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A Didactic Note for Laboratory Determination of Local Acceleration of Gravity

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Abstract
We describe a laboratory experiment in which the local acceleration of gravity, g, was determined. This work is primarily one utility of a theory for conversion of mechanical energy into electrical energy. It involves measurement of impact time between two metallic bodies. The setup consists of a thin copper wire suspending a pendulum metal bob. The wire was passed over into connection with a chosen resistor in series with an electrolytic capacitor. The latter connected to a larger metallic body. At a determined height, the bob was allowed to make an impact with the larger metallic body to induce an electric current of which voltage was recorded by means of a voltmeter connected across the electrolytic capacitor. Errors due to oscillations in the conventional pendulum experiment were avoided because only one impact was allowed for every set of readings. Interesting common analyses were found sufficient for calculations of standard errors on *g*.
Keywords: Didactic, local acceleration of gravity, conversion of mechanical energy, collision time, capacitor, pendulum bob, impact time, error calculation
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I. INTRODUCTION

One of the earliest fundamental constants is the acceleration due to gravity. It is traditionally denoted by g (m/s^2). At least, it is older, in literature, than the Planck's constant, *h*. It is a physical constant that varies from place to place, (Woo, et. al., 2007). Just like the Planck's constant, (Adam, 2000; Jean-Philippe, 2003; Peter & David, 2010; Mercelo & Oscar, 1996), it varies with time and age, (Woo, et. al., 2007; Adam, 2000; Jean-Philippe, 2003; Peter & David, 2010). Possibility is that many other fundamental constants may have similar tendencies of variations, (Peter & David, 2010). Existing endeavours on *g*, may be subsumed under two broad categories: One class is to do with efforts towards teaching of Physics at all level of educational curricula, (Martin, 2009; Robert & Olsson, 1986; Madhur, et. al., 2007; Dupre, & Janssen, 2000; Patrik, et. al., 2011; White, et. al., 2007; Sinacore & Takai, 2010; Messar & Pantaleone, 2010; Robert, 1981). The other area has to do with the purposes of prospecting, (Lowrie, 1997) as in Geophysics. In within the first category, one could find journal articles with contents for determination of *g* (John & Joseph, 2005; Onorato et. al., 2010). The present attempt would fit into the first category.

In within the didactic journal articles (*i.e.*, those that are for the purposes of teaching Physics), one could notice several methods for determining *g*. The familiar approaches are worth noting: the pendulum experiments, (Martin, 2009; White, et. al., 2007) and many experiments that are based on free-fall of objects like balls or bobs, (Robert & Olsson, 1986; Dupre & Janssen, 2000; Patrik et. al., 2011; Messar & Pantaleone, 2010; Robert, 1081). The free fall procedures happen to be earlier. However, the pendulum system is commoner even at the level of secondary school education. Apparently it may be because, the pendulum (the simple pendulum in particular) apart from being a basic importance, it has interesting history, (Martin, 2009). The pendulum is usually utilized in student laboratory for measurement of *g*. In this manner the measured *g*, may be related to local gravity determination, (Martin, 2009). Just as in many efforts, whether didactic or otherwise, accuracy is emphasized, (Martin, 2009; Madhur, et. al., 2007). Here sophiscated analyses are abound, (Martin, 2009; Madhur, et. al., 2007) on calculations of errors. Errors on *g*, have been known to arising from the measurements of the pendulum length *l*, and the oscillation period *T*, appearing in the following equation

$$g = \frac{4\pi^2}{T^2} l \qquad (1)$$

The usual assumption that the angle of oscillations be small is a unique source of errors even in prospecting, (Woo, et. al., 2007; Mercelo & Oscar 1996). If, therefore, an experiment could be designed to avoid oscillations, it may then make an advantage to enable some level of accuracy on *g*. This is one objective of the experiment to be described here shortly.

