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A Study of Scattering Proton by Zirconium Between 50MeV and 65MeV

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Abstract
The experimental data of the scattering proton has been analysis by using one of the methods for optical dispersion model which depending on the afferent between the real and imaginary parts and this led to a derivation decrease in determining the optical parameters from the experimental data. Also on the stripe expending of the real potential parameters from high energy to low energy to the close area of the Coulomb barrier which characterized the lack of information about the experimental data for each, using the program SPI-GINOA in order to determine the value of the volume integral for the real and imaginary parts (surface and volume) for the interaction ($P + Zr_{40}^{90}$) in the of energy ($20 \leq E_p \leq 65$ MeV). Which resulted in the perfect value of integrals for the real and imaginary parts (surface – volume) and has obtained by comparing them with experimental data in the references [1], [2], [3], [4].The Value of the volume integral for the real part and integrals oh "Hartree – Fock" was pointed and then determined the value of real part of the potential Hartree – fock potential. In addition we also has been determined the imaginary potential (two parts the surface and the volume) and studied on function of energy for all the specific pointed ingredients.The potential dispersion was determined (surface – volume) and studied their functional energy. Therefore, we determined the radius proton optical model and also we found its energy way match close to what reveal the correctness of method of dispersive optical model at one hand, and the accuracy in the determination of optical model parameters at other hand.
Keywords: SPI-GINOA, imaginary potential, functional energy

The Dispersion Optical Model

The dispersion optical model origins from processing of energy dependences of optical potential parameters depending on data analysis that related to differential cross section, total cross section and polarization in terms of classic optical modelOur paper concern on using one new methods of the dispersion optical model that aids to know and understand the properties that related to dependence of energy and mass with optical potential parameters at low and high energy. This method is called Variation Moment Approach (VMA).
In this method the real part of optical potential divided in two parts:

$$V(r,E) = V_{HF}(r,E) + \Delta V(r,E) \qquad \dots\dots(1)$$

Where: $V_{HF}(r,E)$ the potential which is represented the equivalent local potential of the unlocal mean field. It had built on the base that it had smooth energy dependence on large energy range which it included Fermi energy. The second term is evaluated from the dispersion relation which is connected between the real and imaginary potentials. The dependences of the this term, which is called dispersion correction, on the energy is complicated especially near the Fermi energy. The Hartee-Fock potential had been evaluated at the positive area of the energy by using the elastic differential cross section and analysis of energy data. After that it can be extrapolated to the negative energies by the helping of the first level energies of single particle and the first level of single hole near the Fermi energy. Then it can be used the mean field to evaluate the binding energies of single particle levels and single hole levels where there is not any experimental data. The second term $\Delta V(r,E)$ is divided into two terms. These two terms are surface imaginary term$\Delta V_d(r,E)$ and volume imaginary$\Delta V_w(r,E)$. The parameters of optical potential, which are depending on the energy, vary when it was done the best fitting of experimental cross section and energy analysis during the searching process. This is given a reasonable result for the experimental data. One coefficient would led to reduce the number of parameters which we are trying to find them if these parameters are depending on the energy of the incident proton. This is called the Volume integrals of optical potential "(J_V, J_{W_w}, J_{W_d})", which are determined by the following relations :

$$J_W(E) = J_{W_w}(E) + J_{W_d}(E)$$
$$= \frac{4\pi}{A}\int_0^\infty (W_d(E,r) + W_w(E,r))r^2.dr \qquad \dots\dots(2)$$
$$J_V(E) = \frac{4\pi}{A}\int_0^\infty (V(r,E))r^2.dr$$

There are parameters of those volume integrals$(E_0, \beta_2, \rho_2, \rho_w)$, which we could calculate by using Brown – Roa relation:

$$J_W(E) = \beta_2 \frac{(E-E_0)^2}{(E-E_0)^2+\rho_2^2} \qquad \dots\dots\dots(3)$$

