

Numerical Computation of Electric Field and Charge Structure of a Typical Tropical Thunderstorm

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Abstract

Cloud charge dynamics are usually determined from ground measurements. Where in-situ cloud measurements are available they sometimes modify the cloud structure; and this can be expensive, and merely diagnostic at best. In this work, Poisson integral formula is simplified by adapting the Bessel functions to obtain the cascaded potential gradients at any point on ground as a result of charge distribution in cylindrical volumes. The model represents the cloud charges as a vertical sequence of axially aligned, uniformly-charged densities of cylindrical disk(slices). The resulting expressions for the potential gradient is solved numerically as a functions of normalized vertical and horizontal distances to model the vertical electric field and potential profiles below a typical thunderstorm.

Keywords: Electric field, Bessel functions, Charge structure, Cylindrical charge model, Lower positive charge center.

Introduction

The scale and complexity of processes in the thunderstorms makes them almost impossible to observe in details at a glance, hence there is need for modeling. A realistic model has been difficult to obtain due to lack of direct measurement within a mature thundercloud, except with aircraft and balloon sounding. When an aircraft is employed, the hazard associated with the penetration and the instruments for obtaining measurement under such severe conditions will not give the accurate result and the profile of electric potential inferred from such balloon sounding only presents the changes with height along the track of balloon aloft, and hence the instantaneous vertical distribution of space charge and potential gradient changes at a particular height cannot be obtained. At the same time, this procedure cannot realize the arrangement of the opposite charge deposit IC flashes and how the space charge is mixed to produce stronger electric field and their reconstruction which terminate IC flashes. Hence, studies of intra-cloud and cloud-to-ground lightning using charge configuration in the thundercloud is significant to understand better our crude models of thunderstorm charge distributions and also how the charges are separated in clouds. Merceret et al.(2007) measured the electric field in and near clouds as a function of position with respect to the cloud edge to provide an observational basis for modifications to the lightning launch commit criteria (LLCC) used by the U.S. space program to protect vehicle from natural and triggered lightning during ascent.

It is a known fact that a normal electrified thunderstorms have a dominant dipolar electrical structure with a mid-level negative and upper-level positive charges, augmented by lower positive charge and possibly negative screening charge at the upper cloud boundary (Krehbiel, P.R., 1986; Williams, E.R., 1988; Krehbiel, P.R., 2008). Based on the work of Simpson and Scrase, 1937, a proposal of a basic tripolar charge structure was given. This proposal is widely supported (Williams 1988). Stolzenburg et al.(1998) added a negative screening charge layer at the upper boundary, which results in four charge regions within convective updrafts of thunderstorm. Soula (1994) obtained the vertical electric field profile deduced from simultaneous field measurements at several levels below a thundercloud shows the development of a space charge layer from $0m$ up to $600m$ and the average charge density in the whole layer is about $1nCm^{-3}$. Similarly the studies conducted by Hayashi (2006), the electrical charge reversal temperature causes the change of electrical charge polarity, at around $-15^{\circ}C$. He obtained total space charge density of about $1nCm^{-3}$ which depends on place and conditions, but at land stations its magnitude is typically of order of $10pCm^{-3}$.

The work is concern with the electric fields within and near clouds associated with thunderstorms. It is shown that electric field, which may be tens or even hundreds of kVm^{-1} in cloud, becomes small, even at a short distance outside the cloud, then the standoff distances of Merceret et al.(2007) may be safely reduced.

Theory : The Cylindrical Charge Model

A cylindrical model was employed by Oriksa and Magono (1961) to estimate the effect of space charge distributed uniformly in a cylindrical form due to charged raindrops on the surface electric field, the model depends strictly upon the estimation of the space charge density and the size of those cylindrical spaces. Such a

representation cannot validly reproduce the potential gradient pattern due to the charges within the entire volume of a cloud nor yield any realistic horizontal or point-discharge current profile at the ground (Ette and Olaofe, 1980). Therefore the objective of this study is to employ the available data obtained from ground-based and hydrometeor measurements of potential gradient/and point discharge current profiles to model the charge configuration within and outside the cloud. The limitations in simple point-charge models can be eliminated to a large extent by assuming the cloud to be developed in absence of pronounced horizontal airflow, such that its generation may be viewed as taking place in situ (the so-called ‘cylinder model’ because of its similarity to an upended cylinder), then by building up the thundercloud from horizontal cylindrical slices –an artifice which facilitates investigations of the effects of volume distribution of charges both inside and outside the cloud (Krehbiel, P.R., 2008).

