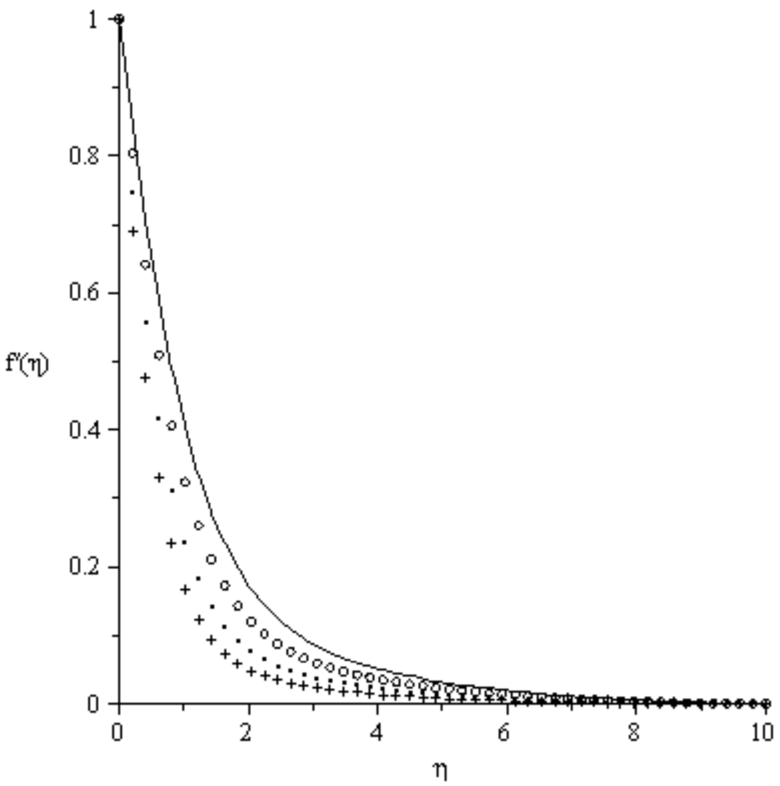

Figure 2: Velocity profiles for varying values of suction parameter

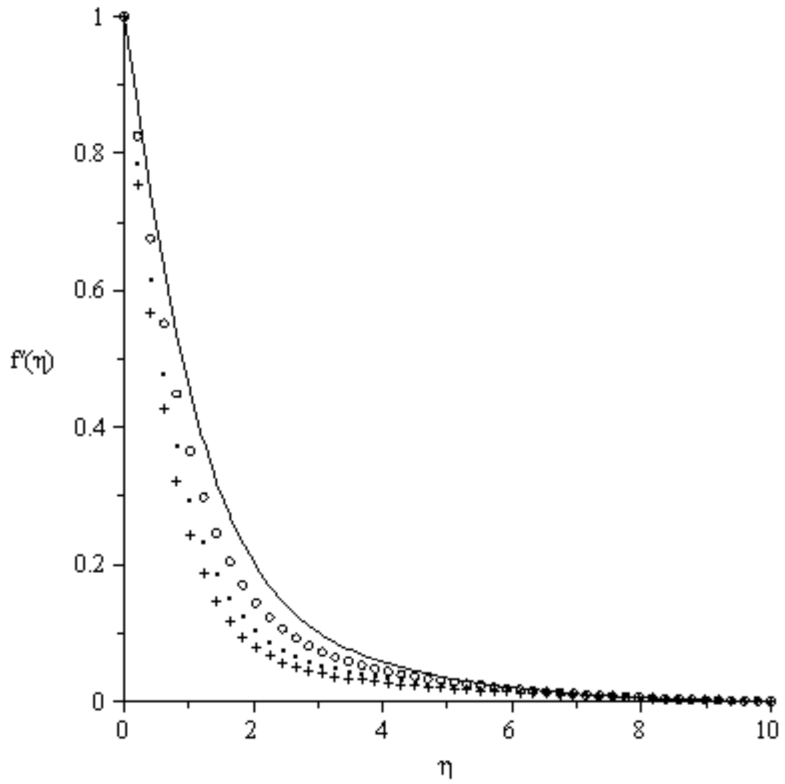

Figure 3: Velocity profiles for varying values of the permeability parameter

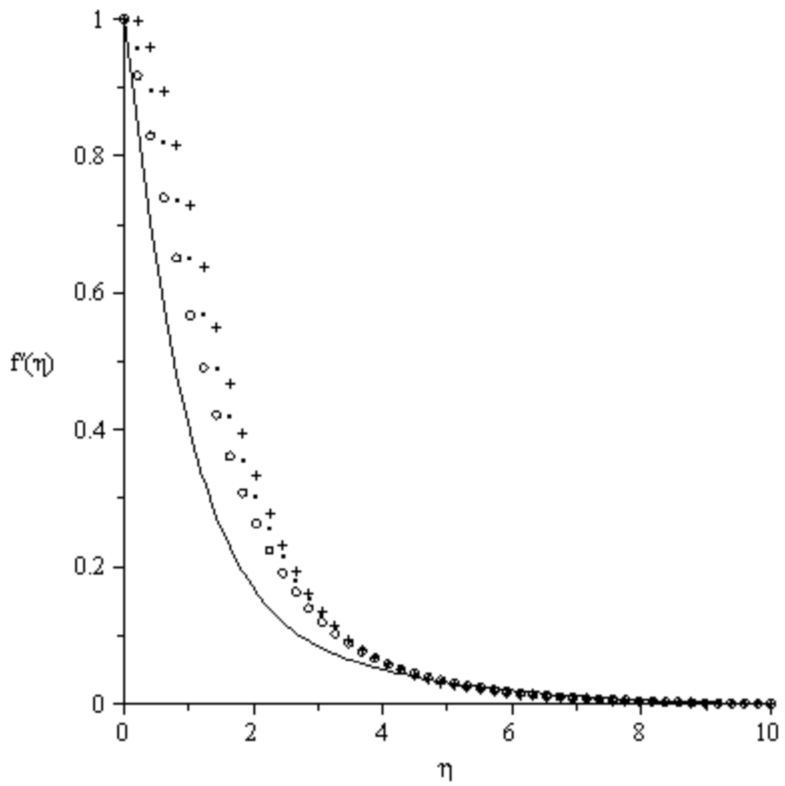

Figure 4: Velocity profiles for varying values of local thermal Grashof number

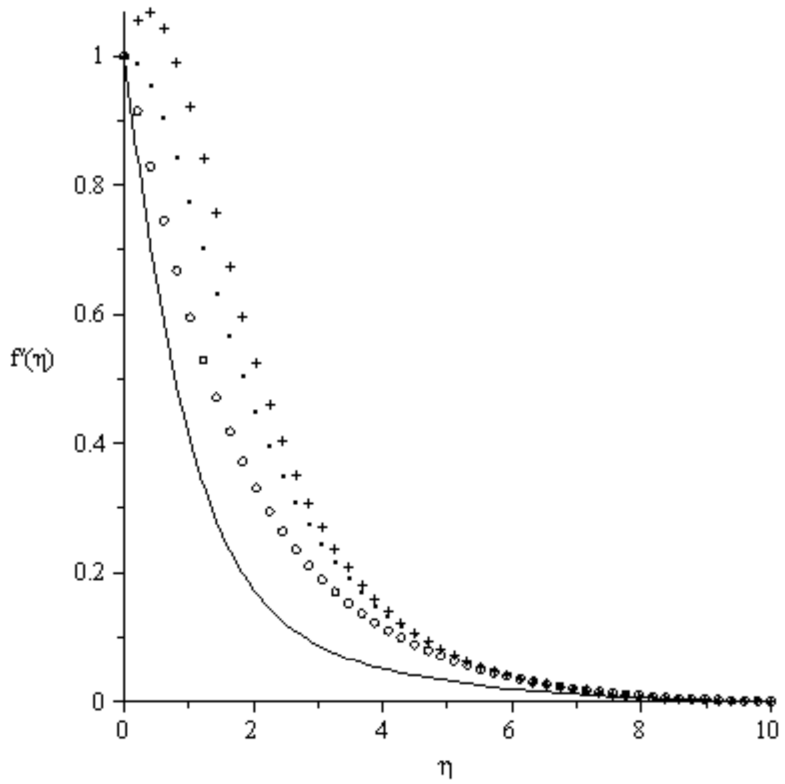

Figure 5: Velocity profiles for varying values of local solutal Grashof number

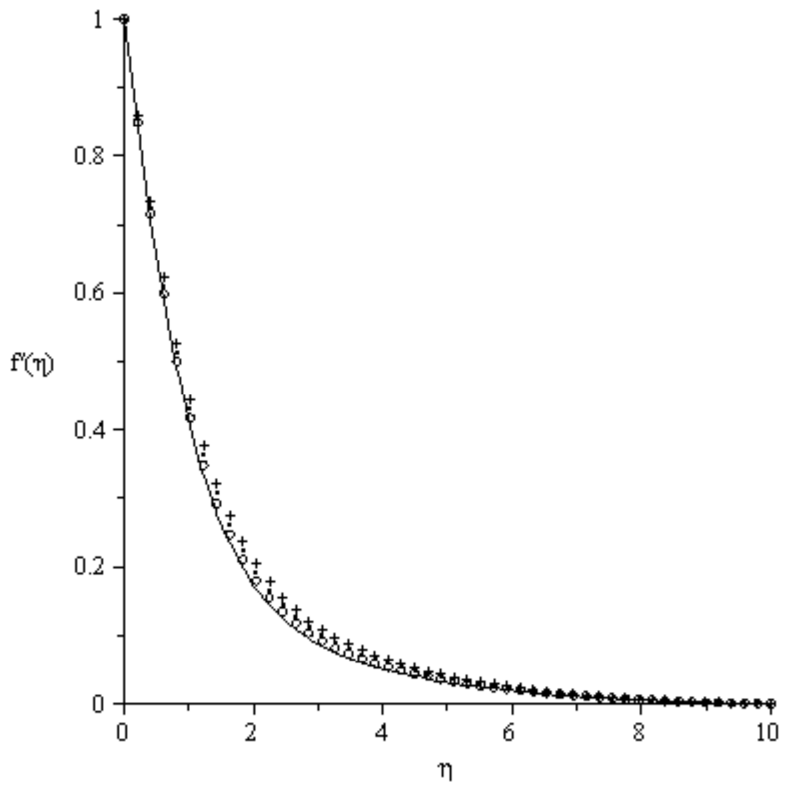

Figure 6: Velocity profiles for varying values of Brinkmann Number

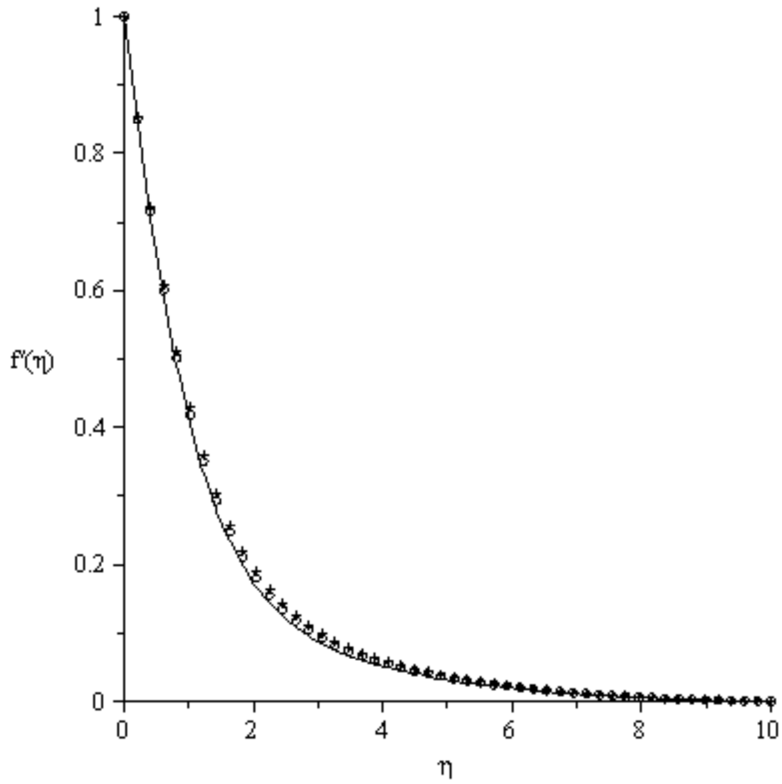

Figure 7: Velocity profiles for varying values of local Biot number

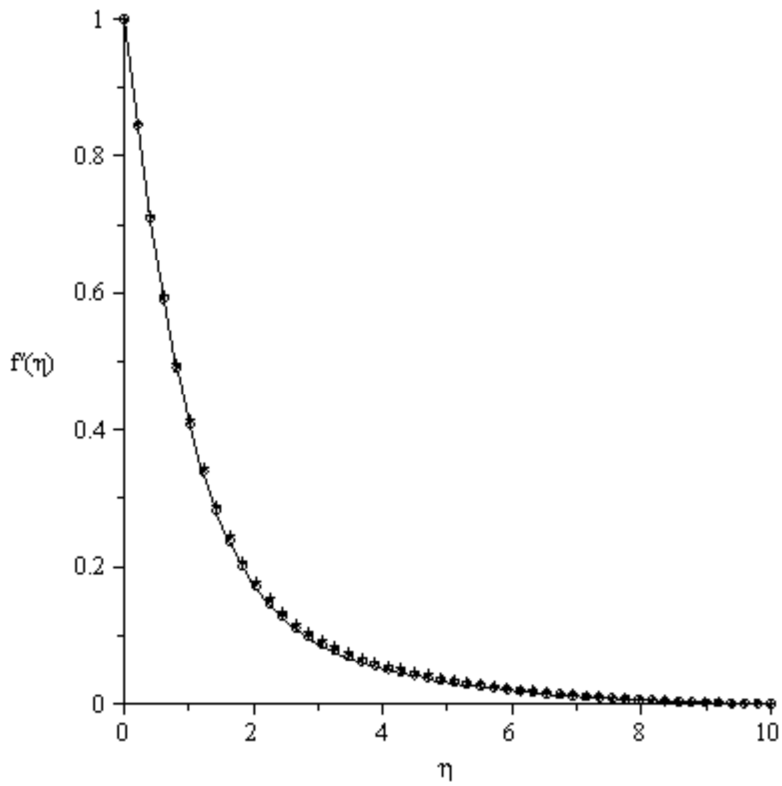

Figure 8: Velocity profiles for varying values of radiation parameter

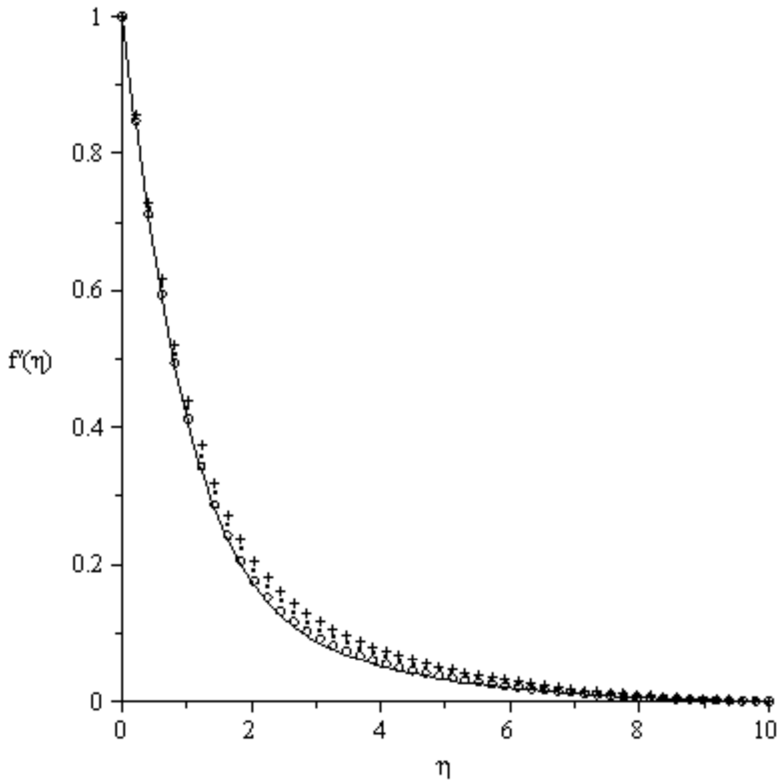

Figure 9: Velocity profiles for varying values Duffor number

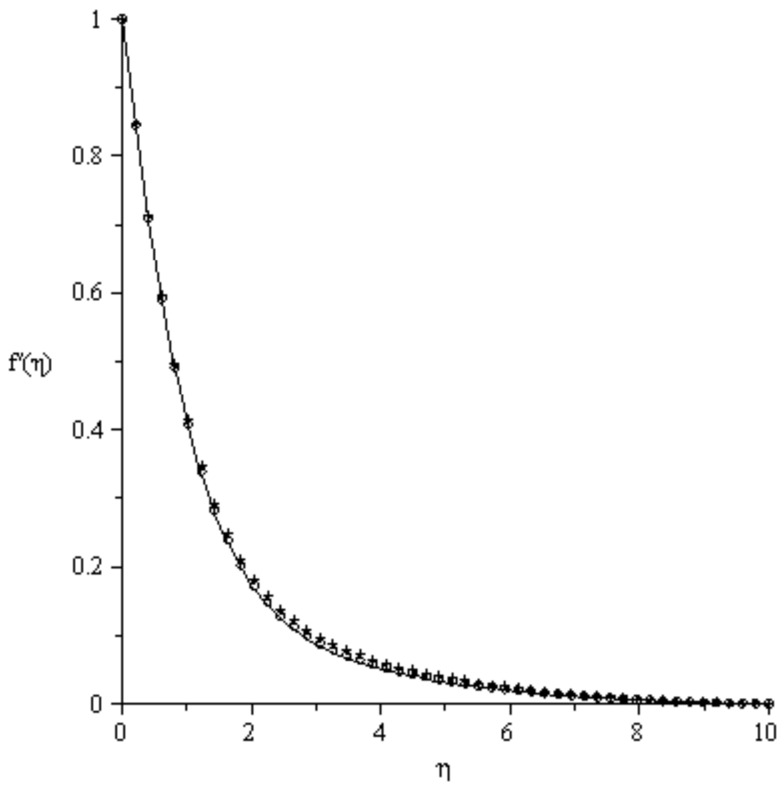

