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Nonplanar Geometry in Multi-Component Dust-Ion-Acoustic Shock Waves for Adiabatic Dusty Plasma

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Abstract
The modified Burgers' equation of nonlinear propagation of Dust-Ion Acoustic Shock Waves (DIASWs) for multicomponent unmagnetized dusty plasma consisting of adiabatic ion fluid, Boltzmann distributed electrons and positrons and static negatively charged dust fluid has been derived using the standard reductive perturbation method. The solution of modified Burgers' equation in nonplanar geometry is numerically analyzed and it has been found that, the nonplanar geometry effects have a very significant role in the formation of shock waves. Further more; it is found that the planar geometry shock structure with higher amplitude is the strongest, followed by cylindrical and spherical shock waves amplitudes respectively. It is also observed that; an increase in positron concentration decreases the amplitude of the DIASWs.
Keywords: Nonplanar Geometry multicomponent, Dust- ion-Acoustic Shocks Adiabatic Dusty Plasma

1. INTRODUCTION

Plasmas, in general, consisting of electrons and positrons of equal masses and ion are usually characterized as electron-positron-ion plasma [1, 2]. The presence of ions brings about the existence of several low frequency waves which other wise do not propagate in electron-positron plasmas. There are electron-positron in astrophysical plasmas such as in magnetosphere of plasmas, in active galactive nuclei, in early universe and in the region of the accretion disks surrounding the central black holes [3-8]. Due to impressive developments in plasma, there has been considerable interest in different type of linear and nonlinear wave structure such as solitons, double layers, vortices etc, in electron-positron plasmas [9-15] as well as in multi-component electron-positron-ion plasmas [16-18]. Most of the astrophysical plasmas usually contain highly charged (negative/positive) impurities or dust particulates in addition to the electrons, positrons and ions. It is a well known fact that, the presence of static charged dust grains modifies the existing plasma wave spectra [19-21].

Dust ion acoustic shocks in an unmagnetized dusty plasma many arise when there is a balance between the nonlinearity (associated with the harmonic generation) and the kinetic viscosity introduced by the dust ion drag. The formation of Dust Ion Acoustic Shock waves (DIASWs) was observed by Nakamura et al [22]. They found out that, both monotonic and oscillatory shock waves exist and the dust density has vital role on the shock waves and phase velocity of the wave. A number of studies have been made on the propagation of DIASWs in dusty plasma by several investigators [23-26].

Recently, several theoretical investigations [27-31] on the properties of dust-ion acoustic and dust acoustic solitary waves in nonplanar geometry had been carried out. Sahu [32] of recent, have carried out theoretical investigation on the effect of nonplanar DIASWs in an Adiabatic Dusty Plasma. Since, the properties of wave motions-electron-positron-ion-dust plasma should be different from those in three component electron-ion-dust plasma.

In this present work, we have considered an unmagnetized multi-component adiabatic dusty plasma; and have studied the basic properties of DIASWs in such a dusty plasma which was not considered in the earlier investigation [32]. The paper is organized in the following manner. The basic equations governing the adiabatic dusty plasma system under consideration are given in section 2. The nonplanar DIASWs are investigated by the reduction perturbation method (RPM) in section 3. In section 4 we present the numerical results and discussion. Section 5 is the conclusion.

2. GOVERNING EQUATIONS

A four-component unmagnetized plasma comprising of static negatively charged dust fluid, Boltzmann distributed electrons and positrons, and adiabatic ion fluid. The dynamics of the DIAWs – nonplanar geometry for such a dusty plasma, is governed by the following normalized fluid equations:

$$\frac{\partial n_i}{\partial t}+\frac{1}{r^{\nu}}\frac{\partial}{\partial r}\left(r^{\nu}n_iu_i\right)=0 \qquad (1)$$

$$n_i\frac{\partial u_i}{\partial t}+n_iu_i=-n_i\frac{\partial \phi}{\partial r}-\alpha\frac{\partial p_l}{\partial r}+\eta_i n_i\left[\frac{1}{r^{\nu}}\right.$$

$$\frac{\partial}{\partial r}\left(r^{\nu}\frac{\partial u_i}{\partial r}\right)-\frac{\nu u_i}{r^2}\Bigg] \tag{2}$$

$$\frac{\partial p_i}{\partial t}+u_i\frac{\partial p_i}{\partial r}+3p_i\frac{1}{r^{\nu}}\frac{\partial}{\partial r}\left(r^{\nu}u_i\right)=0 \tag{3}$$

$$\mu\exp(\phi)-\delta_p\exp(-\phi)-n_i+\left(\delta_p+\frac{1}{1+\mu}\right)=0 \tag{4}$$

