The Analytical Solution and Numerical Simulation for Ytterbium-Doped Silica Glass Fiber Laser Output Power

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Abstract
In this paper as the first the rate equations in end pump fiber laser have been solved analytically with negligible the scattering loss and the output power versus input power has been derived. The result were applied for a single and double clad Yb^{3+}- doped silica glass fiber laser, for lasing transition it acts as a quasi-four-level system the effect of the, type concentration core radius fiber length output reflectivity, pump power and figure of inner cladding on the output lasing power have been studied.
Keywords: Rare-Earth; Fiber Laser; Rate Equation; Quasi four levels .

1. Introduction

Fiber lasers have many advantages such as high conversion efficiency immunity from thermal lasing effect, simplicity of optical construction and excellent beam quality. Ytterbium doping is attractive for high power fiber laser because of its high efficiency and strong pump absorption.Yb^{3+}-doped silica glass fiber exhibit very broad absorption and emission band, from (800) nm to (1064) nm for absorption and (970) nm to (1200) nm for (1200) nm for emission [1,2]. The simplicity of the level structure provides freedom unwanted processes such as excited state absorption, multi-phonon nonradioactive decay, and concentration quenching [3, 4]. However, pumping doped silica fiber with high concentrations can result in excess loss at the pump and lasing wavelengths owing to photo darkening, which can significantly reduce the overall conversion efficiency and degrade the long-term performance [5]. For high power applications, Single Clad Fiber (SCF) is not suitable because of the very low injection efficiency of large stripe laser diodes. Therefore, Double Clad- Fiber (DCF) was design with an attractive medium to gain high power high brightness and broad wavelength tuning. Development of high power laser diode source with advance in design and fabrication of (DCF) have successfully demonstrated kilowatt level fiber lasers and amplifiers pump by laser diode [6].

In this paper, as the first, we analytically solved the rate equations in single end-pumped Yb^{3+}-doped silica fiber laser so; we improved the analytical rate equation solutions and found the output laser power versus input parameters. The results were applied of Yb^{3+}-doped silica (SCF) and (DCF) for lasing transitions it acts as a quasi-four-level system.

2. Theory Models

2.1 Energy Level Structure

Fig (1) shows the energy level of (Yb^{3+}) ion in silica fiber. (Yb^{3+}) processes a simple atomic structure with only two principal manifolds .i.e. ground state ($^2F_{7/2}$) and excited state ($^2F_{5/2}$) separated by~(10000) cm^{-1},which makes it an ideal rare–earth element for lasing[7].therefore sublevels of upper ($^2F_{5/2}$) are labeled as (e, f, g) and the four sublevels of lower ($^2F_{7/2}$) manifold are labeled as (a, b, c, d). Weak multi-phonon decay is practically the only nonradioactive channel that exists. The excited state has a lifetime of ~ (1×10^{-3}sec) and acts as metastable level .the absence of higher energy levels near the upper manifold reduces the occurrence of multi-photon relaxation and excited state absorption (EAS) [7].

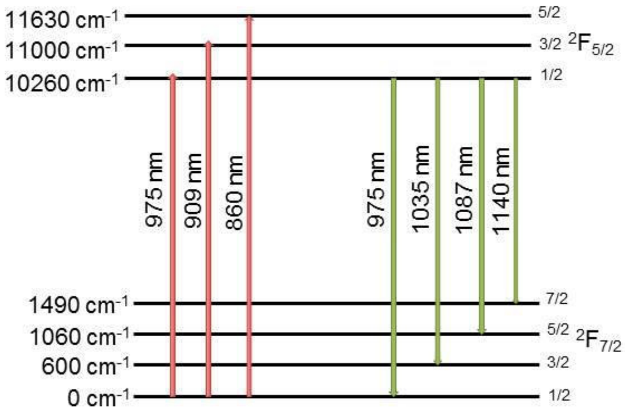

Fig. (1): Energy Level Diagram of Yb^{3+} ion in Silica.

2.2 Cavity design

Our linear cavity is composed of Yb^{3+}-doped silica glass of length (L) and with reflectivity's of R_1 and R_2 at the lasing wavelength Fig (2) shows schematic illustration of laser oscillator [8].