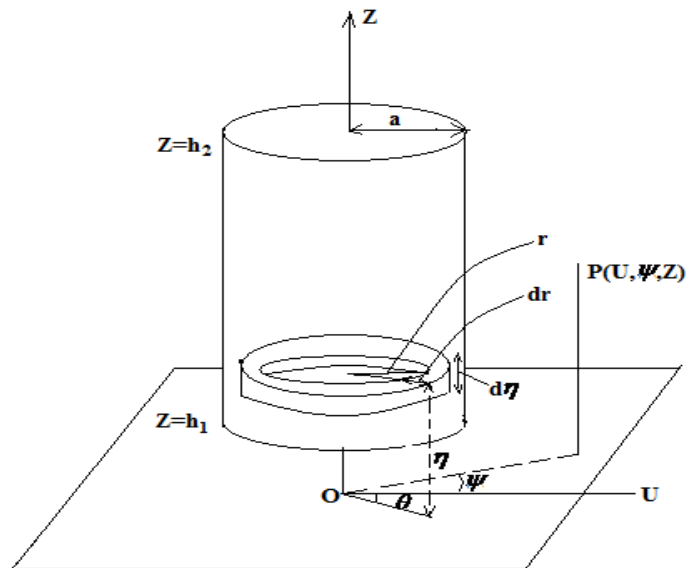


Fig.1: A vertical cylindrical charge volume of radius a , with ends at height h_1 and h_2 .

Fig.1 shows a vertical cylindrical charge volume of radius a , with ends at height h_1 and h_2 in a cylindrical coordinate system (r, θ, z) in which the z -axis coincides with the axis of the cylinder and the origin 0 is the intersection of the axis with the ground. By considering the charge in the volume element $rdrd\eta d\theta$ of the cylinder and the axial symmetry of the problem, the potential at the field point $P(u, \psi, z)$ external to the cylinder may be written as

$$\phi = \frac{1}{4\pi\epsilon_0} \int_{h_1}^{h_2} d\eta \int_0^a r dr \int_0^{2\pi} \frac{\rho d\theta}{[u^2 + r^2 - 2ur \cos \theta + (\eta - z)^2]^{1/2}} \dots\dots\dots(8)$$

where ρ is the constant charge density in the cylinder, and ϵ_0 is the permittivity of free space.

The graph of potential ϕ produced by uniform charge density ρ on the cylinder is displayed in fig. 2(a) and the level curves are shown in fig. 2(b).

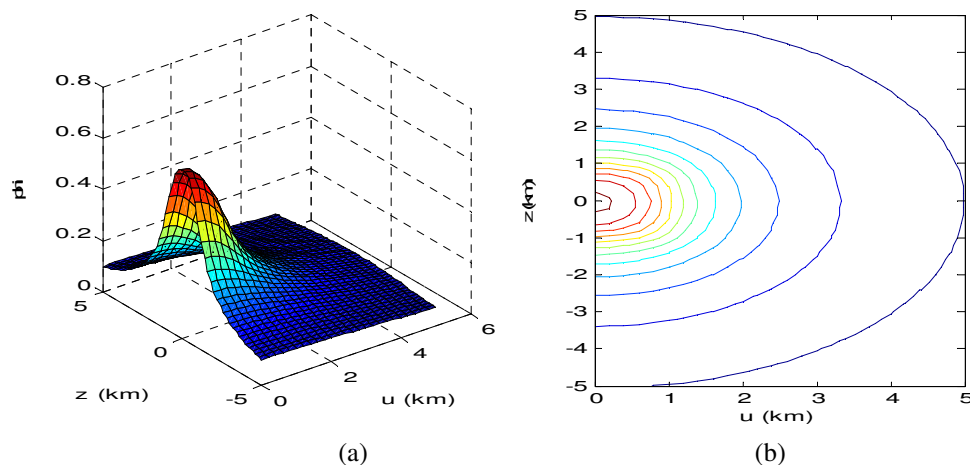


Fig. 2. The profile of ϕ with $\rho = 1$ against horizontal distance. This Poisson integral formula is solved by applying the result of Watson (1922), a treatise on the theory of Bessel functions. We have