In the above equation, n_i is the ion number density normalized by its equilibrium value n_{io}, u_i is the ion fluid speed normalized by $C_i=\left(K_BT_e/m_i\right)^{1/2}, \phi$ is the wave potential normalized by $K_BT_e/e, p_i$ is the ion thermal pressure normalized by $n_{io}K_BT_i, \alpha=T_i/T_e$. The time and space variable are normalized by reciprocal plasma frequency $w_{p_i}^{-1}=\left(m_i/4\pi n_{io}e^2\right)^{1/2}$ and the Debye length $\lambda_D=\left(K_BT_e/4\pi n_{io}e^2\right)^{1/2}$ respectively. Where $\nu=o$, for one-dimensional geometry and $\nu=1,2$ for cylindrical and spherical geometry respectively. η_i is the viscosity coefficient of ion fluid normalized by $w_{pi}\lambda_D^2, \mu=n_{eo}/n_{io}$ and $\delta_p=n_{po}/n_{io}$. We note that in equation (4), we have assumed the quasi-neutrality condition.

3. DERIVATION OF NONPLANAR BURGERS' EQUATION

We derive the Burgers' equation (1)-(4) by employing the reductive perturbation method (RMP) and the stretched coordinates [33] $\xi=\in^{1/2}\left(r-V_ot\right)$ and $\tau=\in^{1/2}t,$ where ε is a smallness parameter measuring the weakness of the nonlinearity and V_0 is the phase speed of DIASWs normalized by C_i. Equations (1)-(4) can be expressed in terms of ξ and τ as follows:

$$\in^{3/2}\partial_\tau n_i-V_o\in^{1/2}\partial_\xi\left(n_i\right)+\varepsilon^{\frac{1}{2}}\partial_\xi\left(n_iu_i\right)+\frac{\nu\in^{3/2}}{V_o\tau\left(1+\xi\in/V_o\tau\right)}\left(n_iu_i\right)=0 \tag{5}$$

$$\begin{aligned}&\in^{3/2}n_i\partial_\tau u_i-V_0\in^{1/2}n_i\partial_\xi u_i+\in^{1/2}n_iu_i\partial_\xi u_i=-\in^{1/2}n_i\partial_\xi\phi_1\\&-\in^{1/2}\alpha\partial_\xi p_i+\in^{1/2}\eta_0n_i\Bigg[\in^2\partial_\xi^2u_i+\\&\frac{\nu\in^2}{V_0\tau\left(1+\frac{\xi\in}{V_0\tau}\right)}\partial_\xi u_i-\frac{\nu\in^3}{V_0^2\tau^2\left(1+\frac{\xi\in}{V_0\tau}\right)^2}u_i\Bigg]\end{aligned} \tag{6}$$

$$\begin{aligned}&\in^{3/2}\partial_\tau p_i-V_0\in^{1/2}\partial_\xi p_i+\in^{1/2}u_i\partial_\xi p_i+3p_i\\&\left[\in^{1/2}\partial_\xi u_i+\frac{\nu\in^{3/2}}{V_0\tau\left(1+\frac{\in\xi}{V_0\tau}\right)}u_i\right]=0\end{aligned} \tag{7}$$

$$\begin{aligned}&\mu\left(1+\phi+\frac{1}{2}\phi^2...\right)-\delta_p\left(1-\phi+\frac{1}{2}\phi^2+...\right)\\&+\left(\delta_p+1-\mu\right)=0\end{aligned} \tag{8}$$

where $\eta=\in^{1/2}\eta_0$ is assumed [34]. We can expand the dependent variables n_i, u_i, p_i and ϕ in power series of $\in$ as follows:

$$n_i=1+\in n_{i1}+\in^2n_{i2}+...$$

$$u_i = 0 + \in u_{i1} + \in^2 u_{i2}$$
$$p_i = 1 + \in p_{i1} + \in^2 p_{i2} \tag{9}$$
$$\phi = 0 + \in \phi_1 + \in^2 \phi_2$$