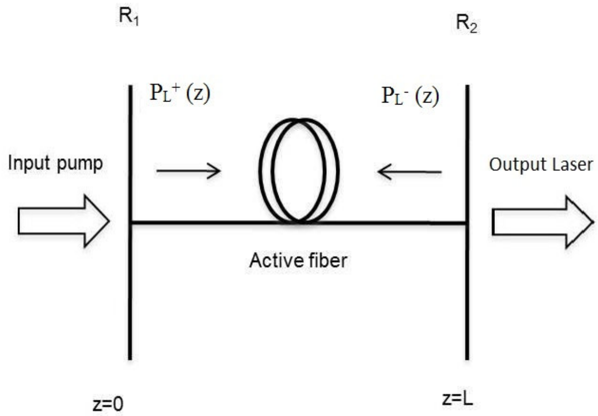

Fig. (2): Schematic of End Pumped Linear Cavity Fiber laser.

2.3 Rate Equations

The relative population of (Yb^{3+})ions in upper and lower energy levels are governed by the local rate equation [9, 10].

$$\frac{dN_1}{dt} = -(R_{12} + W_{12})N_1 + (R_{21} + W_{21} + A_{21})N_2 \quad (1)$$

$$\frac{dN_2}{dt} = (R_{12} + W_{12})N_1 - (R_{21} + W_{21} + R_{21})N_2 \quad (2)$$

Where (N) is the total number of ions per unit volume, (N_i) is the number of ions per unit volume in level (i), R is the pump transition, (W) is the lasing transition and (A_{21}) the spontaneous emission transition rate coefficient. The pump and seed transition rate are governed by absorption and emission cross section for ion in the host medium and can be written as [8, 10].

$$R_{12} = \frac{\sigma_{aP}\Gamma_P}{h\nu_P A_{effc}} P_P \quad (3)$$

$$R_{21} = \frac{\sigma_{eP}\Gamma_P}{h\nu_P A_{effc}} P_P \quad (4)$$

$$W_{12} = \frac{\sigma_{aL}\Gamma_L}{h\nu_L A_{effc}} P_L \quad (5)$$

$$W_{21} = \frac{\sigma_{eL}\Gamma_L}{h\nu_L A_{effc}} P_L \quad (6)$$

The pump cross-sections ($\sigma_P = \sigma_{aP} + \sigma_{eP}$),Vp is the pump frequency ($V_p = c/\lambda_P$) and lasing cross-sections ($\sigma_L = \sigma_{aL} + \sigma_{eL}$),$V_L$is the lasing frequency ($v_L = c/\lambda_L$), the spontaneous emission rate write as following:-

$$A_{21} = \frac{1}{t} \quad (7)$$

Where (t) is the lifetime of Yb^{3+}ion in excited state, (P_p), (P_L) are the pump and lasing power respectively [8].

The power filling factor for pump (Γ_P) is the ratio of core area and cladding area $\Gamma_P = \frac{A_{effc.}}{A_{clad.}}$. Where ($A_{effc.} = \pi a^2$)in Multi-Mode Fiber and ($A_{effc.} = \pi w_P^2$) in Single Mode Fiber (W_p)is the mode field radius, for fundamental mode defined by [11]

$$w_P = a[0.761 + \frac{1.237}{V^{1.5}} + \frac{1.429}{V^6}] \quad (8)$$

Where (V) is normalized frequency at pump wavelength ($V = 2\pi a N_a/\lambda_P$), N_a is the numerical aperture and ($A_{clad.} = \pi b^2$)where (a) and (b) is the radius of core and cladding respectively [12]
And the power filling factor for lasing [Γ_L= 1-exp[-2(r/W_L)^2]], Where (r) shows the radius of doped area, (W_L) is the mode field radius, for a fundamental mode at (r=a) is defined by [12]

$$w_L = a[0.616 + \frac{1.660}{U^{1.5}} + \frac{0.987}{U^6}] \quad (9)$$

Where U is the normalized frequency at lasing wavelength($U = \frac{2\pi a N_a}{\lambda_L}$)
By applying the energy conservation law $N_1 + N_2 = N$ and under steady state condition $\frac{dN_i}{dt} = 0$, (i = 1, 2) the eqs (1 and 2) became as

$$N_1 = N\frac{\frac{\sigma_{eP}\Gamma_P P_P}{h\nu_P A_{effc}} + \frac{\sigma_{eL}\Gamma_L P_L}{h\nu_L A_{effc}} + \frac{1}{t}}{\frac{\sigma_{aP}\Gamma_P P_P}{h\nu_P A_{effc}} + \frac{\sigma_{eP}\Gamma_P P_P}{h\nu_P A_{effc}} + \frac{\sigma_{aL}\Gamma_L P_L}{h\nu_L A_{effc}} + \frac{\sigma_{eL}\Gamma_L P_L}{h\nu_L A_{effc}} + \frac{1}{t}} \quad (10)$$