Figure 10: Velocity profiles for varying values Soret numbers

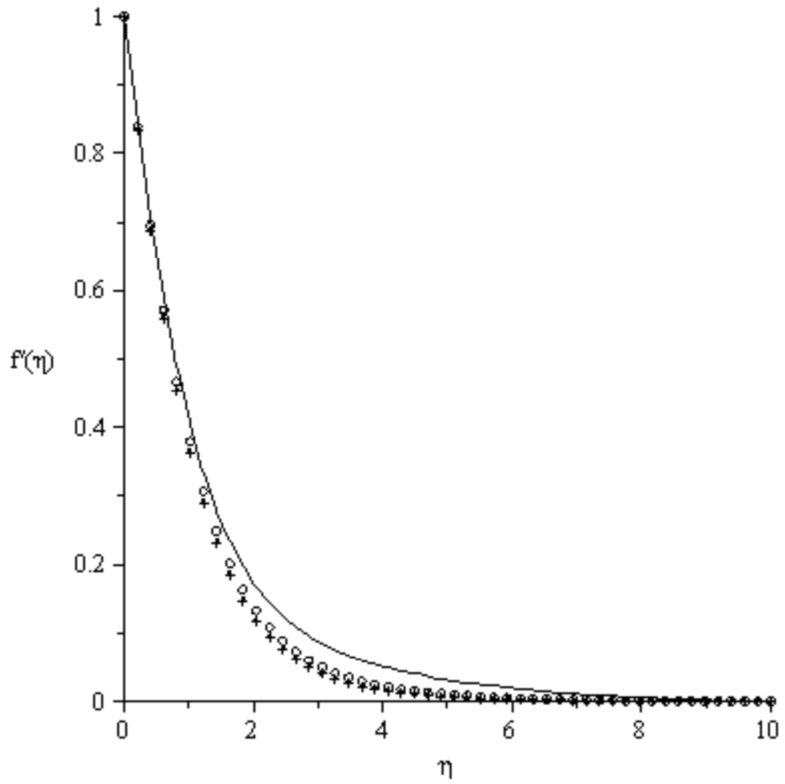

Figure 11: Velocity profiles for varying values of Schmidt number

Effects of Parameter Variation on Temperature Profiles

The effects of parameter variation on temperature profiles are shown in Figures 12-23. Normally, the fluid temperature is highest at the surface of the plate and shrinks to free stream temperature satisfying the boundary conditions. Figure 12 displays the variation of Prandtl number (Pr) on the temperature. The thermal boundary layer thickness decrease as the Prandtl number increases. It is reasonable in the sense that larger Prandtl number corresponds to the weaker thermal diffusivity and thinner boundary layer. It is observed in Figures 13, 14 and 15that increasing viscous dissipation, thermal radiation and convective heat exchange respectively increased the temperature profiles. This is due to the fact that as more heat is generated within the fluid, the fluid temperature increases leading to a sharp inclination of the temperature gradient between the plate surface and the fluid.

Meanwhile, from Figures 16, 17 and 18 it is observed that increasing the values of M_x, K and Sc respectively, reduces the thermal boundary layer thickness. Similarly, from Figures 19 and 20, the temperature profiles are reduced as a result of increasing the buoyancy forces (Gt_x and Gc_x respectively). This means that increasing the buoyancy forces enhance the cooling process.

Figure 21 shows that diffusion thermal effects greatly affect the fluid temperature. As the values of the *Du* increase, the fluid temperature also increases. Meanwhile from Figure 22, increasing values of *Sr* decays the temperature boundary layer thickness at the surface of the plate. It is observed in Figure 18 that, increasing *fw* also reduces the temperature profile for obvious reasons.