We obtain the lowest order of the coefficient $\in$ as follows by substituting equation (9) into equation (5)-(8):

$$n_{i1} = (\mu + \delta_p)\phi_1 \tag{10}$$

$$u_{i1} = V_0(\mu + \delta_p)\phi_1 \tag{11}$$

$$p_{i1} = 3(\mu + \delta_p)\phi_1 \tag{12}$$

$$V_0 = \sqrt{3\alpha + \frac{1}{(\mu + \delta p)}} \tag{13}$$

The next higher order in $\in$, is given as the following set of equations,

$$\partial_\tau n_{i1} - V_0 \partial_\xi n_{i2} + \partial_\xi (n_{i1} u_{i1}) + \partial_\xi u_{i2} + \frac{\nu u_{i1}}{V_0 \tau} = 0 \tag{14}$$

$$\partial_\tau u_{i1} - V_0 \partial_\xi u_{i2} + u_{i1} \partial_\xi u_{i1} - V_0 n_{i1} \partial_\xi u_{i1} + \alpha \partial_\xi p_{i2}$$
$$= -\partial_\xi \phi_2 - n_{i1} \partial_\xi \phi_1 + \eta_0 \partial_\xi^2 u_{i1} \tag{15}$$

$$\partial_\tau p_{i1} - V_0 \partial_\xi p_{i2} + u_{i1} \partial_\xi p_{i1} + 3\partial_\xi u_{i2} + 3 p_{i1} \partial_\xi u_{i1} + \frac{3\nu u_{i1}}{V_0 \tau} = 0 \tag{16}$$

$$n_{i2} = (\mu + \delta_p)\phi_2 + \frac{1}{2}(\mu - \delta_p)\phi_1^2 \tag{17}$$

Now, using equations (10)-(17) and eliminating n_{i2}, u_{i2}, p_{i2} and ϕ_2, we finally obtain a modified Burgers' equation

$$\partial_\tau \phi_1 + \frac{\nu}{2\tau}\phi_1 + A\phi_1 \partial_\xi \phi_1 - C\partial_\xi^2 \phi_1 = 0 \tag{18}$$

where

$$A = \frac{12\alpha(\mu + \delta_p)^3 + 3(\mu + \delta_p)^2 - (\mu - \delta_p)}{2\left(\sqrt{3\alpha + 1/(\mu + \delta_p)}\right)(\mu + \delta_p)^2} \tag{19}$$

$$C = \eta_0 / 2 \tag{20}$$

4. NUMERICAL RESULTS AND DISCUSSION

The modified Burgers' equation describing the nonlinear propagation of the DIASWs for multicomponent unmagnetized dusty plasma consisting of adiabatic ion fluid, Boltzmann distributed electrons and positrons and static negatively charged dust fluid is given in Equation (18). The stationary DIASWs of this modified Burgers' equation for planar geometry $(\nu = 0)$ is

$$\phi_1 = \frac{V}{A}\left[1 - \tanh\left(\frac{V(\xi - V\tau)}{2C}\right)\right] \tag{21}$$

where V is a constant velocity normalized by C_i. For nonplanar geometry, an exact analytical solution of equation (18) is not possible, hence it is solved numerically. The results are shown in Figs. 1-5. The initial condition that we have considered for all the numerical results, is the form of the stationary solution of equation (21) without the geometry term $(\nu/2\tau)$ at $\tau = -10$. The shock wave structure for different geometries with respect to the considered parameters is presented in Fig.1. It is obvious that, the developed shock amplitude in the different geometry are different from each other, while the shock steepness is the same for all geometry. The planar geometry shock structure, with higher amplitude is the strongest. The cylindrical shock wave amplitudes are bigger than that of the spherical shock wave, but smaller than that of the planar geometry shock wave structure.