$$N_2 = N\frac{\frac{\sigma_{aP}\Gamma_P P_P}{h\nu_P A_{effc}} + \frac{\sigma_{aL}\Gamma_L P_L}{h\nu_L A_{effc}} + \frac{1}{t}}{\frac{\sigma_{aP}\Gamma_P P_P}{h\nu_P A_{effc}} + \frac{\sigma_{eP}\Gamma_P P_P}{h\nu_P A_{effc}} + \frac{\sigma_{aL}\Gamma_L P_L}{h\nu_L A_{effc}} + \frac{\sigma_{eL}\Gamma_L P_L}{h\nu_L A_{effc}} + \frac{1}{t}} \quad (11)$$

2.4 Propagation Equation

A linear cavity for Yb^{3+}doped fiber laser can be modeled from the 2-level system equations described by applying the boundary condition for resonator cavity and by setting the net round trip gain to unity. The boundary condition for forward $\mathbf{P_L^+(L)}$ and back-word $\mathbf{P_L^-(L)}$ propagation lasing power respectively see fig. (2) Are [8].

$$P_L^+(0) = R_1 P_L^-(0) \quad (12)$$

$$P_L^-(L) = R_2 P_L^+(L) \quad (13)$$

The difference of pump and laser power and small signal gain coefficient along the fiber length (N_1= N – N_2)and negligible the scattering loss are given by propagation equation [8, 13].

$$\frac{dP_P(z)}{dZ} = \Gamma_P[(\sigma_{aP} + \sigma_{eP})N_2 - \sigma_{aP}N]P_P(z) \quad (14)$$

$$\frac{dP_L(z)}{dZ} = \Gamma_L[(\sigma_{aL} + \sigma_{eL})N_2 - \sigma_{aL}N]P_L(z) \quad (15)$$

$$g(z) = \Gamma_L[\gamma(\sigma_{aL} + \sigma_{eL})N_2 - (\gamma - 1)\sigma_{aL}N] \quad (16)$$

The term (γ)is given by [13].

$$\gamma = 1 + f_u/f_L \quad (17)$$

Where f_L and f_u are the thermal Boltzmann factor in lower and upper laser levels respectively, are given by [14,15].

$$f_L = \frac{\exp[-\frac{E_i}{kT}]}{\sum_1^4 \exp[-E_j/kT]} \quad (18)$$

$$f_u = \frac{\exp[-\frac{E_i}{kT}]}{\sum_1^3 \exp[-E_j/kT]} \quad (19)$$

When (Y=1) the laser system is a true four-level, (Y=2) is a true three-level, ($\gamma < 1.5$) a quasi four- level, and ($\gamma > 1.5$))is a quasi-three-level since (KT=207cm-1) where (T) is the temperature of host material in (K^o) is Boltzmann constant [13].

2.5 Pump Power

The gain by pumping is given by integrated the eq. (13) along the fiber length (L)

$$G_P = \ln\frac{dP_P(L)}{P_P(0)} = \Gamma_P(\sigma_{aP} + \sigma_{eP})\int_0^L N_2(z)dz - \Gamma_P\sigma_{aP}NL \quad (20)$$

And the gain by laser is obtained by integration small signal gain coefficient g (z)
In equation (15) a long fiber length (L)

$$G_L = \int_0^L g(z)dz = \int_{P_L(0)}^{P_L(L)} \frac{dP_L(z)}{P_L(z)} \quad (21)$$

The stationary condition for the linear cavity fiber laser is given by V

$$R_1R_2\exp(2G_L) = 1 \quad (22)$$

$$G_L = \ln\frac{1}{\sqrt{R_1R_2}}$$

Or can be written
The difference relationship of pump power over the fiber length can be written as:

$$P_P(L) = P_P(0)\exp[\frac{\Gamma_P\sigma_P}{\gamma\Gamma_L\sigma_L}\ln\frac{1}{\sqrt{R_1R_2}} - (\frac{\sigma_{aP}\sigma_{eL}-\sigma_{eP}\sigma_{eL}}{\sigma_L} - \frac{\sigma_{aP}\sigma_{aL}+\sigma_{eP}\sigma_{eL}}{\gamma\sigma_L})\Gamma_P NL \qquad (23)$$