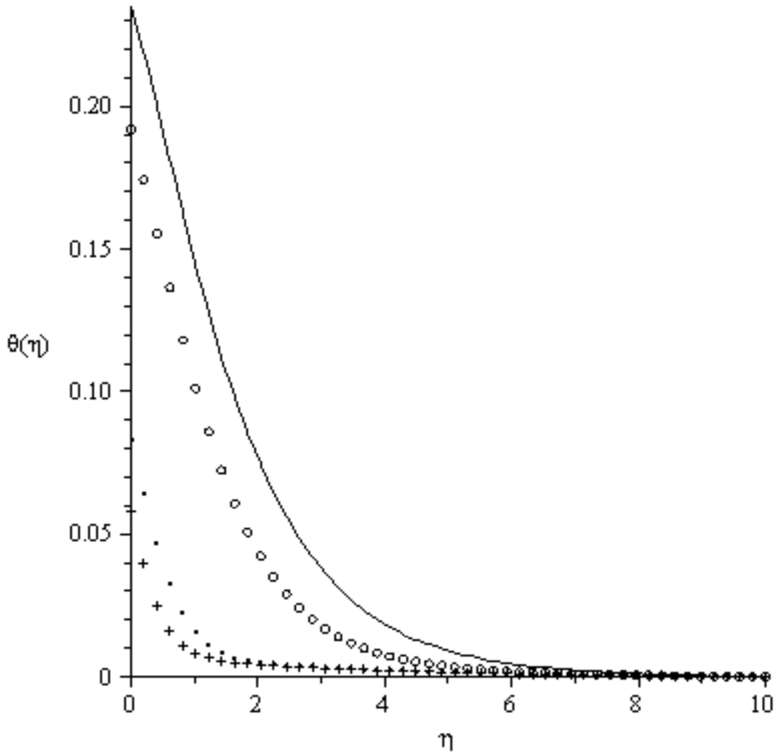

Figure 12: Temperature profiles for varying values of the Prandtl number

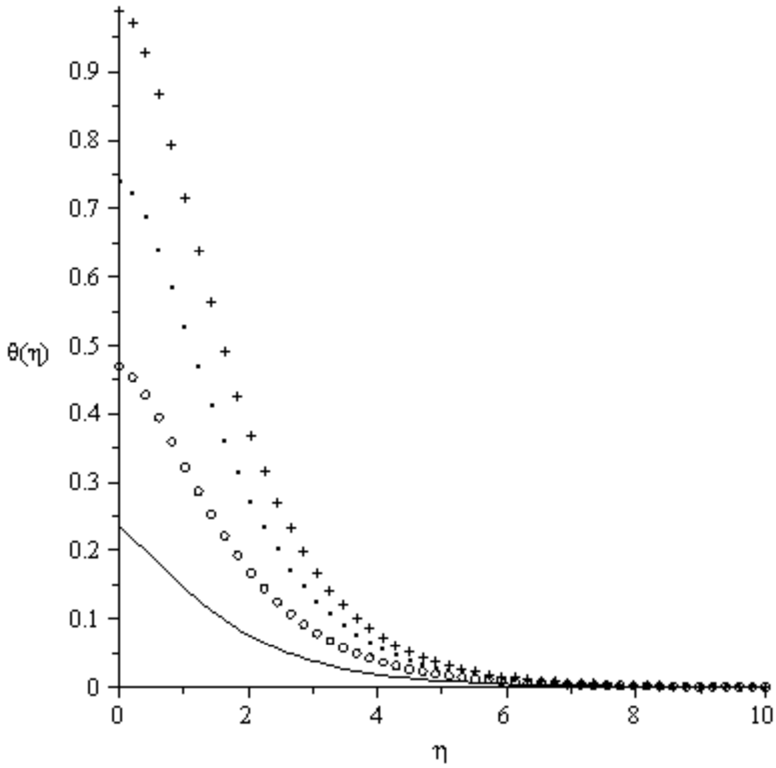

Figure 13: Temperature profiles for varying values of the Brinkman number

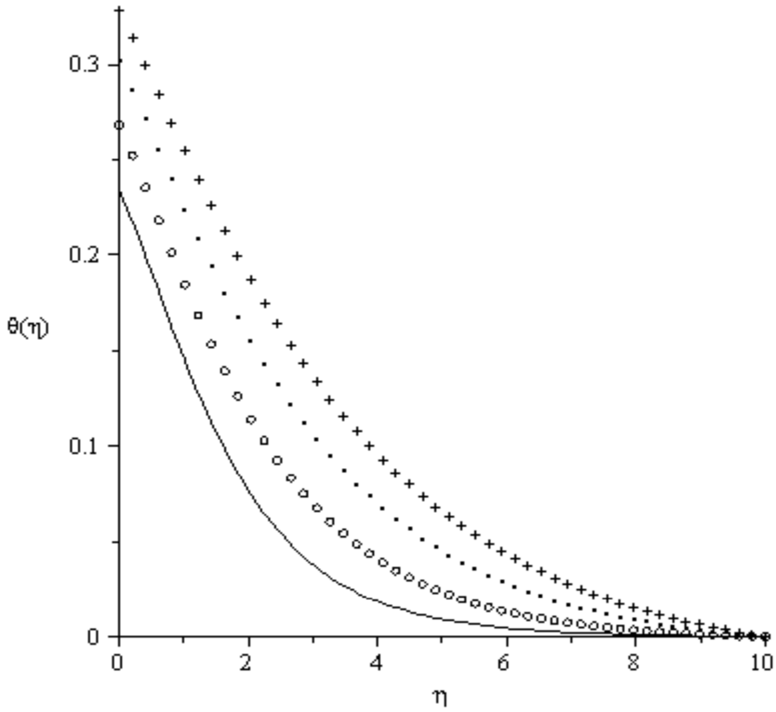

Figure 14: Temperature profiles for varying values of the radiation parameter

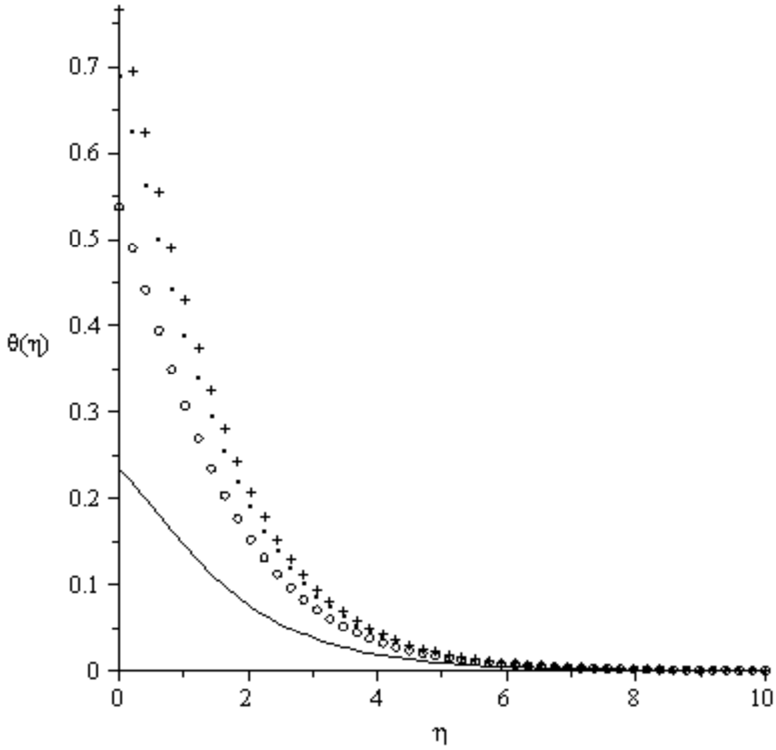

Figure 15: Temperature profiles for varying values of local Biot number

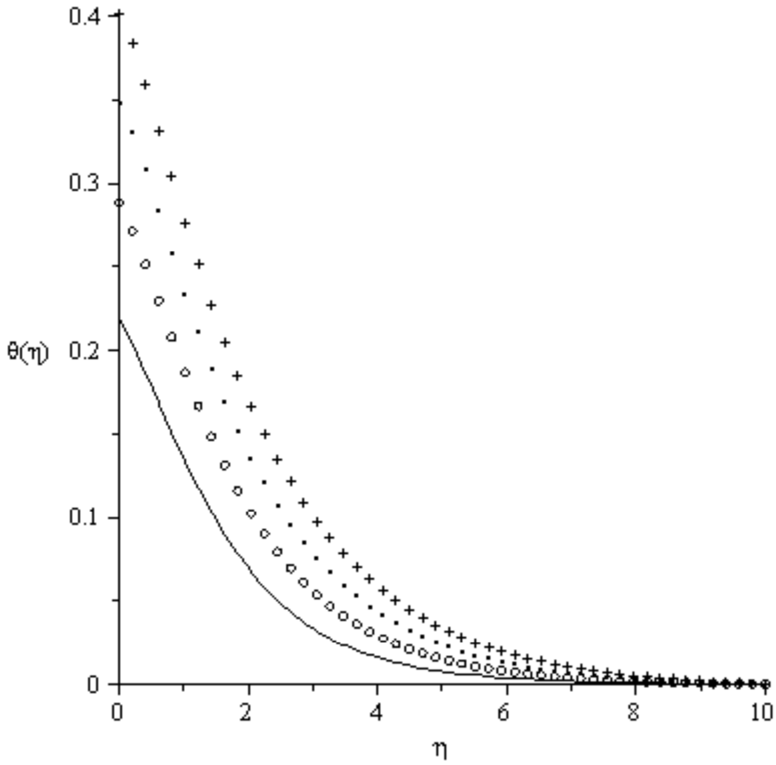

