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Combined Influence of Chemical Reaction, Dissipation and Radiation Absorption on Convective Heat and Mass Transfer Flow in a Non-Uniformly Heated Vertical Channel

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Abstract
In this paper, we investigate the effect of chemical reaction and dissipation on mixed convective heat and mass transfer flow of a viscous, electrically conducting and incompressible fluid in a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. The viscous dissipation is taken into account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, temperature, concentration and the rate of heat and mass transfer have been analyzed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.
Keywords: Chemical Reaction, Dissipation, Radiation absorption, Heat and mass transfer, Vertical channel.

1. INTRODUCTION

Flows which arise due to the interaction of the gravitational force and density differences caused by the simultaneous diffusion of thermal energy and chemical species, have many applications in geophysics and engineering. Such thermal and mass diffusion plays a dominant role in a number of technological and engineering systems. For example, in controlling surface temperature by evaporation, cooling controlling polymerization reaction products by injecting suitable reactants along the porous wall of the reactor, distillation of volatile components from a mixture with non – volatiles, are a few technological process in which mass transfer accompanies the transfer of heat. Likewise such combined heat and mass transfer occurs in natural environmental vaporization of mist and fog, evaporation in terrestrial bodies of water viz. the oceans , rivers and the ponds, drying of food grains, simultaneous diffusion of metabolic heat and perspiration to control body temperature. Obviously the understanding of this transport process is desirable in order to effectively control the overall transport characteristics. The combined effect of thermal and mass diffusion in channel flows has been studied in the recent times by a few authors.

The analysis of heat transfer in a viscous heat generating fluid is important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids. The Volumetric heat generation has been assumed to be constant or a function of space variable. For example a hypothetical core-disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris as horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate. The heat losses from the geothermal system in some cases can be treated as if the heat comes from the heat generating sources. In the above processes the bounding walls are maintained at constant temperature. However, there are a few physical situations which warrant the boundary temperature to be maintained non-uniform.

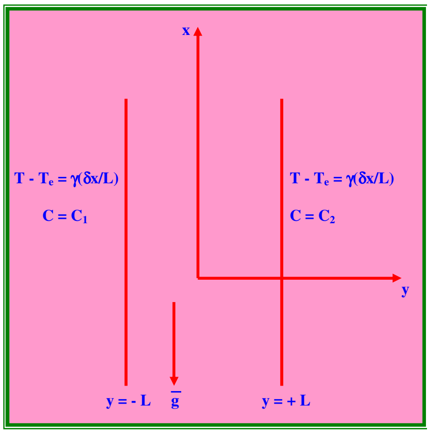
It is evident that in forced or free convection flow in a channel (pipe) a secondary flow can be created either by corrugating the boundaries or by maintaining non-uniform wall temperature such a secondary flow may be of interest in a few technological processes. For example in drawing optical glass fibers of extremely low loss and wide bandwidth, the process of modified chemical vapour deposition (MCVD) has been suggested in recent times. Performs from which these fibers are drawn are made by passing a gaseous mixture into a fused – silica tube which is heated locally by an oxy-hydrogen flame. Particulates of SiO_2- GeO_2 composition are formed from the mixture and collect on the interior of the tube. Subsequently these are fined to form a vitreous deposit as the flame traversed along the tube. The deposition is carried out in the radial direction as the flame traversed along the tube. The deposition is carried out in the radial direction through the secondary flow created due to non-uniform. Keeping these applications in view several authors have studied the heat transfer and heat and mass transfer in different configurations under varied conditions.

Rajeswara Rao [1986] has studied the combined free and forced convection flow of a viscous incompressible electrically conducting fluid in a vertical channel whose boundaries are maintained at non-uniform temperatures. Sailaja et.al. [2012] have discussed the convective heat transfer flow of a viscous electrically conducting fluid in a non-uniformly heated axially varying pipe. Ravindranth et.al. [2010] have analyzed the computational hydro magnetic mixed convective heat and mass transfer flow through a porous medium in a non-uniformly heated vertical channel with heat sources and dissipation. Sreenivasa Reddy [2006] has discussed the convective heat and mass transfer flow of a viscous fluid through a porous medium in a horizontal wavy channel maintained at non –uniform temperature. Ramakrishna Reddy [2007] has investigated the effect of thermo diffusion on hydro magnetic convective heat and mass transfer flow through a porous medium in a vertical channel maintained at non uniform temperature. Reddaiah et.al.[2012] have discussed the effect of radiation on hydro magnetic convective heat transfer flow of a viscous electrically conducting fluid in a non- uniformly heated vertical channel. Vijayabhaskar Reddy et.al.[2009] has analyzed the combined influence of radiation and thermo-diffusion on convective heat and mass transfer flow of a viscous fluid through a porous medium in vertical channel whose walls are maintained at non-uniform temperatures. Suneela [2012] has discussed the effect of non-uniform wall temperature on convective heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium in a vertical channel with dissipative effects. Vekatesulu [2012] has considered the Effect of chemical reaction and thermo-diffusion on convective heat and mass transfer flow of a viscous fluid through a porous medium in a non-uniformly heated vertical channel. Umadevi et.al. [2012] have investigated the effect of chemical reaction on double diffusive flow in a non-uniformly heated vertical channel.

In this paper, we discuss the effect of chemical reaction and dissipation on mixed convective heat and mass transfer flow of a viscous, electrically conducting, and incompressible fluid in a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. The viscous dissipation is taken into account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, temperature, concentration and the rate of heat and mass transfer have been analyzed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.