2.6 Lasing Power $P_L(L)$
We combine equations (11and 12) to obtained as

$$\frac{dP_L(z)}{dz} + \frac{v_L}{v_P}\frac{dP_P(z)}{dz} + \frac{hv_LA_{effc}}{t}N_2 = 0 \qquad (24)$$

The equation may be integrated a long fiber length (L) to obtained as

$$[P_L(L) - P_L(0)] + \frac{v_L}{v_P}[P_P(L) - P_P(0)] + \frac{hv_LA_{effc}}{t}\int_0^L N_2(z) = 0 \qquad (25)$$

Which is used $P_P(L) = P_P(0)\exp G_P$ and

$$\int_0^L N_2(z)dz = \frac{1}{\Gamma_L\sigma_P}[G_P + \Gamma_P\sigma_{aP}NL] \qquad (26)$$

$$P_P^{sat} = \frac{hv_PA_{effc}}{\Gamma_P\sigma_Pt} \qquad (27)$$

Then, equation (24) can be rewritten as

$$P_L(L) - P_L(0) = \frac{v_LP_P^{sat}}{v_P}[\frac{P_P(0)}{P_P^{sat}}(1-\exp G_P) - (\Gamma_P\sigma_{aP}NL)] \qquad (28)$$

2.7 Out Lasing Power (P_{out})
The output power of the lasing in Fig. (2) To obtained as

$$P_{out} = (1-R_2)P_L^+(L) \qquad (29)$$

When $G_L = \ln\frac{P_L(L)}{P_L(0)} = \ln\frac{1}{\sqrt{R_1R_2}}$ then

$$P_L^+(0) = P_L^+(L)\sqrt{R_1R_2} \qquad (30)$$

$$P_L(L) - P_L(0) = P_L^+(L)[1 + R_2 - \sqrt{R_1R_2} - \frac{\sqrt{R_2}}{\sqrt{R_1}} \qquad (31)$$

Which are used the equations. (11 and 12 and 29) to be obtained as
However, we combine equations (30 and 27) the eq. (28) become as

$$P_{out} = \frac{\lambda_P}{\lambda_L}\frac{(1-R_2)}{1+R_2-\sqrt{R_1R_2}-\sqrt{\frac{R_2}{R_1}}}(1-\exp G_P)[P_P(0) - P_P^{sat}\frac{G_P+\Gamma_P\sigma_{aP}NL}{(1-\exp G_P)} \qquad (32)$$

When $P_{out} = 0$, the laser threshold power obtained as

$$P_{th} = P_P^{sat}\frac{G_P+\Gamma_PN\sigma_{aP}L}{(1-\exp G_P)} \qquad (33)$$

And we combine equations (31) from ($P_{out} = \eta(P_{abs} - P_{th})$) efficiency (η)is obtained as

$$P_{abs} = P_P(0) \qquad (35)$$

3. Result and discussion
Firstly In this research we calculated (ɣ) by the equation (16) when the pumping of the Yb^{3+} doped silica fiber laser with a wavelength (λ_P=920nm), where through the parameter value (Ɣ) we can determine the type of the pumping plan for the laser emission (λ_L)The table (3-1) shows parameter values (Ɣ) in each laser wavelength emitted at the above (λ_P).

Table (3-1): Lasing wavelength (λ_L) and transition factor at pumping wavelength (λ_P=920 nm).

Transition	Lasing Wavelength λ_L*10^{-9} (m)	Ɣ-Factor
e ⟶ a	975	1.9694
e ⟶ b	1035	1.0534
e ⟶ c	1090	1.0058
e ⟶ d	1140	1.0007

We relied in the search on (λ_L=109nm) as a typical example of a quasi-four level pumping plan so table (3-2) shows All transactions with both (λ_P) and (λ_L) [8, 12].

Table (3-2): Default values for used parameters.

Parameter	value	Unit
λ_p	$920 * 10^{-9}$	m
σ_{ap}	$6*10^{-25}$	m^2 m^2
σ_{ep}	$0.25 * 10^{-25}$	m^2
λ_L	$1090*10^{-9}$	m
σ_{a_L}	$0.014 * 10^{-25}$	m^2
σ_{e_L}	$2 * 10^{-25}$	m^2
T_o	$1 *10^{-3}$	sec

In the event that the optic fiber from the Single - Clad Fiber (SCF) type, the laser design that was used in this research is (Lycom) specifically, since all of this design transactions that were used in the numerical simulation through matlab program (8.1) to calculate (Pout) shown in table (3-3).where (*) in this table means that the parameter is calculated in the present study

Table (3-3): Default values for used parameters (Lycom design).