We will start from these relations to calculate volume integrals of volume imaginary, surface and total part. Then we can evaluated both parts of imaginary potential named volume and surface imaginary potential due to the

presence of correlation between them as following:

$$J_{W_d}(E) = \frac{4\pi}{3} 12.a_d.\frac{R_d^2}{A}.\left[1+\frac{1}{3}\left(\frac{\pi a_d}{R_d}\right)^2\right].W_d(E) = g_d.W_d(E) \dots\dots(4)$$

$$J_{W_w}(E) = \frac{4\pi}{3}\frac{R_w^3}{A}.\left[1+\frac{1}{3}\left(\frac{\pi a_w}{R_w}\right)^2\right].W_w(E) = g_w.W_w(E) \qquad (5)$$

Where:

$$g_d = \frac{4\pi}{3} 12.a_d.\frac{R_d^2}{A}.\left[1+\frac{1}{3}\left(\frac{\pi a_d}{R_d}\right)^2\right]$$

$$g_w = \frac{R_w^3}{A}.\left[1+\frac{1}{3}\left(\frac{\pi a_w}{R_w}\right)^2\right]$$

It can be notice that the finding of the energy dependence of $W_d(E)$ and $W_w(E)$, rely on the determination g_d and g_w which considered constant values. While the values of parameters (r_w, r_d, a_w, a_d) are maintained as a mean value.

The dispersion component of real optical potential was determined by using the dispersion relation (DR) that connect between the real and imaginary parts of optical potential. It can be given by the following relation :

$$\Delta V(r,E) = \frac{2}{\pi}(E-E_f)\int_{E_0}^{\infty}\frac{W(r,E')dE'}{(E-E_f)^2-(E'-E_f)} \qquad \dots\dots(6)$$

$$\Delta V(r,E) = \Delta V_w(E).f(r_w) + \Delta V_d(E).g(r_d)$$

We can observe that the dispersion part consist of surface dispersion part $\Delta V_d(E)$ and volume imaginary part $\Delta V_w(E)$ which we can determine them by:

$$\Delta V_d(E) = \frac{2}{\pi}(E-E_f)\int_{E_0}^{\infty}\frac{J_{W_d}(E')\,dE'}{g_d(E)\left[(E-E_f)^2-(E'-E_f)^2\right]} \qquad \dots\dots(7)$$

$$\Delta V_w(E) = \frac{2}{\pi}(E-E_f)\int_{E_0}^{\infty}\frac{J_{W_w}(E')\,dE'}{g_w(E)\left[(E-E_f)^2-(E'-E_f)^2\right]} \qquad \dots\dots(8)$$

After determination of the dispersion component of real part we tend to define the Hartee-Fock component which takes of the form of Wood-Saxon function according to the following formula

$$V_{HF}(r,E) = U_{HF}(E).f(r_{HF}) \qquad \dots\dots(9)$$

The depth of Hartee-Fock potential is given by the relation as following :

$$U_{HF}(E) = U_{HF}(E_f)\exp(\alpha_{HF}(E-E_f)) \qquad \dots\dots(10)$$

Since the volume integral of real potential is given by

$$J_V(E) = \frac{4\pi}{3}\frac{R_V^3}{A}.\left[1+\left(\frac{\pi a_V}{R_V}\right)^2\right].V(E) = g_V.V(E) \qquad (11)$$

Then:

$$J_{HF}(E=E_f) = g_{HF}.V(E=E_f) \qquad (*)$$

Substituting Equation (10) into (9) and taking the (*) expression into account, we have,

$$V_{HF}(r,E) = \frac{J_{HF}(E_f)}{g_{HF}}\left[\exp(\alpha_{HF}(E-E_f))\right]f(r_{HF}) \qquad (12)$$