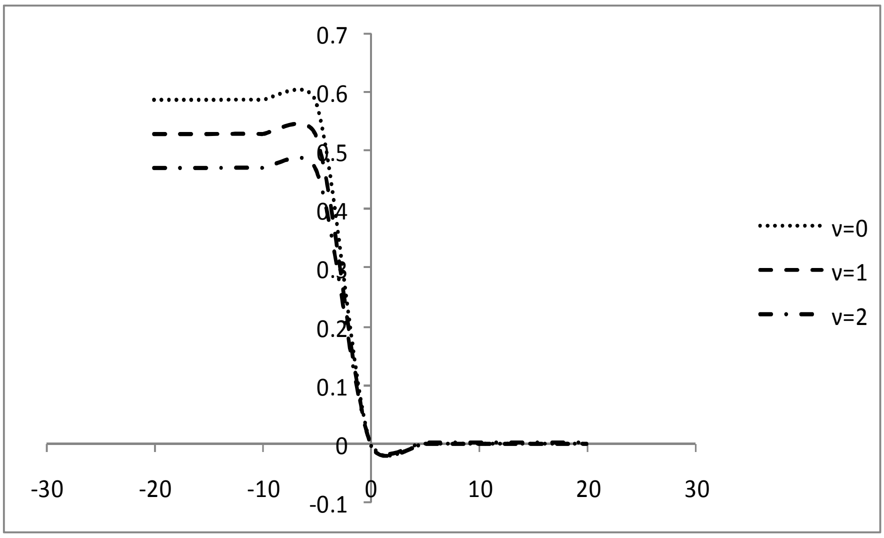

Fig. 1: Showing how the shock profile ($\phi_1 V_s \xi$ curves) varies in different geometries for $\tau = -5,\ \eta_0 = 0.5,\ \alpha = 0.5,\ \delta_p = 0.3$ and $\mu = 0.4$.

In Figures 2 and 3, we present the variation of cylindrical shock structures and spherical shock structures for different values of τ respectively. It can be observed that as time increases (τ), the amplitude of the cylindrical shock waves also increases. It can also be observed that, the shock wave profile for the nonplanar (Fig. 2 and Fig.3) are similar to what is obtained for the planar geometry of fig. 1 in terms of the amplitude for larger value of $|\tau|$. This result confirms the fact that, large value of the nonplanar geometrical effect $(\nu/2\tau)$ is no longer dominant.

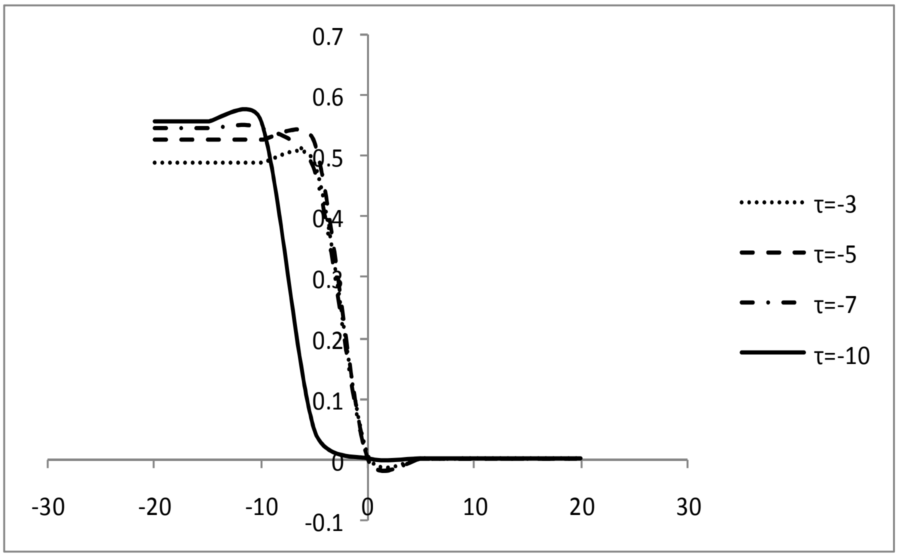

Fig. 2: Variation of ϕ_1 with respect to ξ at different values of τ for the cylindrical geometry $(\nu = 1)$ and the other parameters being the same as figure 1.