There are five sections in this note. The forgone introduction is the first. Section 2 will briefly give the principles for the basis of the experiment of which procedures are described in section 3. Section 4 is for the analyses. Here, simple error calculations are given since the measurement of *g* still refers to local gravity. That is, competition is not intended with many abounding complex error calculations associated with prospecting endeavours. Thus, section 5 will do the conclusions.

2. THEORETICAL IDEAS (PRINCIPLES)

Determination of *g*, is actually a by-product of an experiment meant for illustration of time of impact between two metal objects, (see Fig. 1). Background required refers to thorough knowledge of charging and discharging in an RC circuit of a capacitor, (Alexander, 2002; Halilday, 1978; Tyler, 1979). For the capacitor, C in Fig. 1, the charging characteristic is governed by equation

$$q = Q_t\left\{1 - e^{-t/RC}\right\} \qquad (2)$$

where *C*, in Farads, is the capacitance of the capacitor, the emf of the battery, D, will be equal to the voltage across the resistor, *R*, at time $t = 0$. Q_t is the final value of the charge in the capacitor.

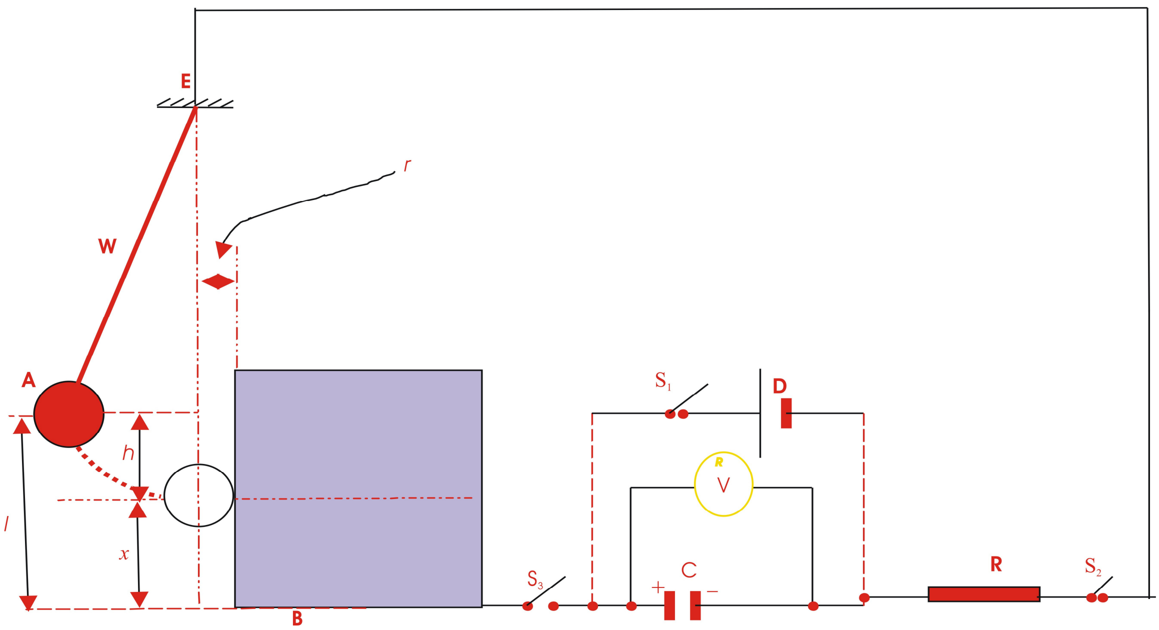

FIGURE 1. Experimental setup. A is the metal bob, B is the large metal object, C = 220 μF, is the capacitor, D is the battery, E is the suspending point connected to A via copper wire W, R = 4 Ω is the chosen resistor. Switches S_1, S_2, and S_3 should be noted with parameters *x*, *h*, and *l* described in the text.