The relation (2) reduced to :

$$V(r,E) = \frac{J_{HF}(E_f)}{g_{HF}}\left[\exp(\alpha_{HF}(E-E_f))\right]f(r_{HF}) + \Delta V_w(E).f(r_w) + \Delta V_d(E).g(r_d) \qquad \dots\dots(13)$$

The volume integral of real potential represents the sum of the volume integral of Hartee-Fock potential and the volume integral of the dispersion potential as following :

$$J_V(E) = J_{HF}(E) + J_{\Delta V}(E) \qquad \dots\dots(14)$$

The volume integral of Hartee-Fock potential is evaluated by :

$$J_{HF}(E) = J_{HF}(E_f)\exp(\alpha_{HF}(E-E_f)) \qquad (15)$$

The determination of Hartee-Fock component requires calculation of g_{HF} magnitude from equation (11)and definition the parameters $(U_{HF}(E_f), r_{HF}, a_{HF}, \alpha_{HF})$.

The determination of component (Hartree – Fock) requires calculating from the eq after determining the parameters. The $U_{HF}(E_f)$ potential is determined according to the relation:

$$U_{HF}(E_f) = V_0 + \frac{N-Z}{A}V_1 \qquad \dots\dots(16)$$

The parameters r_{HF}, a_{HF} are taken as a mean value and constant, After that it can be find both $J_{HF}(E)$ and g_{HF} by using the equations (15), (11). The parameter α_{HF} is determined by comparison of tow values of $J_V(E)$ that are calculated from equations (11) and (15). after the determination of the parts of the mean nuclear field is finished, it can be calculated the differential cross section $\sigma(\theta)$ and the reaction cross section $\sigma_r(E)$ by using a program that is written by a Pascal language, In this case the real optical potential must be expressed by the parameters of Wood-Saxon optical potential as we are mentioned later. After that we must be used the linear extrapolation method of

the Wood-Saxon potential as following :

$$V(r,E) = U_V(E)f(r_V)$$
$$U_V = V(r=0,E) \qquad \ldots\ldots(17)$$

The results and discussion:

The parameters of optical dispersion potential have been obtained by using VMA method for $(P + Zr^{90})$ nuclei. These values are listed in the table 1.

Table (1): the parameters of the optical potential for $(P + Zr^{90})$

E (MeV)	U_v (MeV)	r_v (Fm)	a_v (Fm)	W_w)MeV(	r_w (Fm)	a_w (Fm)	W_d)MeV(	r_d (Fm)	a_d (Fm)
30	50.44	1.188	0.62	3.733	1.22	0.57	6,754	1.22	0.57
40	46.44	1.176	0.62	5.198	1.22	0.57	5.942	1.22	0.57
49.7	43.08	1.168	0.62	6.389	1.22	0.57	5.076	1.22	0.57
60.8	39.52	1.162	0.62	7.53	1.22	0.57	4.151	1.22	0.57

$$V_{so} = 6.2, r_{so} = 1.01, a_{so} = 0.75, r_c = 1.25, E_0 = -2.65$$
$$E_f = -5.3, \rho_2 = 13.825, \rho_w = 50, \beta_2 = 115$$

1- The determination of the values of volume integrals of the Surface and volume imaginary potential :

The values of volume integrals of the Surface and volume imaginary potential were determined by using the Brown- raw relations.

Figure 1 shows comparison of volume integrals with reference values versus energy.