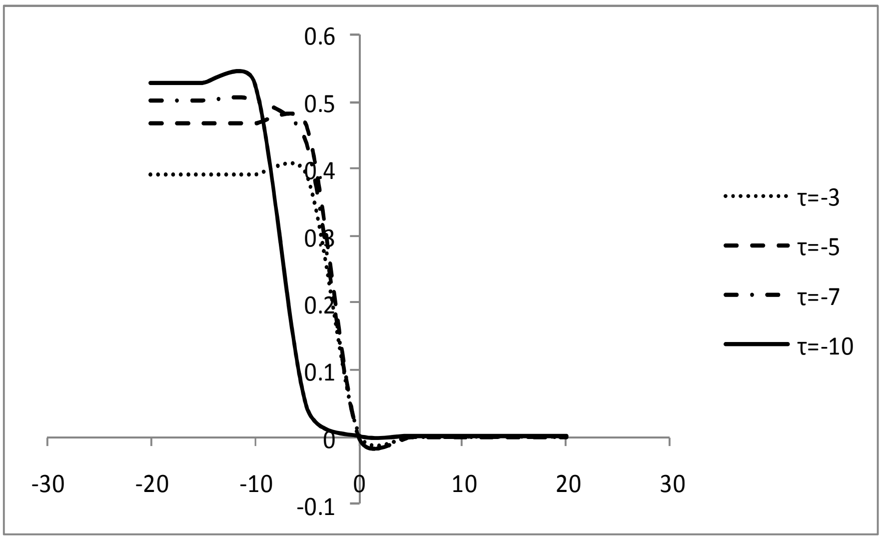

Fig. 3: Variation of ϕ_1 with respect to ξ at different values of τ for the spherical geometry $(\nu = 2)$ and the other parameters being the same as figure 1.

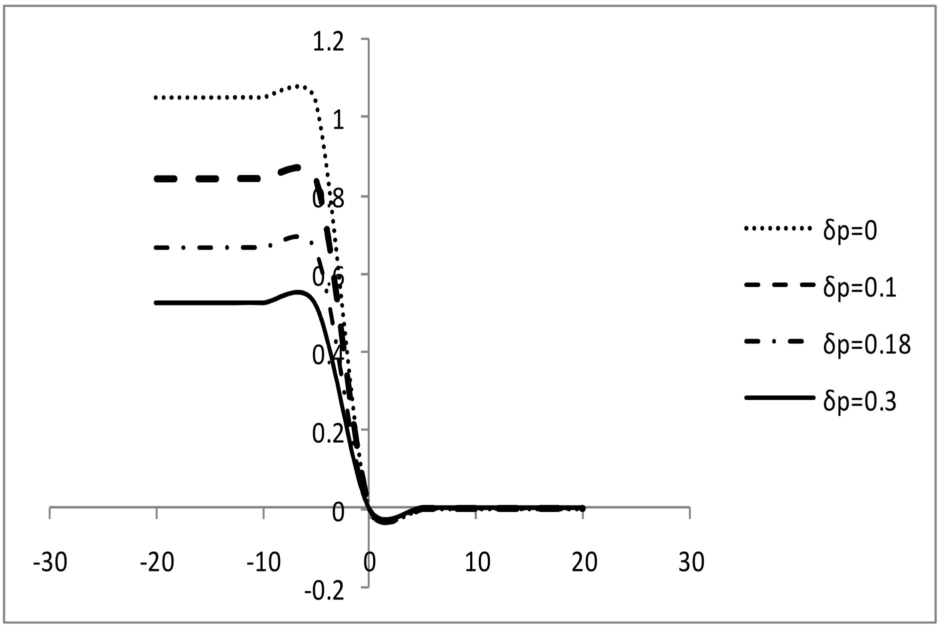

Fig. 4: Variation of ϕ_1 with respect to ξ at different values of the positron concentration for the cylindrical geometry $(\nu = 1)$ and the other parameters being the same as figure 1.