2. FORMULATION OF THE PROBLEM

We analyse the steady motion of viscous electrically conducting incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source and the concentration on these walls are taken to be constant. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy dissipations and the joule heating are taken into account in the energy equation. Also the kinematic viscosity ν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system O(x ,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y = ± L. The equations governing the steady flow, heat and mass transfer are



Configuration of the Problem

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Equation of linear momentum:

$$\rho_e\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g-(\sigma\mu_e^2 H_o^2)u \tag{2}$$

$$\rho_e\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Equation of Energy:

$$\rho_e C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q+\mu\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right)+$$
$$+(\sigma\mu_e^2 H_o^2/\lambda)(u^2+v^2)+Q_1^{'}(C-C_e) \tag{4}$$

Equation of Diffusion:

$$(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y})=D_1(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2})-k_1'C \tag{5}$$

Equation of State:

$$\rho-\rho_e=-\beta\rho_e(T-T_e)-\beta^{\bullet}\rho_e(C-C_e) \tag{6}$$

where ρ_e is the density of the fluid in the equilibrium state, Te ,Ce are the temperature and Concentration in the equilibrium state, (u,v) are the velocity components along O(x ,y) directions, p is the pressure, T ,C are the temperature and Concentration in the flow region, ρ is the density of the fluid, μ is the constant coefficient of viscosity , Cp is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, μe is the magnetic permeability of the medium , σ is the electrical conductivity, β is the coefficient of thermal expansion, $\beta^{\bullet}$ is the coefficient of expansion with mass fraction , D1 is the molecular diffusivity , Q is the strength of the constant internal heat source , and k11 is the cross diffusivity and k1 is chemical reaction coefficient.
The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q=\frac{1}{L}\int_{-L}^{L}u\,dy \quad . \tag{8}$$

The boundary conditions for the velocity and temperature fields are

$$u=0\ ,\ v=0\ ,\ T-T_e=\gamma(\delta x/L) \quad \text{on } y=\pm L$$
$$C=C1 \quad \text{on } y=-L\ ,\ C=C2 \quad \text{on } y=+L \tag{9}$$

γ is chosen to be twice differentiable function, δ is a small parameter characterizing the slope of the temperature variation on the boundary.
In view of the continuity equation we define the stream function ψ as

$$u=-\psi y,\ v=\psi x \tag{10}$$

the equation governing the flow in terms of ψ are

$$[\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}]=\nu(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2})^2-\beta g\frac{\partial T}{\partial y}-$$
$$-\beta^{\bullet}g\frac{\partial C}{\partial y}-(\frac{\sigma\mu_e^2H_o^2}{\rho})\frac{\partial^2\psi}{\partial y^2} \tag{11}$$

$$\rho_eC_p(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\lambda\nabla^2\theta+Q+\mu((\frac{\partial^2\psi}{\partial y^2})^2+(\frac{\partial^2\psi}{\partial x^2})^2)+$$
$$+(\frac{\mu}{k}+\sigma\mu_e^2H_o^2)((\frac{\partial\psi}{\partial x})^2+(\frac{\partial\psi}{\partial y})^2))+Q_1'(C-C_e) \tag{12}$$

$$(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})=D_1(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2})-k_1'C \tag{13}$$

Introducing the non-dimensional variables in (11) - (13) as

$$(x',\ y')=(x,y)/L,\ (u',v')=(u,v)/q,\theta=\frac{T-T_e}{\Delta T_e},C^{\bullet}=\frac{C-C_1}{C_2-C_1}$$
$$p'=\frac{p_D}{\rho_e q^2},\quad \gamma'=\frac{\gamma}{\Delta T_e} \tag{14}$$

(under the equilibrium state $\Delta T_e=T_e(L)-T_e(-L)=\frac{QL^2}{\lambda}$) the governing equations in the non-dimensional form (after dropping the dashes) are

$$R\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}=\nabla^4\psi-\frac{G}{R}(\theta_y+NCy)-M^2\frac{\partial^2\psi}{\partial y^2} \tag{15}$$

and the energy and diffusion equations in the non-dimensional form are

$$PR\left(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\nabla^2\theta+\alpha+\left(\frac{PR^2E_c}{G}\right)\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right)+ +(M^2)\left(\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right) \tag{16}$$

$$RSc\left(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right)=\left(\frac{\partial^2C}{\partial x^2}+\frac{\partial^2C}{\partial y^2}\right)-k_1C \tag{17}$$

where

$R=\frac{qL}{\nu}$ (Reynolds number), $G=\frac{\beta g\Delta T_eL^3}{\nu^2}$ (Grashof number),

$M^2=\frac{\sigma\mu_e^2H_o^2L^2}{\upsilon^2}$ (Hartmann Number), $\mathrm{P}=\frac{\mu c_p}{\lambda}$ (Prandtl number) ,

$E_c=\frac{\beta gL^3}{C_p}$ (Eckert number), $N=\frac{\beta^\bullet\Delta C}{\beta\Delta T}$ (Buoyancy Number)

$Sc=\frac{\nu}{D_1}$ (Schmidt number), $\alpha=\frac{QL^2}{\Delta TC_p}$ (Heat source parameter)

$Q_1=\frac{Q_1'(C_w-C_e)L^2}{\lambda(T_w-T_e)}$ (Radiation absorption parameter),

$k_1=\frac{k_1'L^2}{D_1}$ (Chemical reaction parameter)

The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=-1,\quad \frac{\partial\psi}{\partial x}=0,\ \frac{\partial\psi}{\partial y}=0\quad at\ y=\pm1 \tag{18a}$$

$$\theta(x,y)=\gamma(\delta x)\qquad on\quad y=\pm1 \tag{18b}$$

$$C=0\ \text{ on } y=-1,\qquad C=1\ \text{ on } y=1 \tag{18c}$$

$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0\qquad at\quad y=0 \tag{19}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function γ(x) .