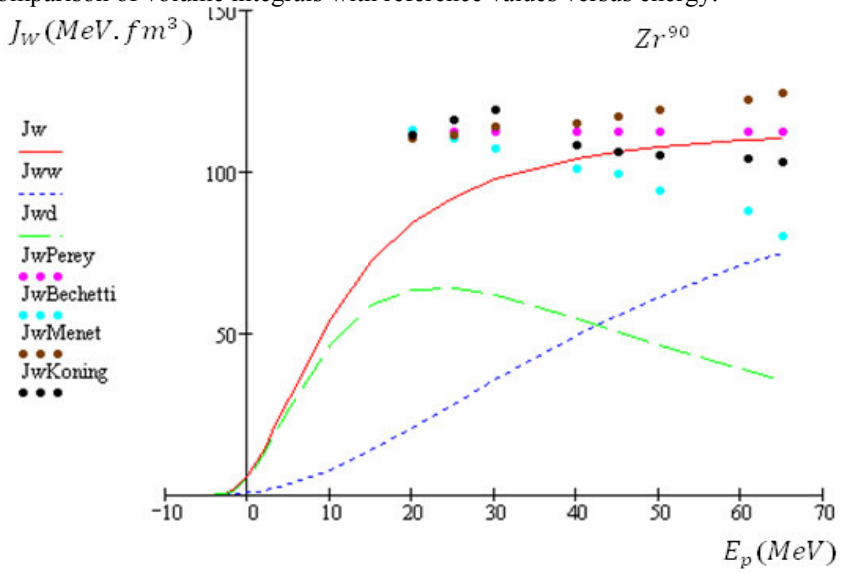
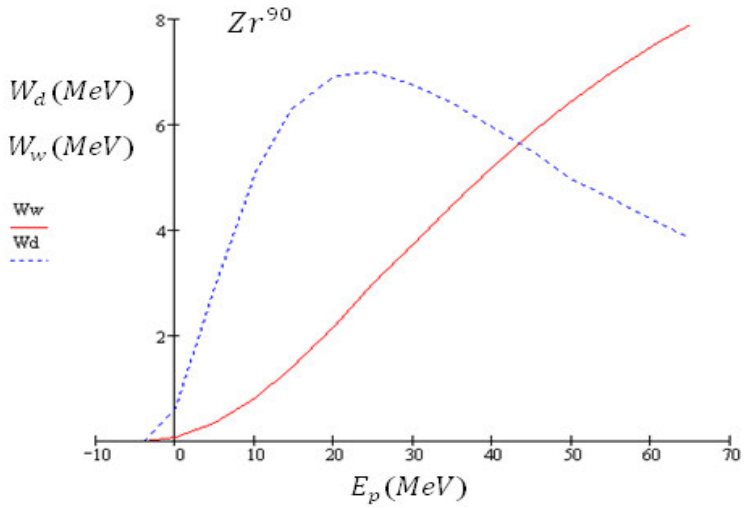


Figure 1 reveals the way that it was used to get the calculated values of volume integral of imaginary potential. The constants of equation 3 had adjusted to obtain the calculated values which had mean values for reference values. It can be noticed from fig 1 that the calculated values by using VMA method agree with reference values, but there is a deviation at certain ranges of energy because the reference values are effective at a just certain rang of energy.

2 - The determination of the values of the volume and surface imaginary potential according to VMA method :

By using the values of parameters in table 1 and equations 4 and 5 we had been determined the surface imaginary potential W_d and the volume imaginary potential W_w. The figure (2) shows the variation of the W_w and W_d versus the energy of incident proton.



It is revealed from fig (2) that there is a deviation near the Coulomb barrier. This is agree with the reference (4).

3. The determination of values of the volume integral of the dispersion component according to VMA method:

By using the values of parameters from table (1) and the eqs (10),(11),(12), we have been calculated the values of the volume integral of the dispersion component according to VMA method.

