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Flow and Heat Transfer Analysis of Casson Fluid due to A Stretching Sheet: An Analytical Solution

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Abstract
In this paper, an analysis is carried out to study the flow and heat transfer characteristics of Casson fluid due to stretching sheet. The governing partial differential equations of boundary layer are converted into ordinary differential equations by means of suitable similarity transformations. The results are obtained by solving the momentum and energy equations of Casson fluid analytically for two heating conditions (PST and PHF). The results are analyzed by plotted graphs and numerical tables of the wall temperature and wall temperature gradient.
Keywords: Casson fluid; Fluid flow; Heat Transfer; Prescribed surface temperature, Prescribed wall heat flux; Wall temperature, Wall temperature gradient

Nomenclature

b	stretching rate
A,B	constants
x	horizontal coordinate
y	vertical coordinate
u	horizontal velocity component
v	vertical velocity component
T	temperature
c_p	specific heat
f	dimensionless stream function
g	dimensionless temperature for PHF Case
Pr	Prandtl number
l	Characteristic length
'	differentiation with respect to η

Greek symbols:

η	similarity variable
θ	dimensionless temperature for PST Case
k	thermal conductivity
μ	viscosity
υ	kinematic viscosity
ρ	density
β	Casson parameter

Subscripts:

w	properties at the plate
∞	free stream condition

1. Introduction

As we know the Non-Newtonian flows generated by a stretching sheet have been widely analyzed due to their application to several manufacturing processes such as extrusion of molten polymers through a slit die for the production of plastic sheets, processing of food stuffs, paper production and fiber coating. The quality of the final product in such processes greatly depends on the rate of cooling in the heat transfer process. Crane [1] gave a closed form solution for the steady two dimensional incompressible boundary layer flow of a viscous fluid generated by stretching sheet. This flow problem has been extended for various diverse physical aspects. Further many researches analyzed the flow and heat transfer problems of various fluids.

Nadeem et al. [2] considered the flow analysis of Casson fluid with uniform magnetic field under the influence of exponentially stretching sheet and obtained the solution by Adomain decomposition method using the Pade's approximation. Mohammed [3] studied the magnetohydrodynamic Casson fluid flow with heat and mass transfer through the porous medium over a stretching sheet. Hayat el al. [4] studied the Soret and Duofour effects on magneto hydrodynamic flow of Casson fluid. Bhattacharya et al.[5] studied the slip effect on parametric space and solution for the boundary layer flow due to non-porous stretching and shrinking sheet. Mukhopadyaya et al.[6] studied the flow of Casson fluid over an unsteady stretching sheet. Qasim and Noreen [7] studied the boundary layer flow of Casson fluid over a permeable shrinking sheet with viscous dissipation. Mukhopadhyaya and Vajravelu [8] studied the diffusion of chemically reactive species in a casson fluid flow over an unsteady permeable stretching sheet. Pramanik [9] studied the flow and heat transfer past an exponential porous stretching surface in presence of thermal radiation.

On analyzing all above works, no authors considered the two heating conditions (i.e. PST and PHF) in their study, which are very important. This motivates us to study the flow and heat transfer of Casson fluid due to stretching sheet with two heating conditions (a) Prescribed Surface Temperature (PST case) (b) Prescribed Wall heat Flux (PHF Case).

2. Flow analysis:

Consider the flow of an incompressible casson fluid past a stretching sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. Assuming the rheological equation of Casson fluid (reported in [refs 2-9]). The steady two-dimensional boundary layer equations for the flow can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}\,, \tag{2}$$

Where u and v are the velocity components of the fluid in x and y directions respectively and υ is kinematic viscosity and β is the Casson parameter (non-Newtonian parameter).

