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Mixed Convection Heat Transfer of MHD Flow Due to Permeable Sheet: An Analytical Solution

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Abstract
In this paper we investigate the analytical solution for MHD flow and heat transfer of electrically conducting fluid due to vertical starching surface. Here the diffusion thermo (Dufour) and thermal diffusion (Soret) effects are considered. It is shown that the porosity, magnetic, convection, concentrationand buoyancy effects can be combined with a new parameter called porous- magneto-convection-concentration parameters .The effect of physical parameter influencing the flow and heat transfer are studied and results are plotted and discussed.
Keywords: Heat Transfer, Porous-Magneto-Convection-Concentration parameter, Buoyancy, Soret, Dufour, Lewis;

Nomenclature

Pr	Prandtl number
Le	Lewis number
s	wall mass transfer parameter
M	Magnetic parameter
N	concentration buoyancy parameter
γ	porosity parameter
Df	Dufour number
Sr	Soret number
Λ	constant mixed convection
G_{rx}	local Grashof number
R_{ex}	Local Reynolds number
Γ	Magneto-convection concentration parameter
Nu_x	Local Nusselt number
Sh_x	Local shear wood number

1. Introduction

The study of magneto hydrodynamics of viscous fluid with heat and mass transfer through a porous media is a great deal in industrial application such as chemical and drying process,crystal magnetic damping control, chromatography, heat and mass transfer characteristic in Nano fluid for power plants etc.Ponnamma rani and Chang [1] investigate the Soret and Dufour effects on unsteady laminar viscous or viscoelastic flow and found solution by numerical method. Beg [2] has Investigated combined effect of Soret and Dufour number on free convection of MHD flow.Tsai andHuang [3] have studied certain kinds of mixtures such as light, medium molecular weight, where Soret and Dufour effects are influenced. Hayat et al. [4] investigated the Soret and Dufour effects on mixed convection due verticalstretching surface in a porous medium filled with a viscoelastic fluid. Gamal [5] Studied Thermal-diffusion, Soret and Dufour effects on Hiemenz flow and mass transfer through porous medium due to stretching surface.Pal and Chatterjee[6] Analyzed the Mixed convection effect onmagneto hydrodynamic heat and mass transfer past a stretching surface in a micro polar fluid-through saturated porous medium under the influence of Ohmic heating. Pal. et al[7] and Mondal.et al[8]investigated the effects of variable thermal conductivity, Soret and Dufour effects on MHD non-Darcy mixed convection heat and mass transfer over a non-linear stretching sheet with viscous dissipation.Kandasamy et al. [9] studied the effects of Soret and Dufour in the presence of thermophoresis deposition with chemical reaction, which plays very important role in industrial. Pal and Mondal[10]analyzed Soret and Dufour effects on non-Darcian mixed convection heat and mass transfer of an incompressible,electrically conducting fluid flow due to a stretching sheet in a porous medium in the presence of magnetic field and non-uniform heat source/sink. Nawaz.et al.[11] Investigated Soret and Dufour effects on the two-dimensional MHD steady flow of an electrically conducting, viscous bounded by infinite sheets. Pal and Talukdar [12] studied the Influence of Soret effects and first order chemical reaction on unsteady MHD convective flow of viscous incompressible electrically conducting fluid.Scott [13] studied the effect of saturated Darcy porous medium with an exothermic chemical reaction on lower boundary, here Dufour effect is neglected and the influence of Soret effect ishighlighted. Pal andMondal [14] investigated the thermophoresis particledeposition and Soret–Dufour effects on the convective flow on heat and mass transfer of an incompressible Newtonian electrically conducting fluid and found that the skin friction and Sherwood number increases with increase of thermal radiation parameter. Pal. et al [15] studied the effects of thermal radiation and viscous dissipation on hydro magnetic mixed convection flow over non-linearly stretching and shrinking sheets in Nano fluids, the effect of local skin friction and local Nusselt number are discussed. RamReddy.et al[16] Investigated the Soret effects on mixed convection in the boundary layer of a semi-infinite vertical flat plate in a nanofluids,the numerical results of local skin-friction, local wall temperature, local nanoparticle concentration and local wall concentration are discussed. Turkyilmazoglu [17] studied the Soret and Dufour effect on MHD flow and heat and mass transfer of an electrically conducting viscoelastic fluid past a vertical stretching surface in a porous media. Hsiao.et al [18] investigated the heat and mass transfer of an electrically conducting non-Newtonian power-law fluid. Nayak.et al [19] investigated the Soret and Dufour effects in the hydro-magnetic unsteady flow by a mixed convection past a stretching sheet in a porous medium in the presence of chemical reaction. In all above investigation some numerical method like Runge-kutta, Finite difference etc… and analytical methods are used to solve the BVP's of fluid flow, heat and mass transfer problems. But here we have given a very simple and new analytical method for solving mixed convection problem.