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the transformation

$$\bar{x}=\delta x$$

With this transformation the equations (15) - (17) reduce to

$$R\delta\frac{\partial(\psi,F^2\psi)}{\partial(x,y)}=F^4\psi-\frac{G}{R}(\theta_y+NC_y)-M^2\frac{\partial^2\psi}{\partial y^2} \tag{20}$$

and the energy &diffusion equations in the non-dimensional form are

$$PR\delta(-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=F^2\theta+\alpha+(\frac{PR^2E_c}{G})((\frac{\partial^2\psi}{\partial y^2})^2+\delta^2(\frac{\partial^2\psi}{\partial x^2})^2)+$$
$$+(M^2)(\delta^2(\frac{\partial\psi}{\partial x})^2+(\frac{\partial\psi}{\partial y})^2))+Q_1C \qquad (21)$$

$$\delta RSc(-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})=F^2C-k_1C \qquad (22)$$

where

$$F^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

for small values of the slope δ,the flow develops slowly with axial gradient of order δ and hence we take

$$\frac{\partial}{\partial\overline{x}}\approx O(1)$$

We follow the perturbation scheme and analyse through first order as a regular perturbation problem at finite values of R, G, P, Sc and D-1

Introducing the asymptotic expansions

$\psi(x,y)=\psi_0(x,y)+\delta\psi_1(x,y)+\delta^2\psi_2(x,y)+\ldots\ldots$

$\theta(x,y)=\theta_0(x,y)+\delta\theta_1(x,y)+\delta^2\theta_2(x,y)+$

$C(x,y)=C_0(x,y)+\delta C_1(x,y)+\delta^2 C_2(x,y)+\ldots.$ (23)

On substituting (22) in (19) – (21) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyy}-M_1^2\psi_{0,yy}=\frac{G}{R}(\theta_{0,y}+NC_{0,y}) \qquad (24)$$

$$\theta_{0,yy}=-\alpha-\frac{PR^2Ec}{G}\psi_{0,yy}^2-\frac{PM_1^2Ec}{G}\psi_{0,y}^2-Q_1C_0 \qquad (25)$$

$C_{0,yy}-k\,C_0=0$ (26)

with

$\psi_0(+1)-\psi_0(-1)=-1$,

$\psi_{0,y}=0$, $\psi_{0,x}=0$ at $y=\pm1$ (27)

$\theta_0(\pm1)=\gamma(x)$ at $y=\pm1$

$C_0(-1)=0$ $C_0(+1)=1$ (28)

and to the first order are

$$\psi_{1,yyyy}-M_1^2\psi_{1,yy}=-\frac{G}{R}(\theta_{1,y}+NC_{1,y})+R(\psi_{0,y}\psi_{0,xyy}-\psi_{0,x}\psi_{0,yyy}) \qquad (29)$$

$$\theta_{1,yy}=P\,R(\psi_{0,x}\theta_{0,y}-\psi_{0,y}\theta_{0,x})-\frac{PEc}{G}(R^2\psi_{1,yy}^2+M_1^2\psi_{1,y}^2)-Q_1C_1 \qquad (30)$$

$C_{1,yy}-(k\,Sc)C_1=R\,Sc(\psi_{0,y}C_{0,x}-\psi_{0,x}C_{0,y})$ (31)

$\psi_1(+1)-\psi_1(-1)=0$,

$\psi_{1,y}=0$, $\psi_{1,x}=0$ at $y=\pm1$ (32)

$\theta_1(\pm1)=0$ at $y=\pm1$ (33)

$C_1(-1)=0$, $C_1(+1)=0$ (34)

Assuming Ec<<1 to be small we take the asymptotic expansions as

$$\psi_0(x,y)=\psi_{00}(x,y)+Ec\,\psi_{01}(x,y)+\ldots\ldots\ldots\ldots$$
$$\psi_1(x,y)=\psi_{10}(x,y)+Ec\,\psi_{11}(x,y)+\ \ldots\ldots\ldots\ldots$$
$$\theta_0(x,y)=\theta_{00}(x,y)+Ec\,\theta_{01}(x,y)+\ldots\ldots\ldots\ldots\ldots$$
$$\theta_1(x,y)=\theta_{10}(x,y)+Ec\,\theta_{11}(x,y)+\ldots\ldots\ldots\ldots\ldots$$
$$C_0(x,y)=C_{00}(x,y)+Ec\,C_{01}(x,y)+\ldots\ldots\ldots\ldots\ldots$$

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$$C_1(x,y)=C_{10}(x,y)+EcC_{11}(x,y)+.............. \tag{35}$$

Substituting the expansions (34) in equations (23)-(33) and separating the like powers of Ec we get the following equations

$$\theta_{00,yy}=-\alpha-Q_1C_{00} \quad , \quad \theta_{00}(\pm1)=\gamma(\bar{x}) \tag{36}$$

$$C_{00,yy}-(kSc)C_{00}=0 \ , \quad C_{00}(-1)=0, C_{00}(+1)=1 \tag{37}$$

$$\left.\begin{array}{l}\psi_{00,yyyy}-M_1^2\psi_{00,yy}=-\dfrac{G}{R}(\theta_{00,y}+NC_{00,y}) \ , \\ \psi_{00}(+1)-\psi_{00}(-1)=1, \psi_{00,y}=0, \psi_{00,x}=0 \ \ at \ \ y=\pm1\end{array}\right| \tag{38}$$

$$\theta_{01,yy}=-\frac{PM_1^2}{G}\psi^2{}_{00,y}-\frac{PR^2}{G}\psi^2{}_{00,yy}-Q_1C_{01} \quad , \quad \theta_{01}(\pm1)=0 \tag{39}$$

$$C_{01,yy}-(kSc)C_{01}=0 \quad , \quad C_{01}(\pm1)=0 \tag{40}$$

$$\left.\begin{array}{l}\psi_{01,yyyy}-M_1^2\psi_{01,yy}=-\dfrac{G}{R}(\theta_{01,y}+NC_{01,y}) \ , \\ \psi_{01}(+1)-\psi_{01}(-1)=0, \\ \psi_{01,y}=0, \psi_{01,x}=0 \ \ at \ \ y=\pm1\end{array}\right| \tag{41}$$

$$\theta_{10,yy}=RP(\psi_{00,y}\theta_{00,x}-\psi_{00,x}\theta_{00,y})-Q_1C_{10} \quad \theta_{10}(\pm1)=0 \tag{42}$$

$$\left.\begin{array}{l}C_{10,yy}-(kSc)C_{10}=RP_1(\psi_{00,y}C_{00,x}-\psi_{00,x}C_{00,y}) \\ \qquad\qquad C_{10}(\pm1)=0\end{array}\right| \tag{43}$$