The boundary conditions for the problem are

$$\left.\begin{array}{lll} u_w(x)=bx,\; v=0, & & y=0 \\ u\to 0, & as & y\to\infty \end{array}\right\} \tag{3}$$

with $b > 0$, the stretching rate. The Eqns. (1) and (2), subjected to the boundary condition (3), admit a self-similar solution in terms of the similarity function f and the similarity variable η defined by

$$u = b\,x\,f'(\eta),\quad v=-\sqrt{b\upsilon}\,,\; \eta=\sqrt{\frac{b}{\upsilon}}\,y. \tag{4}$$

It can be easily verified that Eq. (1) is identically satisfied and substituting the above transformations in Eq. (2) we obtain

$$f'^2-f''f=\left(1+\frac{1}{\beta}\right)f'''. \tag{5}$$

Similarly the boundary conditions (3) can be written as:

$$\left.\begin{array}{llll} f'(\eta)=1\,, & f(\eta)=0 & at & \eta=0 \\ f'(\eta)\to 0, & & as & \eta\to\infty \end{array}\right\}. \tag{6}$$

The exact solution of (5), satisfying the boundary conditions (6) is given by:

$$f=\sqrt{1+\frac{1}{\beta}}\left(1-e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\quad (\beta \text{ is positive}) \tag{7}$$

3. Solution of Heat transfer analysis:

The Energy equations with boundary layer approximations can be written as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}\,, \tag{8}$$

where k is the thermal conductivity, T is the temperature, ρ is the density of the fluid, C_p is the specific heat at constant pressure .

3.1: Prescribed Surface temperature case (PST Case):

The PST boundary conditions are:

$$\left.\begin{array}{lll} T=A\left(\dfrac{x}{l}\right)^2, & at & y=0 \\ T\to T_\infty & as & y\to\infty \end{array}\right\}, \tag{9}$$

where T_∞ is the temperature of the fluid far away from the sheet. Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \tag{10}$$

Where T_w the temperature of the sheet.

Using Eqn. (10), Eqs. (8) and (9) can be written as

$$\theta''+\Pr f\,\theta'-2f'\Pr\theta=0\,, \tag{11}$$

$$\left.\begin{array}{lll} \theta(\eta)=1 & \text{at} & \eta=0, \\ \theta(\eta)\to 0 & \text{as} & \eta\to\infty. \end{array}\right\} \tag{12}$$

Where

$\Pr=\dfrac{\mu C_p}{k}$ is the Prandtl number.

Substitute (7) in (11), we obtain

$$\theta''+\Pr\left(\sqrt{1+\frac{1}{\beta}}\left(1-e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\right)\theta'-2\Pr\left(e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\theta=0 \tag{13}$$

Since this is a second order differential equation with variable coefficients, to solve this differential equation (13), we introduce a new variable:

$$\xi=-\Pr\left(1+\frac{1}{\beta}\right)\left(e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right). \tag{14}$$

Using (14) in (13), we obtain:

$$\xi\frac{d^2\theta}{d\xi^2}+\left(1-\left(\frac{\Pr}{\left(1+\frac{1}{\beta}\right)}\right)-\xi\right)\frac{d\theta}{d\xi}+2\theta=0 \tag{15}$$

The boundary conditions (12) reduced to

$$\theta\left(-\Pr\left(1+\frac{1}{\beta}\right)\left(e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\right)=1;\quad \theta(\xi\to\infty)\to 0 \tag{16}$$

The solution of (15) in terms of confluent hypergeometric function is given by

$$\theta(\xi)=\mathrm{C}_1\,\xi^{\Pr(1+\frac{1}{\beta})}F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\ \ \Pr\left(1+\frac{1}{\beta}\right)+1,\ \xi\right). \tag{17}$$

Making use of the boundary conditions (16) and re-writing the solution in variable η we get:

$$\theta(\eta)=\mathrm{C}_1\,e^{-\frac{\Pr}{\sqrt{1+\frac{1}{\beta}}}\eta}F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\ \ \Pr\left(1+\frac{1}{\beta}\right)+1,\ \ -\Pr\left(1+\frac{1}{\beta}\right)e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right) \tag{18}$$

