

Effect of Dissipation, Thermal Radiation, Radiation Absorption on Convective Heat and Mass Transfer Flow in a Non-Uniformly Heated Vertical Channel

C. Sulochana* G.N. Ramakrishna
Department of Mathematics, Gulbarga University, Gulbarga-585106, India
Email: math.sulochana@gmail.com, ramakrishnagn@gmail.com

Abstract
We analyze the effect of thermal radiation on mixed convective heat and mass transfer flow of a viscous, electrically conducting incompressible fluid through a porous medium in a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. Assuming the slope δ of the boundary temperature to be small, we solve the governing equations by a perturbation technique. The velocity, the temperature, the concentration, the rate of heat and mass transfer has been analyzed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.
Keywords: Non-uniform temperature, Porous medium, Thermal Radiation, Radiation Absorption, Dissipation

1. INTRODUCTION

The analysis of heat transfer in a viscous heat generating fluid is important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids. The Volumetric heat generation has been assumed to be constant [Ajay Kumar Singh(2003), Bejamin Gebhort et.al.(1988), Bejan et.al.(1985),Cheng (1979), Ostrach(1954), Palm(1975)] or a function of space variable [Chen et.al.(1980), Low(1955)]. For example a hypothetical core-disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris as horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate [Gabor et.al.(1974),]. Keeping this in view, porous medium with internal heat source have been discussed by several authors [Buretta(1972), Gobar et.al.(1974), Palm(1975)].

In the above mentioned investigations the bounding walls are maintained at constant temperature. However, there are a few physical situations which warrant the boundary temperature to be maintained non-uniform. It is evident that in forced or free convection flow in a channel (pipe) a secondary flow can be created either by corrugating the boundaries or by maintaining non-uniform wall temperature such a secondary flow may be of interest in a few technological processes. For example in drawing optical glass fibers of extremely low loss and wide bandwidth, the process of modified chemical vapour deposition (MCVD) [Low(1955), Simpikins et.al.(1979)] has been suggested in recent times. Ravindranath et al (2010) have studied the combined effect of convective heat and mass transfer on hydro magnetic electrically conducting viscous incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at non-uniform temperatures.

All the above mentioned studies are based on the hypothesis that the effect of dissipation is neglected. This is possible in case of ordinary fluid flow like air and water under gravitational force. But this effect is expected to be relevant for fluids with high values of the dynamic viscous flows. Moreover Gebhart(1962), Gebhart and Mollen dorf(1969) have shown that that viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size or at extremely low temperature or in high gravitational filed. On the other hand Barletta has pointed out that relevant effect of viscous dissipation on the temperature profiles and on the Nusselt numbers may occur in the fully developed forced convection in tubes. In view of this several authors notably, Soudalgekar and Pop, Raptis etc al, Barletta(1997, 1998). El-hakeing(2000), Bulent Yesilata (2002) and Israel et al (2003) have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation has been studied by Nakayama and Pop for steady free convection boundary layer over non-isothermal bodies of arbitrary shape embedded in porous media. They used integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. Costa has analyzed a natural convection in enclosures with viscous dissipation. Recently Prasad has discussed the effect of dissipation on the mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel bounded by flat walls. Vijayabhaskar Reddy (2009) has analyzed the combined influence of radiation and thermo-diffusion on convective heat and mass transfer flow of a viscous fluid through a porous medium in vertical channel whose walls are maintained at non-uniform temperatures.

In this paper, we discuss the Effect of dissipation; thermal radiation and radiation absorption on convective heat and mass transfer flow of a viscous electrically conducting chemically reacting fluid through a porous medium in a Non linear coupled equation governing flow heat and mass transfer have been solved with the slope δ of the boundary temperature as a perturbation parameter. The effect of various forces on the flow characteristics have been discussed graphically.

2. FORMULATION OF THE PROBLEM

We analyze the steady motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source and the concentration on these walls are taken to be constant. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy dissipations and the joule heating are taken into account in the energy equation. Also the kinematic viscosityν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0(x, y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y = ± L. The equations governing the steady flow, heat and mass transfer in terms of stream function ψ are

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Equation of linear momentum:

$$\rho_e\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g-\left(\frac{\mu}{k}\right)u \tag{2}$$

$$\rho_e\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\left(\frac{\mu}{k}\right)v \tag{3}$$

Equation of Energy:

$$\begin{aligned}\rho_e C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)&=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q+\mu\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right)+\\&+\left(\frac{\mu}{\lambda k}+\sigma\mu_e^2H_o^2/\lambda\right)(u^2+v^2)-\frac{\partial(q_r)}{\partial y}+Q_1'(C-C_e)\end{aligned} \tag{4}$$

Equation of Diffusion:

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1'C \tag{5}$$

Equation of State:

$$\rho-\rho_e=-\beta\rho_e(T-T_e)-\beta^\bullet\rho_e(C-C_e) \tag{6}$$

where ρ_e is the density of the fluid in the equilibrium state, Te , Ce are the temperature and Concentration in the equilibrium state (u, v) are the velocity components along O(x ,y) directions, p is the pressure, T ,C are the temperature and Concentration in the flow region,ρis the density of the fluid,μ is the constant coefficient of viscosity , Cp is the specific heat at constant pressure,λis the coefficient of thermal conductivity ,k is the magnetic permeability of the porous medium , β is the coefficient of thermal expansion, $\beta^\bullet$ is the coefficient of expansion with mass fraction ,D_1 is the molecular diffusivity ,Q is the strength of the constant internal heat source , q_r is the radiative heat flux diffusivity and k_1^1 is chemical reaction coefficient.

Invoking Rosseland approximation for radiation

$$q_r=-\frac{4\sigma\bullet}{3\beta_R}\frac{\partial(T'^4)}{\partial y}$$

Expanding T'^4 in Taylor's series about Te neglecting higher order terms

$$T'^4\cong 4T_e^3T'-3T_e^4$$

where $\sigma^\bullet$ is the Stefan-Boltzmann constant β_R is the Extinction coefficient.

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \tag{7}$$

Where $p = p_e + p_D, p_D$ being the hydrodynamic pressure.
The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q = \frac{1}{L}\int_{-L}^{L} u\, d\, y \,. \tag{8}$$

The boundary conditions for the velocity and temperature fields are

u = 0, v = 0 on y = ±L

$T - T_e = \gamma(\delta x / L)$ on y = ±L

C = C_1 on y = -L

C = C_2 on y = +L (9)

γ is chosen to be twice differentiable function,δ is a small parameter characterizing the slope of the temperature variation on the boundary.