In the discharging operation, the *RC* circuit is governed by equation

$$q = Q_0 e^{-t/RC} \qquad (3)$$

where $Q_0 \cong Q_t$ now implies the initial charge on the capacitor, C. The voltage, V_t, that is being built up as time *t,* goes on can be obtained as

$$V_t = V_0 e^{-t/RC} \qquad (4)$$

where V_0 is the initial voltage. If V_t is recorded for each impact of the bob A with the metallic object B then the time *t* can be calculated from Eq. (4) now written as

$$t = -RC\ln\left(\frac{V_t}{V_0}\right) \qquad (5)$$

The product, *RC*, in Eq. (5) has dimensions of time as may be verified by the student. If interested, a student may search the web for standard manuals on actual experimental procedures for charging and discharging laboratory exercises. One interesting laboratory exercise, appropriate for second year level, is found in (Tyler, 1979).

It is reasonable to suppose that the potential energy (p.e) of the bob is converted to kinetic energy (k.e) as the bob falls the height *h*. At the impact, the k.e is converted into electrical energy. This electrical energy is equal to the potential energy difference stored in the capacitor. The initial electrical potential energy is $V_0^2 C/2$, and at the impact time, *t*, the corresponding electrical potential energy would be $V_t^2 C/2$. Thus, the potential energy difference of the bob would yield

$$\frac{1}{2}V_t^2C = mgh + \frac{1}{2}V_0^2C \qquad (6)$$

where m is the mass of the pendulum bob. To enable one determine g, Eq. (6) is rewritten as

$$V_t^2 = \frac{2mg}{C}h + V_0^2 \qquad (7)$$

3. DESCRIPTIONS OF EXPERIMENTAL SETUP

Pieces of apparatus used are enumerated in the caption of Fig. 1. The key S_2 was kept open when the capacitor C was charging by closing the key S_1. The capacitor would be charged to the maximum voltage, V_0, of the battery, D, with a suitable value of the resistor R ≈ 4.0 Ω. To measure the time impact, a particular height l, was recorded. At this height, the steel bob was raised and then released. When it made an impact with the large metal object B, discharging of the capacitor occurred. The amount of discharge would correspond to a potential difference, p.d., V_t, indicated by the voltmeter, V across the capacitor. Duration of collision which is the time of impact was next calculated using Eq. (5).

Some ingenuity is necessary to reproduce the setup. For more insight, it should be noticed that the pieces of apparatus included three pairs of clamps and stands, placed at suitable distances apart. One was used to suspend the steel bob. Another one was used to hold the copper wire thus passing over to key S_2. S_2 eventually connected adjacent to the charging/discharging circuit via the resistor R. The third clamp with stand held the large metal object B fixed in position, and by this, the height x, for point of impact of A with B from horizontal table was carefully determined. With a meter-rule, having measured x (cm) once, measurement of l (cm) began from 10 cm and in step-size of 5.0 cm up to 0.5 m (~ 50.0 cm), as recorded in Table I. One should observe the calculation of h (cm) which is the actual height for the falling steel pendulum bob.

One of the precautions was the insertion of key S_3 between the large metal object B, and the charging/discharging circuit of the capacitor. This prevented the occurrence of short circuit when S_1 was closed. As mentioned earlier, anyone set of readings would begin by keeping S_2 opened and keeping S_1 closed. This charged the capacitor C, to the maximum voltage V_0, of the charging battery D. This was maintained constant. Now, after a particular height l, was measured for the steel bob, the key S_1, was opened and S_3 closed before releasing the steel bob. Four consecutive readings were made for each value of V_t, to corresponding to one value of l. Each set of readings ended with the calculations of mean value of V_t and impact time, t using Eq. (5).

TABLE I. Data collected for measurement of g using charging/discharging circuit of Fig. 1. Note that the impact times t, were calculated using Eq. (5). Fixed parameter values are: x = 7.5 cm, mass of the metal bob, m = 0.21 g, and V_0 = 5.5 V.

