

## Neti-Neti-Onomasiological Onerousness And E Pluribus Unum La Escallonia Adventitious Ad Infinitum Model

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**ABSTRACT:** In knowledge, there are logical contradictions and a logical incompatibilities, divergent affirmation and dialectic transformation. Brahman is not the universe, nor sky, nor senses, nor mind. By Brahman we understand the one agency that stimulates the human being in to action. Good. And Anti Brahman is a part of Brahman and this is the agency of stimulus response of individuals based on their Rajas and Tamás equilibrium or proportion of disequilibrium (Kind attention drawn to the author's articles on the subject matter. And we have to recapitulate the bank example given wherein the conservativeness generalisationality do not abnegate or revoke applicability to the individual systems which are conservation or which might be unconservational, like in self transformational, system mutational, syllogistically disintegrational, configurationally changeable. Anti Brahman stimulates one to do bad things. All individual records of commission and omission of Gratification or deprivation to others are assumed to be recorded in Neuron DNA (in space). We will not repeat this explanation again and holds good for the entire monograph. Separate Papers are written (Free will and Destiny, Natures general Ledger, A Model for God etc..) and bad. This depends on freewill and destiny. We have already defined freewill as the in discretionary usage of power be it financial embezzlement, pecuniary misappropriation or fiscal defalcation. The other person might have been desensitized for such actions and might have been taught by reference groups (See Merton) to take things in his "stride". But our General Ledger of all actionable and actions accounts.

The General Ledger is the General Theory Of all actions of all individual human beings. It is also assumed that the reactionary potentiality depends on the genes (See Neurobiology by John Smith). This is a very important aspect not to be overlooked. Because Schrodinger's equation is applicable to general system of the Universe, it does not mean its inapplicability to individual systems and the conservativeness and /or non conservativeness notwithstanding. In consideration to the fact that we have already built a model encompassing Brahman-Anti Brahman, Rajas-Tamás, we shall not include this in the model. Instead we take "Neti Neti concept" by its representation in the Schrodinger's form (See Penrose on Schrödinger's cat) and stimulus-response as the further constituents to tie up with the cosmic thread. One sentence about association need be said. Einstein said that matter is the one that gives rise to space time world lines. In our case human beings with their pen chance, predilections, proclivities, and propensities "svabhava" are responsible for the world lines of space time. The characteristics we have already stated depend on Rajas-Tamás-Sattva combination of the individual (all the three qualities are in the individual and in disequilibrium state, albeit we are not sure about the conservativeness of the states holistically. This we have codified in the General Ledger which is the General Theory of actions in the Universe. So we have the structure of space and time is determined by "Svabhava". Like they say: "That area is not OK". This is true of all characteristics, inertia, static, dynamic (Rajas, Tamás, Sattva means those words). These are most salient and perennial concepts that

are to be remembered in the read of monograph. This is 'neti/ neti' process, repeating which one becomes thoughtless. In jnana the entire world is illusory like a dream; only Brahmn is real- 'Brahma satya jagat mithya.' The world is neither unreal nor real. This means that reality is equivalent to unreality. Here let us go by the usual definition of "reality" and "unreality". Unreality is pathological. Many philosophers like Constantine and Lester have said, it is schizophrenics who are suffering from the Truth. we shall marginalize the proposition in so fact as the study of monograph is concerned. This implies that Unreality is a part of reality. Further we assume that Objective reality and subjective experience are necessary to realize the Truth. Lest probably a courtesan would have become "enlightened" or a Professor had attained "Nirvikalpa Samadhi". The world is mere projection of self. Jnana requires strong discriminating ability. Krishna says, "It is more difficult to realize the formless God than One with form". We reiterate that formlessness is nothing but more than mere loss of style and substance and is well within space time. This is "subjective experience "most recruitment agency calls for. You can see without staring, and can hear without cupping your ears. Dealings with fellow people, bonhomie, and joie de vivre are the ones that could tackle a tricky situation involving a belligerent, cantankerous and recalcitrant person. Here the importance of zero is to be emphasized." Experience "is nothing but the "experience" so assiduously and avidly asked by recruiter agencies while they recruit people. When they say that whether you know the proficiency in anagrammatism like addition, subtraction, multiplication, division And may be differentiation and integration, which a probationer does, but something more, a knack to deal with people, know them, adjustable temperament, which comes by subjective experience. Both "Objective Reality" and "Subjective Experience" are necessary for the determination of Truth. And Schrödinger and Wagner have said that Truth is always elusive. That we state in Model form. It is not "emptiness", but "Nothing" which is zero. And this zero is the cancellation of two opposite terms or particle-antiparticle duality, and the zephyra that is written with so much of gusto and aplomb in every equation.

An equation equivalent to zero means that all the constituents cancel out each other in some form. We shall for our purpose choose one permutation and combination. It is to be mentioned in this connection that waking state is one of the three states of dream state and dreamless state and space time is only projection on the screen of consciousness (which we have defined as collection of all knowledge – individuals-his peer group references, belief systems etc., including visual representations, with anagrammatic mind acting on its to fabricate concoct realistic, consistent, surrealistic, unreal dreams and actual "thinking". Like a movie being watched. Mind doth act upon it and weaves a web like a spider. Lastly we state that "association" is the part and parcel of space and time and once taught. The "association" fails to leave you. Even eminent scientists fall prey for this defending their theories till the end instead of applying their minds to the twists their own theories have taken. This statement is made in a dispassionate manner of witness consciousness and not participatory consciousness. When over thinking about association takes place in the spatio temporal actualization of events it becomes pathological. Be it love, murder, mayhem, we have various psychiatric cases. And we want to "renormalize them". So there is reality, surrealism, unreality, inconsistencies, contradictions in the waking state, dream state-note you experience in dream state also- and dreamless state.)

## **PREFACE:**

### **Section 1**

Since the beginning of the human race, there has been a tug of war between the corporeal depth and sonorous continuum, locus of essence of expression and sense, propositional subsistence and corporeality, perceptual field ,form background, theme potentialities, transitive states and substantive subsystems ,determinate orientation, heights of cognition, evaluative integrational collective cultural pamaoutacy, generalized strain of rationalized consistency, essential cognitive orientation and choice variables, cathartic evaluative impressionistic mechanisms of individual minds, and componential clustering's principal frontier of diurnal dynamics, principal concept of determinate apriori, and differential aposteori, aphorism ,anecdote, syndromes, internal differentiation, comparative variability, structural morphology and normative aspect of credibility of both religion and science. Many riveting riposte and recrimination and dialectic deliberations, polemical conversations and argumentative confabulations have taken place as to which is superior or not. But the best of minds have taken the spirit of the Eastern Science and have adapted to the materialistic science which has helped the science and progressed the frontiers, with generalization manifestations and personalized significations. Krishna

in Bhagavad Geeta explicitly states that he who is carried away by the alliterations, epistemological engendered propositions, phenomenological transcendental constitutional ties, and metaphorical constitutive conditionalities, radical cleavage of the disjunctive syllogisms, non modalized root forms ,apotheosis of willful injustices and ignominies, dissententious resentient and dejurational resentient, apodictic knowledge of successful effectuality, extrinsic predications and non consummative abstractions would never be able to understand Geetha in its wide ranging manifestations and he shall be consigned to dustbins of history despite knowing it y heart. In recent days Penrose has been in the forefront in the study of Consciousness and without the inclusion and occlusion of Quantum neuropsychology, has been able to use some of the concepts to the scientific advantage. Walker albeit little ambiguous, in His Physics of Consciousness mentions certain equations of consciousness one may not completely agree with albeit Quantum Mechanics has become a subjective science, and the necessarily compulsion of Quantum Computation has lead many a people in to forays in to eastern mysticisms. Fritzaf Capra is another name that has done much collaborative work in the field of consciousness and Buddhism

And western physics. William James one of the respected exponents of Consciousness states: There is no science of the soul without a metaphysical basis to it and without spiritual remedies at its disposal. Penrose uses Schrödinger's equation in the context of superposition of the cat and there have been criticisms over that especially by Hawking. We have in a series of papers (Theory of hidden variable: perception is not reality; A model for destiny and free will, and God does not put signature-a Brahman and Anti Brahman model for god) have tried to provide a scientific interpretation of our own age old predications and postulation alcovishness which every Indian knows but unfortunately fails to consummate either in practice or in theory due to irresolute concern of the mundane musings and day to day drooling. If perception is not reality what is it? What you see is not what you see, what you do not see is not what you do not see, what you see is what you do not see and what you do not see is what you see. Quintessentially some variables have to be added to perception to know the truth. For the physicist and the instrument are governed by the same laws as is dictated by the universe to the system under investigation. Infact recently, on a discourse on Paramahansa Yogananda, author found Von Neumann grudgingly acceded that some factor is to be added to perception and this is consciousness. Unless you know that there is a cricket match how do you explain the rush at the stadium? You go on and on without any end. In this monograph we apply the concepts of both time dependent and time independent equations of Schrodinger, which define the state of perception of the Universe defined by the "collapse of the wave function" and apply to the fundamental concept of Brahman namely Neti Neti, meaning neither this nor that. We only hope that his shall be a small contribution to further distanced invigoration and ramified resonations in the field. Counter actualisation of consciousness is also ridden with condensation of singularities and we write to state that the evolutionary concept of integrative structure of system of moral value orientation patterns on the relational and transcendental level is relevant to the institutionalization of the widest commonalty spread. Lord does not need either gratification or mortification. A dispassionate view is one which God shows and a sensitive passionate observer would come to know what reality of "Grand design" is all about. All it takes is to keep the eyes and ears open.