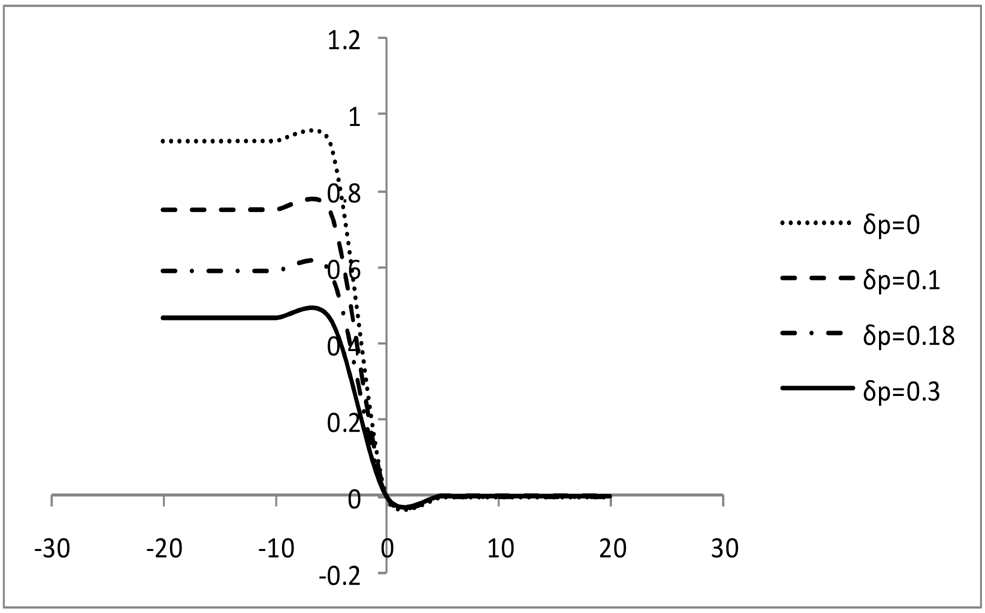

Fig. 5: Variation of ϕ_1 with respect to ξ at different values of the positron concentration for the spherical geometry $(\nu = 2)$ and the other parameters being the same as figure 1.

The effect of positron concentration on the shock wave structures for cylindrical and spherical geometries are shown in figures 4 and 5 respectively. From these two figures (Fig.4 and Fig.5), it is found that increase in positron concentration decreases the amplitude of the DIASWs for nonplanar geometry. It is obvious that; in physical situation, any increase in positron concentration decreases the ion concentration. Since, we are dealing with DIASWs; the amplitude of the shock structure will decrease for increase in positron concentration.

5. CONCLUSION

We have derived the nonplanar Burgers' equation for Dust-ion-acoustic shock waves in a four component unmagnetized plasma comprising of static negatively charged dust fluid, Boltzmann distributed electrons and positrons, and adiabatic ion fluid by using the standard reductive perturbation method. We have found out that; the developed shock amplitude are different in the different geometries. With the planar geometry presently a higher amplitude in the shock structure, followed by the cylindrical and spherical shock structure respectively. For large negative values of $|\tau|$, it is observed that the nonplanar geometries approaches the planar geometry. Finally, increase in positron concentration decreases the amplitude of the DIASWs. This research findings is very important for the understanding of the propagation characteristics of DIASWs in plasma applications as well as laboratory Plasma.

REFERENCES

1. Tandberg-Hansen, E. and Emisle, A. G. (1988). The Physics of Solar Flares, Cambridge University Press, Cambridge.
2. Ghosh Samiran and Bharuthram, R. (2008) Ion acoustic solitons and double layers in electron-positron-ion plasmas with dust particulates. Astrophys. Space Sci. 314: 121-127.
3. Mille, H. R. and Witta, P. (1987). Active Galactic Nuclei, Springer. Berlin. p. 202
4. Michel, F. C. (1982). Theory of Pulsar Magnetospheres. Rev. Mod. Phys. 54, 1.
5. G. W. Gibbons, S. W. Hawking and S. Siklos (edited). The Very Early Universe (1983). Cambridge University Press, Cambridge, England.
6. Rees, M. J.: In Gibbons, G. W., Hawkings, S. W., Siklos, S. (eds.). The Early Universe. Cambridge University Press, Cambridge (1983).
7. Michel, F. C. (1991). Theory of Neutron Star Magnetospheres. University of Chicago Press, Chicago.
8. Misher W., Thorne, K. and Wheeler J. D. (1973). Gravitation. San Francisco, C. A. Freeman, p. 763.
9. Sabry, R. (2009). Large amplitude ion-acoustic solitary waves and double layers in multicomponent plasma with positron. Phys. Plasmas 16, 072307
10. Shukla, P. K., (2001). A Survey of dusty plasma physics. Phys. Plasmas 8(5). 1791-1803.
11. Bliokh, P., Sinitsin, V. and Yaroshenko, V. Dusty and Self-Gravitational Plasmas in Space (Kluwer Academic, Dordrecht, 1995).