$$\left.\begin{array}{l}\psi_{10,yyyy}-M_1^2\psi_{10,yy}=-\dfrac{G}{R}(\theta_{10,y}+NC_{10,y})+ \\ +R(\psi_{00,y}\psi_{00,xyy}-\psi_{00,x}\psi_{00,yyy}) \ , \\ \psi_{10}(+1)-\psi_{10}(-1)=0, \psi_{10,y}=0, \psi_{10,x}=0 \ \ at \ \ y=\pm1\end{array}\right| \tag{44}$$

$$\theta_{11,yy}=RP(\psi_{00,y}\theta_{1,x}-\psi_{1,x}\theta_{00,y})-Q_1C_{11} \quad , \quad \theta_1(\pm1)=0 \tag{45}$$

$$C_{11,yy}-(kSc)C_{11}=RSc(\psi_{00,y}C_{00,x}-\psi_{1,x}C_{00,y}), C_{11}(\pm1)=0 \tag{46}$$

$$\begin{array}{l}\psi_{11,yyyy}-M_1^2\psi_{1,yy}=-\dfrac{G}{R}(\theta_{11,y}+NC_{11,y})+R(\psi_{00,y}\psi_{11,xyy}- \\ -\psi_{00,x}\psi_{01,yyy}+\psi_{01,y}\psi_{00,xyy}-\psi_{01.x}\psi_{00,yyy}) \ , \\ \psi_{11}(+1)-\psi_{11}(-1)=0, \psi_{11,y}=0, \psi_{11,x}=0 \ \ at \ \ y=\pm1\end{array} \tag{47}$$

where M12 = M2

Solving the equations (35) - (43) subject to the relevant boundary conditions we obtain

$$C_{00}=0.5(\frac{Ch(\beta_1 y)}{Ch(\beta_1)}+\frac{Sh(\beta_1 y)}{Sh(\beta_1)}))$$

$$\theta_{00}=0.5(1-y^2)+a_3(\cosh\beta_1-\cosh\beta_1 y)+a_4(\sinh\beta_1 y-y\sinh\beta_1)+\gamma(x)$$

$$\left.\begin{array}{l}\psi_{00}=a_9Ch(M_1y)+a_{10}Sh(M_1y)+a_{11}y+\phi_1(y) \\ \phi_1(y)=-a_6y^3-a_7Sh(\beta_1y)-a_8Ch(\beta_1y)\end{array}\right|$$

$$\theta_{01} = 0.5a_{25}(1-y^2) + \frac{a_{26}}{4M_1^2}(Ch(2M_1) - Ch(2M_1 y)) + \frac{a_{27}}{4\beta_1^2}(Ch(2\beta_1) - Ch(2\beta_1 y)) +$$
$$+\frac{a_{28}}{\beta_2^2}(Ch(\beta_2 y) - Ch(\beta_2)) - \frac{a_{29}}{\beta_2^2}(Ch(\beta_3 y) - Ch(\beta_3)) + \frac{a_{30}}{30}(1-y^6) + \frac{a_{31}}{12}(y^4 - 1)$$

$$C_{01} = a_{33}(Ch(2M_1 y) - Ch(2M_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{34}(Ch(2\beta_1 y) - Ch(2\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) +$$
$$- a_{35}(Ch(\beta_2 y) - Ch(\beta_2)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{36}(Ch(\beta_3 y) - Ch(\beta_3)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) +$$
$$- a_{37}(y^4 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)}) - a_{38}(y^2 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$

$$\psi_{01} = a_{55}Ch(M_1 y) + a_{56}Sh(M_1 y) + a_{57}y + +a_{58} + \phi_2(y)$$
$$\phi_2(y) = a_{47}y^2 + a_{48}y^3 + a_{49}y^5 + a_{50}Sh(2M_1 y) + a_{51}Sh(2\beta_1 y) + a_{52}Sh(\beta_2 y) +$$
$$+ a_{53}Sh(\beta_3 y) + a_{54}Sh(\beta_1 y)$$

$$\theta_{10} = a_{71}y^2 + a_{72}y^3 + a_{73}y^4 + a_{74}y^6 + a_{75}Ch(M_1 y) + a_{76}Sh(M_1 y) +$$
$$+ a_{77}Sh(\beta_1 y) + a_{78}Ch(\beta_1 y) + a_{79}yCh(M_1 y) + a_{80}ySh(M_1 y) +$$
$$+ a_{81}ySh(\beta_1 y) - a_{82}yCh(\beta_1 y) + a_{83}y + a_{84}$$

$$C_{10} = b_{33}Ch(\beta_1 y) + b_{34}Sh(\beta_1 y) + \phi_3(y)$$
$$\phi_3(y) = -b_{13} - b_{14}y - b_{15}y^2 - b_{16}y^4 + b_{17}Ch(M_1 y) + b_{18}Sh(M_1 y) + b_1 ySh(\beta_1 y) +$$
$$+ b_{20}Ch(\beta_2 y) + b_{21}Ch(\beta_3 y) + b_{22}Sh(\beta_2 y) + b_{23}Sh(\beta_3 y) + b_{24}Ch(2\beta_1 y) +$$
$$+ b_{25}Sh(2\beta_1 y) + b_{26}yCh(M_1 y) + b_{27}y^2 Ch(\beta_1 y) + b_{28}y^2 Sh(\beta_1 y) + b_{29}y^2 Sh(M_1 y) +$$
$$+ b_{30}y^2 y^2 Ch(M_1 y) + b_{31}y^3 Ch(\beta_1 y) + b_{32}yCh(\beta_1 y)$$

$$\psi_{10} = db_8 Ch(M_1 y) + b_9 Sh(M_1 y) + b_{10}y + b_{11} + \phi(y)$$
$$\phi_4(y) = d_{28}y + d_{29}y^2 + d_{30}y^3 + d_{31}y^4 + d_{32}y^5 + d_{33}y^6 + d_{34}y^7 + (d_{35}y + d_{39}y^2$$
$$+ d_{53}y^3)Ch(M_1 y) + (d_{36}y + d_{40}y^2 + d_{52}y^3)Sh(M_1 y) + (d_{37} + d_{42}y^2 + d_{50}y^3 +$$
$$+ d_{54}y^4)Ch((\beta_1 y) + (d_{38} + d_{41}y^2 + d_{51}y^3 + d_{55}y^4)Sh(\beta_1 y) + d_{45}y^S Ch(\beta_2 y) +$$
$$+ d_{44}Sh(\beta_2 y) + d_{45}Sh(\beta_3 y) + d_{46}Ch(\beta_2 y) + d_{47}yCh(\beta_2 y) + d_{48}Sh(2\beta_1 y) + d_{49}Ch(2\beta_1 y)$$