2. Mathematical formulation:

Let us consider the steady two-dimensional boundary layer flow due to the permeable vertical stretching sheet.Electrically conducting fluid. A uniform magnetic field is further applied along the y-axis. The sheet is assumed to be stretched with the velocity. The governing equations are given

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}-\sigma B_0^2 u-\frac{\vartheta}{\gamma_1}u+g[\beta_T(T-T_\infty)+\beta_C(C-C_\infty)] \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{DeK_T}{C_sC_p} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=De\frac{\partial^2 C}{\partial y^2}+\frac{De\,K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \quad (4)$$

Subject to the boundary conditions:

$$u=u_w(x)=ax\,,\quad v=v_{w,}\quad T=T_w(x)=T_\infty+bx,\qquad C=C_w(x)=C_\infty+cx\ \ at\ \ y=0$$
$$u\to 0,\ u_y\to 0,\ T\to T_{\infty,}\quad C\to C_{\infty,}\quad y\to\infty \quad (5)$$

Here u and v are the velocity components in x and y directions respectively, ϑ is the kinematic viscosity, σ is the electrical conductivity,Moreover, the fluid temperature is T,the concentration is C, the acceleration due to gravity is g , the thermal diffusivity isα , the coefficient of thermal expansion isβ_T , the coefficient of concentration is β_C. Moreoverv_w is the mass flux velocity with $v_w<0$ for suction and $v_w>0$ for injection respectively. The induced magnetic field is neglected. Further $a(>0), b$ and c are constants.

3. Solution of the flow & Heat transfer BVP's:

Using the following similarity transformation and dimensionless variables

$$\eta=y\sqrt{\frac{a}{\vartheta}}\ ,\ u=axf'(\eta),\ \ v=-\sqrt{a\vartheta}f(\eta),\quad \theta=\frac{T-T_\infty}{T_w-T_\infty}\ ,\qquad \phi=\frac{C-C_\infty}{C_w-C_\infty} \quad (6)$$

in Eqs (2)– (4) we get the following partial differential equations:

$$f'''+ff''-f'^2-Mf'-\gamma f'+\Lambda(\theta+N\phi)=0 \quad (7)$$

$$\theta''+\Pr\left(f\theta'-f'\theta+D_f\phi''\right)=0 \quad (8)$$

$$\phi''+\mathrm{Le}[\Pr(f\phi'-f'\phi)+Sr\,\theta'']=0 \quad (9)$$

Subject to the boundary conditions are

$$f(0)=s\,,\quad f'(0)=1,\quad \theta(0)=1,\quad \varphi(0)=1$$
$$f'(\infty)=0,\quad f''(\infty)=0\quad \theta(\infty)=0,\quad \varphi(\infty)=0 \quad (10)$$

