

Pulsed second harmonic generation in plasma with a magnetic wiggler

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Abstract

Magnetic wiggler assisted phase matched second harmonic of a Gaussian laser pulse in a plasma is studied. The wiggler provides additional momentum required for phase matching, however this is only for a particular instant. The required wiggler wave number increases with pulse duration and plasma density. The efficiency of the process drops sharply away from the phase matching instant.

Keywords: Harmonic generation, wiggler magnetic field, phase matching

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Introduction

Laser –plasma interaction is an area of significant research activity from past several decades [1-3]. The advancements in laser technology has made it possible to generate ultrashort laser pulses (fs) with intensity above 10^{20}W/cm^2. At such high nonlinear due to relativistic effects. A host of nonlinear effects are observed viz. generation of large amplitude plasma waves, tunnel ionized plasma, plasma channel formation, modulational instability, laser self focusing, harmonic generation, electron cooling etc. Amongst these, harmonic generation is one of the prominent nonlinear effect with applications in plasma diagnostics, shorter wavelength generation

and in theoretical understanding of nonlinear effects in plasma [5-12]. In a second harmonic generation process two photons of fundamental wave combine to generate a photon of twice the frequency of fundamental wave. The phase matching conditions for a second harmonic generation process demand,

$\omega_2 = 2\omega_1$, and $\hbar\vec{k}_2 = 2\hbar\vec{k}_1$,

where $\omega_{1(2)}, \vec{k}_{1(2)}$ are frequency and wave vectors of fundamental (second harmonic) wave, and $\hbar$ is Planck's constant.

Since plasma is a dispersive medium, $k_2 > 2k_1$ and the above mentioned conditions for phase matching are not satisfied, thereby making the process a non-resonant one. If the process is made a resonant – one the efficiency of the process can be enhanced significantly. Various schemes are proposed to make the process a resonant one. Ivanov et al. [13] have proposed a phase matching process by cascading of two phase – matched third order processes for fifth harmonic generation. Dimmock et al. [14] have analyzed phase matched second harmonic generation and optical parametric oscillations in bireferingent semiconductor waveguides by exploiting the waveguide geometry. Balcou et al.[15] have reviewed high order harmonic generation process and proposed a new scheme for phase matching by using the effect of the spatially varying atomic phase displayed by the high harmonics. Parashar and Pandey [16,17] have proposed to employ a density ripple or a magnetic wiggler of wave number $\vec{k}_0$, to compensate for momentum mismatch i.e., $\hbar\vec{k}_0 = \hbar\vec{k}_2 - 2\hbar\vec{k}_1$. Their studies showed significant enhancement in second harmonic generation efficiency. In this communication we extend their work to study second harmonic generation of a Gaussian laser pulse in the presence of a magnetic wiggler including relativistic effects. The physics of the process is as follows: The electron oscillatory velocity at $(\omega, \vec{k})$ couples with wiggler magnetic field to exert a force at $(\omega, \vec{k} + \vec{k}_w)$. The electron density oscillations due to ponderomotive force couples with electron velocity at $(\omega, \vec{k})$ to produce a nonlinear current density at $(2\omega, 2\vec{k} + \vec{k}_w)$ which produces the second harmonic radiation.

Nonlinear current density

Consider the propagation of a Gaussian laser pulse through a plasma of electron density n_0^0. The electric and magnetic fields of laser pulse are,

$$\vec{E} = A(\hat{x} + i\,\hat{y})\, e^{-i(\omega t - kz)},$$

$$\vec{B} = \frac{c}{\omega}\vec{k} \times \vec{E},$$

$$A^2 = A_0^2\, e^{-(t - z/v_g)^2/\tau^2}, \qquad (1)$$

where $k = (\omega/c)\eta$, η is the refractive index of the plasma, and $v_g = c\eta \approx c$ is the group velocity. The oscillatory velocity of electrons due to laser on solving the equation of motion $m(d\vec{v}/dt) = -e\vec{E} - (e/c)\vec{v} \times \vec{B}$ is

$$\vec{v}_{\omega,k} = \frac{e\vec{E}}{mi\omega\gamma_0} e^{-i(\omega t - kz)}, \qquad (2)$$

where - e and m are electronic charge and mass respectively, $\gamma_0 \approx \left(1+a^2/2\right)^{1/2}$,$a = eA/m\omega c$ and $a<1$. In terms of γ_0 and the plasma frequency $\omega_p = \left(4\pi n_0 e^2/m\right)^{1/2}$, the refractive index, in the limit $\omega_p^2/\omega^2 << 1$, can be written as