Parameter	Value	Unit
a	$2.5 * 10^{-6}$	m
b	$62.5 * 10^{-6}$	m
N_A	0.15	-
R_1	0.998	-
R_2	0.1	-
L	$(4 – 100) * 10^{-2}$	m
$P_P(o)$	(2 – 150)	w
V(*)	2. 561	-
M(*)	3	-
A_{effc}(*)	$1.964 * 10^{-11}$	m^2
A_{cld}(*)	$1.227 * 10^{-8}$	m^2
Γ_p(*)	0.0016	-
Γ_L(*)	0.78	-

Figure (3) shows how (pout) calculated by equation (31) increases linearly with the increase of pumping power $P_P(o)$, and note that both of the laser output (Pout) and efficiency (η) would be higher in case the concentration Is of (CorActive) type of this laser design, as for Figure (4), it explains how the (P_{out}) linearly decreases with the increases in the radius of the core of the optical fiber (a), while Figures (5), (6) and (7) show How (P_{out}) increases linearly with the increase in the length of the optical fiber (L), and the reflectivity of the output laser mirror (R_2) and the pumping power $P_P(o)$ respectively, and we also note in the latter form that all of (Pout) and (η) would be higher if the(SCF)is of the (Single Mode) type .

While, In the case that the optic fiber is of the Double – Clad Fiber (DCF) type and for a laser design of the type of Large Mode Area (LMA), where all transactions with this design shown in table (3-4) we note in Figure (8) how (P_{out}) increases linearly with the increase of $P_P(o)$, whether the form of the inner cladding is a rectangle or square or circle. We also find that both of (Pout) and (η) would be higher for the rectangular shape because (A_{cld}) of the rectangle is less than they are in the other shape.

Table (3-4): large mode area design.

Parameter	Value	Unit
N	$4*10^{25}$	ion/m^3
N_A	0.05	-
T	$1*10^{-3}$	sec
R_1	0.988	-
a	$10*10^{-6}$	m
b	$200*10^{-6}$	m
d	$400*10^{-6}$	m

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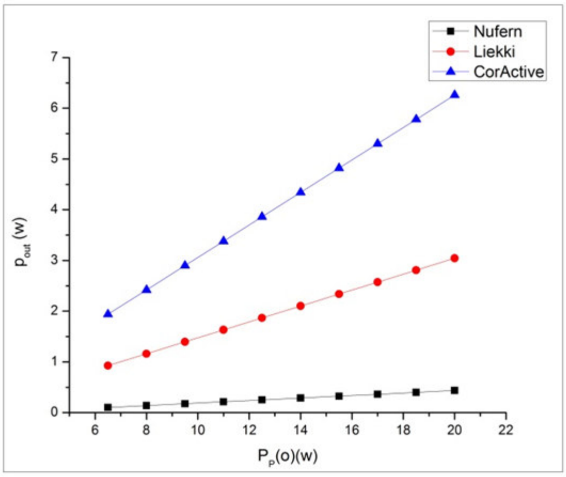

Fig. (3)The laser output Power Vs pump power for SCF.

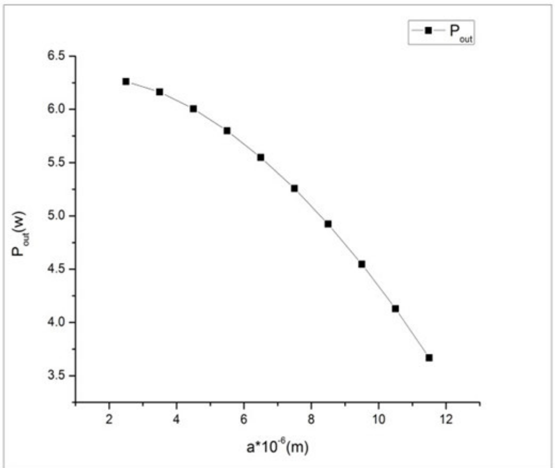

Fig. (4) The laser output Power Vs Core radius for SCF.