Here$Pr=\nu/\alpha$ is the Prandtl number, $Le=\alpha/De$ Lewis numbers, $M=\sigma B^2/a\rho$ is the magnetic parameter, $\gamma=\nu/a\gamma_1$is the porosity parameter,The Dufour number D_f and the Soret number $S_r$and The concentration buoyancy parameter N,which are defined as $N=\frac{\beta_C C_w-C_\infty}{\beta_T T_w-T_\infty}$, $D_f=\frac{Dek_T C_w-C_\infty}{C_sC_p\nu T_w-T_\infty}$, $Sr=\frac{Dek_T T_w-T_\infty}{T_m\alpha C_w-C_\infty}$,$v_{w,}=-\sqrt{a\nu s}$ is the wall mass transfer velocity, The constant mixed convection parameter $\Lambda=\frac{dGr_x}{Re_x^2}$ with $Gr_x=\frac{(g\beta_T(T_w-T_\infty)x^3}{\upsilon^2}$ being the local Grashof number with $\Lambda=\mathrm{d}g\beta_T$, where $\mathrm{d}=\mathrm{d}/a$,with$\Lambda=0$ shows forced convection flow, where as $\Lambda>0$ means assisting flow and $\Lambda<0$ indicates cooled plate respectively.

$R_{ex} = \frac{u_w(x)x}{\gamma}$ is a local Raynold's number.

The exact solutions of flow, heat and mass transfer are given as:

$$f(\eta) = s + \frac{1-e^{-\lambda\eta}}{\lambda} \quad (11)$$

$$\theta(\eta) = f'(\eta) = e^{-\lambda\eta} \quad (12)$$

$$\phi(\eta) = \theta(\eta) = e^{-\lambda\eta} \quad (13)$$

That enables us to unify the parameters $\gamma, \mathrm{M}, \Lambda$ and N appearing in Eq. (7, 8 & 9) under a newly defined parameter $\Gamma = -\gamma - \mathrm{M} + \Lambda(1+\mathrm{N})$, which is here after named as the porous magneto-convection concentration parameter. Consequently, the energy equation (8) and concentration equation in (9), gives the relations

$$\lambda^2 + pr\left(-1 - s\lambda + D_f\lambda^2\right) = 0 \quad (14)$$

$$\lambda^2 - Le(Pr + Prs\lambda - Sr\,\lambda^2) = 0 \quad (15)$$

Further substituting Eqs (12) into first of Eqs (7) gives therelation

$$-1 + \Gamma + \lambda^2 - s\lambda = 0 \quad (16)$$

Eqs. (13), (14) and (15) can be combinedas

$$\Gamma\mathrm{Pr} - \lambda^2 + Pr\lambda^2 = 0 \quad (17)$$

With the Prandtl number Pr given by

$$\mathrm{Pr} = \frac{1+\mathrm{Le}(-1+\mathrm{Sr})}{D_f Le} \quad (18)$$

The physically feasible positive real root λ of Eqs (17), which plays critical role in determining the flow or temperature fields. It is note that Eq. (18) inserts a restriction on the Prandtl number, which is always positive. In another view, for a prescribed value ofPr, a vast variety of parameters Le, Sr and D_f exist. Thus, Soret and Dufour's effects are connected to the Prandtl number by the relation Eqs (18) so that the pure exact solution of the form Eqs (11) exists. Further the skin friction parameter is defined by

$$C_f = \frac{\mu u_y(0)}{\frac{\rho u_w^2}{2}} \quad (19)$$

The local Nusselt number $Nu_x = -xkT_y(0)$ and the local Sherwood number $Sh_x = -(\frac{x}{De})C_y(0)$Can be determined differentiating from Eqs.(11)

$$\sqrt{Re_x}C_f = \lambda \quad (20)$$

$$\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) = \lambda, \quad (21)$$

$$\frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0) = \lambda \quad (22)$$

It is addressed that due to the form of the solution (11), the local Nusselt number and Sherwood number are equal in the sequel. The exact solutions to the present physical problem come from the real roots λ of Eq. (17), which are positive and have very significant role in determining the mixed fields. Hence, depending on the number of roots of Eqs (17) the solution might be unique or multiple. In the case of two solutions we will call them branch 1 and branch 2 in the subsequent analysis and which are:

$$\lambda_1 = \frac{\sqrt{\mathrm{pr}}\sqrt{T}}{\sqrt{1-\mathrm{pr}}} \text{ and } \lambda_2 = -\frac{\sqrt{\mathrm{pr}}\sqrt{T}}{\sqrt{1-\mathrm{pr}}} \quad (23)$$

4. Result & Discussion

The flow and heat transfer of an electrically conducting fluid due to vertical stretching surface in a porous mediam has been considered with the effects of the soret and dufour parameters. Here an attempt is made to extract an analytical solution for the highly complicated and highly coupled system of equations.