I am grateful to professor B.S.Kiranagi and Professor C.S.Bagewadi for their invaluable suggestions on the subject matter in question

Consciousness is in its original nature quiet, pure, and above the dualism of subject and object./But here appears a principle of particularization., and with the rise of wind of action the waves are agitated over the tranquil surface of mind . It is now differentiated and evolved in to various levels. --D.T.Suzuki

There is an incessant multiplication of the inexhaustible one and the Unification of the indefinitely many. Such are the beginnings and endings of the worlds and of the individual beings, expanded from a point without position or dimensions and a "Now" without date or duration.--Ananada Coomaraswamy

Consciousness is Maya (Please see model on Hidden variable Theory)

-Schrodinger

Neti, Neti principle we write in mathematical form as follows (See Schrodinger’s cat in Penrose’s The Big, the Small and The Human Mind”) Observer and the observed (Here we take both accentuation coefficient and dissipations coefficients as zero)

- (2).  $(\Psi) = -e_1 (\Psi) + e_2 (\Psi) - \dots -en (\Psi) n$
- (3).  $(\Psi) = +e_1 (\Psi) + e_2 (\Psi) - \dots (-)+-en (\Psi) n$
- (4).  $(\Psi) = +e_1 (\Psi) - e_2 (\Psi) - \dots +(-)-en (\Psi) n$
- (5).  $(\Psi) = -e_1 (\Psi) + e_2 (\Psi) - \dots -en (\Psi) n$

(6) Svabhava and space time world lines

(7) Consciousness (Storage of all information and visual representations with mind acting upon it anagrammatically. Note we eschew using words “being” and “becoming” We are infact beings with sense of consciousness and individual consciousness always tries to do”high jump” like body wants to. This is an evolutionary process. Leviathans’ literature has done more damage to the subject than addressing quintessential. We leave metaphorical and take the categorical and classification.)

(8) In quantum mechanics, the Schrödinger equation is an equation that describes how the quantum state of a physical system changes with time. It was formulated in late 1925, and published in 1926, by the Austrian physicist Erwin Schrödinger

Simulations and dissimulations by brahman and AntiBrahman agency (the stimulus providers to the response of the human beings) and concomitant misconception of it to be real by the human being (if two persons walk with you when you go out, it is accident. if twenty people walk with you every day and every time you go out it is coordination, a “design” created by the agency of Brahman-AntiBrahman category. be very clear about the association is nothing but what has been taught to you by reference groups and in the eventuality of the “association” say green” for good and “red” for danger.

Representation of neti neti in Schrödinger’s equation first term on the above equation when added to other terms on the RHS leads to the same situation of addition of milk to water. Here we shall not take the terms separately but assume that they are added to the lhs on one shot basis

### **SCHRÖDINGER’S WAVE EQUATION, RELATIVISTIC THEORIES AND WERNER HEISENBERG MATRIX**

Following variables are taken in to consideration:

- (1) Theory of Schrodinger’s Wave Equation(It is to be reiterated that we are classifying the universally applicable Schrödinger’s Wave equation based on the individual systems or clusters of systems and their characteristics,paramaters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Schrödinger’s equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself. )
- (2) Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits, which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behavior of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalization consummation and realization of the Werner Heisenberg Matrix itself
- (3) Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We

recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits, which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behavior of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalization consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

- (4) Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, Penchance, predilection, proclivities, propensities, Solutional behavior conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behavior of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalization consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself in the foregoing).

## PLATONIC WORLD-PHYSICAL WORLD AND MENTAL WORLD

Following factors are taken in to consideration:

- (1) Category one of Platonic World (No one has seen entire world or universe. It is based on this concept that classification is done. Everything needs to be explored)
- (2) Category Two of Platonic World(or the part thereof)
- (3) Category three of Platonic world
- (4) Human mind corresponding to the above classification category one
- (5) Human mind corresponding to the above classification category Two
- (6) Human mind corresponding to the above classification category Three
- (7)  $G_{16}$  : Category one of Physical World
- (8)  $G_{17}$  : Category Two Of Physical World
- (9)  $G_{18}$  : Category Three Of Physical World
- (10)  $T_{16}$  : Individuals With Mind Corresponding To Category One In The Foregoing
- (11)  $T_{17}$  : Individuals With Mind Corresponding To Category two In The Foregoing
- (12)  $T_{18}$  : Individuals With Mind Corresponding To Category Three In The Foregoing
- (13)  $G_{20}$  : Category One Of Mental World
- (14)  $G_{21}$  : Category Two Of Mental World
- (15)  $G_{22}$  :Category Three Of Mental World
- (16)  $T_{20}$  : Individuals With Mind Corresponding To Category One In The Foregoing
- (17)  $T_{21}$  : Individuals With Mind Corresponding To Category One In The Foregoing
- (18)  $T_{22}$  : Individuals With Mind Corresponding To Category One In The Foregoing

## TIME DEPENDENT SCHRODINGER'S EQUATION:

$$E\Psi = \hat{H}\Psi$$

## AFFECTIVE CONSCIOUSNESS AND QUANTUM FIELD

Following variables are taken in to consideration:

1. Change is a potentially produces and transforms as an emotional event as people anticipate or



experience its outcomes and processes.

2. Emotional aspects of organizational change, promote acceptance of change or resistance to it. Cognitive, affective and behavioral responses to organizational change indicates outcomes, scale, temporal issues and justice
3. Emotional intelligence, disposition, previous experience of change, and change and stress outside the workplace affect those in the employee's perceptions of the leaders/managers/agents (their leadership ability, emotional intelligence and trustworthiness); and those in the employee's perception of the organization (its culture and change context)
4. Cognitive appraisal theory takes the position that emotion derives from cognition as people contemplate the importance of events (such as organizational change) to their wellbeing and consider how they will cope.
5. Social constructionism was used as a theoretical platform because it combines the individual experience of emotions during change with the social forces that help shape them.

Following are the second line variables:

1. The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself.
2. The particle Higgs Boson
3. Higgs Quantum Field
4. Mass of the matter
5. Quantum Field Theory
6. Electro-Weak Unification and Broken Symmetry
7. Grand Unification Theory
8. Super symmetry
9. Quantum Gravity and Super gravity
10. The Heterotic String
11. Unified Quantum Field Theories
12. Principle of spontaneously broken symmetry, locates (eb) deeply hidden symmetries of nature at fundamental space-time scales.
13. Principle of broken symmetry explains the emergence of diverse forces from an initially unified field.
14. Profound symmetry principle called super symmetry, which has the potential of unifying force fields and matter fields in the context of a single field.
15. Superstring theories.
16. Quantum field theory

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**We have given a model for Brahman (Neti Neti) as defined in the scriptures using Schrödinger's Equation. There do not seem to be any literature on this. This is a mathematical statement of whatever has been said since long time and no compactified model seems to be there in literature.**

## **INTRODUCTION:**

Neti Neti: Neither This, Nor That -- In (transpersonal/non-dual) Reality "WE" are more than that! WE are THEE! Thee WE BE! Merry BE!( We have used Wikipedia in the preparation of the Introductory remarks)

### **Neti Neti: What it is not**

Neti -- neti is a well known Sanskrit expression that is found in the Upanishads, but also is widely misunderstood (both in the West as well as in classic Indian dualistic philosophic traditions as well). That is, it is subject to much varied interpretations. The common misunderstanding comes from a false assumption (which is nothing other than a dualistic preference of the mind) in which the saying is

interpreted as a presumption of a programmed alienated context (an illusory assumption where God, Spirit, or Brahman is absent, separate in our daily life, or "elsewhere"). This limited backdrop occurs within the assumed context of a predilection toward the pre-existing false assumption of spiritual alienation (and may I add corruption), where a state of sin/separation from spirit is taken as the underlying "reality". Certainly, if a spiritual seeker finds themselves spiritually alienated and estranged from spirit, which is lacking, then this sadhak (spiritual seeker) can strive for re-union/re-integration. However, it is valuable to not set the quest within this corrupt and alienated framework less we amplify it as a condition paralysis/stickiness or spiritual stasis. Such is a limited/ignorant view of reality. In REALITY, this false coloring of spiritual alienation can be likened to grasping in the wind, being an error of the programmed dualistic mind. Within this mind-made conditioned schism that separates "spiritual principles or the sacred" from man, Beingness, and from the earth in an oppositional manner, is the error that underlies all the corrupted brutalities that man lays upon himself, his fellow creatures, and that of future generations. Thus, the Upanishad saying, neti-neti (neither this nor that), will be used to disclose the lie and reveal the truth. Neti-Neti in the Non-Dual Context will posit the opposite jumping off place of interpretation than dualism (dvaita). We will reveal and be grounded within that Reality, which true and spirited vision and clarity naturally expresses. That Spirit is the ever-present all pervasive Untouched Eternal Reality, as nisprapanca, acintya, aprapanca, and non-dual (advaita). It is Never-ending, Beginning less Reality, which is Eternally Present as sacred presence. Therefore, knowing and affirming the true context of "reality" from the beginning, as a non-dual assumption, then the term neti-neti can be applied simultaneously as a negation of subject/object duality, and hence as an affirmation of the non-dual unitive state.