Where

$$C_1=\frac{1}{F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\ \Pr\left(1+\frac{1}{\beta}\right)+1,\ -\Pr\left(1+\frac{1}{\beta}\right)\right)}, \tag{19}$$

Where Kummer's function F is defined by [Ref(10)]:

$$F(\mathrm{a},\ \mathrm{b},\ \mathrm{z})=1+\sum_{n=1}^{\infty}\left(\frac{(a)_n\,z^n}{(b)_n\,n!}\right) \tag{20}$$

where

$$(a)_n=a(a+1)(a+2).....(a+n-1) \tag{21}$$

$$(\mathrm{b})_n=b(\mathrm{b}+1)(\mathrm{b}+2).....(\mathrm{b}+n-1) \tag{22}$$

The dimensionless wall temperature gradient $\theta'(0)$ is given by:

$$\theta'(0)=\frac{\left[-\Pr\left(1+\frac{1}{\beta}\right)F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\Pr\left(1+\frac{1}{\beta}\right)+1,-\Pr\left(1+\frac{1}{\beta}\right)\right)+\left(\frac{\Pr\left(1+\frac{1}{\beta}\right)-2}{\Pr\left(1+\frac{1}{\beta}\right)+1}\right)\left(\Pr\sqrt{1+\frac{1}{\beta}}\right)F\left(\Pr\left(1+\frac{1}{\beta}\right)-1,\Pr\left(1+\frac{1}{\beta}\right)+2,-\Pr\left(1+\frac{1}{\beta}\right)\right)\right]}{F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\Pr\left(1+\frac{1}{\beta}\right)+1,-\Pr\left(1+\frac{1}{\beta}\right)\right)}$$

(22)

3.1: Prescribed Surface Wall Heat Flux (PHF Case):

Consider the power law heat flux on the wall surface is considered as:

$$\left.\begin{aligned}-k\frac{\partial T}{\partial y}=B\left(\frac{x}{l}\right)^2 &\quad at\quad y=0\\ T\to T_\infty &\quad at\ y\to\infty\end{aligned}\right\} \tag{23}$$

where B is a constant. The scaled temperature is defined as:

$$g(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \tag{24}$$

Making use of the transformation (24) into Eqn(11) and (23) we get

$$\mathrm{g}''+\Pr f\,g'-2f\Pr g=0 \tag{25}$$

$$\left.\begin{aligned} g(\eta)&=1 \quad \text{at} \quad \eta=0, \\ g(\eta)&\to 0 \quad \text{as} \quad \eta\to\infty. \end{aligned}\right\} \tag{26}$$

Introducing a new variable as in the PST case and following the same procedure one can obtain the solution to (25) in terms of Kummer's function [Ref 10] as:

$$\mathrm{g}(\eta)=\mathrm{C}_2\, e^{-\frac{\Pr}{\sqrt{1+\frac{1}{\beta}}}\eta} F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\ \Pr\left(1+\frac{1}{\beta}\right)+1,\ -\Pr\left(1+\frac{1}{\beta}\right)e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right). \tag{27}$$

where

$$\mathrm{C}_2=\left[-\Pr\left(1+\frac{1}{\beta}\right)F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\Pr\left(1+\frac{1}{\beta}\right)+1,-\Pr\left(1+\frac{1}{\beta}\right)\right)+\left(\frac{\Pr\left(1+\frac{1}{\beta}\right)-2}{\Pr\left(1+\frac{1}{\beta}\right)+1}\right)\left(\Pr\sqrt{1+\frac{1}{\beta}}\right)F\left(\Pr\left(1+\frac{1}{\beta}\right)-1,\Pr\left(1+\frac{1}{\beta}\right)+2,-\Pr\left(1+\frac{1}{\beta}\right)\right)\right]. \tag{28}$$

The non-dimensional wall temperature is given by

$$\mathrm{g}(\eta)=\mathrm{C}_2\, F\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\ \Pr\left(1+\frac{1}{\beta}\right)+1,\ -\Pr\left(1+\frac{1}{\beta}\right)\right) \tag{29}$$

4. Results and Discussion

A boundary layer flow and heat transfer of a non-Newtonian fluid has been solved analytically using the hypergeometric series. The velocity and temperature distributions are presented in Figs 1-5; numerical values of skin friction, wall temperature gradient and wall temperature are calculated and presented in the tables1-2.