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y, v = \psi_x \tag{10}$$

the equation governing the flow in terms of ψ are

$$\left[\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}\right] = \nu\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right)^2 - \beta g\frac{\partial T}{\partial y} - -\beta^{\bullet} g\frac{\partial C}{\partial y} - \left(\frac{\nu}{k}\right)\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) \tag{11}$$

$$\rho_e C_p\left(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \lambda\nabla^2\theta + Q + \mu\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right) + + \left(\frac{\mu}{k} + \sigma\mu_e^2 H_o^2\right)\left(\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2\right) + \frac{16\sigma^* T_e^3}{3\beta_R}\frac{\partial^2\theta}{\partial y^2} + Q_1'(C - C_e) \tag{12}$$

$$\left(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = D_1\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - k_1' C \tag{13}$$

Where $u = -\psi_y$, $v = \psi_x$

Introducing the non-dimensional variables in (11)- (13) as

$$(x', y') = (x, y)/L,\ \theta = \frac{T - T_e}{\Delta T_e}, C^{\bullet} = \frac{C - C_1}{C_2 - C_1}$$

$$.\psi^1 = \frac{\psi}{\delta L}\ p' = \frac{p_D}{\rho_e q^2}\quad ,.\gamma' = \frac{\gamma}{\Delta T_e} \tag{14}$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$R\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nabla^4\psi - \frac{G}{R}(\theta_y + NCy) - D^{-1}\nabla^2\psi \tag{15}$$

and the energy and diffusion equations in the non-dimensional form are

$$P_1 R\left(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = (\theta_{yy} + N_2\theta_{xx}) + \alpha_1 + \left(\frac{P_1 R^2 E_c}{G}\right)\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right) + + (D^{-1})\left(\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2\right) \tag{16}$$

$$RSc\left(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right)=\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1 C \tag{17}$$

where

$R=\frac{qL}{\nu}$ (Reynolds number), $G=\frac{\beta g\Delta T_e L^3}{\nu^2}$ (Grashof number)

$\mathrm{P}=\frac{\mu c_p}{\lambda}$ (Prandtl number), $D^{-1}=\frac{L^2}{k}$ (Inverse Darcy parameter),

$E_c=\frac{\beta g L^3}{C_p}$ (Eckert number), $N=\frac{\beta^\bullet\Delta C}{\beta\Delta T}$ (Buoyancy Number)

$Sc=\frac{\nu}{D_1}$ (Schmidt number), $\alpha=\frac{QL^2}{\Delta T C_p}$ (Heat source parameter)

$Q_1=\frac{Q_1'(C_w-C_e)L^2}{\lambda(T_w-T_e)}$ (Radiation absorption parameter),

$k_1=\frac{k_1' L^2}{D_1}$ (Chemical reaction parameter), $N_1=\frac{\beta_R\lambda}{4\sigma_\bullet T_e^3}$ (Radiation parameter)

$N_2=\frac{3N_1}{3N_1+4}$, $P_1=PN_2$, $\alpha_1=\alpha N_2$

The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=-1$$

$$\frac{\partial\psi}{\partial x}=0,\quad \frac{\partial\psi}{\partial y}=0 \quad at\ y=\pm 1 \tag{18}$$

$$\theta(x,y)=\gamma(\delta x) \qquad on\quad y=\pm 1 \tag{19}$$

C= 0 on y = -1

C = 1 on y = 1 (20)

$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0 \qquad at \quad y=0 \tag{21}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis (8) .Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function γ(x).

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the transformation

$$\bar{x}=\delta x$$

With this transformation the equations (15) - (17) reduce to

$$R\delta\frac{\partial(\psi,F^2\psi)}{\partial(x,y)}=F^4\psi-\frac{G}{R}(\theta_y+NC_y)-D^{-1}F^2\psi \tag{22}$$

and the energy &diffusion equations in the non-dimensional form are

$$P_1R\delta\left(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=F^2\theta+N_2+\left(\frac{P_1R^2E_c}{G}\right)\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\delta^2\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right)+$$
$$+(D^{-1}+M^2)\left(\delta^2\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right)+Q_1C \tag{23}$$

$$\delta RSc\left(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right)=F^2C-k_1C \tag{24}$$

where
$$F^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

for small values of the slope δ,the flow develops slowly with axial gradient of order δ and hence we take

$$\frac{\partial}{\partial \bar{x}}\approx O(1)$$

We follow the perturbation scheme and analyze through first order as a regular perturbation problem at finite values of R, G, P, Sc and D^{-1}

Introducing the asymptotic expansions

$\psi(x, y) = \psi_0(x, y) + \delta\psi_1(x, y) + \delta^2\psi_2(x, y) + \ldots\ldots$

$\theta(x, y) = \theta_0(x, y) + \delta\theta_1(x, y) + \delta^2\theta_2(x, y) +$

$C(x, y) = C_0(x, y) + \delta C_1(x, y) + \delta^2 C_2(x, y) + \ldots.$ (25)

On substituting (25) in (22) – (24) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyyy}-M_1^2\psi_{0,yy}=\frac{G}{R}(\theta_{0,y}+NC_{0,y}) \tag{26}$$

$$\theta_{0,yy}=-\alpha_1-\frac{P_1R^2Ec}{G}\psi_{0,yy}^2-\frac{P_1M_1^2Ec}{G}\psi_{0,y}^2-Q_1C_0 \tag{27}$$

$C_{0,yy} - k\,Sc\,C_0 = 0$ (28)

with

$\psi_0(+1) - \psi(-1) = -1,$

$\psi_{0,y} = 0,\ \psi_{0,x} = 0$ at $y = \pm 1$ (29)

$\theta_0(\pm 1) = \gamma(x)$ at $y = \pm 1$

$C_0(-1) = 0\ C_0(+1) = 1$ (30)

and to the first order are

$$\psi_{1,yyyy}-M_1^2\psi_{1,yy}=-\frac{G}{R}(\theta_{1,y}+NC_{1,y})+R(\psi_{0,y}\psi_{0,xyy}-\psi_{0,x}\psi_{0,yyy}) \tag{31}$$

$$\theta_{1,yy}=P_1R(\psi_{0,x}\theta_{0,y}-\psi_{0,y}\theta_{0,x})-\frac{P_1Ec}{G}(R^2\psi_{1,yy}^2+M_1^2\psi_{1,y}^2)-Q_1C_1 \tag{32}$$

$C_{1,yy} - (k\,Sc)\ C_1 = RSc(\psi_{0,y}C_{0,x} - \psi_{0,x}C_{0,y})$ (33)

$\psi_{1(+1)} - \psi_{1(-1)} = 0$

$\psi_{1,y} = 0,\ \psi_{1,x} = 0$ at $y = \pm 1$ (34)

$\theta_1(\pm 1) = 0$ at $y = \pm 1$ (35)

$C_1(-1) = 0,\ C_1(+1) = 0$ (36)

Assuming Ec<<1 to be small we take the asymptotic expansions as

$$\psi_0(x,y)=\psi_{00}(x,y)+Ec\,\psi_{01}(x,y)+\ldots\ldots\ldots..$$
$$\psi_1(x,y)=\psi_{10}(x,y)+Ec\,\psi_{11}(x,y)+\ \ldots\ldots\ldots..$$
$$\theta_0(x,y)=\theta_{00}(x,y)+Ec\,\theta_{01}(x,y)+\ldots\ldots\ldots\ldots..$$
$$\theta_1(x,y)=\theta_{10}(x,y)+Ec\,\theta_{11}(x,y)+\ldots\ldots\ldots\ldots..$$
$$C_0(x,y)=C_{00}(x,y)+Ec\,C_{01}(x,y)+\ldots\ldots\ldots\ldots..$$
$$C_1(x,y)=C_{10}(x,y)+Ec\,C_{11}(x,y)+\ldots\ldots\ldots\ldots.. \tag{37}$$