Although primordial presence appears to be absent or lacking from "normal" dualistic materialistic everyday life (as chronic ignorance), never-the-less we will view this "normalcy" as an error of the conditioned "normal" mind -- as a result of negative programming, where the gross physicality of sense "objects" as well as similar I/it dualistic obsessions are perceived as fragmented and disorganized. That mental and energetic obsession/fixation occludes universal sacred presence out of context of the whole. Thus this materialistic/dualistic chronic "normalcy" will be considered as the normal icy process of that establishes ignorance (avidya), as a fragmented self limiting skew, as a prejudicial and superficial bias of what really is in its natural universal and unbiased way. As such, chronic dualistic referencing (the reification process), as an illusory and an aberrant, corrupt, and perverted way of seeing and being, maintains a hallucinatory mindset, which when accepted by our peers becomes institutionalized ignorance, social stasis, bastion, pillar, post, stylobate and sentinel and a force that is counter evolutionary.

The Sanskrit expression, neti neti, as expressed by the realized sages points to that vast reality, which is more than dualistic fixation. As such "not this, not that", which is the common translation, becomes transformed to, "we belong to a boundless all inclusive realm of Great Integrity and Continuity-the WE realm of All Our Relations being therefore more than just a fragmented limitation or corruption of the whole, but rather the phrase, neti neti, also implicitly implies "tat tvam asi" -- That eternal spirit thou art. Yoga practice is designed to open that living book where all beings and things are relatives and kin as Vasudev Kutumbhkam --the entire Universe is One Family. This is an exploration of vast interconnectedness, a vast diversity within a coherent unity, and a vast unity (hologram) found in the form of a vast holographic diversity, both.

#### Many levels of Interpretation

The expression, neti neti, literally means "neither this, nor that" or not this and not that. In the first level this is the rejection of a separate self or ego. It is a rejection of fragmentation or split from universal spirit which is embedded within all beings and things. In the holistic multidimensional context, it means that human beings are not just separate egos; we cannot ever be adequately defined as being separate from spirit without introducing a delusion -- spiritual self-alienation, counteractualisation of consciousness. The projection of an individual self or observer apart from the object, which the observer observes defines a limited dualistic mental framework. In turn such occludes the larger picture, which includes all time, space, and knowledge. Rather we, as human beings, are part of an immense sacred process, not apart from it.

Thus "neti neti", as a statement, means that we are not anything separate as in the disparate dualistic

framework of a separate "I/ it" subject/object duality context (versus the sacred non-dual and transpersonal "I-Thou" context) wherein we identify as a finite expression integrally part of a boundless spirit (like a wave on the ocean). We are neither the ego, nor are we nothing at all. We are neither the all, nor nothing at all. Neither just this observer, nor just that (the observed). Neither eternal nor finite, neither eternalism nor nihilism, neither empty nor solid. Rather it is the great non-dual or advaita statement of both/and -- both, but neither by itself. "And" in the sense of a greater synthesis or unity -- call it the tantric Siva/Shakti if one likes, albeit nihilists will take it as a negation of reality.

Although negative dialectics (such as found in Nagarjuna's Buddhist Madhyamika philosophy, which is considered as non-dual philosophy by Buddhists) , neti neti, not only negates the validity of a perception which imputes a separate/independent self existence (an ego) which includes the perception of a valid existence of a separate/independent object as well as the egoic observer, but also net-neti, by disowning duality per se, one is affirming a holographic non-dual dynamic holistic system, where "things" as sense objects, the senses, the process of perception, mental dispositions, and intellectual function, all operate interdependently, becoming organized by an inherent vast intrinsic transconceptual schemata within the sphere of beginning less primordial awareness totally free from concepts of sequential time and limited space. Neti-neti is thus not a statement of existence per se, negating existence per se, affirming non-existence, negating the existence of both, or the negation of the negation of both, and so on infinitum, but of negating any dualistic view. Rather it affirms the interdependence of all things and beings -- the holographic relationship. Negative dialectics (negation without affirmation) most often serves to go down a slippery slope of the extremes of nihilism and cynicism.

Although there are volumes of pages written about Buddhist non-dual philosophy (Madhyamika), we will briefly describe it in terms of neti neti, stating that phenomena, things, objects, the objective world, or "existence per se" do not exist by themselves independently. In short objects or observers have no independent or exclusive self existence other than in the dualistic imputation of the egoic mental formulation. Such things, objects, phenomena, or objective world per se, cannot rightly be said to exist, to not exist, both simultaneously, or neither. There are many logical reasons given. The easiest reason is that they never existed in the first place, so there is no self existing reference point (referent) to some "thing" which is not existent as compared to something else which does not exist. Secondly, nothing can be said to truly "not exist" exclusively in relationship to something else (to that which is considered to truly exist exclusively or independently). Likewise things, phenomena, objects, the objective world, etc., do not both exclusively not exist and exist at the same time, as it would violate the previous statements. Lastly, according to Nagarjuna things, objects Phenomena, or the objective world cannot be said to both not exist, and both not not exist, ad infinitum.

Philosophical speculation, dialectics, logic, and reductionism, all have been used to resolve the abstract problematic structure and apodictic knowledge and functional topology of the neti neti. While extracting it from the realm of philosophical speculation, with multiplicity of contrarian presuppositions, and logical and dialectical attributes there is both intellectual gratification and divergent affirmation on one hand and disjunctive syllogism and dialectic transformation and disjunction of synthesis on the other. This however is done not at the cost of predicational integrity and character constitution as stated in the original texts.

The four (and more) negations

- the negation of solid independent existence
- the negation of the negation of existence
- the negation of both existence and non-existence
- the negation of the negation of both existence and non-existence

We have written these forms in Schrodinger's equations following Penrose's interpretation of Schrödinger's cat

None of these imputations can stand on their own, so they are negated. In short, things, objects, phenomena, events, or the world per se do not exist as separate selves possessing an intrinsic independent existence, rather phenomena can only be said to truly exist when taken as parts of a mutually interdependent whole, hence the inseparable nature of differentiated reality (temporal ever-



changing relativity) with undifferentiated absolute reality (the unchanging Beginning less a-temporal and primordial formless realm). Both are married as the inseparable unity of dharmakaya (empty space) and rupakaya (form), wisdom and compassion, emptiness and clarity, or emptiness and essential nature, which are different formulations of the same non-dual truth. The crucial key is that the non-dual view is equivalent to the inseparable interdependent/interconnected state of naljor (yoga) which is the great natural state of perfect wealth. Things do not exist separate as exclusive independent entities as solid or permanent, nor do they not exist at all as in an inane nihilistic state. This is the essential non-dual meaning of the statement of neti/neti. By both/and, it is meant that neti neti refers to the illusion that things/objects exist by in and by themselves, as well as the illusion that the observer or ego truly exists as a separate independent observer. The both/and affirmation is the affirmative statement of interconnectedness and interdependent co-arising, where things as well as the intelligent evolutionary life force behind all things can be said to truly co-exist interdependently in the context of primordial time and wisdom.

Neti Neti as the body too -- the body too! We are the intimate limbs and expression of the body of cosmic consciousness

So on one level neti neti says, "not self not self", not small mind self, not just individual mind, "not ego-not ego", not duality, not separateness, not division, not only this or only that, but in doing so it affirms its own negation in a greater affirmation in an unlimited all encompassing holographic integration as we will see. Too often dualistic interpreters (see introduction above) interpret net neti as a dualistic statement hearing separate as if saying "separate" from the body (as ego identifies with the body), separate from nature, the earth, the senses or even entire world of form as in an infinite and final nihilistic withdrawal. But that is an oversimplified logical mistake, throwing the baby out with the bath water, unless one understands that this statement is an affirmation of the integrative non-dual state where nothing is separate. But dropping the ego is only one step within the integrative step (re-integrating with the whole).

As a spiritual saying within the framework of yoga being union, net-neti, is understood correctly within a spiritual context of union with Universal all pervading Spirit, (Quantum Field pervading the entire universe) not within a dualistic alienated context. Anyone who has practiced yoga as a spiritual practice consistently over time begins to be familiar with that territory. They begin to recognize that they exist within a larger relational boundless framework, where nothing exists in and by itself as if in a vacuum as an ego or entity. Indeed these interconnections between all beings, all of creation in All Our Relations are the realization of the non-dual Great Integrity which is yoga. By the body too, it is meant that the all pervasive consciousness includes all, including the body, but it is not limited to the body. In that experiential state of pure being and pure consciousness nothing is excluded and nothing clamors for inclusion.

The common error of the dualistic mind that must be pointed out is that a negation must also be an affirmation of another avenue of perception. If that new context does not exclude anything and is thus does not need to include anything; if it is not lacking in any way, is NOT separate, isolated, or independent, but is universal, all pervading, and primordial, then it satisfies the non-dual criteria. If not negation becomes part of an aversion process, where one craves escape, creates negative karma by trying to escape, avoid, isolate, or separate oneself from an imputed isolated autonomous "thing" (usually the body, natural feelings, the natural world, and evolutionary process). The latter is not yoga. In yoga we are looking for integration. Rather when a yogi starts to realize the greater wholeness of an expanded process oriented realm of evolutionary consciousness and being, he/she gladly surrenders his/her previously conditioned limited sense of "self" consciousness (asmita), recognizing it as an impediment/obscuration (klesha). Thus, suffering (dukha) is avoided and happiness and bliss eventually realized in Sat Chit Ananda. This is not to negate that at physical death, the physical body is relinquished, but that is part and parcel of a greater continuity and evolutionary integration process which has already been affirmed.

Thus authentic spiritual practice in yoga is aimed at realizing the "Reality" and truth of this non-dual transpersonal omnipresent and all knowing universal boundless Mind which is expressed through every cell in the body subjectively as well as objectively (united). Intelligent spirit is not only seen as manifest in the physical body but also in Cosmic consciousness nature, and the entire manifest universe

simultaneously as sacred presence --who we really are (swarupa) in context of total boundless space and absolute time.