$$\phi = \frac{\rho}{\pi\epsilon_0} \int_{h_2}^{h_1} d\eta \int_0^a r dr \int_0^\infty \cos(\eta - z) I_0(ut) K_0(rt) dt, \quad u < a \quad \dots\dots\dots(9a)$$

$$= \frac{\rho}{\pi\epsilon_0} \int_{h_1}^{h_2} d\eta \int_0^a r dr \int_0^\infty \cos(\eta - z) I_0(rt) K_0(ut) dt, \quad u > a \quad \dots\dots\dots(9b)$$

Upon integrating equations (9a) and (9b), we get

$$\phi = \frac{\rho}{\pi\epsilon_0} \int_0^\infty \frac{\sin(h_2 - z)t - \sin(h_1 - z)t}{t} I_0(ut) K_1(at) dt, \quad u < a \quad \dots\dots\dots(10a)$$

$$= \frac{\rho}{\pi\epsilon_0} \int_0^\infty \frac{\sin(h_2 - z)t - \sin(h_1 - z)t}{t} K_0(ut) I_1(at) dt, \quad u > a \quad \dots\dots\dots(10b)$$

where $I_0(\xi), I_1(\xi), K_0(\xi), K_1(\xi)$ are modified Bessel functions.

By considering equation(10) as a functions of the axial coordinate z , the equipotential surfaces of a pair of equal point charges called the image potential $\phi_1 = \phi(-z)$ must be added to these equations in order to satisfy the condition $\phi = 0$ at the conducting boundary $z=0$. The vertical potential gradient $F_{z=0}$ at points on ground is given by

$$F_{z=0} = \left. \frac{\partial \phi}{\partial z} \right|_{z=0}$$

By employing Bessel relations and the integral rules, we obtain

$$F_{z=0} = \frac{2\rho a}{\pi\epsilon_0} \left[\frac{\pi h}{2a} - \int_0^{\frac{\pi}{2}} \frac{a + u \cos 2x}{\chi^2} (h^2 + \chi^2)^{1/2} dx \right]_{h_1}^{h_2}, \quad u < a \quad \dots\dots\dots(11a)$$

Similarly, by substituting equation (21) into equation (11b), we have

$$F_{z=0} = \frac{2\rho a}{\pi\epsilon} \left[\int_0^{\frac{\pi}{2}} \frac{a + u \cos 2x}{\chi^2} (h^2 + \chi^2)^{1/2} dx \right]_{h_1}^{h_2}, \quad u > a \quad \dots\dots\dots(11b)$$

At the point of intersection of cloud and the ground, i.e. $x = 0$ and $u = 0$, equation (11a) becomes

$$F_{z=0} = \frac{\rho}{\epsilon_0} \left[(a^2 + h_1^2)^{1/2} - h_1 - (a^2 + h_2^2)^{1/2} + h_2 \right] \dots\dots\dots(12)$$

This expression was derived by Magono and Orikasa (1961) to calculate the ‘net surface electric field’ due to space charge and lower charge region of cylindrical clouds.

From basic electromagnetic theory, a cloud of positive polarity with a charge +Q at a height H and a charge -Q at a height h, then if observations are made at a horizontal distance u from the cloud, the vertical potential gradients are

$$\frac{-2QH}{4\pi\epsilon_0(H^2 + u^2)^{3/2}} \quad \text{and} \quad \frac{+2Qh}{4\pi\epsilon_0(h^2 + u^2)^{3/2}}$$

the factor 2 arising from the electrical image charges. The sum of these two expressions is zero when $u^2 = h^{2/3}H^{2/3}(h^{2/3} + H^{2/3})$. Similarly the potential must be continuous at point $u = a$ which is reversal distance, hence the vertical potential gradient at this point is

$$F = \frac{2\rho a}{\pi\epsilon_0} \left[\frac{\pi h}{4a} - \int_0^{\pi/2} \frac{a + u \cos 2x}{\chi^2} (h^2 + \psi)^{1/2} dx \right]_{h_1}^{h_2}, \quad u = a \dots\dots\dots(13)$$