12. Shukla, P. K., Mendis, D. A. and Chow, V. W. The Physics of Dusty Plasmas (World Scientific, Singapore, 1996).
13. Mendis, D. A. and Rosenberg, M. (1992). Some aspects of dusty-plasma interaction in cosmic environment IEEE Trans. Plasma Sci. 20 (1): 929 – 934
14. Mendis, D. A. In Advances in Dusty Plasmas edited by P. K. Shukla, D. A. Mendis and t. Desai (world Scientific, Singapore, 1997).
15. Verheest, F. Waves in Dusty Space Plasmas (Kluwer Academic, Dordrecht, 2000).
16. Geortz, C. K. (1989): Dusty Plasma in the solar system. Rev. Geophys. 27, 271.
17. Mendis, D. A. and Rosenberg, M. (1994). Cosmic Dusty Plasmas. Annu. Rev. Astrophys. 32, 419.
18. D'Angelo, N. (1990). Low frequency electrostatic waves in dusty Plasma Planet. Space Sci. 38 (9): 1142 – 1146.
19. Bliokh, P. V. and Yaroshenko, V. V. (1985). Electrostatic Waves in Saturn's rings. Sov. Astron. 29, 330 – 336.
20. De Angelis, U., Formisano, V., Giordano, M., (1988). Ion Plasma waves in dusty plasmas. Halley's comet. J. Plasma Phys. 40, 399 – 406.
21. Shukla, P. K. and Sihin, V. P., (1992). Dust ion-acoustic waves. Phys. Scripta 45, 508.
22. Nakamura, Y., Bailung, H., Shukla, P. K. (1999). Observation of ion-acoustic shocks in a dusty plasmas. Phys. Rev. Lett. 83, 1602
23. Shukla, P. K. (2000). Dust ion-acoustic shocks and holes. Phys. Plasmas 7, 1044.
24. Shukla, P. K. and Mamun, A. A. Introduction to Dusty Plasma Physics (Bristol: IOP Publishing Ltd. 2002).
25. Shukla, P. K. and Mamun, A. A. (2003). Solitons, Shocks and Vortices in dusty plasmas. New J. Phys. 5, 17.2 – 17.37.
26. Rahman, A., Sayed, F. and Mamun, A. A. (207). Dust ion-acoustic shock waves in an adiabatic dusty Plasma. Phys Plasma 14, 034503.
27. Mamun, A. A. and Shukla, P. K. (2002). Solitary Potentials in cometry dusty Plasma. Geophys. Res. Lett. 29, 1870. doi:10.1029/2002 GLO15219.
28. Ju-kui, X. and He, L. (2003). Modulational instability of Cylindrical and spherical dust ion-acoust waves. Phys. Plasmas 10, 339.
29. Xue, Ju-kui (2003). A Spherical KP equation for dust acoustic waves. Phy. Lett. A 314, 479.
30. Xue, Ju-kui (2003). Cylindrical dust acoustic waves with transverse perturbation. Phys. Plasmas 10, 3430.
31. Xue, Ju-kui (2005). Nonplanar dust ion-acoustic shock waves with transverse perturbation. Phys. Plasmas 2, 012314.
32. Sahu, b. (2011). Cylindrical or Spherical Dust ion Acoustic Shocks in an Adiabatic Dusty Plasma. Bulg. J. Phys. 38, 175 – 183.
33. Washimi, H. and Taniuti, T. (1996). Propagation of ion-acoustic solitary wave of small amplitude. Phys. Rev. Lett. 17, 996 – 998.