Substituting the expansions (37) in equations (26)-(36) and separating the like powers of Ec we get the following equations

$$\theta_{00,yy}=-\alpha_1-Q_1C_{00}\quad,\quad \theta_{00}(\pm1)=\gamma(\bar{x}) \tag{38}$$

$$C_{00,yy}-(kSc)C_{00}=0\ ,\quad C_{00}(-1)=0, C_{00}(+1)=1 \tag{39}$$

$$\psi_{00,yyyy} - M_1^2\psi_{00,yy} = -\frac{G}{R}(\theta_{00,y} + NC_{00,y}) \ ,$$
$$\psi_{00}(+1) - \psi_{00}(-1) = 1, \psi_{00,y} = 0, \psi_{00,x} = 0 \ \ at \ \ y = \pm 1 \qquad (40)$$

$$\theta_{01,yy} = -\frac{P_1M_1^2}{G}\psi^2{}_{00,y} - \frac{P_1R^2}{G}\psi^2{}_{00,yy} - Q_1C_{01} \quad , \quad \theta_{01}(\pm 1) = 0 \qquad (41)$$

$$C_{01,yy} - (kSc)C_{01} = 0 \qquad , \qquad C_{01}(\pm 1) = 0 \qquad (42)$$

The equations (39) – (48) are solved algebraically subject to the relevant boundary conditions. For the sake of brevity the solutions are not presented here.

$$\psi_{01,yyyy} - M_1^2\psi_{01,yy} = -\frac{G}{R}(\theta_{01,y} + NC_{01,y}) \ ,$$
$$\psi_{01}(+1) - \psi_{01}(-1) = 0,$$
$$\psi_{01,y} = 0, \psi_{01,x} = 0 \ \ at \ \ y = \pm 1 \qquad (43)$$

$$\theta_{10,yy} = RP_1(\psi_{00,y}\theta_{00,x} - \psi_{00,x}\theta_{00,y}) - Q_1C_{10} \qquad \theta_{10}(\pm 1) = 0 \qquad (44)$$

$$C_{10,yy} - (kSc)C_{10} = RP_1(\psi_{00,y}C_{00,x} - \psi_{00,x}C_{00,y})$$
$$C_{10}(\pm 1) = 0 \qquad (45)$$

$$\psi_{10,yyyy} - M_1^2\psi_{10,yy} = -\frac{G}{R}(\theta_{10,y} + NC_{10,y}) +$$
$$+ R(\psi_{00,y}\psi_{00,xyy} - \psi_{00,x}\psi_{00,yyy}) \ ,$$
$$\psi_{10}(+1) - \psi_{10}(-1) = 0, \psi_{10,y} = 0, \psi_{10,x} = 0 \ \ at \ \ y = \pm 1 \qquad (46)$$

$$\theta_{11,yy} = RP_1(\psi_{00,y}\theta_{1,x} - \psi_{1,x}\theta_{00,y}) - Q_1C_{11} \quad , \quad \theta_1(\pm 1) = 0 \qquad (47)$$

$$C_{11,yy} - (kSc)C_{11} = RSc(\psi_{00,y}C_{00,x} - \psi_{1,x}C_{00,y}), C_{11}(\pm 1) = 0 \qquad (48)$$

$$\psi_{11,yyyy} - M_1^2\psi_{1,yy} = -\frac{G}{R}(\theta_{11,y} + NC_{11,y}) + R(\psi_{00,y}\psi_{11,xyy} -$$
$$- \psi_{00,x}\psi_{01,yyy} + \psi_{01,y}\psi_{00,xyy} - \psi_{01.x}\psi_{00,yyy}) \ ,$$
$$\psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,y} = 0, \psi_{11,x} = 0 \ \ at \ \ y = \pm 1 \qquad (49)$$

4. NUSSELT NUMBER and SHERWOOD NUMBER

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w}\left(\frac{\partial\theta}{\partial y}\right)_{y=\pm 1}$$, where $$\theta_m = 0.5\int_{-1}^{1}\theta\, dy$$

The local rate of mass transfer coefficient (Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm 1}$$, where $$C_m = 0.5\int_{-1}^{1}C\, dy$$

6. DISCUSSION OF THE NUMERICAL RESULTS

We discuss the effect of chemical reaction, and radiation absorption on the mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel whose walls are maintained at non-uniform temperature. The non-linear, coupled equations have been solved by using a regular perturbation technique with the slope δ of the boundary temperature as a perturbation parameter.

The axial velocity (u) is exhibited in figs 1 – 6 for different values of γ, Q_1, N, α_1 N_1 and Ec. With reference to radiation absorption parameter Q_1 we find that u exhibits a reversal flow for higher $Q_1 \geq 3.5$. |u| enhances with increase in $Q_1 \geq 3.5$ (fig. 1). Fig. 2 represents u with buoyancy ratio N. It is found that when the molecular buoyancy force dominates over the thermal buoyancy force the axial velocity enhances irrespective of the directions of the buoyancy forces. Fig. 3 represents u with chemical reaction parameterγ. We find an enhancement in the axial velocity in the degenerating chemical reaction case. An increase in the amplitude α_1 of the boundary temperature results in a depreciation in u (fig. 4). From figs. 5&6 we find that higher the radiative heat flux/dissipative heat larger |u| in the flow region.

The secondary velocity (v) which arises due to the non-uniformity of the boundary temperature is exhibited in figs. 7 – 12 for different parametric values. An in increase in the radiation absorption parameter Q_1 results in a depreciation in |v| in the left half and enhances in the right half of the channel (fig. 7) With reference to the buoyancy ratio N it can be seen that when the molecular buoyancy force dominates over the thermal buoyancy force the secondary velocity depreciates in the left half and enhances in the right half when the buoyancy forces act in the same direction and for the forces acting in opposite directions, it enhances in the entire flow region(fig. 8). An increase in the chemical reaction parameter γ results in depreciation in the left half and enhances in the right half of the channel (fig. 9). |v| reduces in the left half and enhances in the right half with an increase in the amplitude α_1 of the boundary temperature (fig. 10). From figs. 11&12 we find that higher the radiative heat flux/dissipative heat smaller |v| in the left half and larger in the right half.