Figure 16: Temperature profiles for varying values of local magnetic parameter

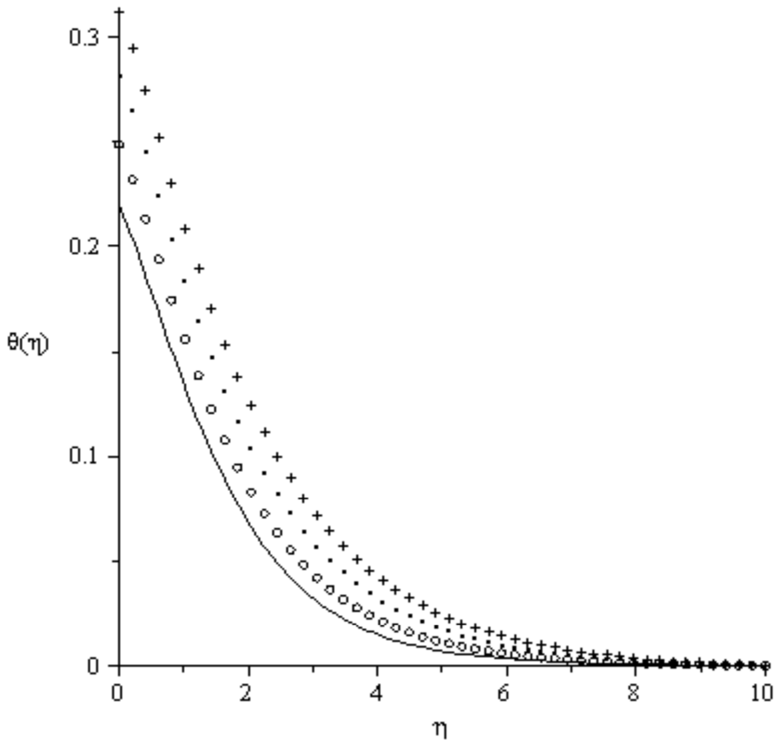

Figure 17: Temperature profiles for varying values of the permeability parameter

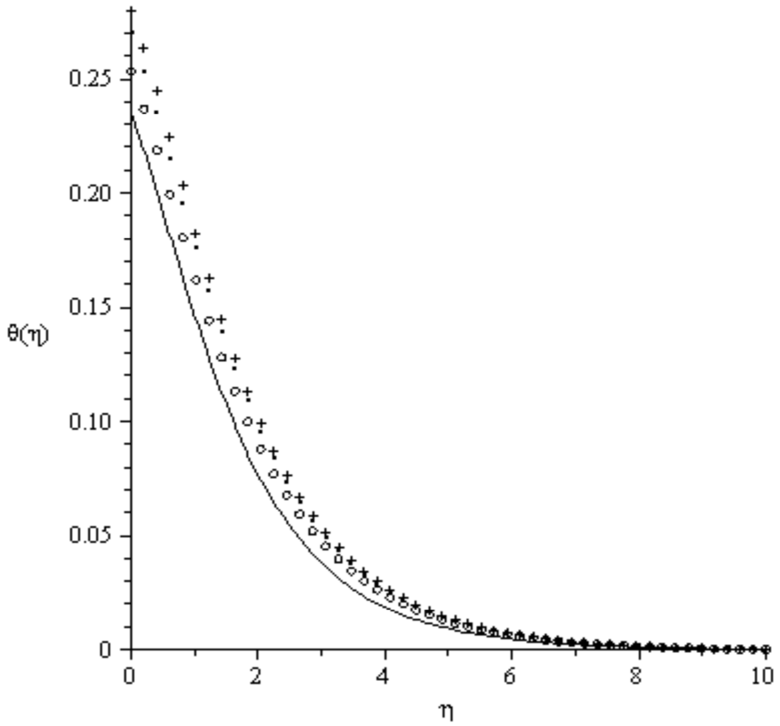

Figure 18: Temperature profiles for varying values of the Schmidt number

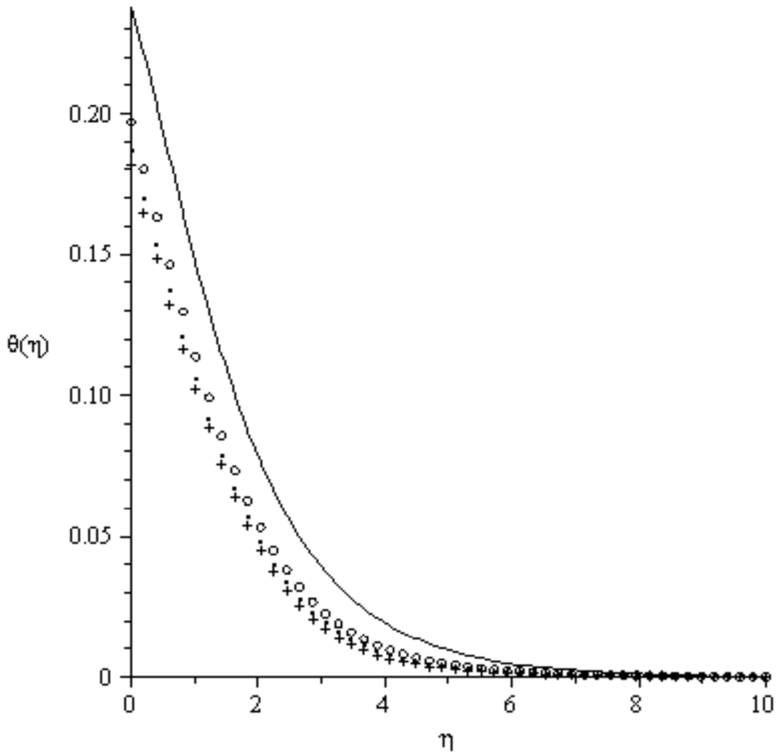

Figure 19: Temperature profiles for varying values of local thermal Grashof number

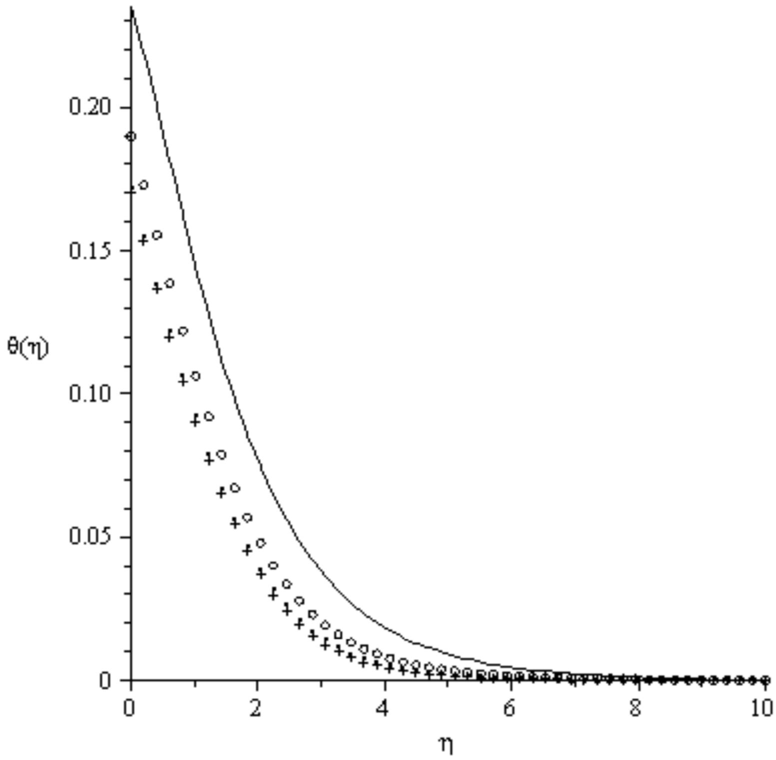

Figure 20: Temperature profiles for varying values of the local solutal Grashof number