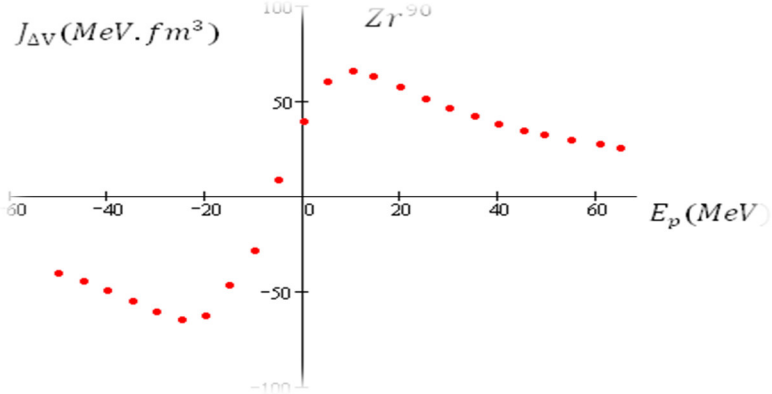


Fig (3) shows the behavior of the volume integral of the dispersion component of the reaction ($p + Zr^{90}$) and this is agree with the expected behavior in the references [7], [8], [9], and It is a clear evidence that our parameters, which are showed in table (1) are accurate.

4. The determination of volume integral of real potential $J_V(E)$ and the volume integral of the (Hartree – Fock) potential $J_{hf}(E)$ according to VMA method:

We had been determine the volume integral of real potential $J_V(E)$ and the volume integral of the (Hartree – Fock) potential $J_{hf}(E)$ by using the eqs (11),(14), and These values are plotted in fig 4.

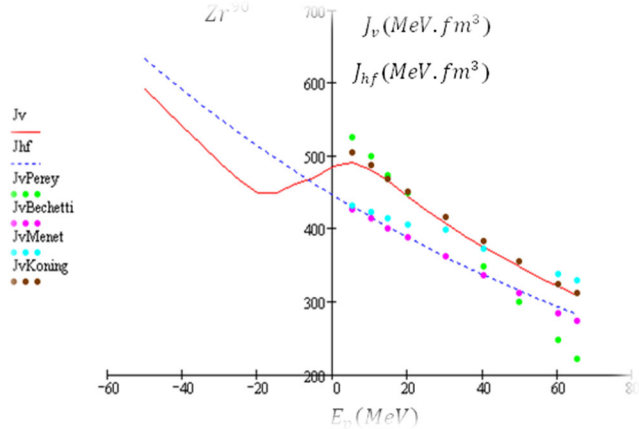


Fig. (4) variations $J_V(E)$ and $J_{hf}(E)$ versus energy. It can be noticed from fig. (4) that there are a good agreement between our values and the values of reference [3].

5. The determination of the real part of the optical potential V and (Hartree – Fock) potential V_{hf}:

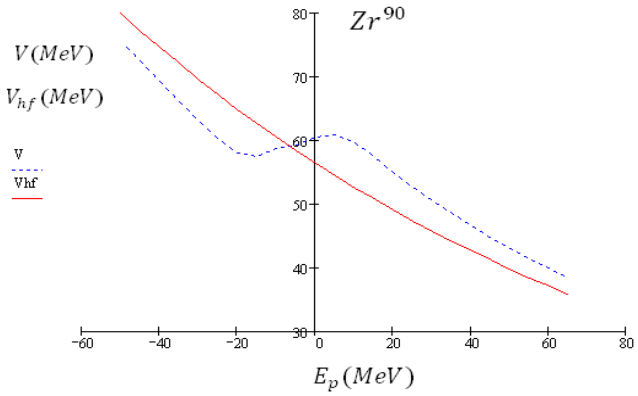


Fig (5), we had determined the values of the real part of optical potential V and (Hartree – Fock) potential V_{hf} by using the equations (15),(16) The obtained values are plotted in fig (5).By helping of Excel program we could express these values in terms of the mean values to obtain straight lines which have certain equations as following:

$$V = -0.273E + 57.67$$

$$V_{hf} = -0.378E + 57.68$$

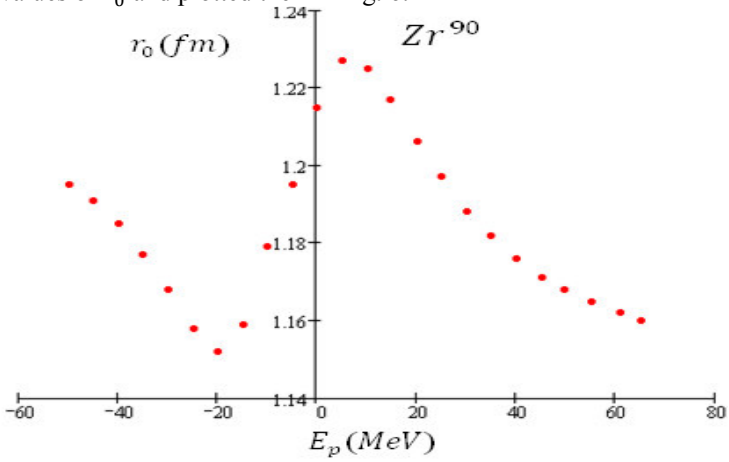
We are observed that the tow last equations are agreed with reference [16].

6. The study of radius of real optical potential according to VMA method:

The values of radius of real optical potential r_0 have been determined in terms of the energy. . This is done by solving third order equation r_0^3 from equation (11), we then have:

$$r_0^3 = \frac{J_V(E)}{\left[1+\frac{9.872}{A^{\frac{2}{3}}}\left(\frac{a_0}{r_0}\right)^2\right]\times(V\times 4.156)}$$

Then we are found the values of r_0 and plotted them in fig. 6.



It was revealed that the variation of radius of real optical potential r_0 versus energy takes the same plot which is described in reference [16].

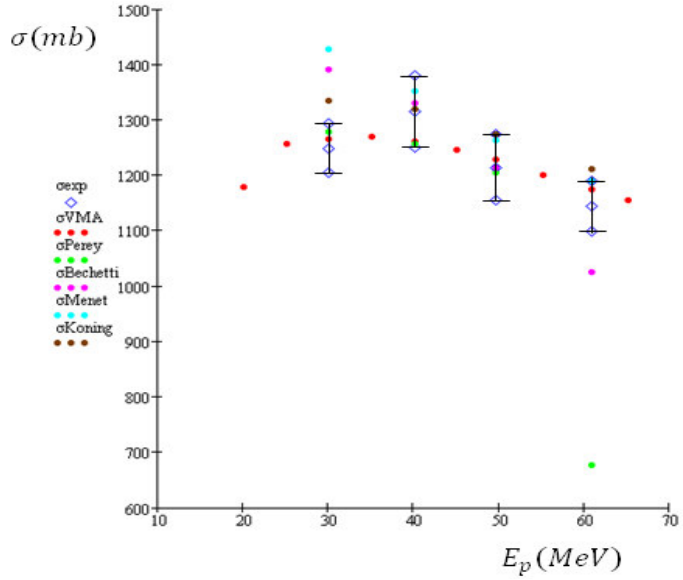
7 – The proof of the accuracy of parameters of optical potential :

Finally, the values of parameters of optical potential that were calculated according to VMA method, could be checked. This is done by calculating the cross section by using these parameters and comparing these values with the experimental data. These result are listed in Table 2.

E_p (MeV)	σ_{VMA} (mb)	σ_{Perey} (mb)	$\sigma_{Becchetti}$ (mb)	$\sigma_{Menetel\ al}$ (mb)	σ_{Koning} (mb)
30	1256	1278	1391	1428	1334
40	1263	1257	1330	1351	1320
49.5	1228	1204	1213	1264	1273
60.8	1174	676	1025	1189	1211

Table (2): the values of cross section that were calculated by VMA method and the reference values.

The variation of cross section versus energy are plotted in fig. 7.



It is obvious form fig. 7 that there is a good agreement between the available experimental and theoretical data.