where $\beta_1 2 = k\ Sc1$

4. NUSSELT NUMBER and SHERWOOD NUMBER

The local rate of heat transfer coefficient (Nusselt number (Nu)) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w}\left(\frac{\partial \theta}{\partial y}\right)_{y=\pm 1} \qquad \theta_m = 0.5\int_{-1}^{1} \theta\, dy$$

and the corresponding expressions are

$$(Nu)_{y=+1}=\frac{(-1+Ec\,d_{64}+\delta\,d_{66})}{(\theta_{m8}-\gamma(x))}\qquad (Nu)_{y=-1}=\frac{(1+Ec\,d_{64}+\delta\,d_{67})}{(\theta_m-\gamma(x))}$$

$$\theta_m=d_{68}+Ec\,d_{69}+\delta\,d_{70}$$

,

The local rate of mass transfer coefficient (Sherwood Number (Sh)) on the walls has been calculated using the formula

$$Sh=\frac{1}{C_m-C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm1}\quad \text{where}\quad C_m=0.5\int_{-1}^{1}C\,dy$$

and the corresponding expressions are

$$(Sh)_{y=+1}=\frac{(d_{71}+Ec\,d_{73}+\delta\,d_{75})}{(C_m-1)}\qquad (Sh)_{y=-1}=\frac{(d_{72}+Ec\,d_{74}+\delta\,d_{76})}{(C_m)}$$

where $C_m=d_{77}+Ec\,d_{78}+\delta\,d_{79}$

5. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis, we investigate the combined influence of chemical reaction, radiation absorption on hydromagnetic convective heat and mass transfer flow of a viscous, electrically conducting fluid in a non-uniformly heated vertical channel in the presence of heat sources. We take $\gamma(x)=\alpha_1\sin(\bar{x})$. The non-linear coupled equations governing the flow, heat and mass transfer are solved by using a regular perturbation technique with the slope δ, of the boundary temperature as a perturbation parameter.

The axial velocity (u) is exhibited in figs. 1-3 for different values of k, Q_1, Ec, α_1. Fig. 1 represents u with chemical reaction parameter k and radiation absorption parameter Q_1. An increase in k results in an enhancement in |u|. Also u exhibits a reversal flow in the right half at $Q_1 = 2.0$ and for higher $Q_1 = 3.0$, it spreads to the entire flow region, |u| reduces with increase in $Q_1 \le 2.5$ and enhances with higher $Q_1 \ge 3.0$. The variation of u with the amplitude α_1 of the boundary temperature shows that higher the amplitude $\alpha_1 \le 0.7$, smaller the axial velocity and for higher $\alpha_1 \ge 0.9$ we notice an enhancement in |u| (fig. 2). With reference to Eckert number Ec we find that higher the dissipative heat larger |u| in the entire flow region (fig. 3).

The secondary velocity (v) which is due to the non-uniform boundary temperature is shown in figs. 4-7 for different parametric values. From fig. 4 we find that |v| enhances with increase in $k \le 1.5$ and enhances with higher $k \ge 2.5$. An increase in the radiation absorption parameter Q_1 results in an enhancement in |v| (fig. 5). With reference to the amplitude α_1 we find that the higher the amplitude lesser the magnitude of v. (fig.6).The variation of v with Ec reveals that higher the dissipative heat larger the secondary velocity (fig. 7).

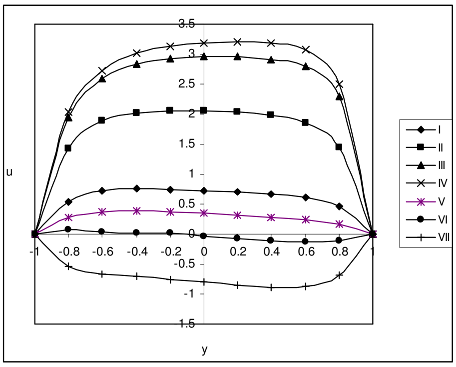

Fig. 1 : Variation of u with k & Q_1

	I	II	III	IV	V	VI	VII
k	0.5	1.5	2.5	3.5	0.5	0.5	0.5
Q_1	1	1	1	1	1.5	2	3

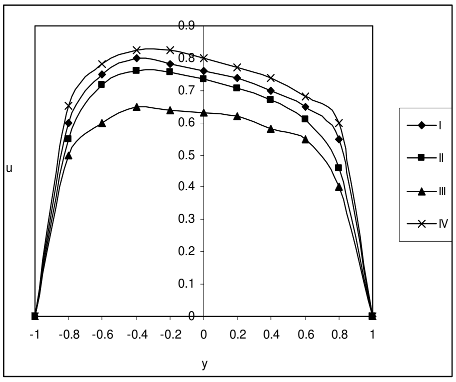

Fig. 2 : Variation of u with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

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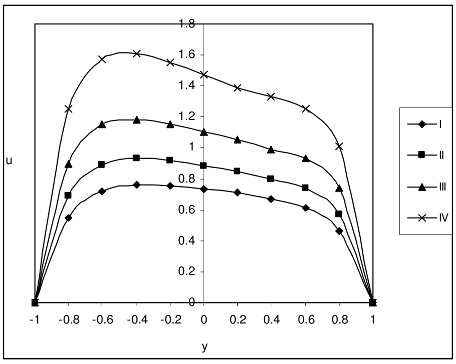