For an authentic yogi, knowledge, jnana, or gnosis, is not derived intellectually or conception ally -- realization is not just words or adherence to a memorized philosophy, but rather it is a direct expression of direct yoga experience. Hence, it is fundamentally non-dual (based on an experiential union/integration) as distinct from an objectification process, which separates the observer from "something". Here we must be careful to make this distinction, and thus not perpetuate this common confusion. Neti-neti means not that, not words, not apparent isolated events that are frozen in time, but rather an experience grounded in the Great river of endless all encompassing continuity and everything short of that.

It is in this sense thus "neti -neti" is an affirmation of the all inclusive, omnipresent, non-dual, universal, and transpersonal identity which knows no bounds -- sometimes called Brahman in Sanskrit, sometimes called God in the west, or nirbija samadhi in traditional yoga, but at the same time we are warned not to think of it as an independent entity, a religious doctrine, belief system, or ideology, to which could be capable of obedience or conformity. Neither is neti-neti a negation of the physical relative world (as is often assumed). Rather on a higher level neti-neti means that we are neither just Brahman separate from the world, nor just the world, neither exclusively absolute nor exclusively relative, but both/and much more. Note that we have taken Brahman as the stimulus provider. He who knows Brahman becomes Brahman. It is just as simple as he who knows his subject becomes Professor. Or he who knows Supari mafia, crime syndicates, hoodlum mugger aggregates, loutish, bearish elements, raucous ribaldry congregation becomes a Don. This we have termed as Anti Brahman When Brahman is truly realized, it is not separate from atman (self), but this is not the glorified ego which is the common illusion. This Self is not an independent entity. The separate self (atman) is not the universal Self (Brahman) by definition. Brahman permeates and contains all, both, while it is known as the essence of all. All pervasive it cannot be isolated. Once realized not as a separate object, but one's true self, only then does atman and Brahman become one. Taken as a whole, Brahman transforms the delusion of a separate self (atman). As such neti-neti is not just a negation, isolation, refutation, nor exclusion, but more so, a great affirmation of sacred non-dual universal presence -- the boundless realm of All Our Relations. It's more than human words can adequately express. There is duality in Brahman or for that matter Anti Brahman. All are equal. They do not care if you are a commissioner or a commissionaire. The show must go on. There is nothing deep to read in these statements.

However the academician, the orthodox scholar, the pandit, the self gratuitous intellectual, the religionist, fundamentalist, the normal left brained dominated (unbalanced), but unfortunately "normal" person (who is submersed chronically in the duality, bias, and fragmented thought processes of the normal "i-it" limited context) carries with them the biased filter of a pre-programmed self perpetuating inability or ignorance (avidya) preventing them from expanding beyond their self imposed dualistic mindset/prison -- held together by rigid mind constructs, concepts, artificial half truth belief systems, preconceptions, mechanisms of analytical reductionism, prejudice, pride, arrogance, etc. Such (albeit normal) people who have become inured to such negative programming having adopted habitual distorted standpoints, their minds have become chronically artificially alienated from living spirit in a tenacious way (citta-vrtti). This is condensation of nomadic singularities and counteractualisation of consciousness. Thus they see spirit or God outside of everyday life -- separate from this world and life. Once that alienation sets in then, it is an easy assumption that God or spirit exists in death or outside of nature. God becomes associated with death. Yet the opposite is of course what is needed. Ideologues and fundamentalists simply need to bring spirit back into their lives rather than further pushing it away by attachment to overly simplistic false assumptions. What is needed hence is to release/relax these mental attachments.

This is not to say that intellectual or rational function is "bad" per se. Rather, it is a specific technical tool that can be misused/overused to create lifeless, contrived, narrow, and artificial mind constructs no matter how logical they may appear, that preclude spirit and dishonor life. This is what we meant by the statement that space and time is a part and parcel of the symphony and anecdote of life and aphorism of thought like that of waking state or dream state. These ignorant mind fabrications can box man in to very limit conceptual frameworks and self limiting beliefs which compound his suffering and alienation

if not surrendered to the larger innate intelligence behind the intellect (buddhi) and which animates it. Logic without inspiration thus gets in the way of universal boundless mind, thus creating an artificial split (duality) between the one who views (a false identification of separateness or ego) and the object that is being viewed, and thus the process of viewing (or consciousness) becomes severely obscured, conditioned, biased, frozen, and self limiting. This is the common assumptive trap of extractive reductionist mentation or over objectification of the right brain imbalanced person (a widespread epidemic in the modern world). Simply the Buddhists call this subject/object duality, or one may simply call it cerebral over dominance or fixation upon a false identification. It is necessary to release (vairagya) this fixation if one is a sincere seeker. This is what we meant by the fact that the fecund imagination, unannihilatory manifestation and nonlinear franticness of Objective Reality and Subjective Experience are one. We have put it mildly and politely by stating if "Subjective Experience" is only enough probably a courtesan would have been a Brahmangyanai. Or "Objective Reality" or storage of knowledge is Brahman probably a Quantum computer would have been one.

A yoga practitioner (sadhak) or true spiritual seeker asks the fundamental question of the spiritual seeker -- the one who inquires, explores, searches, and observes -- who practices self observation and self study (swadhyaya), thus removing the bias and prejudice of the a separate observer. Such a seeker is not interested in the small answers and the inquiry is not necessarily put into words, other than into describing the nature of inquiry itself. The essential practice is trans conceptual. By inquiring, "who am I" conceptually, one defines oneself in terms of everything else. As such it is only the first preliminary question, through whose ultimate answer everything else understood. The answer is not separate, not independent. Ultimately, the non-dual realization is beyond the ability of human intellectual/conceptual thought constructs; rather it is truly timeless primordial wisdom, which must be allowed to express itself. Neti neti says it is not in the books, nor not "not in the words, belief systems, or conceptual frames. Neither is gnosis found solely in the physical body, nor not only in the body, rather the selfless unselfish state of pure being (Sat) both the body and everything else, and primordial wisdom, including vast time. Hence in this yogic light we experience a holographic embodied spirituality where phenomena is permeated with timeless wisdom, where phenomena and being co-emerges spontaneously as one when we allow it to happen -- when the wisdom eye opens, the true nature of phenomena is known. Yoga is a practice, which allows us to realize that great integral synchronicity. Otherwise, we exist spiritually estranged and disempowered in chronic spiritual self-alienation (error or sin).

Spiritual practice as such then is the remedial adjustment for the skew introduced by the conditioned small self (limited viewer which introduces time/space bias) in favor for the transpersonal universal spiritual boundless view. Sadhana thus brings us back into balance and harmony. That this REALITY is possible and that it involves an inner awareness; i.e., an awareness of "self" is the assumption that separates the spiritual explorer from the assumptive limitations and beliefs of an ordinary philosophic inquirer.

So this again leads one to ask the spiritual question, who am I? Thus in authentic yoga sadhanas of swadhyaya (self study) this exact process of inquiry into Self nature is engaged, not the study of what books, scripture, priests, or external authority figures or status quo structures tell us who we are. The successful answer to this question depends on LISTENING/ASKING going deep into that very process of inquiry itself. This is called, BEING OPEN, in dhyana (emptiness meditation) practice.

Knowing our own mind (instrument of knowing) is a wonderful gift that we can give ourselves. At first, one may encounter demons (phantoms of the dualistic mind which one can apply "neti-neti", neither this nor that -- not self not not self. Something like mathematic sing or anthropomorphizing of mathematics with all rational Leibneizism, rational relativity and Socratic Subjectivity. Going into that light of knowledge it leads to what is called self realization in the spiritual context. If the answer is found in a book, external authority, scripture, or consensus reality then the book answer is temporal, prejudiced, contrived, and "religious" (conforming to an external authority) versus spiritual and universal which assumes that the answer has come about experientially from a subjective inner realization. The former conclusion (which is the norm) is achieved through conformity and memorization, while the latter is achieved at first through critical and creative thought (neti-neti) and then from their non-dual integration is based upon the highest Universal Eternal Omnipresent Source which is eternally present -- which is omniscient as well as omnipresent -- eternally here and now as

beginning less and endless boundless universal Mind.

Thus the first step in this spiritual process is to make the inquiry which must challenge all sacred cows and institutionalized authority. The second step when this external knowledge (knowledge about something) and programming has been discarded, then one has space for the inner clarity and grace to arise (prasadam) which is subtle beyond the most subtle -- objectless. How else could one know the true Self who is not bound by any singular object, if not from every cell in your very body and every cell in the universe and every non-cell, and non-atom? These points to beautiful completion (where the bonds of karma are exhausted --to ultimate fulfillment and Grace). Kind attention is drawn to the coextensive representation, a priori determination and discursive addendum of consciousness which is defined as the storage of all images, information, information fields, mind acting on it and the resultant being calculated by ASCII .It is just that when you think another sentence is formed.