$$\eta(\omega)=1-\omega_p^2/2\omega^2\,\omega^2\gamma_0 . \qquad (3)$$

There also exists a wiggler magnetic field given by

$$\vec{B}_w = B_0\,(\hat{x}-i\,\hat{y})\ e^{i k_w z} . \qquad (4)$$

For second harmonic generation, the second harmonic wave vector $k_2 > 2k_1$. For the process to be a resonant one, the phase matching condition demand

$$\omega_2 = 2\omega_1 ,$$

$$\hbar k_2 = 2\hbar k_1 + \hbar k_w . \qquad (5)$$

To satisfy the phase matching conditions in Eq. (5), the required wiggler wave number k_w is

$$k_w \approx \frac{3}{4}\frac{\omega_{p0}}{\omega\gamma} . \qquad (6)$$

In Figures (1) & (2), we have shown variation of normalized wiggler wave number ck_w/ω_p with t'/τ $(t' = t - z/c)$ at different values of a_0 for $\omega_p/\omega = 0.1$ and 0.2 respectively. The wiggler wave number required for phase matching increases with time and plasma density. It decreases with pulse amplitude.

The electron velocity $\vec{v}_{\omega,k}$ beats with laser magnetic field $\vec{B}$ to produce a ponderomotive force $\vec{F}_p'$ at $(\omega_1, k+k_w)$,

$$\vec{F}_p' = -\frac{e}{2c}\vec{v}_{\omega,k}\times\vec{B}_w = -\hat{z}\frac{e^2 A\,B_0}{2m\omega c\gamma_0}e^{-i[\omega t-(k+k_w)z]} . \qquad (7)$$

The electron velocity $\vec{v}'$ due to $\vec{F}_p'$ is

$$\vec{v}' = \hat{z}\frac{e^2 A B_0}{m^2 i\omega^2 c\gamma_0}e^{-i[\omega t-(k+k_w)z]} \qquad (8)$$

Using Eq.(8) in equation of continuity $\partial n/\partial t + \nabla.(n\vec{v}) = 0$, the electron density perturbation n' at $(\omega_1, k+k_w)$ is obtained as

$$n' = \frac{(\vec{k}+\vec{k}_w).\vec{v}'\, n_0}{\omega} = \frac{e^2 A B_0 (k+k_w) n_0}{m^2 i \omega^3 c \gamma_0} e^{-i[\omega t-(k+k_w)z]}. \tag{9}$$

This electron density perturbation beats with $\vec{v}_{\omega,k}$ to give second harmonic nonlinear current density

$$\vec{J}^{NL}_{2\omega, 2k+k_w} = -\frac{1}{2} n' e \vec{v}_{\omega,k}$$

$$= (\hat{x} + i\hat{y}) \frac{n_0 e^4 A^2 B_0}{2 m^3 \gamma_0^2 \omega^4 c} (k+k_w) e^{-i[2\omega t-(2k+k_w)z]}. \tag{10}$$

There also exists a self consistent second harmonic field $\vec{E}_{2\omega} = (\hat{x}+i\hat{y}) A_2\, e^{-i[2\omega t-(2k+k_w)z]}$. The linear current density $\vec{J}^L_{2\omega}$ due to $\vec{E}_{2\omega}$ is,

$$\vec{J}^L_{2\omega} = -\frac{n_0 e^2 \vec{E}_{2\omega}}{2 i m \omega}. \tag{11}$$

Second harmonic field

The wave equation governing the third harmonic field is

$$\frac{\partial^2 \vec{E}_{2\omega}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \vec{E}_{2\omega}}{\partial t^2} - \frac{1}{c^2}\frac{\partial \vec{J}^L_{2\omega}}{\partial t} = \frac{-4\pi}{c^2} 2i\omega \vec{J}^{NL}_{2\omega} = (\hat{x} + i y) Q_2\, e^{-i[2\omega t-(2k+k_w)z]}, \tag{12}$$

where , $Q_2 = -i \dfrac{\omega_p^2}{\omega^2} \dfrac{a A}{\gamma_0^2} \dfrac{\omega_c}{c} (k+k_w)$, $\omega_c = \dfrac{eB_0}{mc}$,and $a = \dfrac{eA}{m\omega c}$.