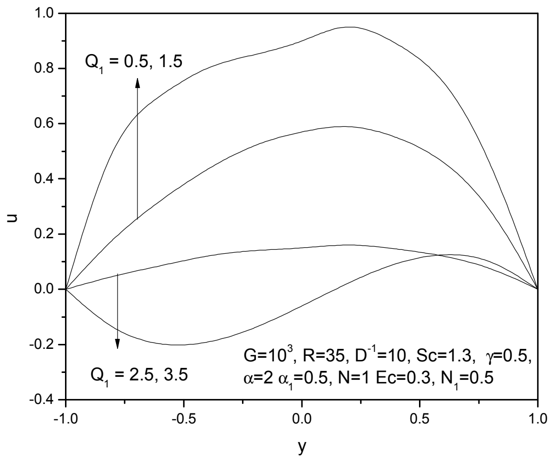

Fig. 1 : Variation of u with Q_1

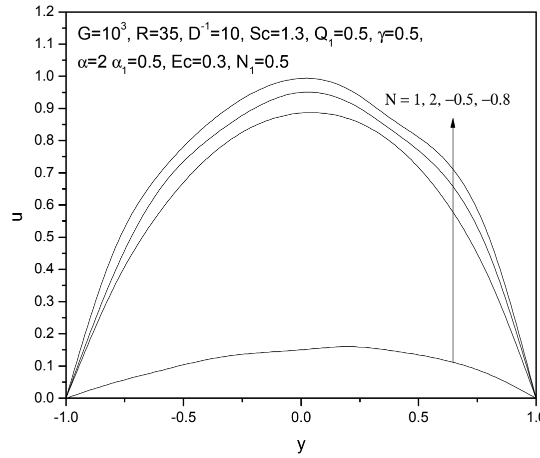

Fig. 2 : Variation of u with N

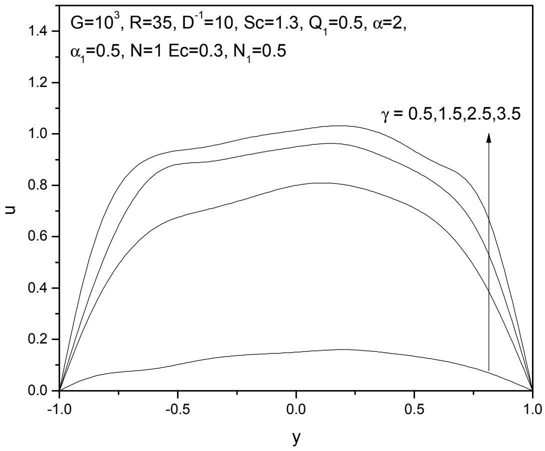

Fig. 3 : Variation of u with γ

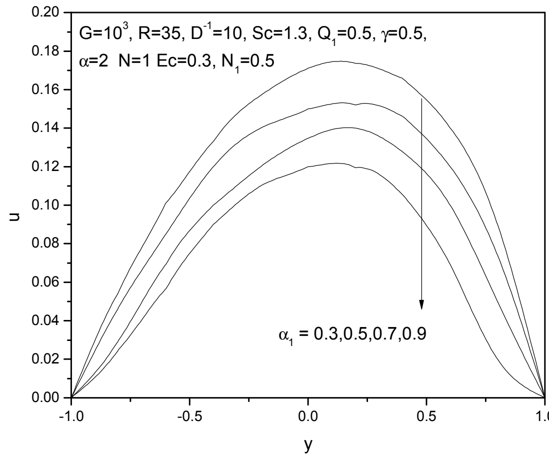

Fig. 4 : Variation of u with α_1

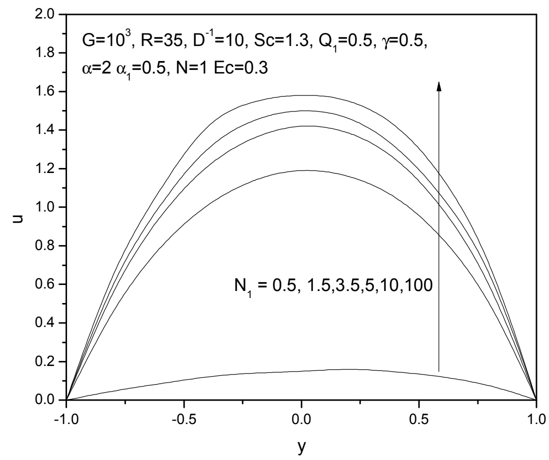

Fig. 5 : Variation of u with N_1

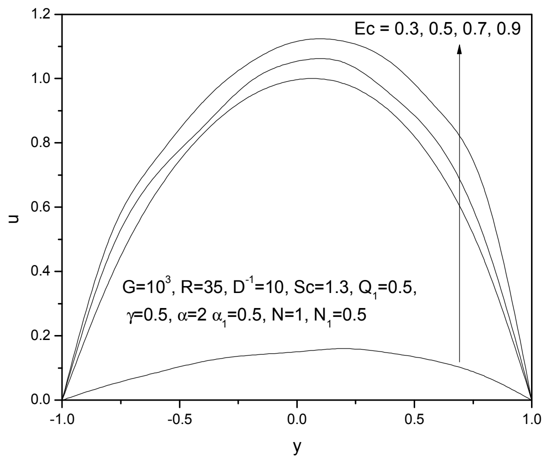

Fig. 6 : Variation of u with Ec

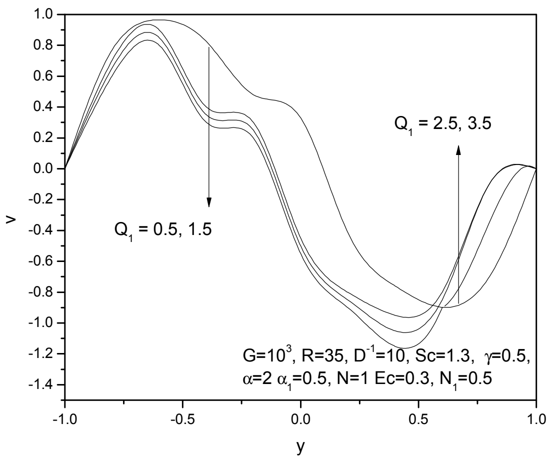

Fig. 7 : Variation of v with Q_1

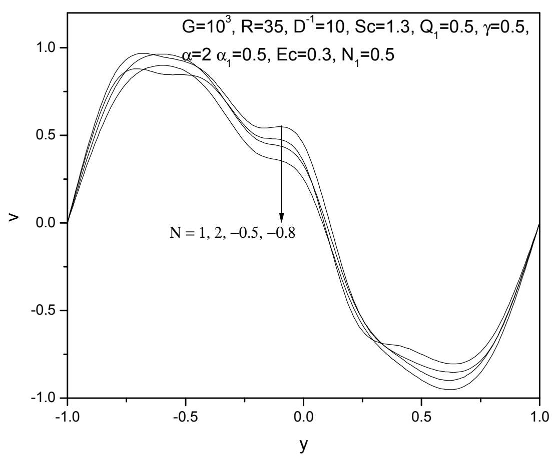

Fig. 8 : Variation of v with N

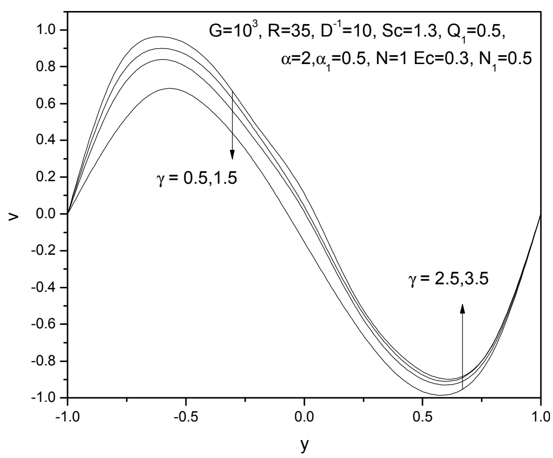

Fig. 9 : Variation of v with γ

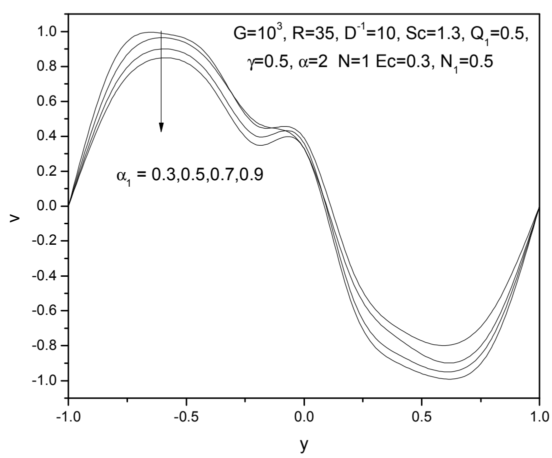

Fig. 10: Variation of v with α_1

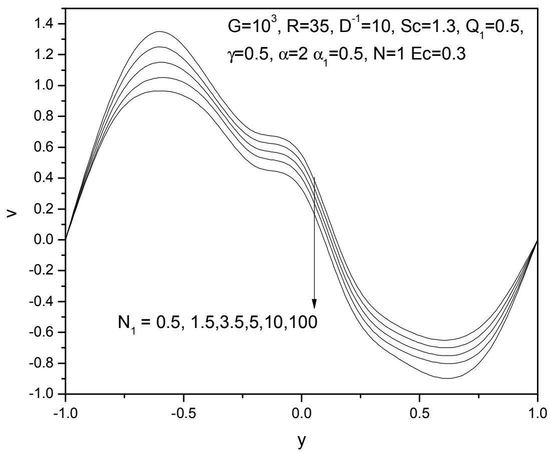

Fig. 11 : Variation of v with N_1

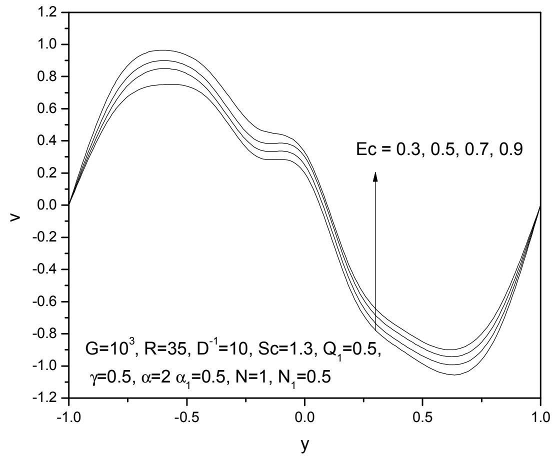

Fig. 12 : Variation of v with Ec

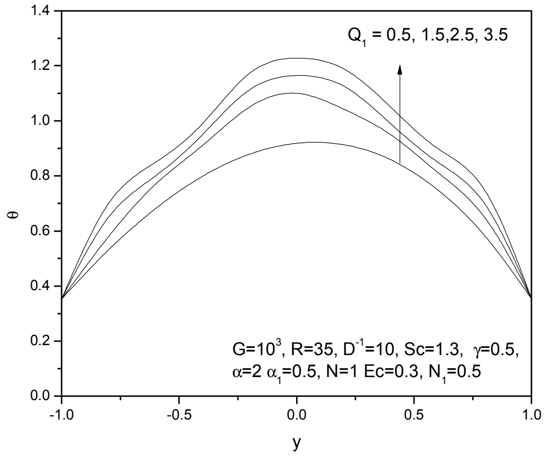

Fig. 13 : Variation of θ with Q_1

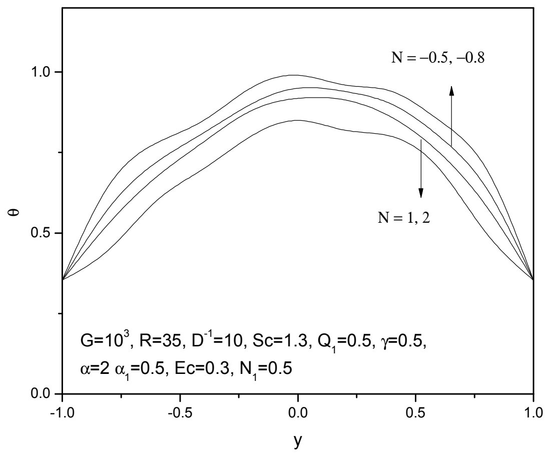

Fig. 14 : Variation of θ with N

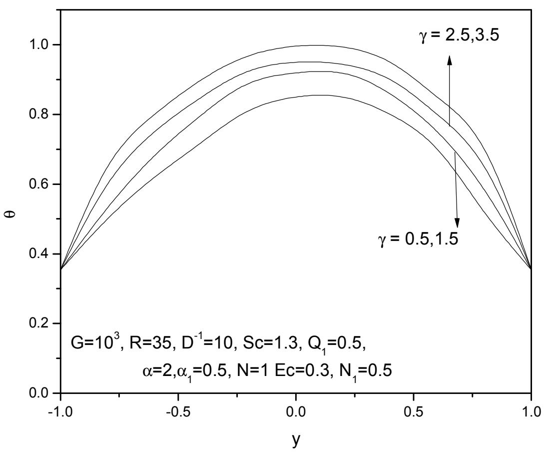

Fig. 15 : Variation of θ with γ

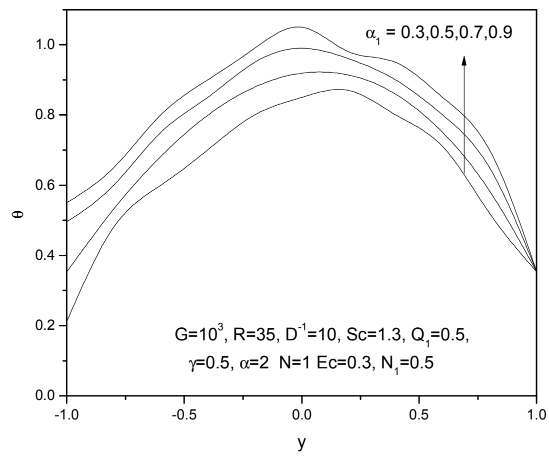

Fig. 16 : Variation of θ with α_1

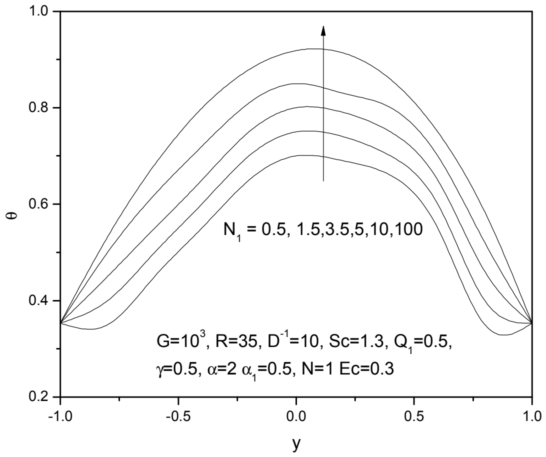

Fig. 17 : Variation of θ with N_1

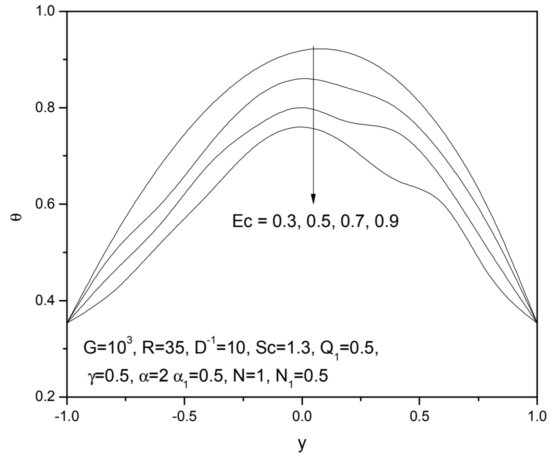

Fig. 18 : Variation of θ with Ec

The non-dimensional temperature (θ) is shown in figs. 13 – 18 for different parametric values. An increase in the radiation absorption parameter Q_1 results in an enhancement in actual temperature (fig. 13). The variation of θ with N indicates that the actual temperature depreciates with N>0 and enhances with |N| (<0) (fig. 14). With reference to the chemical reaction parameter γ it can be seen that the actual temperature depreciates with increase in $\gamma \leq 1.5$ and enhances with higher $\gamma \geq 2.5$ (fig. 15).Higher the amplitude α_1 of the boundary temperature larger the actual temperature (fig. 16). From fig. 17&18 we find that higher the radiative heat flux/dissipative heat smaller the actual temperature.

The non-dimensional concentration (C) is shown in figs. 19-24 for different values larger the radiation absorption parameter Q_1 smaller the actual concentration (fig. 19). With respect to N, we notice that when the molecular buoyancy force dominates over the thermal buoyancy force the actual concentration enhances when the buoyancy forces act in the same direction and for the forces acting in opposite directions it reduces in the entire flow region (fig. 20). An increase in γ reduces the concentration while it enhances with α_1 (figs.21, 22). From figs. 23&24 it can be observed that higher the radiative heat flux/dissipative heat larger the actual concentration.