Conclusions

- It was found that the volume integrals of surface and volume imaginary potential which were calculated according to VMA method agree with the references values that obtained from [1], [2], [3] at the range of the energy $20 \leq E_P \leq 65$ MeV.
- During determining the volume integrals of the real and Hartree – Fock parts of optical potential we found agree with the obtained values from the reference [3]. Then we calculated the values of a real part of the optical potential and Hartree – Fock potential then we calculated the surface and volume imaginary potential. we had been studied the energy dependence of all these values. After that we determined a relation that described the behavior of a real and Hartree – Fock potential versus the energy.
- We had been determined the values of the volume integrals of the dispersion potential and had been studied the energy dependences. Then, we calculated the values of a radius of optical potential.
- We had calculated the cross section by using the obtained parameters and we had compared them with the experimental data to check the accuracy of the obtained parameters.
- Finally, we can say that the dispersion optical model give a good method to describe the cross section of the proton scattering by Zr_{40}^{90}within a range of energy reaches to 65 MeV.

So, we can take these values as a base for this nucleus.

References

[1] Becchetti, F. D., and Greenlees, G. W. (Nucleon-nucleus optical model parameters, A>40,E<50 Mev. (1969). Phys. Rev, 182, p.1190 .
[2] Menet, J. J. H., Gross, E. E., Malanify, J. J. al. Total reaction cross section measurements for 30-60 Mev Protons and imaginary optical potential. Phys. Rev . (1971),V.C4,P. 1114-1129.
[3] Koning, A. J., Delarocheb J. P. Local and global nucleon optical models from 1 KeV to 200 Mev .Nuclear physics A713 (2003) 231-310.
[4] Romanovsky, E. A. Botros, S. Becpalova, B. O. (1995). News. Ras. Phys. Vol. 59. No. 10.
[5] Hodgson, P. E. The Dispersive Optical Model, OUNP,1991-1992,p.p.1-7.
[6] Romanovskye, E. A. Becpalova, B. O. Analyzed within the Dispersive Optical Model For the Proton and Zr^{90} within energies 5<E<65 Mev. (2004) Nucl. Phys.
[7] Mahaux, C. Bortingnon, P. F.1985- Dynamics of the Shell Model .(1985),Phys. Rep, V120,P.1-274.
[8] Mahaux, C. Sartor, R. 1989,Nucl.Phys.V.503,P.525.
[9] Mahaux, C. Sartor, R. Advance in Nuclear. (1992). Phys. Edited New York, Vol. 20,P.1.
[10] Belal, A. 2006- Study The Systematic Of The Scattering Protons by Nuclei Which Have Mass Number $50 \leq A \leq 70$ near The Coulomb Barrier $E \leq E_B$ in The Range Of VMA Method. Journal of Al Baath University – Syria-Homs , No.28.
[11] Belal, A. 2009-Using the Dispersion Optical Model for Analysis of Proton Scattering by Nuclei $Zr^{92,94,96}$ Energy Rang close to columbic barrier. Journal of Al Baath University –Syria-Homs , No.31.
[12] Belal, A. , 2010-Analysis of Data Related to the Scattering of Protons bombed by Energy $6 \leq E_p \leq 10$ MeVat

the Target Nucleus $Sn^{116,118,120,122,124}$ in the range of the desperation optical- model Journal of AlBaath University –Syria-Homs , No.32.
[13] Beer, K. Hodgson, P. E. The Systematic of Nuclear Bound States. J. Phys. G. Nucl. Phys. 1978, V.4. N.12, P.P. $1287 - 1291$.
[14] Sakaguchi, H. Nakamura, M. Hatanaka, K. Goto, A. Elastic Scattering of 65MeVpolarized Protons. Phys. Rev., 1982, V. C26, N.3, P.P. 944-960.
[15] Perey, C. M. Perey, F. G. Compilation of Phenomenological model Parameters $1954 - 1975$ Data and nuclear data Tables, 1976, V.17, P. P $1 - 101$.
[16] Romanovsky, E. A. Belal, A. Morzen, L. R. 1993, News. Ras, Phys. Vol. 57, No. 10, P. 179.
[17] Carlson, R. F. Atomic Data and Nuclear Data Tables. (1996), 63, 93-116 .