This is the reason that yogis and true seekers meditate (practice dhyana) and perform spiritual practices (sadhana) in order to penetrate the veil of illusion, to remove conditioned ignorance, to free themselves of conditioned bias, belief systems, prejudice, programmed mental propensities/tendencies, habits, limitations, and karma, and thus to eventually abide in the wisdom expanse of the Great Integrity -- the Great Universal Consciousness and Beingness which is supra-human, i.e., it includes the human realm and earth realm, but is neither anthropocentric nor dualistic. To limit, reduce, or impose human terminology, symbolism, and language into this realm, is exactly the self imposed barrier which the intellectual suffers and becomes imprisoned. Thus neti -neti can also be translated, "not just that, but more than that - more than just that, much more unlimitedly so". Nothing is excluded. This may appear to be a heretical interpretation to the orthodox academician intent on separation and negation, yet maybe some academicians would agree that analytical processes are preliminary and supportive, helping the practitioner to eventually weed out errors of the mind and surpass the limitations of the human brain. Certainly we are more than THAT, which words can describe. In other "words", it is a spiritual statement only if it is an affirmation of the presence of an all inclusive living spirit; i.e., that non-dual teachings calls forth as it discards the ego (false and limited view of a separate "I" observer self). It does not negate nature, and the natural unconditioned mind, but rather it negates false and limited views and delusion. Failing these criteria, it is not a spiritual statement; rather it is intellectual speculation and/or ideology. Zen thus coordinates on the fact that there is no great knowledge to be acquired. "Experience" is what we have defined and there are neither births nor rebirths.

Unfortunately, in both the East and the West the common man substitutes belief, pramana. Academic book knowledge, or such as a displacement for wisdom or gnosis, but the yogi does not fall victim to that trap. Those who are tied to the sterile puppet strings of reductionist thinking of the intellect (immersed in the limitations of symbolic world and duality) interpret the expression neti-neti, as "not this body, not this body" even though the body has nothing to do with the Sanskrit word, neti. Rather this occurs because of the intellectual's ingrained dualism, i.e., that says that God or Spirit has to exist elsewhere -- not here not here they are saying. Somewhere else -- somewhere else! This self defeating estrangement puts the intellectual into self imposed spiritual alienation and by definition distances them from God, Spirit, sacred presence, union, or samadhi -- the living world of "All Our Relations".

Fear of life or pain, is the "normal" and common reaction, but unnatural. It produces an alienated world of institutionalized spiritual Diaspora, corruption, fragmentation, and sin. This is the affliction that authentic yoga attempts to alleviate and eventually remedy, by meeting our mental pain, and thus destroying fear and attachment, instead of attempting escape. This institutionalized anti-body and anti-nature prejudice unfortunately has become the acculturated and authoritative standard interpretation (no matter how absurd it appears to a true sadhak). It has become so entrenched that many humans are destroying themselves and their habitat. Rather in truth, neti neti means that we are not separate or apart from the sacred web of life; that this body does not exist separate from millions of years of co-evolution with the universe; that this body is a manifestation of beginningless love -- of eternal spirit right here in the present as a representative of eternal presence -- of love loving love. Here the true nature of phenomena is disclosed as it discloses its beginningless primordial origin. Such a life becomes a living library in each instant -- in all instances.

The alienation of man from this deep holographic/reflexive relationship is ignorance, where man denies his connectivity with all beings and things. Nothing else will be able to fill this empty gap. Neurotic

consumerism is dependent upon this spilt from living spirit and source. Man's true authority has become institutionalized by greedy and exploitive priests, kings, and their lackeys who thrive on manipulating others. These vampires create confusion on purpose in order to dumb people down and create a fundamental self doubt, which thus creates dependence upon these external authoritative structures, institutions, empires, kingships, priesthood, hierarchies, institutions, religions, etc. Of course those entrenched and dependent on this structure will deny this.

For example, *neti neti* as interpreted by some Vedantists (not all) is a form of denial of and escape from the world, of embodiment, of aversion to and fear of nature, creation, and embodiment. It's an easy way out/copout, but is none-the-less very seductive to the confused masses. The self delusion rap goes like this; "Just pretend that the world doesn't exist, that life is an illusion (*maya*), numb yourself out to it, the body, the senses, and sever all bonds to it, and then all your problems will go away. All suffering is just an illusion, war, torture, disease, poverty etc., is an illusion". In short rationalize out suffering and rationalize in Brahman. Take that as one's acting role as if playing in a theatre. Such is the pretentious play of those lost within inorganic and contrived self fabricated and sterile mindsets based on preferential thinking (*raga/dvesa* as attachment and repulsion). Obviously *kleshas* and ignorance such as *raga/dvesa* cannot be remediated through application of *raga* (preference) or *dvesa* (dislike and escapism), nor can *nirvikalpa samadhi* be realized through utilizing *vikalpa*.

Yes, many attachments such as to the body go away when the physical body dies, but who dies and who dreams on, what *samskaras*, mental attachments, fears, confusion, and *karma* continues. These are the elements that are to be liberated today in order to become liberated today and in the future. What about future generations and the welfare of our children? What about the future of the human species and the planet? These are human questions which beg the question of why human birth in the first place which are too earthly questions for the orthodox.

The narrow minded academic Advaita (non-dual) adherent interpretation of *neti-neti* precludes and escapes nature, embodiment, and differentiated consciousness, focusing on isolation toward Source only or monism. Monism is not non-dual, but rather one way union with the one at the expense of the many. Orthodox escapist oriented Vedanta tends toward monism -- a force moving away from differentiated consciousness while labeling everything Brahman at the expense of everything else.

They may deny or affirm, depending, that they have created a conceptual ideological split between spirit (Brahman) and Nature (embodiment); yet at the same time they may mistakenly call their philosophy non-dual. As such these so called non-dualists are as other worldly as fundamentalist Christians, who believe that God is in heaven (not here). Heaven is where they will go AFTER they die. That is another word, for institutionalized spiritual estrangement -- something Emperor Constantine created in the 4th century at the Nicene Council with a design to own the world. Joshua bin Josef of course never indicated anything as absurd as a distant God. In fact he is reported as saying: "My God is of the Living". In the Nicene Council that established the legitimacy of the Christian bible, only those representatives chosen by the Roman emperor, Constantine, were invited. Even some of these could not be made to agree to a new church based on Paul's doctrines of sin, so they were condemned and banished. The main controversy stemmed from what was called the Nicene Creed, a statement of Christian faith that was adopted by the Nicene Council which clearly affirmed that heaven was elsewhere than here.

It is remarkable that there are elements within Vedanta who also create that same alienation from Spirit (the well known Brahman-Maya split) while still insisting on labeling it non-duality. However it appears that such represents a deep duality, dissociation, and denial. The deepness of that denial is that the Vedantists think that they are non-dualists (most of them), but have in Reality created their own drama which may I add won't end until they drop their own self imposed Vedantic filter. In both systems there exists a fear of nature, the body, and embodiment -- a statement of enduring suffering and denying fulfillment in trade for some distant far off time. In order to deal with the fear and trauma of existence brought upon one's state of confusion, one dissociates from "self" and embodiment -- from even an association with the body or life. Such systems reinforce an alienation from our own feelings. Such dogma makes us believe that sex, pleasure, bodily feelings, connection with the world or our animal nature is bad. Implied in that assumption is that we should ignore or overcome the body and our feelings. In that sense there is the idea of sacrifice without reward or a delayed other world rewards.



However in truth the body is god's temple and our feet and arms are his limbs. True non-duality (call it advaya if you like) synchronizes non-differentiated consciousness or absolute reality with differentiated consciousness (relative reality), one not excluding the other in true non-duality (advaya) with neither extreme existing by itself nor negating/excluding the other. Thus neti/neti is neither an undifferentiated consciousness (Brahman) interpretation, nor a differentiated consciousness (maya), but rather both/and. Maya is the clothing or apparel of Brahman. It both conceals Brahman as well as discloses Brahman. Without the clothing Brahman is invisible. They must be seen as an overall whole. A monist would state that only Brahman or the eternal is real to the exclusion of the temporal, while a non-dualist would recognize the relative and absolute truths as mutually co-arising.

Here we come to the true value of the saying, neti neti, as being neither monist, neither one extreme nor the other, neither up, nor down, but rather non-linear and multidimensional. If one searches inside to the observer - to see how his conditioning and prejudice (vrttis) colors the field of consciousness, one would eventually have to ask who is self and where does it abide? Religionists do not embark in such a genuine inquiry because they are compelled to follow their politically correct external authoritative traditional dictates, and not contradict or question it. However after a thorough spiritual inquiry by true seekers, no one has been able to find a separate self, or ego. Where does it abide? This separate self or ego is the simple dualistic question, but no one has been able to find it. After a logical search most people mistake that the ego is defined by the physical limits of the body; i.e., where the body ends, the ego ends. Hence in this situation, the term, neti neti does mean; no, I am not just the body -- not the physical body only, in the sense we are not the ego, rather we are more than that. It says we are more than a thing that dies but also this thing that dies AND also includes this thing that dies and more than that. The yogi gets this answer not by logic alone, but from direct experience of that expanded boundless universal awareness brought through dhyana and samadhi -- from the open heart. That experience informs then the frontal cortex and thus the logic of it is inspired through direct experience of the opened heart dictating to the open mind.

So that most common mistake by the closed minded, logician, philosopher, religionist, or cortically dominated man is the most common mistake -- the preferential assumption that the body is not real or spiritual because of great pain and dvesa (repulsion). Neti neti really means is that the body is only a small part of who we really are in terms of the big Self of All Our Relations. It is part of the omnipresent magical and intelligent great matrix of Siva/shakti. The Great Integrity is just that, all inclusive, omnipresent, universal, innate, and non-fragmented by definition so how could it not include the body and the entire cosmos? The only thing it does not include is the delusion of the ego -- that somehow the body is the ego or that any thing exists by itself in a vacuum. The "i" in Reality is empty and vacuous -- there is no "i". The "i" is a neurotic substitute -- place holder, for the absence of the sacred relationship. This sacred relationship occurs in the transpersonal. Both/and, non-dual realization that authentic yoga (reunification) brings.