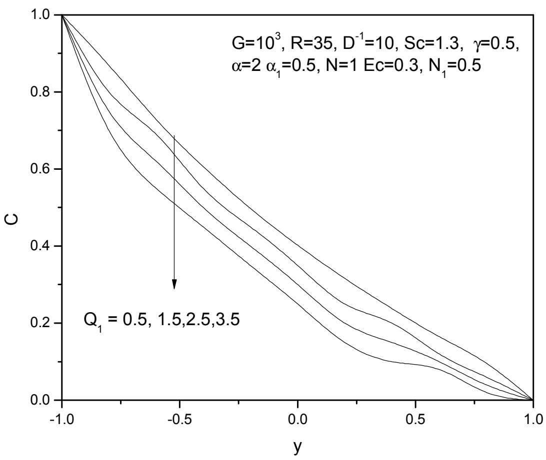

Fig. 19 : Variation of C with Q_1

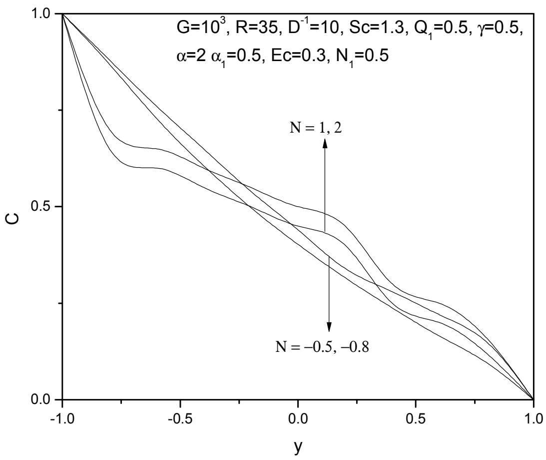

Fig. 20 : Variation of C with N

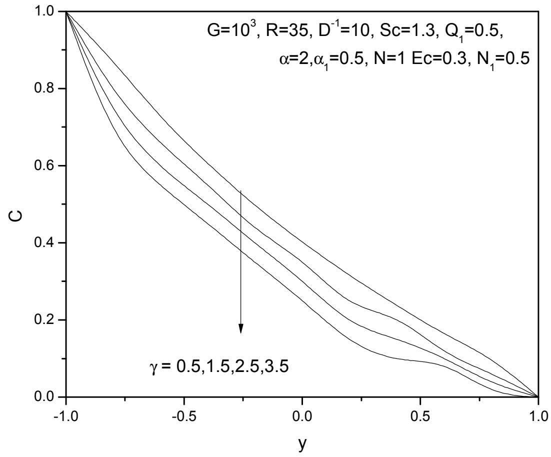

Fig. 21 : Variation of C with γ

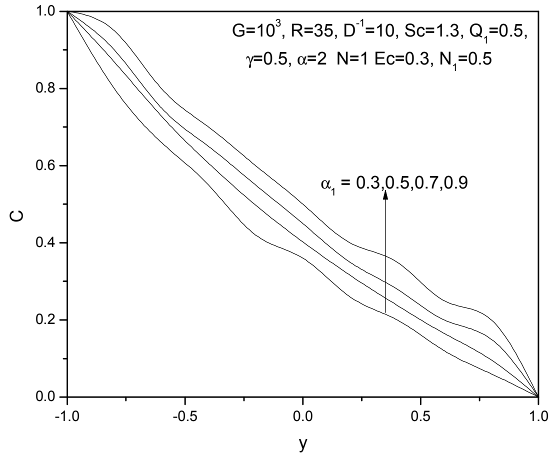

Fig. 22 : Variation of C with α_1

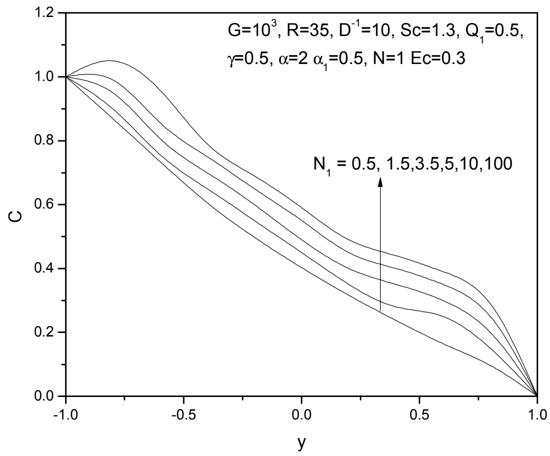

Fig. 23 : Variation of C with N_1

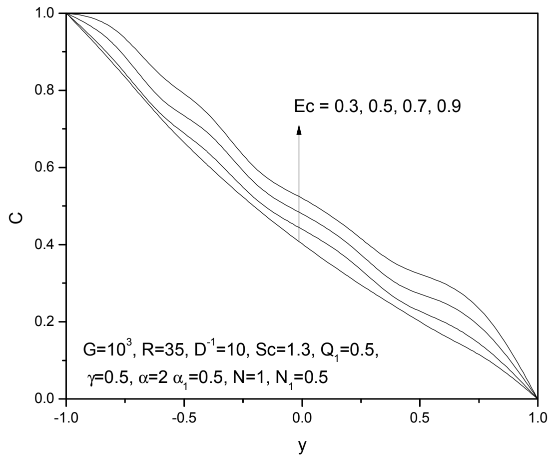

Fig. 24 : Variation of C with Ec

The rate of heat transfer for (Nusselt number) at y ±1 is shown in tables 1-4 for different values of, γ, Q_1, N, α_1, Ec, N_1. An increase in the radiation absorption parameter $Q_1 \leq 1.5$ reduces |Nu| at y = +1 and enhances it at y = -1 while for higher $Q_1 \geq 2.5$, enhances for G>0 and reduces for G<0 at y = +1 and at y = -1, |Nu| reduces with $Q_1 \geq 2.5$. The variation of Nu with chemical reaction parameter γ shows that |Nu| at y = +1 reduces for G>0 and enhances for G<0 increase in with $\gamma \leq 1.5$ and for higher γ = 2.5, it depreciates and for still higher γ = 3.5, |Nu| enhances in the heating case and reduces in the cooling case. At y = -1, |Nu| reduces with increase in $\gamma \leq 2.5$ and for higher γ = 3.5, it enhances in the heating case and reduces in the cooling case. With reference to buoyancy ratio N, we find that when the molecular buoyancy force dominates over the thermal buoyancy force |Nu| reduces for G>0 and enhances for G<0 when the buoyancy forces act in the same direction and for the forces acting in opposite directions |Nu| enhances at both the walls.

Table – 1 : Average Nusselt number (Nu) at y = +1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	2.4966	1.1439	2.5131	17.4401	-2.2929	-3.1258	3.9151	5.0946	5.4813
3×10^3	-35.5803	11.7064	-13.2871	11.6519	-1.7110	-3.7008	-82.6009	-137.023	-161.0368
-10^3	-3.1125	-5.1973	-4.1811	38.8079	-4.3125	-4.0886	-7.1154	-10.8455	-12.2723
-3×10^3	7.3057	38.1538	9.1886	66.7836	-7.3888	-6.4952	10.3065	11.4387	11.6622
Q_1	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	0.5
γ	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5	0.5
N_1	0.5	0.5	0.5	0.5	0.5	0.5	1.5	3.5	10

Table – 2 : Average Nusselt number (Nu) at y = +1

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	2.7777	2.4966	1.9513	1.6986	0.809	1.7605	-2.1393	5.0905	14.4857	-99.9873
$3x10^3$	-21.6458	-35.5803	-99.874	-123.7599	11.603	14.9379	-19.3119	10.0603	-8.2457	-7.2631
-10^3	-3.9538	-3.1125	-2.5664	-2.1833	-103.651	12.4913	16.1066	-14.9092	38.8027	43.5356
$-3x10^3$	5.8023	7.3057	9.8605	15.1630	17.517	43.3018	-187.2302	3.9626	3.3884	3.1053