L (m) 10^{-2}	**$l - x \equiv h$ (m) 10^{-2}**	**V_t (V)**				**Mean of V_t (V)**	**Mean of V_t^2 (V^2)**	**t (s) $\times 10^{-4}$**
		1	**2**	**3**	**4**			
10	2.5	5.04	4.25	4.59	4.75	4.6875	21.9727	1.463
15	7.5	4.69	4.84	4.80	4.70	4.7575	22.6338	1.276
20	12.5	4.84	4.86	4.90	4.83	4.8575	23.9531	1.093
25	17.5	4.67	4.87	5.09	5.25	4.9700	24.7009	0.892
30	22.5	5.02	5.03	5.03	5.05	5.0325	25.3261	0.782
35	27.5	5.14	5.16	5.14	5.13	5.1425	26.4453	0.591
40	32.5	5.24	5.27	5.20	5.23	5.2360	27.4157	0.435
45	37.5	5.33	5.34	5.33	5.34	5.3350	28.4622	0.268
50	42.5	5.40	5.44	5.46	5.43	5.4325	29.5121	0.109

4. RESULTS AND ANALYSES

An interesting observation is that during the impact time, the capacitor discharged, because, the thin copper wire, the steel bob and the large metal object B are all conductors of electricity. Two graphs became necessary: Fig. 2 gives the plot of impact time t (s) on the vertical axis against the height, h (m) of the steel bob; Fig. 3, shows the plot of V_t^2 (V^2) on the vertical axis against h (m) on the horizontal axis.

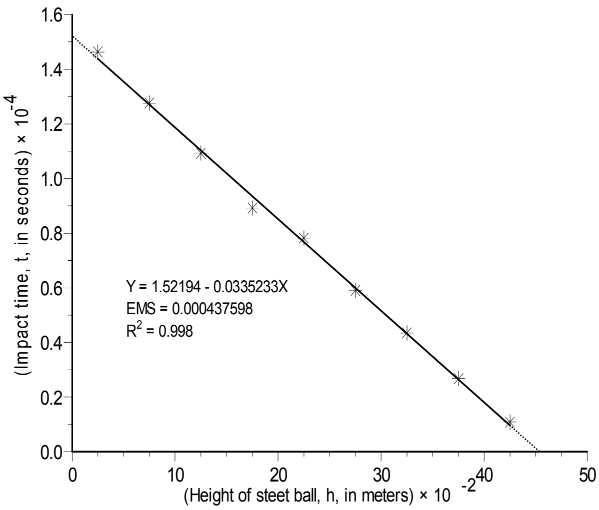

FIGURE 2. Graph of collision time, t (s), versus, height, h (m) of the steel bob.

Fig. 2 verifies the variation of the impact time with the height of release of the steel bob. The nine data points were observed to lie closely on a straight line. Therefore, least squares analyses are appropriate. This was used to fit the line. Although, a plotting package known as PlotIt 3.2 (available at http://www.plotit.com) was used to fit the least squares line, the Excel package could also be used. The PlotIt package gave the line to be $y = 1.52194 - 0.03353x$. This was confirmed by hand calculations to be $y = 1.5220 - 0.0335x$.

Someone might think that hand calculations, for verification, are unnecessary. It should however be noticed that the PlotIt package, just like the Excel package, would give the EMS and R^2. These two parameters help to indicate, among other things, the closeness of the data points to the regression line. Perhaps, they also give assurance that the data must have been carefully realized thus avoiding instrumental and observation errors. Since the data points are not large, (n = 9 in this work), it is encouraging for student laboratory exercise to calculate errors in the slope and intercept. We therefore did the calculations to evaluate the slope by a method available to us (Okeke, 1983) . The method used involves calculations of range, R, along the horizontal axis and vertical scatter, w, of the plotted points. Thus, the standard error, S_{sl}, in the slope is obtained from (Okeke, 1983) as

$$S_{sl} \cong \frac{4w}{nR} \tag{8}$$

The error in the intercept is given (Okeke, 1983) by

$$S_{\text{int}} = \left\{ \left(\frac{w}{n} \right)^2 + \left(\bar{x} S_{sl} \right)^2 \right\}^{1/2} \tag{9}$$

where, $\bar{x}$, is the mean along the horizontal axis.