On further simplification of Eq. (12) considering the group velocity of third harmonic as *c* and $k_2 = (2\omega/c)\left(1-\omega_p^2/8\omega^2\gamma_0\right)$, we obtain

$$\frac{\partial A_2}{\partial z} + \frac{1}{c}\frac{\partial A_2}{\partial t} = \frac{Q_2}{2ik_2}. \tag{13}$$

Introducing a new set of variables $z' = z, t' = t - z/c$, Eq. (13) reduces to

$$\frac{\partial A_2}{\partial z'} = \frac{Q_2}{2ik_2} e^{-i\Delta z'}, \tag{14}$$

where, $\Delta = k_2 - 2k - k_w$. For the Gaussian pulse $a^2 = a_0^2 \exp(-t'^2/\tau^2)$, $a_0 = eA_0/m\omega c$,

$\gamma_0 = \left[1 + \left(a_0^2/2\right)\exp\left(-t'^2/\tau^2\right)\right]^{1/2}$ is a function of time . For a given k_w, one cannot have phase matching $(\Delta = 0)$ for harmonic generation at all times. If one matches the wiggler wave number at the peak of the laser pulse $(t' = 0)$, i.e.

$$k_w \approx \frac{3}{4}\frac{\omega}{c}\frac{\omega_p^2}{\omega^2 \gamma_{00}},$$

(15)

where $\gamma_{00} = \left(1 + a_0^2/2\right)^{1/2}$.

At all other times we have.

$$\Delta = k_w \left(\frac{\gamma_{00}}{\gamma_0} - 1 \right),$$

(16)

and Eq. (14) gives.

$$A_2 = \frac{Q_2 \left[e^{-i\Delta\,(\gamma_{00}/\gamma_0 - 1)z'} - 1 \right]}{2k_2 k_w \left(\gamma_{00}/\gamma_0 - 1 \right)}. \quad (17)$$

At a distance z = L

$$\left| \frac{A_2}{A} \right| = \left| \frac{\omega_p}{2\omega} \frac{a}{\gamma_0^2} \frac{\omega_c}{\omega} \frac{(k + k_w)}{2k_2 k_w (\gamma_{00}/\gamma_0 - 1)} \exp\left[i k_w (\gamma_{00}/\gamma_0 - 1) L - 1 \right] \right|. \quad (18)$$

In Fig.(3) we have shown the variation of $\left|A_2/A\right|$ with t'/τ for $\omega_p/\omega = 0.1$ and 0.25. The other parameters are: $a_0 = 1$, $L\omega_p/c = 10^3$, $\omega_c/\omega = 0.001$. The efficiency increases with plasma density.

Results and Discussion

The application of a wiggler magnetic field provides the additional momentum required to generate resonant second harmonic generation. It also provides the necessary transverse electron velocity for the process. The phase matching condition for a short duration laser pulse can be satisfied only for an instant and for the remaining period the process is off resonant. The peak efficiency is observed only for the instant when the phase condition is satisfied and drops of sharply away from the resonance. The duration of resonance can be increased if a tapered wiggler or a plasma with tapered electron density is used. For the parameters mentioned above, they can be realized by employing a CO_2 laser (10.6μm, 10^{16}W/cm^2) in a plasma of electron density ~10^{17}cm^{-3}, wiggler with λ_w=0.5mm, B_0=100kG and plasma length L=10cm.

Conclusion

A magnetic wiggler can be applied to generate resonant second harmonic radiation from a short pulse Gaussian laser pulse in plasmas. The analysis shows a temporal evolution of the intensity of harmonic radiation and because the phase matching conditions are satisfied only for a very short duration the peak intensity depends upon the wiggler wave number chosen for that instant only. The study is useful for free electron lasers and one can have peak intensity for a larger duration if one applies a staged magnetic field or a tapered magnetic wiggler. A guide magnetic field can also be applied to exploit the cyclotron resonance condition in the millimeter wave range.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

HJ carried out the analysis and drafted the manuscript, JP provide the concept, did the numerical analysis and proof read the manuscript.

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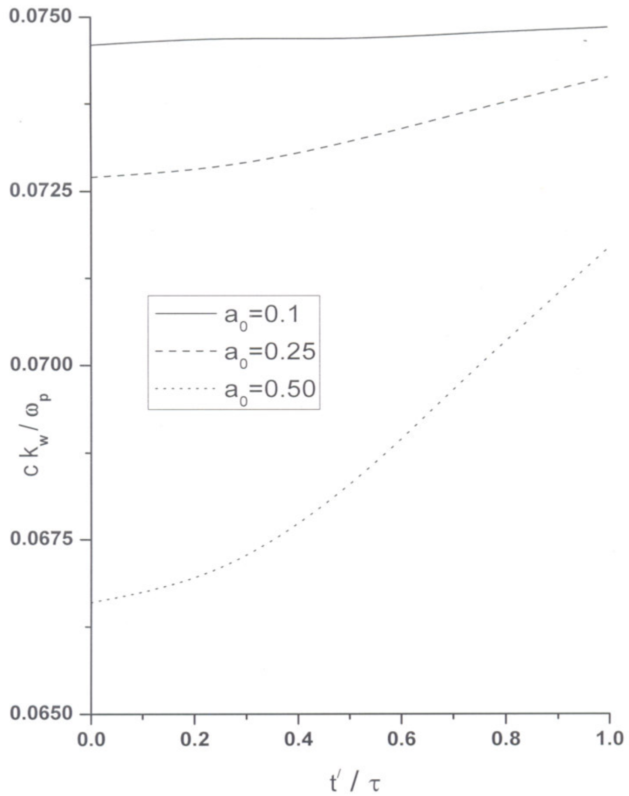

Fig.1 Variation of normalized wiggler wave number with for a0=0.1, 0.25 and 0.50 respectively at .