Thus the true interpretation of neti-neti is that we are not this, not that -- not any "thing" separate. Neti-neti states clearly that we are not "just" that or this, i.e., that we are not separate and limited beings but rather thou are that, i.e., that our true identity is Brahman, not the limited and biased spin of ego (the delusion of being separate). In truth we live in sacred presence, but we are asleep in ignorance, delusion, and forgetfulness. Thus a Sufi affirmation to the divine goes; "I am nothing at all by myself. Only YOU exist -- God is everything"! Ignorance means just that; i.e., that some "observer/I" is ignoring the whole/hologram.

Just as the stars and valley are obscured by clouds and fog, they none-the-less truly exist as they are beyond their appearances, beyond the senses, beyond superficial clothing. The clearing of the clouds and lifting of the fog alludes to the process of purifying the matrix and lifting the veil of ignorance and illusion. Egohood only exists in our mind. It is a shared hallucination although a powerful one. Although many people may "act out" from this neurotic state, and thus mutually serve to amplify the illusion, it does not follow that it is none-the-less more real. Ego reality is "in Reality" an empty illusion -- void, a hallucination. Here we are approaching Buddhist non-dualism (free from all extremes). The ego is a delusion and hence so are the way that the ego normally perceives phenomena. But these limited or superficial ways of knowing does not preclude the true nature of mind or true nature of phenomena.

The non-dual unitive process includes the process of going toward primordial uncreated beginningless and the evolutionary creative manifestation from primordial source. This is only possible to know in the primordial present as instant presence. Primordial space and time has no end, no beginning, nor middle. It is not a thing or entity, and has always been available as sacred presence in the eternal timeless moment Primordial space has always been here, beyond before the beginning and after the end in the Beginning less never ending/ever-new/ever-now-ness), which is both nowhere and everywhere, neither one nor the other, and neither neither (neti neti). This is the ultimate unbiased universal unlimited and complete Reality, of which nobody can step outside of and view objectively.

This non-dual process in which all are kin is non-linear and non-dual BOTH/AND simultaneously like a non-dual pillar of light going up and down, neither up or down, left or right, in or out, neither both, but both/and. In authentic yoga we are this non-dual pillar in the activated core/heart reached through the middle innermost central channel (within the sushumna). Energetically this occurs when the winds moving in the ida and pingala channels (nadis) are subtilized to the extent that they dissolve into the central channel (sushumna) and stilled (sunya) in samadhi (see yoga Sutra III.3). Here samsaric mental afflictions are dissolved

Thus we are not "just" the energy body and not just the physical body which is ever changing, temporary, and corporeal, but at the same our identity must include the form bodies (in the greater context) in each sacred eternal moment of non-dual being. This is the salient point; i.e., that in all pervasive vast space, there is nothing excluded nor nothing that remains to be included. It is the Great Integrity -- the Great Completion -- the Mahamudra as the vast expanse where duality is merely a limited misperception. The orthodox scholar, dualist, and common man mistakes the expression to say that we are not the body, but the yogi takes it to say that we are all inclusive, the body, and the river, and the stars, the sacred mandala, divine creatrix, -- not just one of these limited, isolated, separate, and false identifications. In other words, the words, neti-neti, cannot be viewed successfully as an exclusive statement out of context with Brahman, but rather as an inclusive affirmation of Brahman and Atman -- Atman only truly exists united with Brahman-- of an all inclusive reality where there is no separation. The yogi as a true seeker and fearless rainbow warrior who learns to see living spirit inherent all created things -- learns to see the continuity of eternal spirit in all of creation at all times. As Van Morrison has put it, "Spirit Don't Ever Die". It exists NOW.

On the Connection between Buddhist Anatta/Anatma, Sunyata, Nondualism, Advaita/Advaya, and (Atman/Brahman Unity Utilizing Neti- Neti as the Integrative Key

Neti neti is a saying found in the Upanishads and especially attributed to the Avadhuta Gita. Literally, it means neither this, nor that. Considerable debate has arisen over exactly "what" (if anything specific) does it really refer to? Indeed, the true nature of that "what", as the true nature of reality is a large topic in Hindu Advaita philosophy as well as in Buddhist Madhyamika philosophy. As discussed above, the most classic fundamentalist anti-nature and escapist view is that neti neti is a statement of transcendence (isolation and escape), saying in effect, and one is not the body, not identified with nature, and thus one doesn't belong on the planet. In that disidentification one attempts to dis-identify and avoid death and suffering. Simplistic, but appealing to the escapist/fearful mind none-the-less. Since it is said twice some may say that it says, I am neither the body AND not the mind; i.e., my true identity is not confined to either self existence, or non-existence, but rather a larger non-dual Reality which encompasses both. In that way, the self (atman) is not viewed as independent, isolated, or separate from the all encompassing Brahman, but neither is Brahman absent from existence. Rather "existence" is redefined in non-dual terms. A cogent point to remember is that neti is repeated TWICE, rather than as a single negation "once", so we will take it as meaning, neither and nor.

Classically, neti neti is a refutation of duality, and thus an affirmation of the Great non-dual Integrity, often called Brahman, which affirms one's identity as interdependent with all beings and things, united as one as part and parcel with the hologram.. However, in some orthodox fundamentalist traditions, Brahman is defined as something separate from what they call Maya, which is usually defined as temporal existence. Hence they create an opposing duality (between eternal and temporal), but are they really separate or rather are they parts of a greater whole? In Kashmir Shaivism, maya both clothes/masks Brahman and reveals Brahman, depending upon the depth of one's penetrating wisdom.

So in some other schools and especially tantricism in general, which evolved from the latter Yoga Upanishads and afterwards such as the Buddhist and Kashmir tantricism, neti-neti can be interpreted as an all inclusive statement of true Universal identification, which is both undifferentiated and differentiated -- as not being separate; i.e., I am not an ego or separate "self". Rather I identify and belong to the greater whole. That self or identity is not separate/independent from the whole (Brahman), yet it is omnipresent simultaneously and transpersonally, reflecting our true non-dual nature. All that can be ascribed to the first neti of neti-neti, which negates the identification of the ego or separate self.

The second neti can say I am not just the whole, but also within all the parts of the whole -- I am both at the same time. Thus one may say that it is an affirmation of the Buddhist idea of anatta or anatman, which similarly is the realization of the unreality of a separate self or ego (that the ego is fabricated and delusional, only existing because of unawareness of the Great Integrity, which bind us all to a universal unbiased commonality/Reality. This is to say that one can not be fixated on an egoic mindset at the same time as residing one's consciousness and being in the realm of vast space, vast primordial time, and vast knowledge. No, the latter is the ever present option which the open-mind chooses. The egoic mind is just a cramped, imprisoned, and limited version of the former.

So just as the first neti can say no to a separate observer (ego) who is free from subject/object duality, the second neti can be said to be a statement that affirms the emptiness of the object as viewed as independent and disconnected from the whole, which is nothing other than the an affirmation that "things" do not exist per se, by themselves, independently by themselves. Thus neti neti is best understood as an affirmation statement that we are not just the body, not an ego, not fragmented parts, and not separate -- that the body is part of a far more vast interconnected web of all life both of form (saguna) and the formless (nirguna) which is inseparable. "Neti neti" can be said to be a profoundly deep ultimate statement of non-dual beingness, neither one or the other, but rather an altogether expansive context which is all inclusive where nothing is left out nor needs to be included -- which is neither form nor void in themselves, but both form and void intrinsically, neither just body or separate from body, but viewed within an integrated context where body and spirit both coincide and are synchronized.

If the Buddhist term, anatta (anatma in Sanskrit), is the negation of the fabricated delusional non-existence of the ego (a separate self), then there is no contradiction between Buddhism with Advaita. Confusion only arises when either Buddhists misunderstand the non-dual intention in neti - neti as a disowning of any idea of separate self; or where Hindus become confused that Brahman is a larger ego (soul) or separate self identification that can be conceived or owned by the ego; or when Hindus believe that Buddhist anatta is a nihilistic statement that denies existence completely. Such confusions arise only because of ideological conditioning and self identification which preclude listening fully or thoroughly while studying the other's point of view.

Even in the older Buddhist Pali Canon, anatta (anatman, in Sanskrit) quite clearly confirms the reality of a non-separate (selfless) self simultaneously with the emptiness of any separate object whatsoever (which is the affirmation of emptiness or sunyata). The Vedantic statement that Atman and Brahman are united as one is thus a statement of no separate "self" also. Indeed atman does not exist separate from Brahman, but rather only when integrated with Brahman. Although Shankara, the first systematizer of Advaita (unqualified non-dualistic) Vedanta (culmination of all revelations), is credited with arguing against the Buddhists, if one studies his commentaries, one simply sees that his intent was simply to defend, promote, and reform Hinduism, rather than to attack basic Buddhist ideas. Buddha also did not challenge basic Hindu assumptions, rather than attempt to free people from dependence upon ritual, custom, ceremony, hierarchy, caste, idol worship, and dependence upon ancient scripture. Ancient Buddhism can be seen as a form of Hinduism, which is stripped naked of all its local, provincial, and non-universal ritual and cultural trappings, albeit cultural baggage has cropped up in Buddhism over the past 2600 years. So although rather heretical, one may postulate that there is not a large gulf between the two religions as religious zealots and defenders may claim; rather the same truth is being stated within two different culturally described contexts.