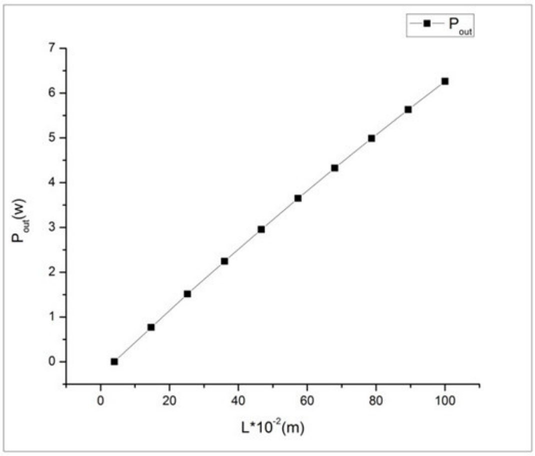

Fig. (5)The laser output Power Vs Fiber Length for SCF.

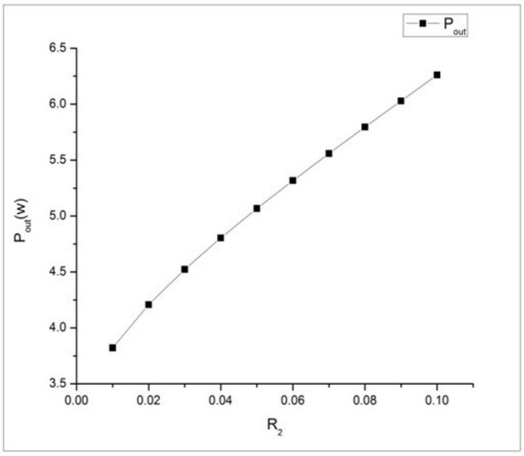

Fig. (6)The laser output Power Vs Output Reflectivity for SCF.

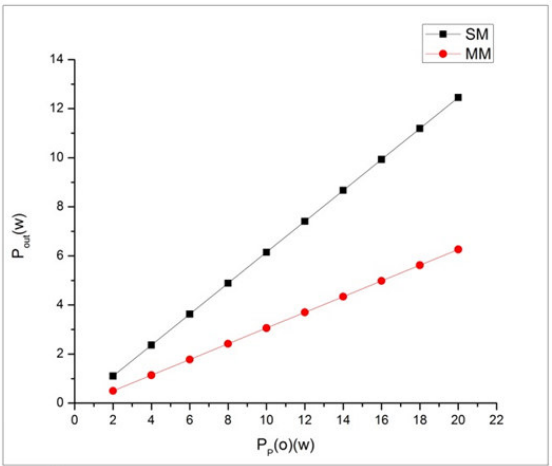

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Fig. (7)The laser output Power Vs Pump Power for SCF.

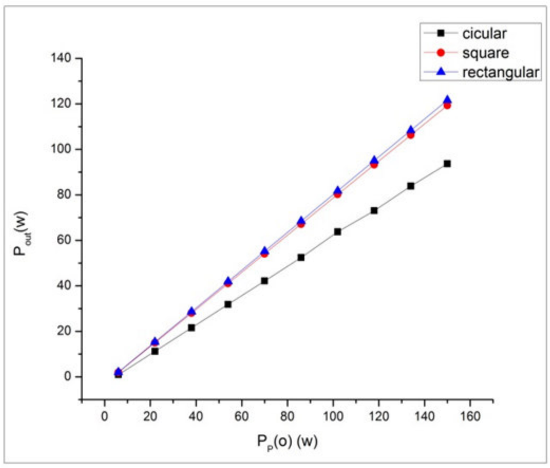

Fig. (8)The laser output Power Vs Pump power for (DCF).

4. Conclusions

In this research the equivalent of the laser output and the calculation of this power was derived through numerical simulation (Yb3+dope silica fiber laser) at (λP=920 nm) and (λL=1090 nm) and for two types of the optical fiber, in the case that the optic fiber is (SCF), and the design of the laser is (Lycom) type We found that the concentration of the (CorActive) type is the best for the purpose of obtaining laser efficiency and output power, and (Pout) decreases with the increases in the radius of the core of the optical fiber while increases with each of the length of optical fiber, reflectively of the output laser mirror and pumping power, and that both the efficiency and the output laser power would be better in the case the (SCF) is of the (Single Mode) type. While, In case the optic fiber is (DCF) and the design of the laser is of the (LMA) type, we found that (Pout) increases with the increase in the pumping power, whether the form of the inner cladding was a rectangle or square or circle, and that both of the efficiency and the laser output would be better in the rectangular shape.

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