Carey et al. (2003) reported a mixed phase zone located between the temperature of 0^0C to -40^0C . The region coincident with presence of large precipitation ice volume, intense updraft strength and likely high supercooled liquid water contents may have resulted in the positive charging of the graupel and hail via the so called ‘charge reversal’ process associated with non-inductive charging (Takahashi, 1978; Saunders et al. 1991).

Numerical Computation of a Electric Field

The effects of variations in charge density, slice radius and horizontal separation between the axis of the various slices into which a given cloud volume may be divided are readily modeled by the superposition of the contributions of the various slices. The space and in-cloud charge densities in thunderstorms are of the order of $10^{-9} Cm^{-3}$ (Winn et al. 1974, Marshall and Winn 1982, Soula 1994, Krehbiel et al. 2008, Marshall et al. 2001 and Davydenko et al. 2009 and Shao et al. 2010) for various storms. The permittivity of free space is $\epsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$ and equations (11a), (11b) and (13) can be normalized in the form

$$F = \frac{2\rho a}{\pi\epsilon_0} \left\{ [f(a_{k1}, b_{k1})]_{h_1}^{\infty} - [f(a_{k2}, b_{k2})]_{h_2}^{\infty} \right\} \dots\dots\dots(14)$$

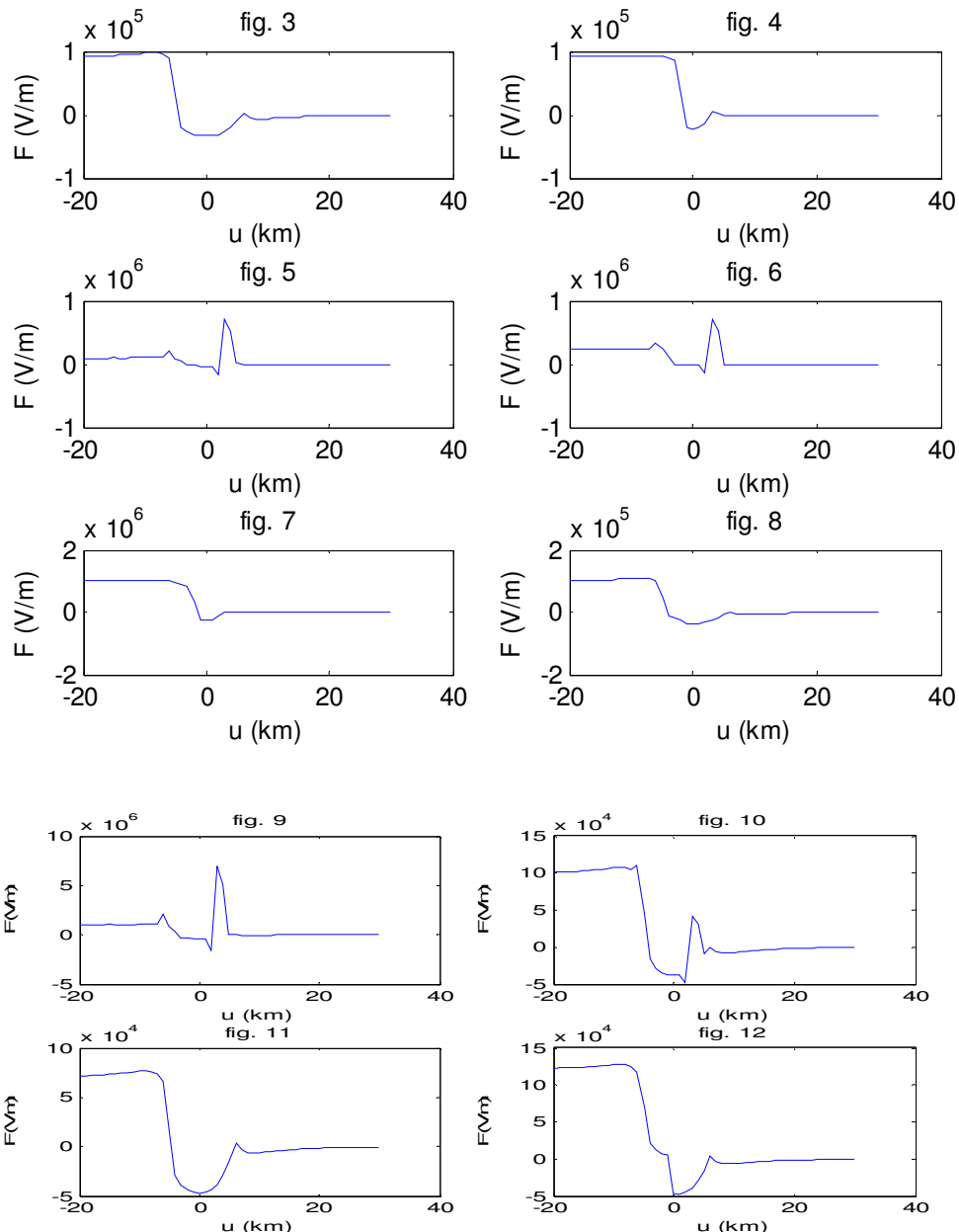
Bipolar Structure of a thunderstorm is considered, where a vertical cylindrical cloud of radius, $a = 5km$, with a uniform negative charge column extending from $2km$ to $4km$ with $\rho_- = -1.0 \times 10^{-9} Cm^{-3}$ and a uniform positive charge column extending from the end of negative charge column to the top of the cloud, i. e. $4km$ to $8km$, also with $\rho_+ = 0.7 \times 10^{-9} Cm^{-3}$ is modeled. The resulting potential gradient profile is shown in fig.2. Various sizes of vertical cylindrical cloud were modeled, their corresponding pattern of field profile were shown in fig.3 and 4 for $a = 4km$ and $a = 2km$ respectively, with same height and charge densities as that of fig.2. When the value of ρ_+ is increased from $0.7 \times 10^{-9} Cm^{-3}$ to $1.0 \times 10^{-9} Cm^{-3}$, within the main positive charge column only fig. 5 is obtained. A vertical cylindrical cloud model with the charge