Here it is shown that all the effects i.e. the porosity, magnetic effect, buoyancy are convection and within a new parameter ' τ ' which is called as porous-magnets-convection concentration profile.Closed analytical solutions were presented for considered problem. The soret and Dufour effects are connected to Pr by the relation (18) so that the pure exact solutions form (11-13) exist.

All other results are discussed as fig.(1) plotted for the velocity and temperature profile for different values of M. due to the effects of Lorentz force the velocity and tem for profile increases with increase in the parametric values of M.

Fig (2): is plotted for the temperature & velocity profile for different values of Prandtle numbers as increase in the parametric value of Pr, Θ & f^1 decreases and thermal boundary layer thickness reduces.

Fig(3): is depicted for different values of Porority parameter γ, as increase in the parameter value of γ, velocity and temperature profiles are decreasing.

Fig(4): is depicted for different value of convection parameter 'λ' with increase in the parametric values of ‘λ’ the velocity Q temperature profile decreases.

Fig (5): is plotted for temperature profile for different values of Df (Dufour no) as M. From this figure it is clear that thermal boundary Layer thickness is increasing with increase in parametric values of Df.

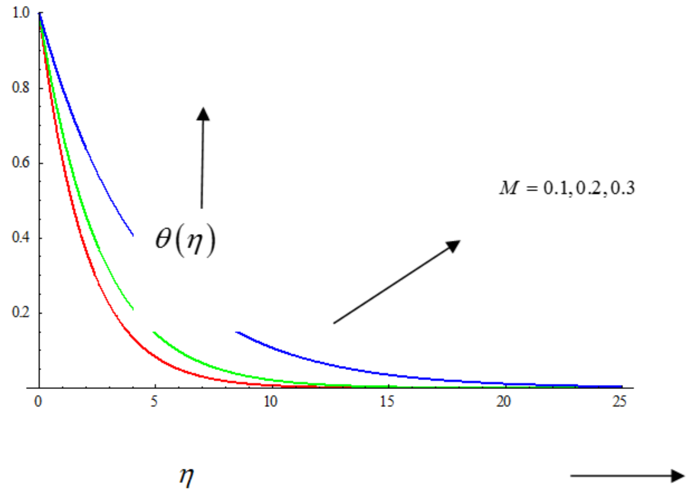
Fig (6): Shows the temperature profile for different values of soret number (Sr) .Here also temperature profile increases as the parametric value of Sr increase and thickness of thermal boundary layer thickness increases.

Fig (7): Shows the temperature profile for different values of Lewis no (Le).the thermal boundary Layer thickness decreases as increase in that parametric value of Le.