Fig. 3 : Variation of u with Ec

	I	II	III	IV
Ec	0.5	0.9	0.15	0.25

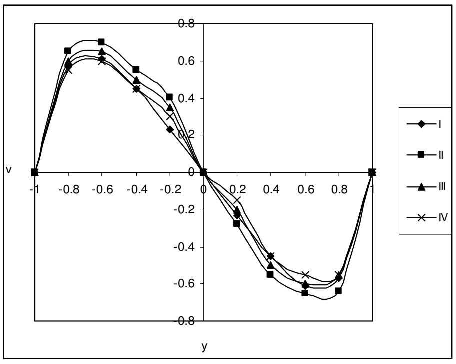

Fig. 4 : Variation of v with k

	I	II	III	IV
k	0.5	1.5	2.5	3.5

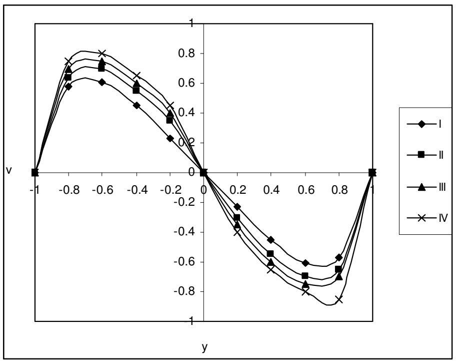

Fig. 5 : Variation of v with Q_1

	I	II	III	IV
Q_1	1	1.5	2	3

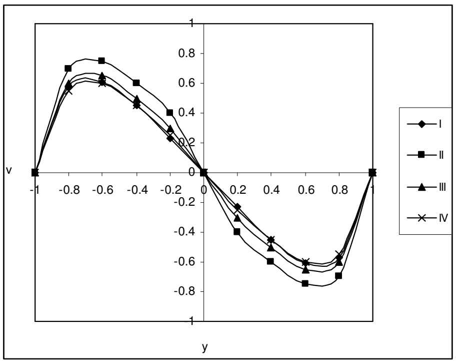

Fig. 6 : Variation of v with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

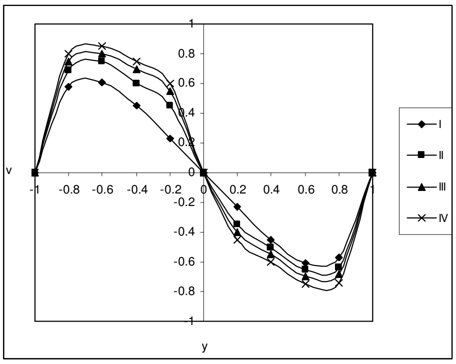

Fig. 7 : Variation of v with Ec

	I	II	III	IV
Ec	0.5	0.9	0.15	0.25

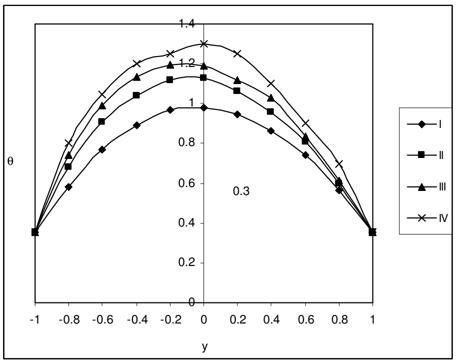

Fig. 8 : Variation of θ with k

	I	II	III	IV
k	0.5	1.5	2.5	3.5

The non-dimensional temperature (θ) is shown in figs. 8-11 for different parametric values. The non-dimensional temperature is positive for all variations. From fig.8, we notice an enhancement in θ with increase in the chemical reaction parameter k. An increase in the radiation absorption parameter Q_1 results in an enhancement in θ (fig.9). An increase in the amplitude α_1 of the boundary temperature leads to an enhancement in the actual temperature (fig. 10). With reference to Eckert number Ec it can be seen that the actual temperature enhances with Ec ≤ 0.07 and reduces with higher Ec ≥ 0.09. Thus higher the dissipative heat larger the actual temperature and for further higher dissipative heat lesser the actual temperature (fig. 11).

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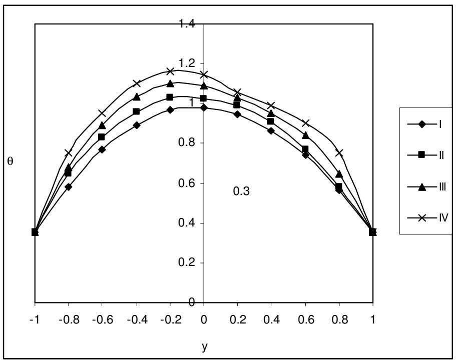

Fig. 9: Variation of θ with Q_1

	I	II	III	IV
Q_1	1	1.5	2	3

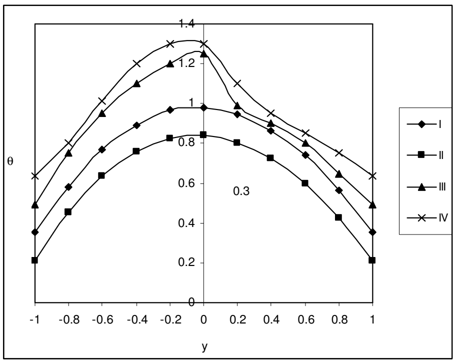

Fig. 10 : Variation of θ with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

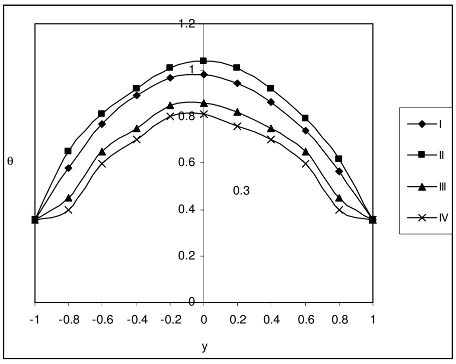

Fig. 11 : Variation of θ with Ec

	I	II	III	IV
Ec	0.5	0.9	0.15	0.25

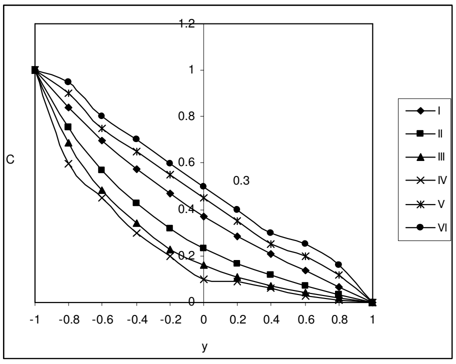

Fig. 12 : Variation of C with k & Q_1

	I	II	III	IV	V	VI
k	0.5	1.5	2.5	3.5	0.5	0.5
Q_1	1	1	1	1	2	3

The concentration distribution (C) is shown in figs. 12-14 for different parametric values. We follow the convention that the non-dimensional concentration is positive or negative according as the actual concentration is greater/lesser than C_2. The variation of C with chemical reaction parameter k and radiation absorption parameter Q_1 shows that the actual concentration reduces with k and enhances with Q_1 (fig. 12). Higher the amplitude of the boundary temperature larger the actual concentration (fig.13). Also higher the dissipative heat smaller the actual concentration (fig.14).