α_1	0.3	0.5	0.7	0.9	0.5	0.5	0.5	0.5	0.5	0.5
N	1	1	1	1	2	-0.5	-0.8	1	1	1
Ec	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.7	0.9

Table – 3 : Average Nusselt number (Nu) at y = -1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-4.3921	12.8216	4.7046	-40.9296	1.9571	2.9216	-6.0366	-7.6208	-8.1276
$3x10^3$	16.8948	-5.0435	-4.3039	11.2166	1.4256	3.4728	26.2766	38.4331	42.9512
-10^3	-33.9033	7.8413	5.2843	-37.7085	3.9435	3.9024	13.7941	25.7996	31.8109
$-3x10^3$	3.5021	-28.6171	-7.1373	-60.4369	6.9501	6.2895	-16.9192	-25.7852	26.9861
Q_1	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	0.5
γ	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5	0.5
N_1	0.5	0.5	0.5	0.5	0.5	0.5	1.5	3.5	10

Table – 4 : Average Nusselt number (Nu) at y = - 1

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	-3.7541	-4.3921	-4.7541	-5.9541	0.1872	-2.9115	3.1958	-13.7739	26.7992	9.2486
$3x10^3$	12.8216	14.8616	15.9216	16.8066	-10.9043	27.8459	61.8918	-10.0026	6.5644	6.0384
-10^3	4.7046	5.7246	6.7646	7.8240	-50.0707	-22.9265	-31.6476	-6.0609	-8.7891	-6.4897
$-3x10^3$	-12.8948	14.2948	-16.3942	-20.8849	-25.4542	31.3683	99.9298	-2.2598	-2.0901	-2.0116
α_1	0.3	0.5	0.7	0.9	0.5	0.5	0.5	0.5	0.5	0.5
N	1	1	1	1	2	-0.5	-0.8	1	1	1
Ec	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.7	0.9

The variation of Nu with amplitude α_1 of the boundary temperature shows that |Nu| at y = +1 depreciates for G>0 and enhances for G<0 with increase in α_1 while at y = -1, |Nu| enhances with α_1 for all G. The variation of Nu with Eckert number indicates that |Nu| at y = +1 enhances with increase in Ec for $|G| = 10^3$ and for $|G| = 3x10^3$ it depreciates with Ec. At y = -1, |Nu| depreciates with Ec for $|G| = 3x10^3$ and for $|G| = 10^3$, the rate of heat transfer enhances with Ec≤0.07 and depreciates with higher Ec≥0.09. The variation of Nu with radiation parameter N_1 indicates that higher the radiative heat flux larger |Nu| at y = ±1.

The rate of mass transfer (Sherwood number) at y = ±1 is shows in tables 5-8 for different parametric values. The variation of Sh with radiation observed parameter Q_1 shows that an increase in Q_1 enhances |Sh| at y=+1 and reduces at y = -1 and enhances at y = -1. With reference to chemical reaction parameterγ, we find that |Sh| at y = +1 reduces with increase in γ≤2.5 and enhances with higher γ≥3.5 and at y = -1 it enhances with γ for all G. w.r.t Eckert number Ec, we observe that higher the dissipative heat smaller (Sh) at y=+1 and larger at y = -1 for all G.