References

1. Hari Ponnamma rani, Chang Nyung Kim, “A numerical study of the Dufour and Soret effects on unsteady natural convection flow past an isothermal vertical cylinder” , Koren J. Chem.Eng ,.2009 :26 : 946-954.
2. O.Anwar Beg, A.Y. Bakire and V. R. Prasad, “Numerical Study of free convection magneto hydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects”, Computational Materials Science, 2009: 46: 57-65 .
3. R.Tsai, J. S. Huang, “Heat and mass transfer for Soret and Dufour’s effects on Hiemenz flow through porous medium onto a stretching surface”, International Journal of Heat and Mass Transfer ,2009 : 52 :2399-2406.
4. T.Hayat, M. Mustafa, I.Pop, “ Heat and mass transfer for Soret and Dufour ‘s Effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid “, Communications in Nonlinear Science and Numerical Simulation ,2010: 15 :1183-1196.
5. Gamal M. Abel- Rahman, “ Thermal-diffusion and MHD for Soret and Dufour’s effects on Hiemenz flow and transfer of fluid flow through porous medium onto a stretching surface”, Physica B: Condensed Matter, 2010 :405 :2560-2569
6. Dulal Pal, Sewli Chatterjee, “Mixed Convection magnetohydrodynamic heat and mass transfer past a stretching surface in a micropolar fluid –saturated porous medium under the influence of ohmic heating, Soret and Dufour effect”, Communications in Nonlinear Science and Numerical Simulation,2011: 16 :1329-1346.
7. Dulal Pal, Gopinath Mandal, “ Mixed convection –radiation on stagnation-point flow of nanofluids over a stretching/shrinking sheet in a porous medium with heat generation and viscous dissaption”, International Communications in Heat and Mass Transfer,2011: 38 :436-467.
8. Dulal Pal, Hiranmoy Mondal , “ Effects of Soret Dufour, chemical reaction and thermal radiation on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet”, Communications in Nonlinear Science and Numerical Simulation,2011: 16 :1942-1958.
9. R.Kandasamy , T.Hayat, S. Obaidat, “Group theory transformation for Soret and Dufour effects on free convective heat and mass transfer with thermophoresis and chemical reaction over a porous stretching surface in the presence of heat source /sink”, Nuclear Engineering and Design, 2011: 241 :2155-2161.
10. Dulal Pal, Hiranmoy Mondal , “Soret and Dufour effects on MHD non-Darcian Mixed convection heat and mass transfer over a stretching sheet with non-uniform heat source /sink” , Physica B: Condensed Matter, 2012 : 407 :642-651.
11. M.Nawaz, T. Hayat, A. Alsaedi, “ Dufour and Soret effect on MHD flow of viscous fluid between radially stretching sheets in porous medium”, Appl .Math. Mech. Engl .Ed.,2012: 33 :1403-1418.
12. Dulal Pal, Babulal Talukdar’ “Influence of fluctuating thermal and mass diffusion on unsteady MHD buoyancy-driven convection past a vertical surface with chemical reaction and Soret effects”, Communications in Nonlinear Science and Numerical Simulation, 2012 : 17 : 1597-1614.
13. Nicola L. Scott, “ Convection in a saturated Darcy porous medium with an exothermic chemical surface reaction and Soret effect”, 2012 : 39 : 1331-1335
14. Dulal Pal, Hiranmoy Mondal, Influence of thermophoresis and Soret – Dufour on magnetohydrodynamic heat and mass transfer over a non-isothermal wedge with thermal radiation and Ohmic dissipation”, Journal of Magnetism and Magnetic Materials,
2013 :331 :250-255.
15. Dulal Pal, Gopinath Mandal, KuppalapalleVajravelu, “MHD convection –dissipation heat transfer over a non-linear stretching and shrinking sheets in nonofluids with thermal radiation”, International Journal of Heat and Mass Transfer,2013: 65 : 481-490.
16. Ch. RamReddy , P.V.S.n.Murthy,Ali J. Chamkha, A.M.Rashad, “ Soret effect on Mixed Convection flow in a nanofluid under convective boundary condition “,International Journal of Heat and Mass Transfer, 2013: 64: 384-392.
17. M.Turkyilmazoglu, “The analytical solution of Mixed convection heat transfer and fluid flow of a MHD viscoelastic fluid over a permeable stretching surface”, International Journal of Mechanical Sciences, 2013 : 77: 263 -268.
18. Chao-Yin Hsiao, Wen-Jengchang , Ming –I Char, Bo-Chen Tai, “ Influence of Thermophoretic particle deposition on MHD free convection flow of non-Newtonian fluids from a vertical plate embedded in porous

media considering Soret and Dufour effects", Applied Mathematics and Computation, 2014 :244 :390-397.
19. A. Nayak, S.Panda, D.K. Phukan, "Soret and Duffer effects on mixed convection unsteady MHD boundary layer flow over stretching sheet in porous medium with chemically reactive species", Appl .Math. Mech. Engl .Ed.,2014: 35 : 849-862.