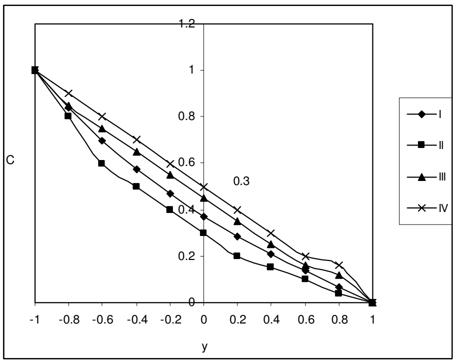

Fig. 13 : Variation of C with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

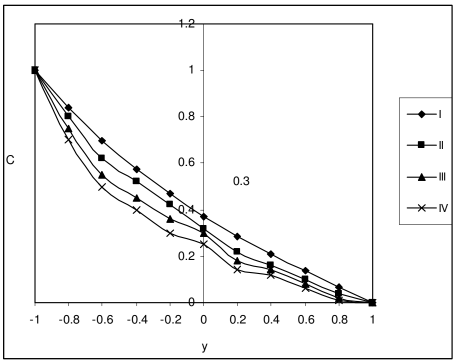

Fig. 14 : Variation of C with Ec

	I	II	III	IV
Ec	0.5	0.9	0.15	0.25

The rate of heat transfer (Nusselt number) at y = ±1 are depicted in tables 1-4 for different values of k, Q_1, α_1, Ec. With reference to the chemical reaction parameter k we find that the rate of heat transfer enhances with increase in $k \le 2.5$ and reduces with $k \ge 3.5$ at y = ±1. An increase in the radiation absorption parameter $Q_1 \le 2$ leads to an enhancement in |Nu| at y=±1 while for higher $Q_1 \ge 4$, |Nu| reduces at y = +1 and at y = -1, it reduces for $|G| \le 10^3$ and enhances for higher $|G| \ge 3x10^3$. The variation of Nu with the amplitude α_1 of the boundary temperature shows that |Nu| enhances with increase in $\alpha_1 \le 1.5$ and reduces with higher $\alpha_1 \ge 2.5$. With reference to Ec we find that |Nu| enhances with increase in Ec. Thus higher the dissipative heat larger the rate of heat transfer at both the walls.

Table – 1
Average Nusselt number (Nu) at y = +1

G	I	II	III	IV	V	VI
10^3	0.42344	7.54367	-9.8386	-4.1539	0.52393	0.4296
$3x10^3$	0.56889	1.84823	8.79494	6.1799	4.5511	3.0264
-10^3	0.42198	9.9823	-6.5881	-3.4435	0.5139	0.4777
$-3x10^3$	0.51530	-1.7942	-3.39194	-2.6419	-16.2858	-9.0214
k	**0.5**	**1.5**	**2.5**	**3.5**	**0.5**	**0.5**
Q_1	**1**	**1**	**1**	**1**	**2**	**4**

Table – 2
Average Nusselt number (Nu) at y = +1

G	I	II	III	IV	V	VI	VII
10^3	0.7115	0.93618	1.1627	1.2642	0.75999	0.63962	0.5522
$3x10^3$	2.4278	3.42817	4.2348	5.1246	2.9351	2.5661	2.2795
-10^3	0.67722	0.88914	1.11188	1.1249	0.71492	0.5977	0.5136
$-3x10^3$	4.7217	2.8337	-3.3117	-4.0212	9.1509	5.8631	4.3134
Ec	**0.03**	**0.05**	**0.07**	**0.09**	**0.05**	**0.05**	**0.05**
α_1	**0.5**	**0.5**	**0.5**	**0.5**	**1.5**	**2.5**	**3.5**

Table – 3
Average Nusselt number (Nu) at y = -1

G	I	II	III	IV	V	VI
10^3	-0.4468	-7.0416	7.1045	1.9498	-0.5691	-0.4835
$3x10^3$	-0.5991	-1.7340	-7.8182	-5.8219	-4.5741	-6.0452
-10^3	-0.4450	-9.2373	4.5605	1.4654	-0.5639	-0.5446
$-3x10^3$	-0.5432	1.6647	2.9469	0.9164	6.3766	9.0510
k	**0.5**	**1.5**	**2.5**	**3.5**	**0.5**	**0.5**
Q_1	**1**	**1**	**1**	**1**	**2**	**4**

Table – 4
Average Nusselt number (Nu) at y = -1

G	I	II	III	IV	V	VI	VII
10^3	-0.7515	-0.39586	-1.1627	-1.8942	0.7599	0.6396	0.5522
$3x10^3$	-2.4608	-3.444	-4.2348	-4.3904	2.9351	2.5661	2.2795
-10^3	-0.7192	-1.1164	-1.1119	-1.3309	0.7149	0.5977	0.5136
$-3x10^3$	-4.802	-20.9577	-34.3117	-13.484	9.1509	5.8631	4.3134
Ec	**0.03**	**0.05**	**0.07**	**0.09**	**0.05**	**0.05**	**0.05**
α_1	**0.5**	**0.5**	**0.5**	**0.5**	**1.5**	**2.5**	**3.5**