Here we discuss the study of effect of the Casson parameter on flow and effect of Casson parameter and Prandtl number on the for both PST and PHF cases.

Fig-1 shows the influence of Casson parameter β on velocity profile. We observe that the magnitude of velocity in the boundary layer decreases with an increase in the Casson fluid parameter β. It is noticed that when Casson parameter approaches infinity the problem will reduce to a Newtonian case. Hence increasing value of Casson parameter β , decreases the velocity and boundary layer thickness.

Figs 2 and 3 show the effect of the casson parameter β on the temperature profiles for the PST and PHF cases respectively. The temperature and the thermal boundary layer thickness are increasing as decreasing function of β

Figs 4 and 5 show the effect of the Prandtl number $\Pr$ on temperature profile in PST and PHF cases respectively. On observing these figurs we can conclude that the temperature and the thermal boundary layer thickness decrease as the Prandtl number increase.

The skin friction values in Table-1, it shows that it decreases with increasing values of β. The values of wall temperature gradient for the PST case and the wall temperature for PHF cases are tabulated in Table-2.

From table-2: we see that the wall temperature gradient and wall temperature values increases with increase in the values of β and decrease with increasing Prandtl number.

5. Conclusions

- Analytical Solutions for flow and heat transfer problems are obtained.
- The effects of the Casson fluid parameter β on flow and temperature are quite opposite.
- The thermal boundary layer thickness decreases with increasing Prandtl number in both PST and PHF cases.
- When β tends to infinity, as reduce to results the Newtonian case
- Skin friction decreases with an increase in the Casson fluid parameter

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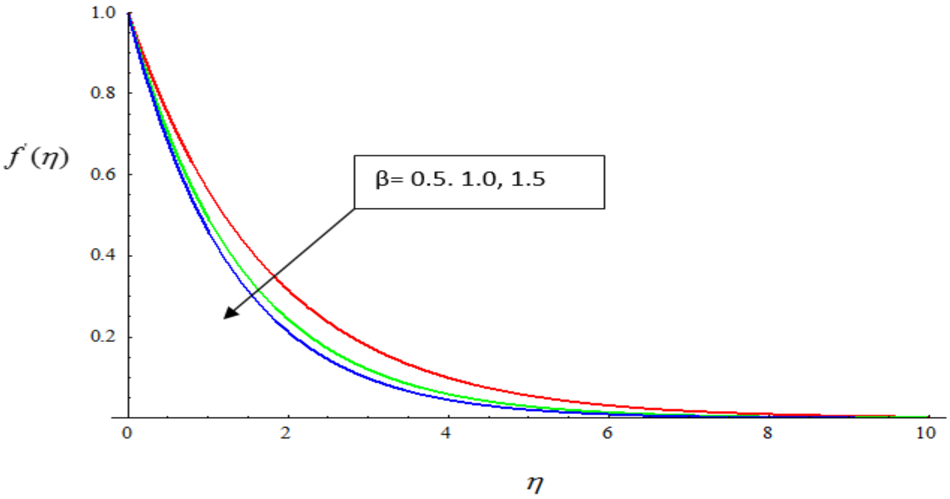