Similarly sunyata (emptiness), is a broader application of the Buddhist statement of anatta, saying that neither the observer nor the observed -- neither the object nor the knower of the object have any basis as separate other than as fabrications modified by conceptual ideations. In short, there remains no objective

basis in Reality, as long as the observation point (observer) is limited or biased. Rather the basis or ground of the observer and observed is their inherent emptiness in themselves of an independent self or thing; i.e., no objects or observers exist in Reality separate from the universal boundless Mind. Without this complete integral boundless universal perspective, all other views are biased, distorted, prejudiced, fragmented and corrupt by definition. Here neti/neti again is remembered neither observer, nor observed, not one or the other, but both/and inside an all inclusive whole. Radical that it may seem to religionists, neti, neti, and sunyata point to the same truth. This is why net is repeated twice. It is the way one says neither one nor the other, but both/and --the many and the one where a bigger complete view is realized in Mahasandhi.

Of course strict dogmatists and ideologues who have become seduced by tradition, will stick to the old doctrine unmoved, unable to think outside their accustomed box. Similarly orthodox Vedantists will interpret this within their own box accordingly, but here we will utilize the Kashmir Tantric view which easily reconciles with the Buddhist view to show that neti/neti is an advanced non-dual statement relevant to both the cave dweller and layperson equally.

To sum up, we have to realize that in Advaita, Atman means as being one with Brahman. It is not the ego, because the ego separates and denies this unity. It can be called the soul only if the soul is defined as united with Brahman. When it is disconnected, then it is the egoic mindset (delusion) which rules, not the union of Brahman and Atman. This soul then is a reflection and embodiment of Brahman, not its negation (not the egoic identification). That Atman can never own or possess Brahman, rather it is entirely owned by Brahman. Thus the statement of neti neti as a statement that one is not different from Brahman is not to be taken as, "the ego is not separate from Brahman", but rather the ego is nothing at all, where the soul is the reflection and expression of the universal all inclusive Brahman. Atman understood this way as not being independent from Brahman, is not an egoic atman, rather it is incapable of standing alone and apart from Brahman. It is anatman in the Buddhist sense of being empty of an intrinsic independent self. This Atman is akin to the Buddhist idea of Buddhature, where all beings contain the evolutionary potential for complete awakening, Buddha nature being intrinsic/inherent as one's true nature. The key in Buddhism is to reflect and express this inherent true nature -- to become a living Buddha. In this way, Buddhature (Tathagatagarbha) is the same as the Advaitic Atman, recognizing that Advaita Vedanta's definition of "Atman" occurred 1000 years after the mahasamadhi of the Buddha.

In the true non-sectarian yogic tradition, that which is put an end to, that which ceases, is ignorance/unawareness, or rather the veil of illusory thought patterns cease. the world of duality ceases in neti neti. When the veil of ignorance is destroyed, then illumination shines forth unadulterated. Conversely, the process of denying or limiting Reality as-it-is through ignoring what is is called avidya (ignorance). When we ignore something, then we lose the big picture and wind up with distorted and biased views. One could just as well say that we are suffering from myopia or self deceit/delusion. When this confusion, delusion, ignorance, unawareness, or illusory bias (labeling it by whatever name that may be useful) is removed/negated, then we see things as they are -- Reality shines forth in net neti -- when "not this and not that" becomes natural and spontaneous.

This Reality in tantra and Mahasandhi is natural, uncontrived, and unconditioned. It is our natural state. What has been removed is the past programming. The result is the profound synchronization of absolute and relative truth, shiva/shakti, right and left channels, pingala/ida, undifferentiated and differentiated consciousness and so forth. It is not a denial of the temporal as if samsara were separate from nirvana, or that nirvana was not found in samsara, or that shiva was not inside shakti, and so forth simply affirming the eternal presence Now abiding in sacred presence - the basis of true spirituality, there is no thing truly solid, separate, or substantial that can be said to exist by itself (independently). Nor can that "something be said to exist (as it never existed separately in the first place. Neither can one say both, that it both truly exists and not exist, because there is no separate thing that can be spoken about. Nor can they be said to both not exist. Again in all these situations the reified separate object of existence, can not freely succeed as a referent or object to refer to. Relative and absolute, differentiated and undifferentiated, form and void. the moon and sun, the left and the right, and the earth and heaven are inseparable in the non-dual context of interdependence. Neti neti therein is the unification of wisdom and compassion, emptiness and bliss, emptiness and clarity, and emptiness and essence, where neti/neti

is the negation of the process of dualistic illusion, fragmentation, and separation itself.

This occurs when awareness and the authentic energetic qualities of awareness are synchronized in the deepest recess of the nondual central channel (the avadhuti, kun dar ma, or sushumna). This occurs when the inner awareness meets its counterpart in outer awareness -- where the breaks in Indra's net is mended -- where what is, is seen by the heart in the heart directly.

From the Hevajra tantra Chapter One

"Buddha replied, 'There are thirty-two nerve-channels. These thirty-two are the bearers of the bodhicitta and flow into the center of Great Bliss. Among them three nerve-channels, Lalana, Rasana, and Avadhuti are the most important. Lalana has the nature of Wisdom and Rasana of Skillful means. In the middle, between them is Avadhuti, free from the duality of subject and object. Lalana is the bearer of Aksobhya and Rasana is the bearer of Rakta. The bearer of both Wisdom and Moon is known as Avadhuti.' "

Not Two (neither not one)

Non-dual (advaita) actually means (divitiam nasti). However purists in Hindu tradition actually mean, eka vastu vada (there is no second substance except that Brahman is the only thing that exists as if it is separate from the physical created world or universe). This is actually a dualistic confusion of course and is really monism, rather than nondualism, where both diversity and unity are celebrated interdependently. Thus within this orthodox definition of Brahman, the phrase, eka vastu vada, is more accurate to describe what they mean, rather than the word, advaita.

Buddhism also uses the word, advaya, more than advaita to connote "not two" i.e. free from the two extremes (skt. dvaya anta mukta) of samarupa (the tendency to see things as really existing) and apavada (the tendency to see things as non-existing). Free from these extremes is the famous middle path (free from eternalism and nihilism) taught by Buddha. Advaya is not of a thing (the one and only thing) like Brahma but a description of the svarupa of samsara) as things really are in their relative connected sense). That is why in tantra samsara which is illusory, but not an illusion is transformed into advaya jnana (nondual wisdom) in Buddhism whereas in Hinduism the illusory samsara vanishes and the true eternal unchanging Brahman dawns, but too often within the orthodox schools at the expense of diversity (excluding or denying the relative world) hence being escapist and unworldly stripping spirituality and God from embodiment.

Further, monism may impute that there exists only the observer, as if the whole is a separate object. That is the common mistake of selfhood (ego) where we ascribe name and form and "self/ego" to that which is unbounded, infinite, ineffable, universal, and totally incapable of being objectified. But really Brahman as the absolute cannot be an object and the "self" cannot identify with "it" as such without falling into dualistic delusion. Such an experience is impossible, otherwise the ego would try to grasp unto it (as it does). Thus monism is really a dualistic system -- an error in which one acknowledges the source/creator by negating or ignoring the creation -- where the observer is separated from the observed. Monism is a one way street. In relative reality "everything" is not the same, rather life is infinitely diverse and rich, but at the same time it is interconnected --part of a greater whole. Rather non-duality in net/neti indicates clearly that creator (as source) and creation (as active manifestation) are not separate, just as a fire was started by a spark -- the spark is implicate to it. They cannot be separated one from the other without creating a dualistic limitation.

Rather the case is that undifferentiated reality (absolute truth) coincides with differentiated reality (pratityasamutpada or relative truth) in sacred presence. Here both are acknowledged as well as linked in yoga. Conceptual formations no longer displace primordial wisdom, rather view is educated by the evidence of our unfiltered/naked awareness and direct experience. In monism however everything is confused with everything else. In other words it is quite different to say that only undifferentiated reality (absolute truth) exists and differentiated reality (relative or conventional truth) does not -- either one or the other, rather in true non-duality both exist AND do not exist -- neti/neti. Existence, phenomena,



things, and events appear to exist, appear to either a rise or fall away, both, or neither, but all such referents are designators, indicators, or imputations toward nothing true, rather it arises from a limited arena of frozen time and contracted space. It is a very small part of the entire picture, like a distorted or occluded lens which obscures the vast hologram and primordial integrity which is rich beyond measure. In truth there is nothing separate from the vast and all pervasive Primordial Mind. This is disclosed to the true seeker. What else can be known? Once the true nature of the Mind is known, so is the true nature of nature also known in its timeless radiant and vibrant wonder? Hence monism thus is a dull sameness, inane, empty, indifferent, and nihilistic. Crucially, non-duality is not monism and it is not negation. *Neti-neti* is not negation of any thing; rather it is all inclusive where nothing needs to be excluded or included. Neither exists separately from each other so in the non-dual Buddhist school it is simply said that no separate thing exists, not even emptiness is a thing. Because we usually use words to communicate, and words are inherently dualistic/symbolic, it is difficult not to speak about an observer and the observed, but we can try to make that extra effort. Ultimately of course we need to give up words and thought constructs completely, like through objectless meditation (*dhyana*) or other transconceptual (*nisprapanca*) yogic arts which allow for direct experience devoid of mere/empty symbolic representations.

Again a quote from Bede Griffiths and Georg Feuerstein, both mature observers of both western and eastern scenes.

"Advaita (nonduality) does not mean "one" in the sense of eliminating all differences. The differences are present in the one in a mysterious way. They are not separated anymore, and yet they are there."