Table – 5 : Average Sherwood number (Sh) at y = +1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	4.8571	5.0294	4.4836	4.7585	4.7109	4.8342	4.5575	4.3667	4.1999
$3x10^3$	4.2451	5.0294	4.4836	4.1986	3.9586	4.2936	5.0026	4.9716	4.9433
-10^3	5.2049	5.7788	12.4838	4.9589	4.8912	5.1296	4.1618	3.8838	3.6411
$-3x10^3$	4.1441	3.4561	2.9364	3.8948	3.7409	4.6485	3.8301	3.4903	3.2069
Q_1	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	0.5
γ	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5	0.5
Ec	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.7	0.9

Table – 7 : Average Sherwood number (Sh) at y = - 1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-1.0659	-2.6862	-3.5983	-1.0949	-1.1256	-1.1509	-1.0819	-1.0933	-1.1049
$3x10^3$	-1.0564	-6.3561	-3.5023	-1.0650	-1.0849	-1.1186	-0.9599	-1.0591	-1.0603
-10^3	-1.0863	-2.1186	-3.4738	-1.0246	-1.1526	-1.1709	-1.1086	-1.1318	-1.1559
$-3x10^3$	-1.1032	-2.4362	-4.9247	-1.1509	-1.1909	-1.2112	-1.1381	-1.1751	-1.2145
Q_1	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	0.5
γ	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5	0.5
Ec	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.7	0.9

7. CONCLUSIONS

In this paper we briefly discussed the Effect of Dissipation, thermal radiation, radiation absorption on convective heat and mass transfer flow in a non-uniformly heated vertical channel.

- An increase in the radiation absorption Q_1 enhance$_s$ the primary velocity u and the temperature θ and reduces the secondary velocity v and concentration C.
- An increase in the buoyancy ratio N>0 enhances u and C and reduces θ and an increase in |N| enhances u and θ and reduces C.
- An increase in the chemical reaction parameter γ enhances u and reduces C. θ reduces with $\gamma \leq 1.5$ and enhances with $\gamma \geq 2.5$. The secondary velocity v reduces in the left half and enhances in the right half of the channel.
- An increase in the amplitude α_1 boundary temperature reduces u and enhances θ and C. The secondary velocity v reduces in the left half and enhances in the right half with increase in $\alpha_{1.}$
- u and C enhances, v and θ reduces with increase in the radiation parameter N_1.
- The primary velocity u and the concentration C enhances with Ec while v and θ reduces with Ec.
- An increase in Q_1 or α_1 or γ reduces the rate of heat transfer at the both walls, while it enhances with N_1 and E_c.
- The rate of mass transfer reduces with G and enhances Q_1 at y ±1. The Sherwood number at Y= +1 reduces with γ and Ec and enhances at y =-1.

8. REFERENCES

Ajay Kumar Singh(2003): MHD free convection and mass transfer flow with Hall Effect, viscous dissipation, Joule heating and thermal diffusion, Ind. J.pure & Application Physics V.41.PP.24-35.

Barletta, A(1997), "Fully developed laminar forced convection in circular ducts for power-law fluids with viscous dissipation", Int.J Heat Mass Transfer, V.40,PP.15-26.

Barletta, A. (1998), "Laminar mixed convection with viscous dissipation in a vertical channel", Int.Heat Mass Transfer, V.41, PP.3501-3513.

Bejamin Gebhort, Yogesh Jaluria, Roop-L Mahjan and Bhagat Samnokia (1988): Buoyancy induced flows and transport, Springer –Verlog, berling.

Bejan ,A and Khair, K.R : (1985) "Heat and Mass transfer by natural convection in a porous medium" , Int. J. Heat and Mass transfer ,V.28,PP.908-918.

Bulent Yesilata (2002), "effect of viscous dissipation on polymeric flows between two rotating coaxial parallel discs", Int. Comns.J.Heat Mass transfer,V.29 ,PP.589-600.

Buretta, R(1972), Thermal convection in a fluid filled porous layer with uniform internal heat sources, Ph.D. dissertation, University of Minnesota.

Chen, T.S, Yuh, C.F and Moutsoglo, A(1980): "Combined heat and mass transfer in mixed convection along vertical and inclined plates", Int. Heat and Mass transfer, V.23, PP.527-537.

Cheng, P(1979): "Heat Transfer in Geothermal Systems", Advances in heat transfer, V.14, PP.1-105.

Costa, V.A.F(2005): Thermodynamics of natural convection in enclosures with viscous dissipation, Int.J.Heat and Mass transfer,V.48,pp.2333-2341.

El. Hakein, M.A., Comns. (2000) "Viscous dissipation effects on MHD free convection flow over a nonisothermal surface in a micropolar fluid" Heat Mass transfer, v.27, PP.581-590.

Gabor, D, Saroa, E.S,Baker, J.L. and Cassulo, J.L. and Cassulo, J.L. (1974) Proc. Fast Reactor Sahly –Mfg California, April 2-4, Conf, 740401 , p.823.

Gebhart , B.J (1962): "Effects of viscous dissipation in natural convection", fluid Mech, V.14,pp.225 -232.

Gebhart ,B and Mollel dorf . (1969): "Viscous dissipation in external natural convection flows", J. Fluid Mech, V.38 ,pp.97-107.

Israel, C, (2003) "Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction", Int. J. Heat Mass transfer, V.40, PP.2305-2311.

Krishna, D.V., (1986), Kolposhcinkov, V.L, Martynenko, O.G, Shabrunya, S.I., Shrip, A.I , and Luikov, A.V. Mixed thermal convection of a vertical tube at non uniform temperature . Accepted for publication in Int. J. Heat Mass transfer.(2002)

Low, G.M(1955): "Stability of compressible laminar boundary layer with internal heat sources or sinks", J. Aero Sci, V.22, P.239.

Nakayama, A and Pop, (1989) "Free convection over a nonisothermal body in a porous medium with viscous dissipation", Int. Comms. Heat Mass Transfer, V.16, P. 173.

Ostrach.S(1954): "Combined natural- and forced-convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperatures", NACA, TN,P.3141.

Palm,E (1975) : “Nonlinear thermal convection”, Ann –Rev: Fluid Mech., V.7 ,P.39-61.

Prasad, P.M.V(2006): Hydromagnetic convective heat and mass transfer through a Porous medium in channels/pipes, Ph.D thesis, S.K. University, Anantapur, India

Raptis, A. Kafousis, N and Massalas : (1981) "Free convection flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux", C.Reg.J, Energy Heat Mass Transfer, V.3, pp.279-283.

Raveendranath. P, Prasad, P.M.V and Prasada Rao, D.R,V (2010): Computational hydromagnetic mixed convective heat and mass transfer through a porous medium in a non-uniformly heated vertical channel with heat sources and dissipation, Computers and Mathematics with Applications, V.59, pp.803-811

Simpikins, P.G (1979), Greenberg, Kosiniki ,S, and Macchesney ,J.B. Thermophoresis. The mass transfer mechanism in modified chemical vapour deposition. J.Appl. Phys . 50 (9) P.5676.

Soundalgekar, V.M and Pop, I. (1974) “Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with variable suction”, Int, J. Heat mass transfer, V.17 ,PP.85-90.

Vijayabhaskar Reddy, P (2009): Combined effect of Radiation and Soret effect on flow of a viscous fluid through a porous medium in a vertical non-uniformly temperature, Jou. Phys and Appl. Phys, V.21,No.3,pp.413-433.