Fig 1. Velocity profile for different values of Casson parameter β

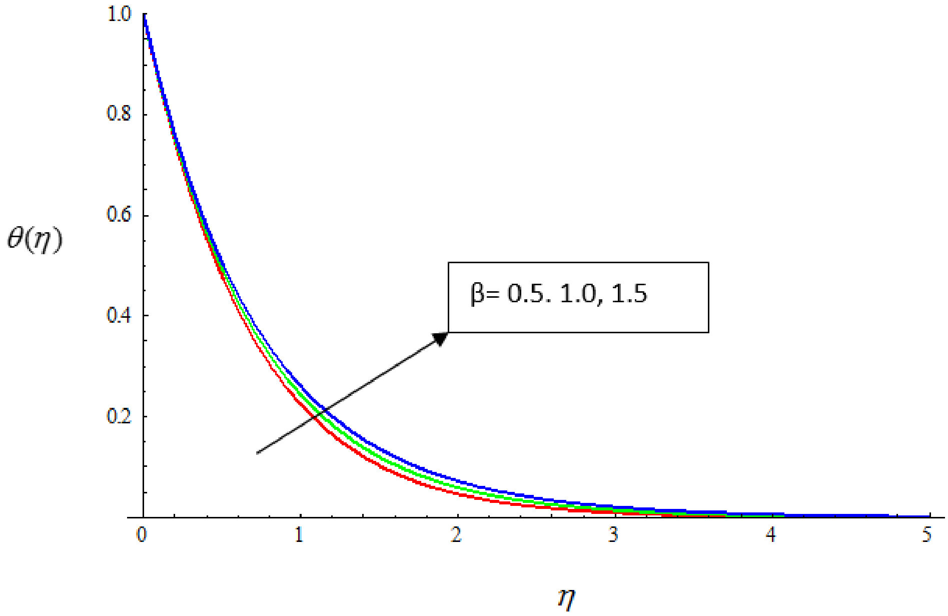

Fig 2: Temperature Profile for different values of Casson parmeter(β) When Pr =1.0 in PST Case

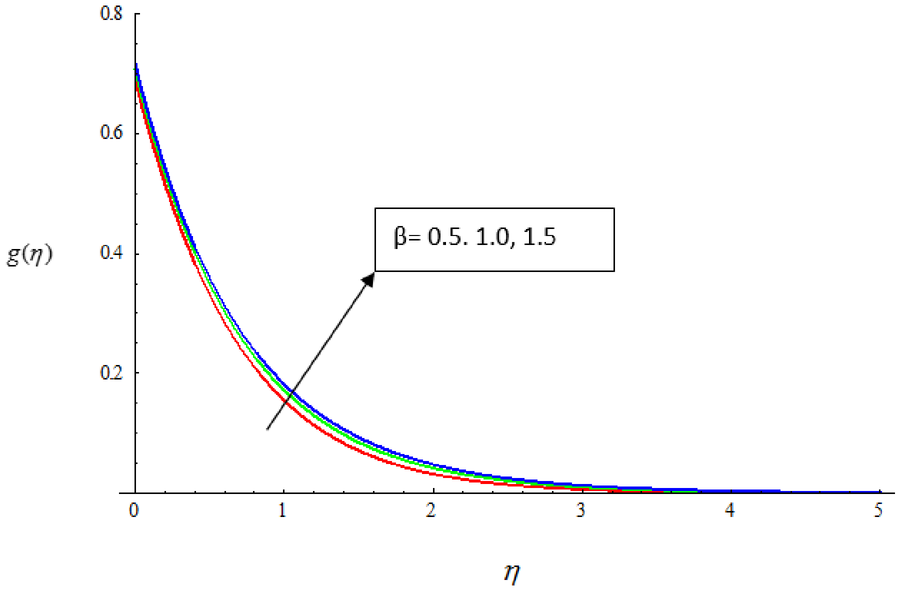

Fig 3: Temperature Profile for different values of Casson parameter (β), When Pr =1.0 in PHF Case