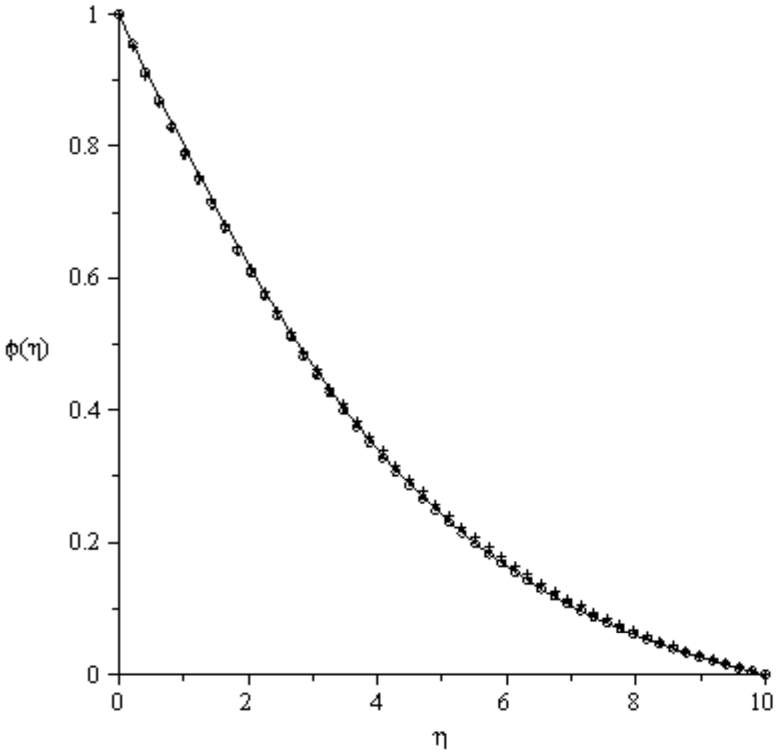

Figure 21: Temperature profiles for varying values of the Dufour number

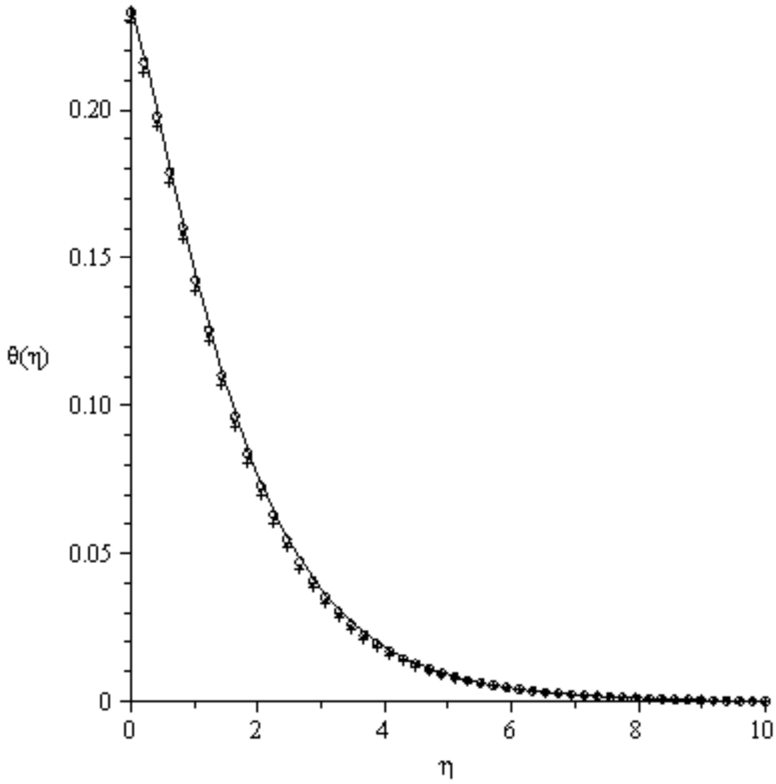

Figure 22: Temperature profiles for varying values of the Soret number

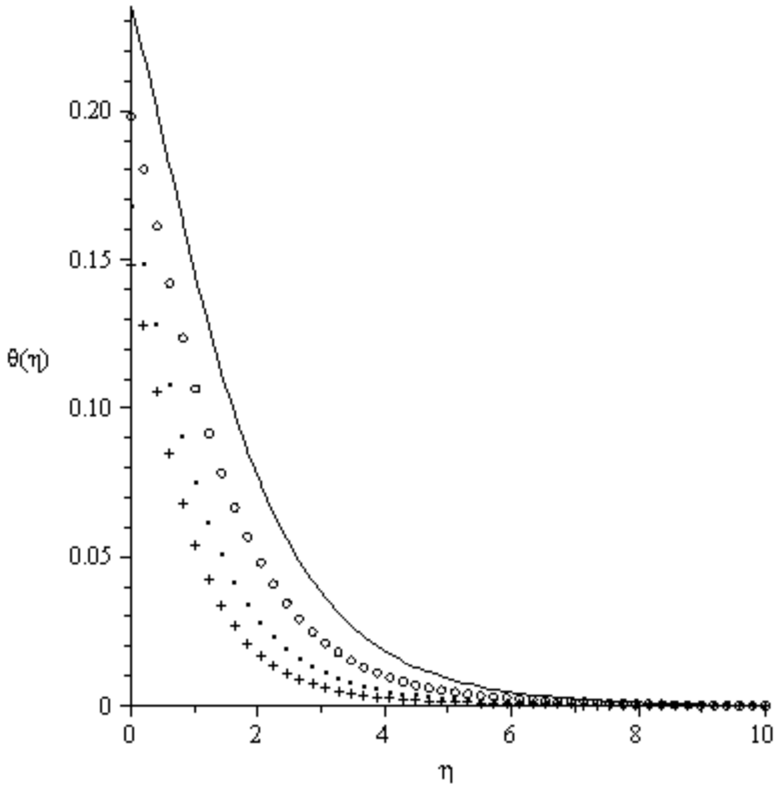

Figure 23: Temperature profiles for varying values of suction parameter

Effects of Parameter Variation on Concentration Profiles

Figures 24-34 depict the effects of varying parameters on the concentration boundary layer thickness. Normally, the chemical species concentration is highest at the plate surface and reduces exponentially to the free stream zero value. It is observed in Figure 24 that the concentration profiles decrease as Gt_x increases. It can be clearly seen that the effect of Gt_x is to decrease the concentration distribution as the concentration distribution is dispersed away largely due to increased temperature gradient. In Figure 25, Gc_x has the same effect as Gt_x on the flow properties. In Figure 26, it is observed that the thickness of the concentration boundary layer increases with increasing values of the M_x.

Figure 27 illustrates the effect of *fw* on the concentration profiles. It is observed that the concentration boundary layer thickness reduces with increasing values fw. Meanwhile in Figure 28, increasing the values of *K* increases the concentration boundary layer thickness.

Figures 29 and 30 depict the effect of *Br* and Bi_x respectively, on the concentration profiles. It is observed that increasing both *Br* and Bi_x increase the concentration boundary layer thickness due to viscous dissipation and convective heat transfer.

In Figures 31 and 32, it is observed that an increase in *Sr* increases the concentration boundary layer thickness while little or no effect was observed with increase in *Du*, respectively. Figures 33 and 34 describe the influence of *Sc* and *Pr* respectively, on the species concentration. It was observed that increasing the *Sc* and *Pr* reduced the species concentration within the boundary layer due to the combined effects of species molecular diffusivity and buoyancy forces.

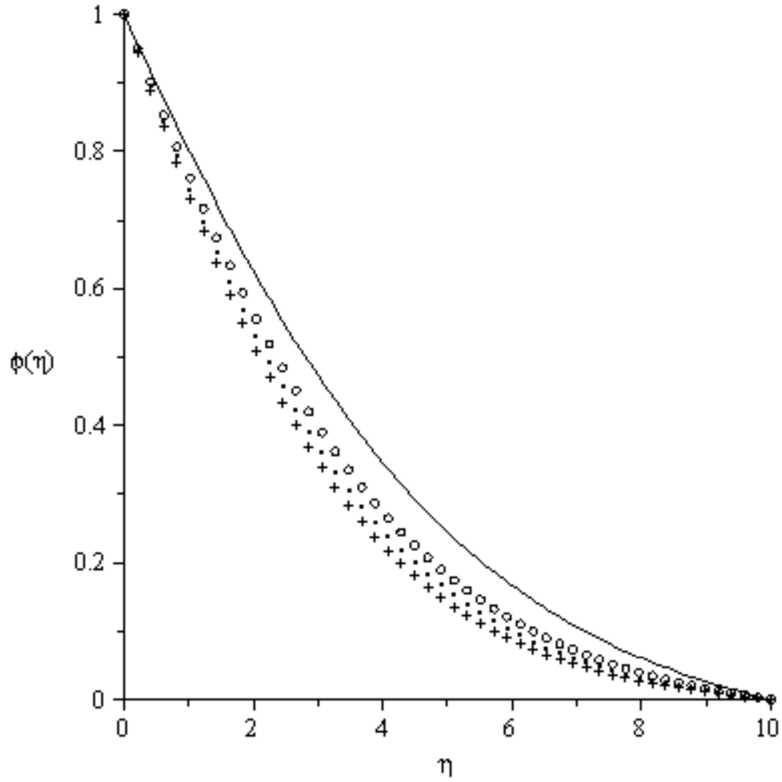

Figure 24: Concentration profiles for varying values of local thermal Grashof number

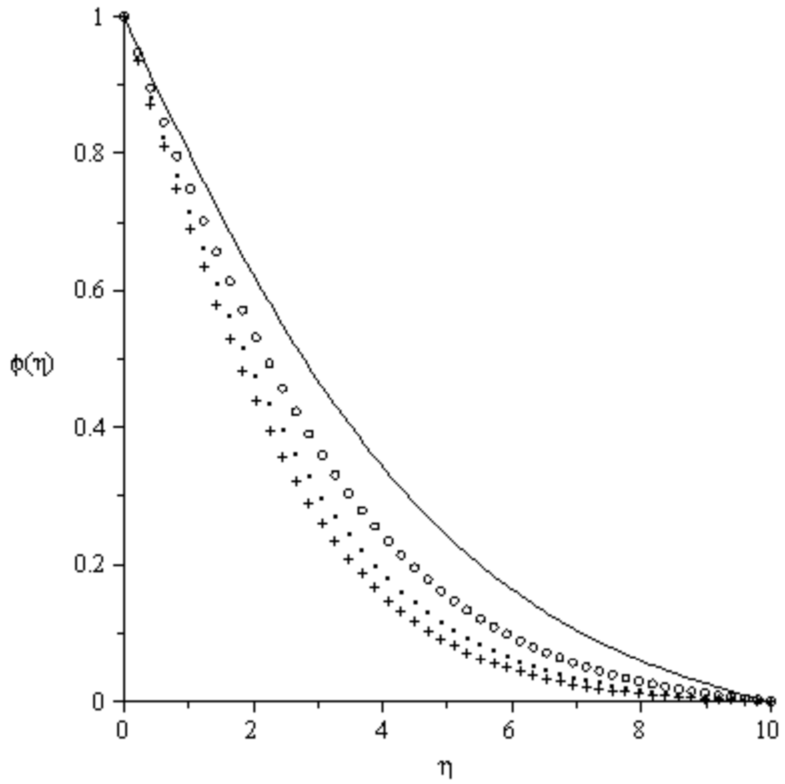

Figure 25: Concentration profiles for varying values of local solutal Grashof number