For the slope, we obtained (- 0.033500 ± 0.000667) s/m. That is, error in the slope is $\pm 6.67 \times 10^{-4}$ s/m. For the intercept, along the t-axis, we had (1.5220 ± 0.0067) s; the error being ± 0.0067 s. It could be seen that the variation of impact time is linear with the height of release of the steel bob. (See Fig. 2).

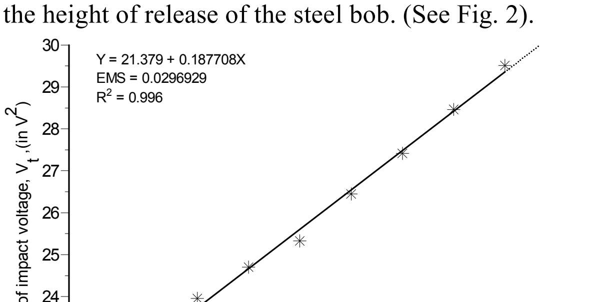

FIGURE 3. Graph of V_t^2 (V^2) versus height, h (m) of the steel bob.

Verifications by hand calculations were inevitable in the case of Fig. 3. Here, error on the slope is more necessary. That is, because, error on *g*, must be calculated, at all, by the simple method available to us, (Okeke, 1983) . We are, however, aware of the sophiscated approaches as given in (Martin, 2009), for ascertaining error on g, referred to as corrections. We found the slope to be $(0.187964866 \pm 0.0061111) \times 10^2$ V^2/m. For the intercept, we got (21.3732 ± 4.2293) V^2. We reverted to Eq. (7) to calculate, *g*, using the slope expression $S_l \equiv 2mg/C$. That is, $g \equiv (0.187964866 \times C)/2m$. The parameter C and *m* were assumed to be constant and error free. Thus $g \approx 9.82484 m/s^2$. If standard error on *g* is S_g and that in the slope, S_{sl} are related to *g* and S_l by

$$\left|\frac{S_g}{g}\right| = \left|\frac{S_{sl}}{S_l}\right| \tag{10}$$

then, the standard error on *g* is obtained as

$$S_g = g\left|\frac{0.0061111}{0.1879649}\right| \approx 0.31942 \text{ m/s}^2 \tag{11}$$

The value of $g \approx (9.82484 \pm 0.31942)$ m/s^2 is within the acceptable figure for class room purposes. One may not worry about the intercept for now. But it may have an interpretation rooted in the capacitor as a transducer or an instrument. The transducer is prone to errors.

5. CONCLUSIONS

One element of surprise that is apt to capture the student's interest is the manner by which the use of stop-watch was avoided in this task. In our didactic experience, students develop confidence in their abilities more with regards to calculations. Besides, the pieces of apparatus are not sophisticated. Moreover, the experiment illustrates the utility of conversion of mechanical energy into electrical energy. The by-product of this illustration is the determination of the local acceleration of gravity *g*.

It should be noticed that several factors that might affect accuracy are possible. These could be referred to as pendulum corrections (Martin, 2009). One other correction that may be of interest later is the elastic nature of the wire W, (see Fig. 1). The elastic correction is due to the weight of the bob. In addition, at the impact, the wire W, could stretch due to passage of current resulting into heat in the wire. Therefore, a starting point for improving accuracy for the setup should be the study of the pendulum corrections (Martin, 2009), one after the other. Also, the electrical resistances of the wire W, the bob A, and that of the large metal body B, were assumed sufficiently small thus facilitating flow of current. In subsequent efforts to improve the setup, these resistances would be considered.

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