Fig(1) : Variations of velocity profile $\Theta(\eta)$ for and different values of M and pr =0.5 , γ=0.4 , α =1 , Λ =0.5

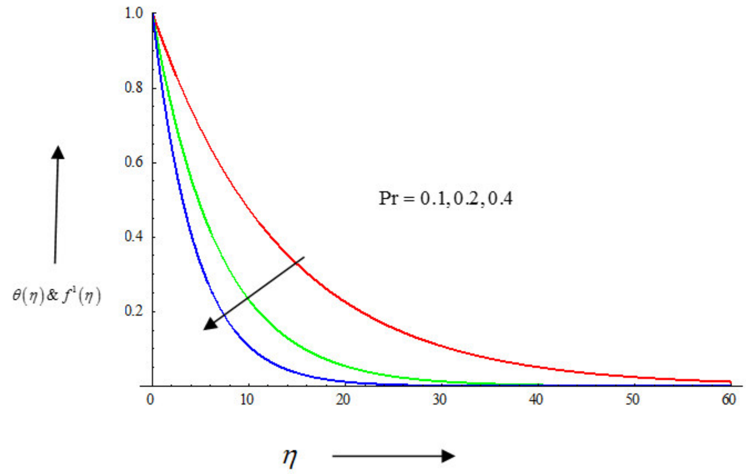


Fig (2): Variations of velocity profile $\theta(\eta) \& f^1(\eta)$ for different values of Pr and for M =0.3 ,γ=0.4 , α =0.5 Λ =0.5

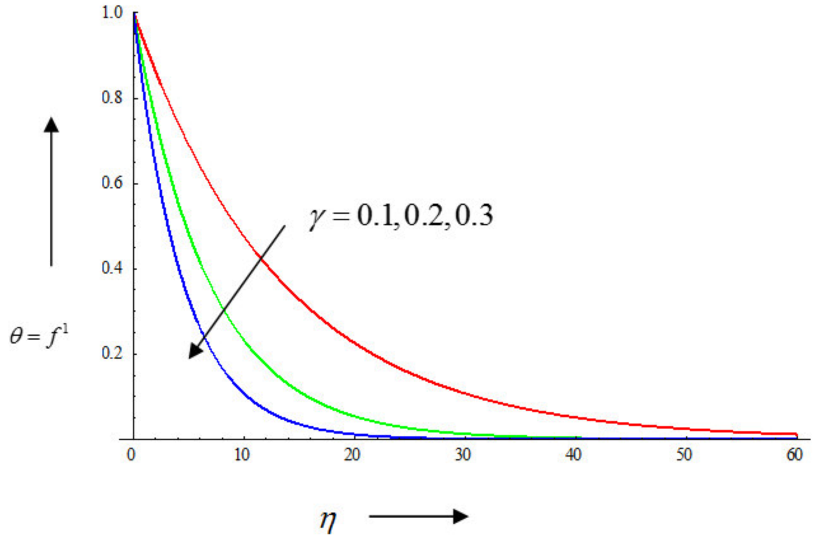


Fig (3):Variations of velocity profile $f'$for different values of $\gamma$and for M =0.3, γ=0.4, α=0.5, Λ =0.5

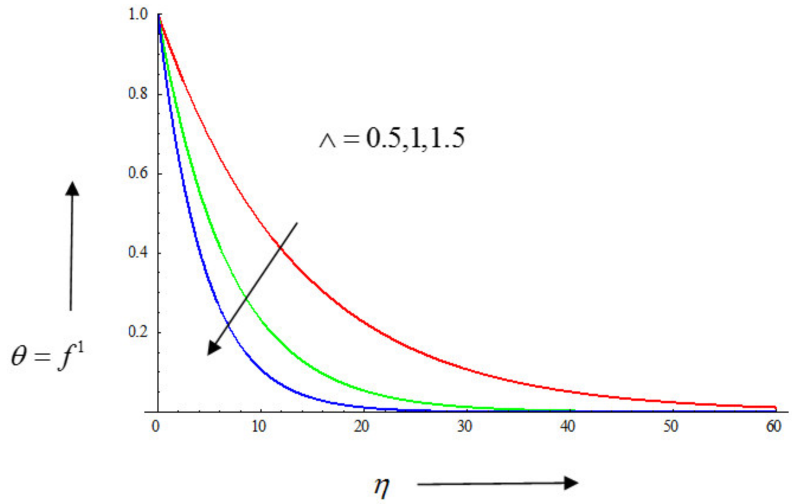


Fig (4):Variations of velocity profile $f'$for different values ofΛand M =0.3, Pr=0.3, α=0.5, γ =0.1