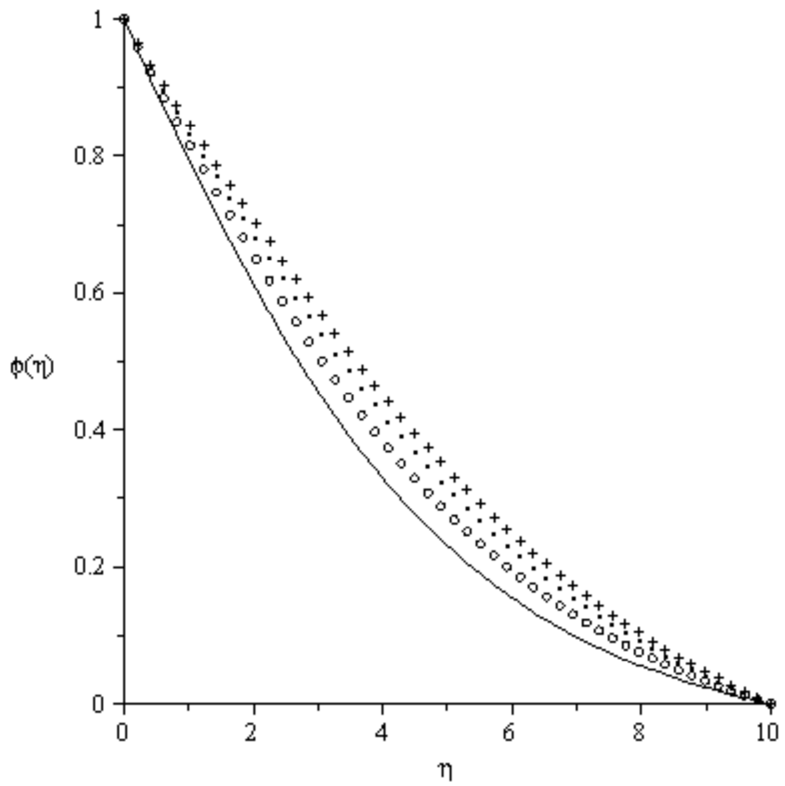

Figure 26: Concentration profiles for varying values of local magnetic parameter

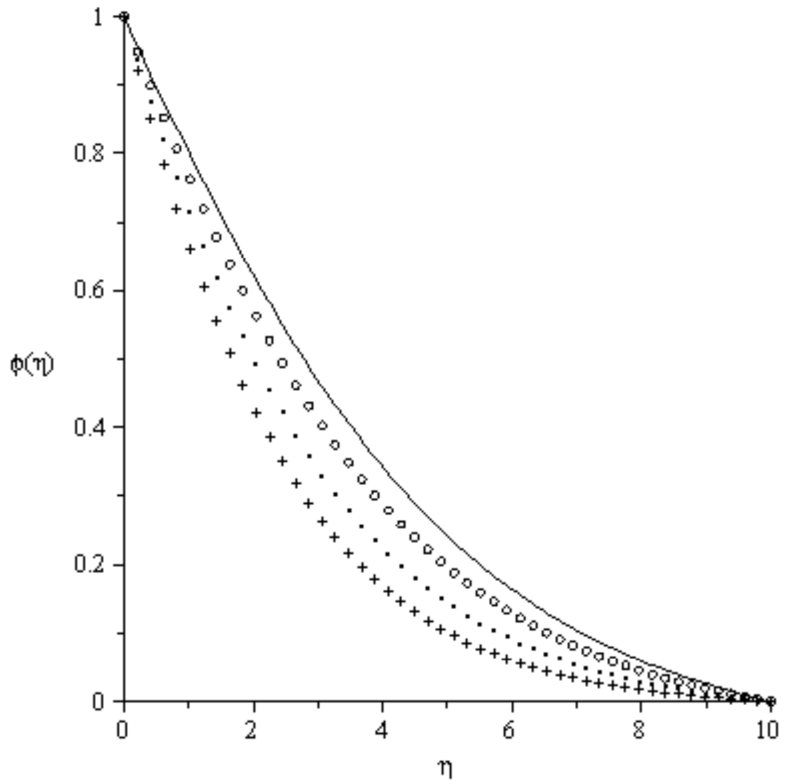

Figure 27: Concentration profiles for varying values of the suction parameter

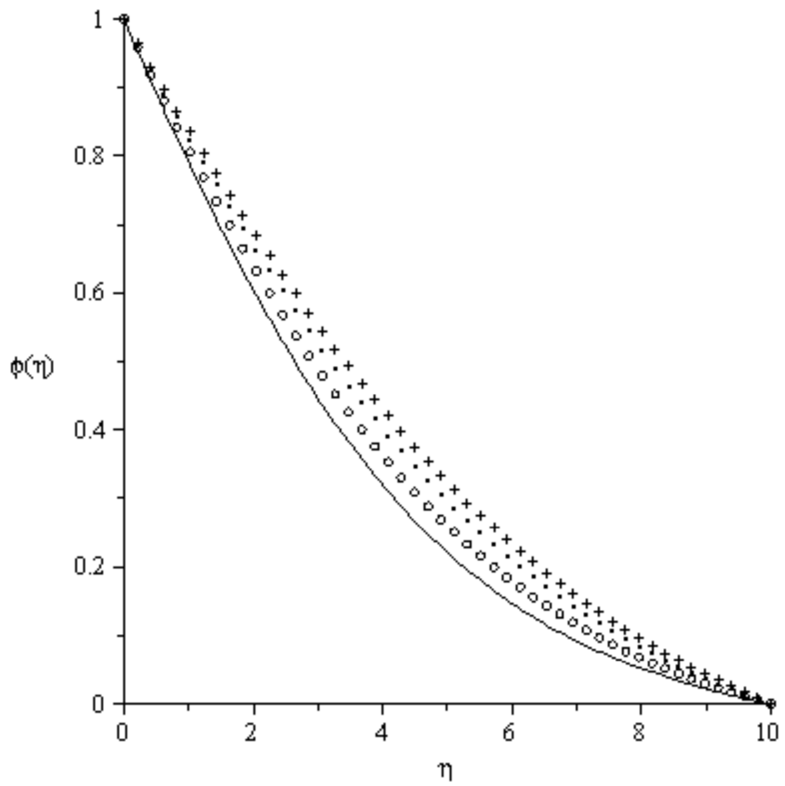

Figure 28: Concentration profiles for varying values of the permeability parameter