Table – 5
Sherwood number (Sh) at y = +1

G	I	II	III	IV	V	VI
10^3	2.5070	3.3492	3.4549	3.5640	2.5243	2.5457
$3x10^3$	1.2041	2.7134	2.9609	3.0377	1.2247	1.3012
-10^3	3.6274	4.3487	4.1435	4.2606	42.6153	57.9199
$-3x10^3$	-3.2312	4.6392	5.0845	5.1460	-3.9082	4.2953
k	**0.5**	**1.5**	**2.5**	**3.5**	**0.5**	**0.5**
Q_1	**1**	**1**	**1**	**1**	**2**	**4**

Table – 6
Sherwood number (Sh) at y = +1

G	I	II	III	IV	V	VI	VII
10^3	3.1395	2.5070	2.0897	0.6897	2.5076	2.4512	2.1249
$3x10^3$	1.7832	1.2041	0.8884	-11.4268	1.2041	1.1546	1.0149
-10^3	38.0366	37.6274	-40.6546	-1.6463	37.6274	35.0642	34.123
$-3x10^3$	-7.7780	-3.2312	-2.1378	2.5076	-8.2312	-3.469	-3.249
Ec	**0.03**	**0.05**	**0.07**	**0.09**	**0.05**	**0.05**	**0.05**
α_1	**0.5**	**0.5**	**0.5**	**0.5**	**1.5**	**2.5**	**3.5**

Table – 7
Sherwood number (Sh) at y = -1

G	I	II	III	IV	V	VI
10^3	-0.6396	-0.6131	-0.8114	-1.1715	-0.6097	-0.5579
$3x10^3$	-1.0393	-0.8218	-1.2474	-1.9398	-0.5556	0.1322
-10^3	-0.3301	-0.2281	-0.3135	-0.4034	-0.3124	-0.2820
$-3x10^3$	-0.0617	0.508	0.7855	-1.5144	0.501	0.0320
k	**0.5**	**1.5**	**2.5**	**3.5**	**0.5**	**0.5**
Q_1	**1**	**1**	**1**	**1**	**2**	**4**

Table – 8
Sherwood number (Sh) at y = -1

G	I	II	III	IV	V	VI	VII
10^3	-0.5314	-0.6396	-0.7742	-0.7751	-0.6394	-0.6396	-0.6399
$3x10^3$	-0.6501	-1.0393	-2.3837	-2.3942	-1.0396	-1.0399	-1.0406
-10^3	-0.3669	-0.3301	-0.3062	-0.3046	-0.3304	-0.3306	-0.3309
$-3x10^3$	-0.1625	-0.0617	0.0128	0.0134	-0.01619	-0.01621	-0.01624
Ec	**0.03**	**0.05**	**0.07**	**0.09**	**0.05**	**0.05**	**0.05**
α_1	**0.5**	**0.5**	**0.5**	**0.5**	**1.5**	**2.5**	**3.5**

The rate of mass transfer (Sherwood number) at y = ±1 are shown in tables 5-8 for different parametric values. The variation of Sh with k shows that the rate of mass transfer enhances with k at y = +1 while at y=-1, |Sh| reduces with k ≤ 1.5 and enhances with k ≥ 2.5. An increase in Q_1 enhances |Sh| at y=+1 and reduces at y = -1. The variation of Sh with the amplitude α_1 of the boundary temperature shows that |Sh| at y = +1 enhances with $\alpha_1 \leq 1.5$ and reduces with $\alpha_1 \geq 2.5$. At y = -1, |Sh| reduces with $\alpha_1 \leq 1.5$ and enhances with higher $\alpha_1 \geq 2.5$. The variation of Sh with Ec shows that |Sh| reduces with Ec ≤ 0.07 and for Ec≥0.09, |Sh| reduces for |G| ≤ 10^3 and enhances for |G| ≥ $3x10^3$. At y = +1, it enhances with Ec in the heating case and reduces in the cooling case.

6. CONCLUSIONS

1. An increase in k results in an enhancement in |u|, θ and depreciation in C. |v| enhances with increase in k ≤ 1.5 and enhances with higher k ≥ 2.5.
2. |u| reduces with increase in $Q_1 \leq 2.5$ and enhances with higher $Q_1 \geq 3.0$. An increase in the radiation absorption parameter Q_1 results in an enhancement in |v|, θ, C.
3. Higher the amplitude $\alpha_1 \leq 0.7$, smaller the axial velocity and for higher $\alpha_1 \geq 0.9$ we notice an enhancement in |u|. Higher the amplitude lesser the magnitude of v, enhances the actual temperature and the actual concentration.
4. Higher the dissipative heat larger |u|, v and smaller the actual concentration in the entire flow region. The actual temperature enhances with Ec ≤ 0.07 and reduces with higher Ec ≥ 0.09.
5. The rate of heat transfer enhances with increase in k ≤ 2.5 and reduces with k ≥ 3.5 at y = ±1. The rate of mass transfer enhances with k at y = +1 while at y=-1, |Sh| reduces with k ≤ 1.5 and enhances with k ≥ 2.5.
6. An increase in the radiation absorption parameter $Q_1 \leq 2$ leads to an enhancement in |Nu| at y=±1 while for higher $Q_1 \geq 4$, |Nu| reduces at y = +1 and at y = -1, it reduces for |G| ≤ 10^3 and enhances for higher |G| ≥ $3x10^3$. An increase in Q_1 enhances |Sh| at y=+1 and reduces at y = -1.
7. |Nu| enhances with increase in $\alpha_1 \leq 1.5$ and reduces with higher $\alpha_1 \geq 2.5$. |Sh| at y= +1 enhances with $\alpha_1 \leq 1.5$ and reduces with $\alpha_1 \geq 2.5$. At y = -1, |Sh| reduces with $\alpha_1 \leq 1.5$ and enhances with higher $\alpha_1 \geq 2.5$.
8. |Nu| enhances with increase in Ec. |Sh| reduces with Ec ≤ 0.07 and for Ec ≥ 0.09, |Sh| reduces for |G| ≤

10^3 and enhances for $|G| \geq 3x10^3$. At y = +1, it enhances with Ec in the heating case and reduces in the cooling case.

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