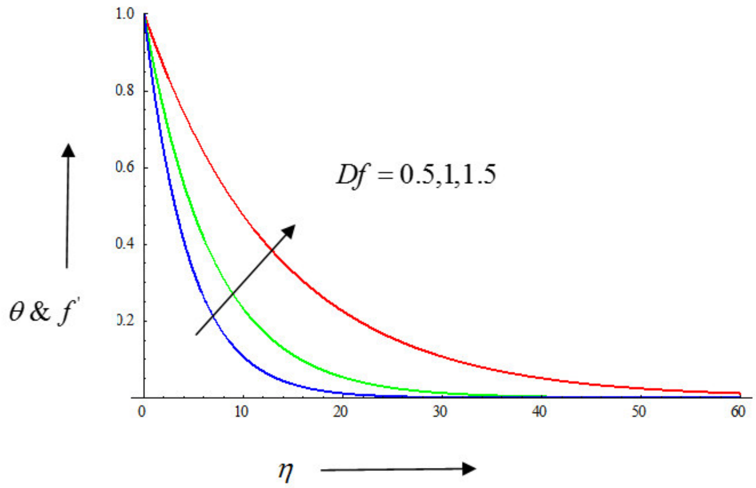


Fig (5): Variations of Temperature and velocity profile for different values of Df and Pr=0.5 and s=0.1

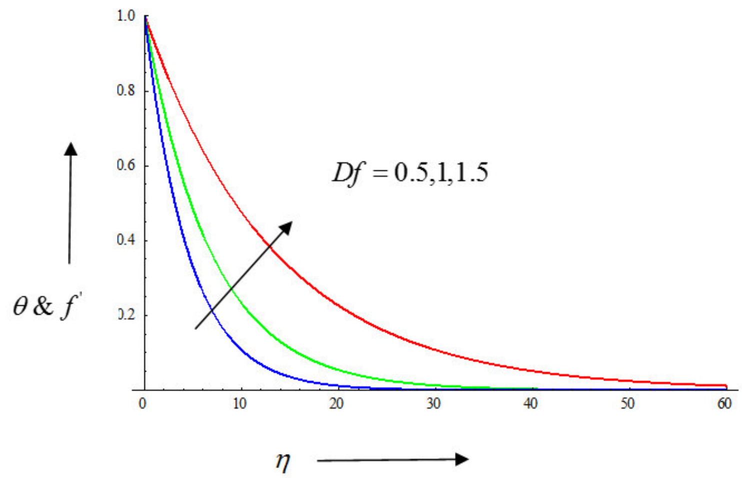


Fig (6):Variations of Temperature profile for different values ofSr, s=0.1, Le=1

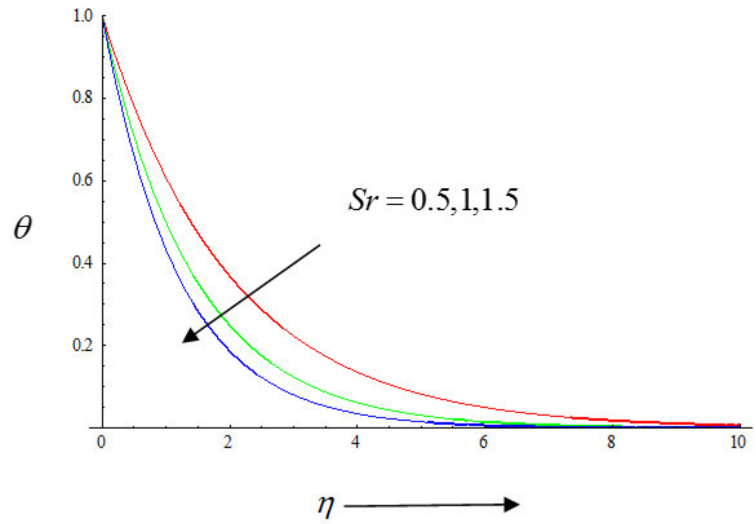


Fig (7):Variations of Temperature for $Pr = 0.5Sr$=0.1 ,s=0.1, and different values ofLe