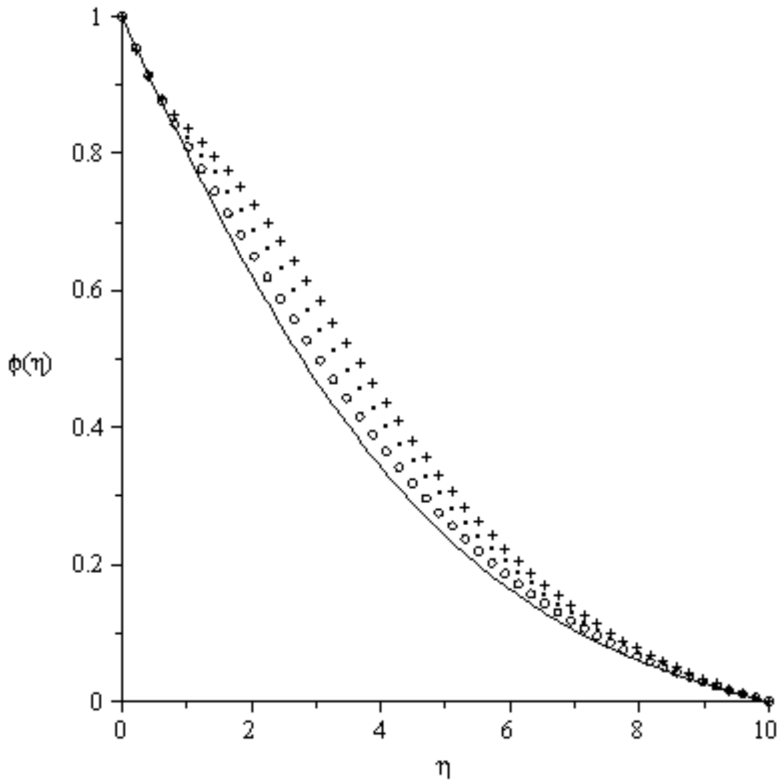

Figure 29: Concentration profiles for varying values of the Brinkmann number

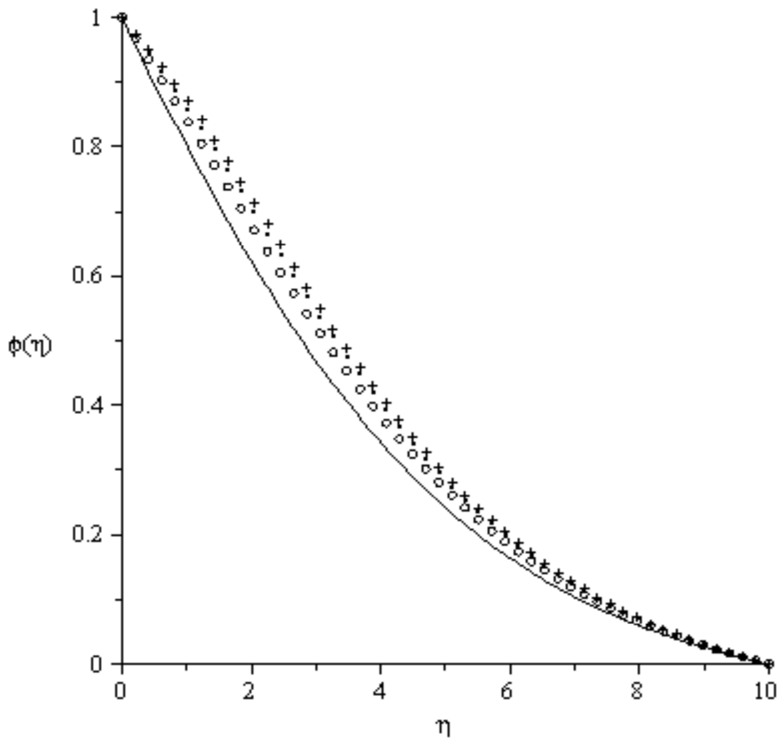

Figure 30: Concentration profiles for varying values of local Biot number

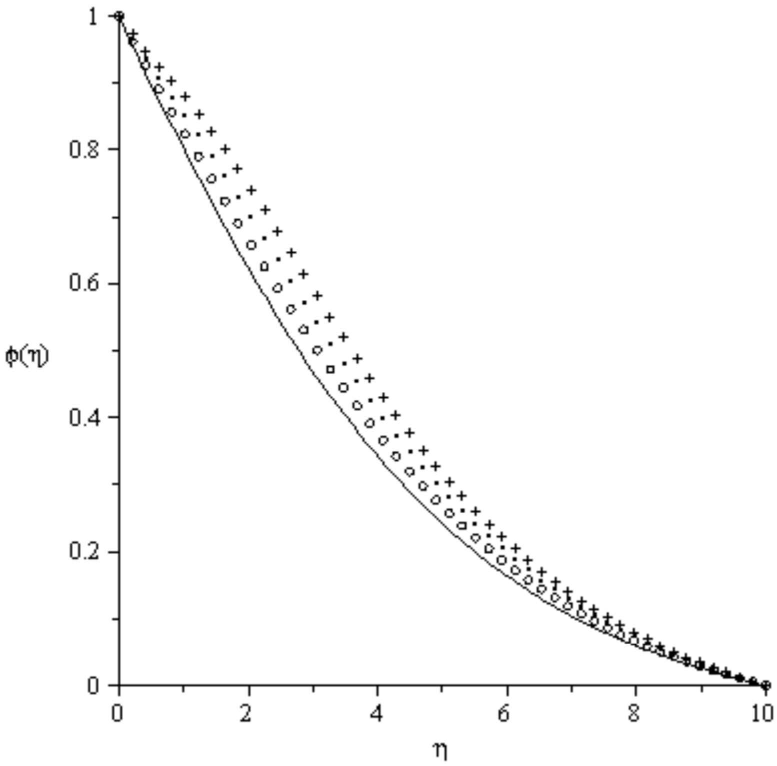

Figure 31: Concentration profiles for varying values of Soret number

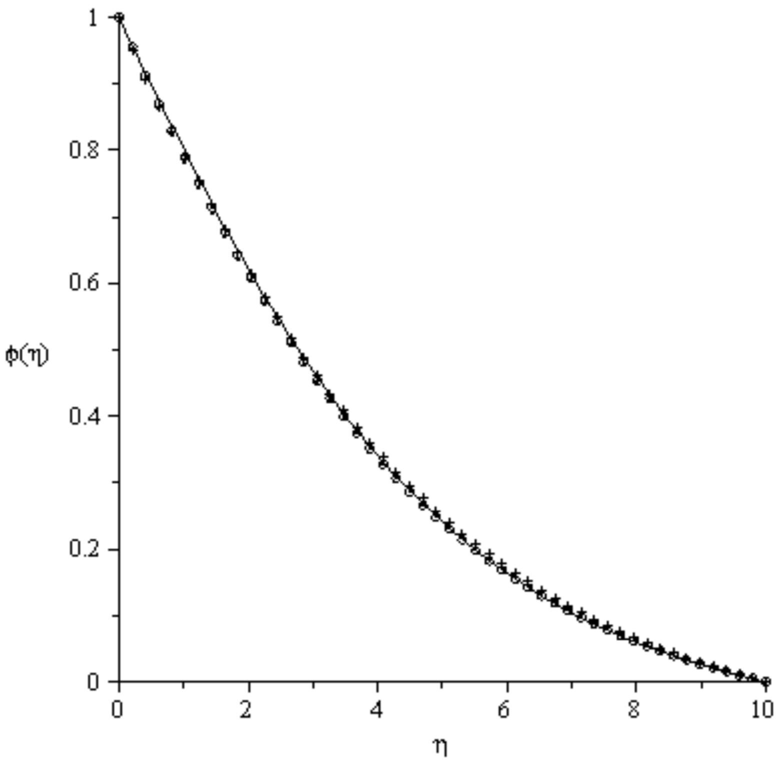

Figure 32: Concentration profiles for varying values of Dufour number

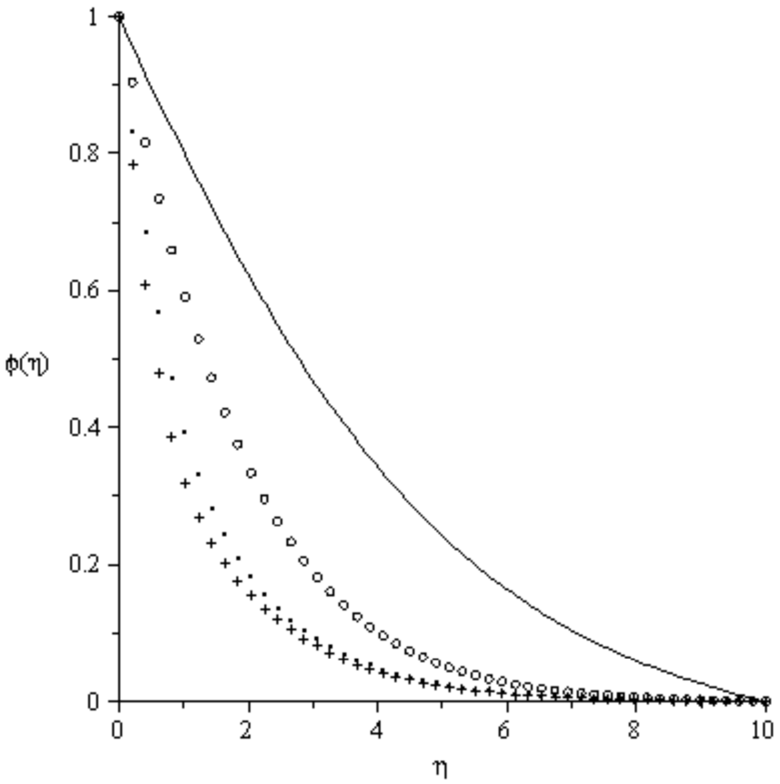

Figure 33: Concentration profiles for varying values of Schmidt number

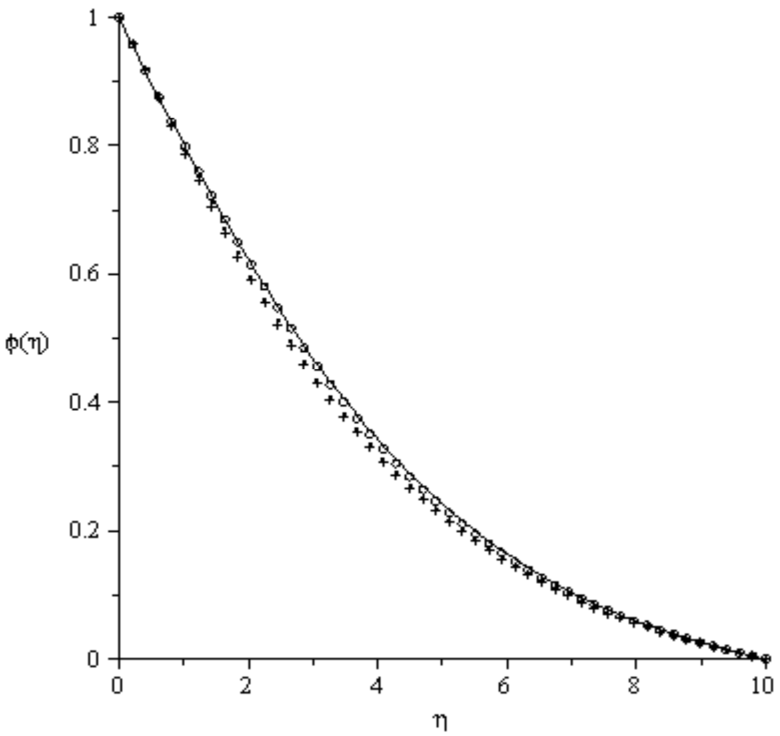

Figure 34: Concentration profiles for varying values of Prandtl number

Conclusion

Analytical solution of Dufour and Soret effects on hydromagnetic flow past a vertical plate embedded in a porous medium has been studied. Numerical results have been compared to earlier results published in the literature and a perfect agreement was achieved. Among others, our results reveal for this particular flow, the combined effects of thermal diffusion and diffusion thermo and the other embedded parameters can help control the flow kinematics and enhances both the heat and mass transfer process.

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