density in the core region of radius $2km$ and $4km$ within the cloud of fig.1 arranged to be ten and one-tenth multiplying those within the corresponding peripheral volumes is shown in curve of figs.6 and 7 and figs.8 and 9 respectively. The effect of the charge densities in the core region have significant effect on the potential gradient profile either in less dense or more dense region of the thunderstorm.

The effect of screening charge densities were computed with $\rho_{s-} = -0.9 \times 10^{-9} Cm^{-3}$ as given by Zhao et al. 2010, but the results showed that their influence is likely to be negligible as in fig.10 as compared with fig.2 (profile without screening charges). The lower positive charge also has significant effect on the

resultant electric field because of its proximity to the ground, it reshape the hump of profile of the field which is feature of interest as evidence in fig.11 as compared with fig.10. With this region , a heavy rain is usually associated and is essential for the initiation of cloud to ground (CG) lightning and in air discharges as well (Marshall et al. 1982, Holden et al. 1983 and Williams, 1985).

Conclusion

A numerical model capable of simulating tropical cloud has been described together with examples of simulation in an unshared environment. The expression derived for the vertical potential gradient at the intersection of the cloud axis and the ground was found to be equal to the expression used by Magono and Orikasa (1961) in calculating the net surface electric field due to the space charge and lower charge regions of cylindrical clouds.



When a tropical size cloud is modeled, a hump appears in the central region of the potential gradient profile, a feature attributed to the presence near the cloud base a pocket of positive charge .The computations done on screening layer showed that it does not appear to play an active role in the thunderstorm electrification. This is in good agreement with the results of the past works which showed that a fully developed screening layer has little effect on the contribution of a thunderstorm to the global circuit (Driscoll et al.,1992, Blakeslee et al., 1989 and Ette and Olaofe, 1980).

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