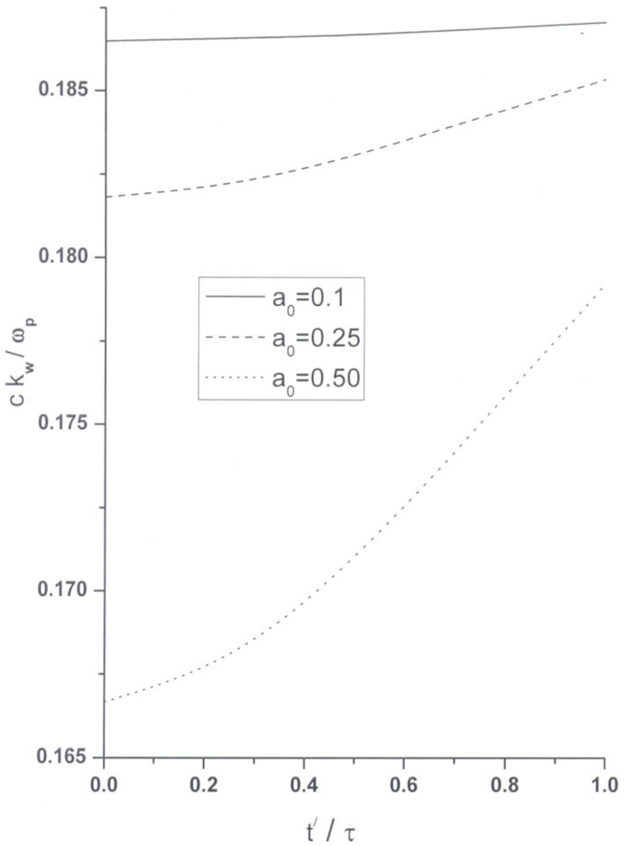

Fig.2 Variation of normalized wiggler wave number $c k_w / \omega_p$ with t' / τ for a_0=0.1, 0.25 and 0.50 respectively at $\omega_p / \omega = 0.25$.

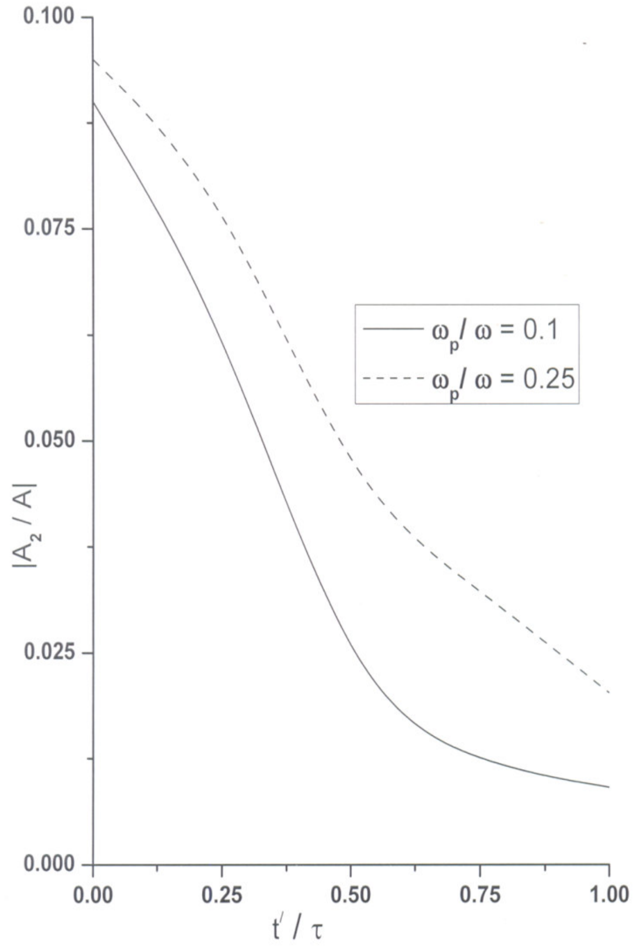

Fig.3 Variation of normalized second harmonic field $\left|A_2 / A\right|$ with $t^{/} / \tau$ for $\omega_p / \omega = 0.1$ and 0.5 respectively, the other parameters are a_0=0.1, $\omega_c / \omega = 0.01$ and $L\omega_p / c = 10^3$.