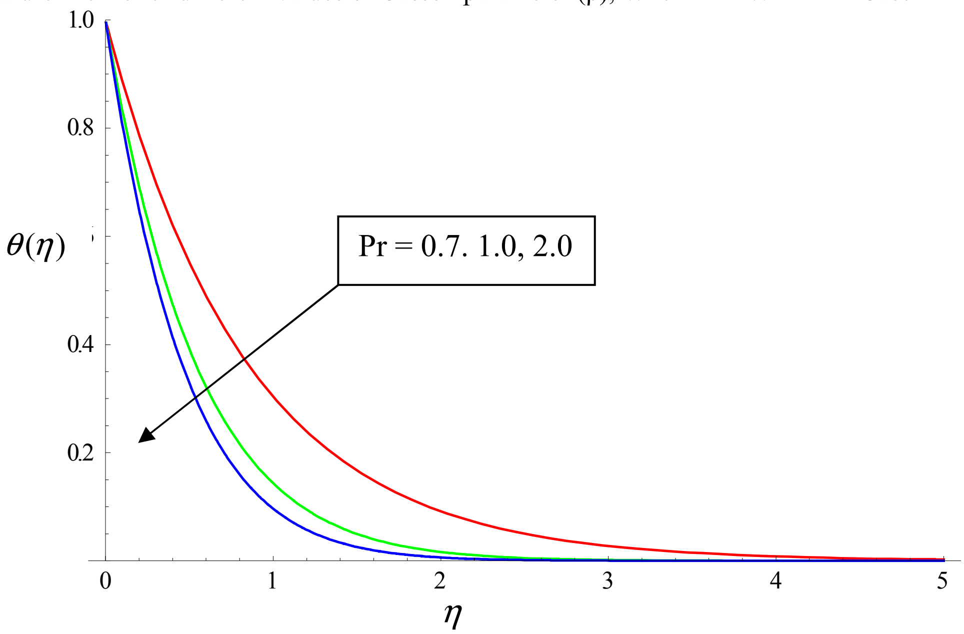

Fig 4: Temperature Profile for different values of Prandtle number (*Pr*) When β =0.5 in PST Case

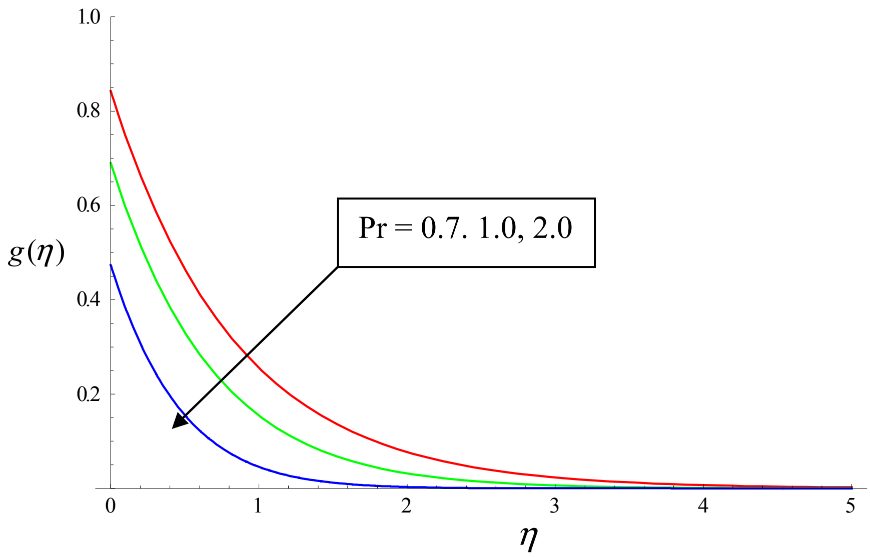

Fig 5: Temperature Profile for different values of Prandtle number (Pr), When β =0.5 in PHF Case

Table-1

Values of $f''(0)$ for different values of β

Parameter value (β)	$-f''(0)$
0.5	0.57735
1.0	0.707107
1.5	0.774597
2.0	0.816497
2.5	0.845154
3.0	0.866025

Table-2

Parameters		$-\theta'(0)$	g(0)
β	Pr		
1.0	1.0	1.41421	0.707107
2.0		1.38437	0.722351
3.0		1.3707	0.729552
0.5	1.0	1.44899	0.690136
	2.0	2.11182	0.473526
	3.0	2.61979	0.38171

Values of $\theta'(0)$ and g(0) for different values of β and *Pr*