Bede Griffiths (1997)

"The difference between monism and Nondualism might appear subtle, but it is important. The question is crucial for functional yoga or not the linguistic expressions by yogins etc. reflect their actual realization. Could it be that an adept has broken through to nondual realization but is still caught in monistic language?"

There is no question in my mind, for instance, that many of the statements in the early Upanishads point to genuine transcendental, transconceptual realization but resort to poetical/mythical/symbolic language that could give an inexperienced reader a different idea. In other words, there is not inevitably a perfect match between experience and language. This is a critical fact, which we need to bear in mind when looking for Upanishadic passages that testify to either monism or nondualism."

George Feuerstein

Although yoga, as an experiential non-dual practice, has been known for millennia, it is not a coincidence that non-duality as elucidated by Nagarjuna (circa 200 AD) and his non-dual/indivisible doctrine of *sunyata* and form (the union of ultimate and relative truths) became further elaborated upon by Shankaracharya with his non-dual doctrine of *advaita vedanta* (circa 7th century CE). This breakthrough context linking form and consciousness set the stage for the innovation of Hindu, and especially Kashmir tantra, hatha yoga, tantra yoga, laya yoga, kundalini yoga, and shaktism. Simultaneously Buddhist *mahasandhi/mahamudra* flourished in Afghanistan, Pakistan, and India eventually going to Tibet, China, and Japan. Such preliminaries posited the non-dual ground for such advanced practices. Both Hindu and Buddhist tantric and hatha yoga certainly explored new ways of self realization in the backdrop of a non-dual (*advaya*) providing a rich all-inclusive vibrant organic context, wherein primordial wisdom was co-evolving with the evolutionary force into an enlightened age, where wakin-up (while in this very body) was deemed accessible. Here the body, energy body, and mind were recognized as active co-participants. The sharpening of that indivisible relationship became the skillful instrument/vehicle for the exploration.

Numerous practices of integration evolved such as Sri Vidya, the body as a magical yantra (Trulkhor in Tibetan), the *sadhanas* of inner heat (kundalini or tummo), the Six Yogas of Naropa, the doctrine of harmonizing the five koshas (sheaths), working consciously with the inner winds, channels, and bindu, and so forth (to name a few yogic techniques which are outlined in tantra and yoga). Similarly, the Tibetan word for yoga is *naljor*. *Nal* means natural and *jor* is wealth. Hence, yoga or *naljor* brings forth

a natural innate state of wealth, health, and fulfillment. This enlightened age of body mind holistic integrated practices, continued to evolve in Medieval India up until the Mogul invasions of India (8th-13th centuries), wherein India's innovative genius became feared, suppressed, repressed, poisoned, and supplanted by alien invaders from the West.

Monism fears diversity, because it is a threat to egoic order and its attempt at imputational domination/control. It is thus an egoic grasping/craving which maintains the rend which substitutes monism as an inadequate compensation of a natural organic non-dual open-ended interdependence free from conflation (the superimposition of egoic confusion). Shankaracharya's non-dual "Self" (Brahman) is not an independent/separate self identity, but a genuine integration as a description of an indivisible interdependence where everything is connected. That Self is all-rich, all encompassing, infinitely differentiated and inseparably married as nature is married to awareness, Shakti to shiva, Samantabhadra/Samantabhadra, the true nature of the mind and the true nature of nature, or simply said as mind is married to the body, so too is heaven married to the earth. The problem, if any are to be identified, is that human beings tend to habitually ignore their truly innate connection with the infinite -- as part of the whole, thus wallowing in a state of chronic egoic confusion (samsara).

Since the human mind, being part of a larger primordial system, cannot grasp the entire process intellectually, the folly is to think otherwise; i.e., that the intellect is an efficacious tool for spiritual knowledge -- knowledge of mind itself. Infact, mind acts upon the storage of information, visual representations, anecdotes, aphorisms of thought, belief systems, and most important is that the anagrammatic mind connotes and concocts and fabricates a thesis for itself mostly the same. This result in various illusions of states that of waking state, dreaming state, and dreamless state. Without knowing the instrument of knowing, then chronic errors will continue to be repeated. So the major "problem" which arises for those mired in intellectual, fundamentalist, philosopher, and others lost in dualistic thinking is that they have become addicted to navigating "reality" through a veil of deadened and numbed out chronic processes of chronic extraction and over objectification --where anecdotes of truth does not meet aphorisms of thought, where the rocky razzmatazz of falsehood and illusion drowns the angst saga of taciturn bay and enduring silence. through separation -- an exclusiveness (reductionism, analytical mentation, and comparative analysis) without placing their data within the holistic context -- in integration and harmony with all time -- as exhibitions of gnosis of the Great Integrity where all the seemingly dualistic parts are seen in relationship with all others in all of time. Indeed, such a world view appears broken and fragmented due to self sabotage. Such folly! Even if a yogi should speak, if the listener has no direct experience, then the listener will try to understand the "yogic speak" through their intellect, which will only confuse the listener more. It will have the effect of bringing the listener even more so in their logical mind in a futile attempt to understand the "yogi speech". It might appear that the yogi is speaking intellectually, because the words are not logical and appear to be "foreign". Thus yogis are rare, but rarer still are teachers of yoga. It is said that the teacher comes when the student is ready, but "who" is ready? A true spiritual seeker places his/her predilections and prejudices aside -- offering it up as a fire offering while making an affirmation that all of his/her activities be reordered and placed in alignment and harmony with the universal timeless guidance of a sacred transpersonal non-duality -- the world of Grace where Maheshwara (the param-purusha resides). In indigenous terms the Great Integrity of "All Our Relations") is a holographic realm of profound and sacred suchness. A true sadhak (practitioner) will not allow themselves to be swayed, distracted, nor dissipated by the intellect or conception processes. "Here" in this sacred presence, which is not based on intellectual reductionism, the eternal timeless reality (creator) bleeds through and is revealed in every action of creation -- as divine Shakti and as such in yogic terms Siva/Shakti is in perfect balance, harmony, and synchronicity. "HERE" yin/yang is in perfect balance and thus the complete Tai-Chi is accomplished. "HERE" father/mother, sky and earth, spirit and nature, pure consciousness and pure Beingness exist as a self supporting mutuality -- as embodied LOVE

In this non-dual/transpersonal world of an all inclusive Brahman, divine Shakti is seen in the tree, the river, the flowers, the stars, the ocean, and even in the human animal. It is beyond objectification or elaboration (nisprapanca/Nirvikalpa). "It" as such cannot be known or owned. The only place it is not seen is in that which is unreal, make believe, symbolic -- that which is illusory, false, or what is considered to exist in isolation in the frozen and deluded corrupted fragmented sphere of the egoic mindset (limitation and individual "self hood"). This latter sphere (the delusional reality of the ego) only

exists in the deluded mind, so it is not considered real or true (although it colors "reality" for those who are deluded in superficial appearances).

"Neti-neti" does not mean that we are not this body, only that we are more than this body -- We are the Long Body -- vast Being, the Big Universal Unbiased Boundless Primordial Mind in which all things see from vast space and vast time as a synchronistic and holographic simultaneity are interdependent, interconnected, and mutually co-emerging. Here everything is alive and vivid.

Things, objects, or phenomena are not interpreted through only the five or six sense organs, but rather through the universal MIND. The human animal, being a vital part of creation, it is up to humans to claim their rightful place in total harmony with the evolutionary force and no longer repress/deny it. This human body is not separate from this great body of timeless being -- in context of the larger body of an all inclusive interpretation of Brahman where we are all related. While in this unbiased and unlimited "Reality", no separate "i" exists. This "reality" is neither exclusively eternal. Nor exclusively temporal, rather it is both/and (neti/neti). It is the result of the marriage of crown and root, sahasrara and muladhara, pingala and ida, Shiva/Shakti, spirit/nature, objective/subjective, consciousness/Beingness. Such is truly non-dual and synergistically synchronistic. It cannot be realized through while remaining bound by the limitations of logic or analytical thought, but rather it comes from a more inclusive intelligent direct Connection/Union.

Indeed we would be misled to identify with just the physical body, the sense world, or the process of perception out of context with how the body arrived here or the origin of thoughts or the process of perception arise as separate from the process of creation/evolution or primordial wisdom-- who and what "it" really is -- we would be lost in an illusion and ignorant limitation, false identification subject to narcissism, delusional self aggrandizement of the illusion of separate self (ego), and all the rest of the many demons of ego delusion born of unawareness such as self pride, nationalism, racism, fame, status, envy, hatred, greed, lust, and the afflictions which stem from this false identification (the unfortunate common milieu). So within a primordial natural context, in light of "reality" and "truth", then ignorance is negated, the world of separateness is viewed not only an illusion, it doesn't really exist per se, i.e., it is empty of independent substantiality outside from the non-dual context of interdependence. Existence viewed from the fragmented context of apparently separate senses, separate sense objects, separate self/perceiver conspire toward a limited and biased view. It is "not" as it appears in terms of what is or what it is not. This "not self" or non-existence also does not exist. Both "not-self" and not "not-self" do not exist alone. Neither both, nor other. Nothing exists separate as "itself" except the universal heart/core -- LOVE.

No separate self

No independent "I"

Rather an interdependent, indivisible, inseparable WE

No I am that, but rather WE ARE THEE

The true nature of our mind, is beyond a stasis of a separate/independent mentation/intellectual conception or artifice

The true nature of phenomena is thus revealed within the appearance of a body as well as outside of the appearance of the body,

In all phenomena as co-participants of all pervading omnipresent omniscience

WE ARE THEE -- Om Tat Sat- Tat Tvam Asi

Neither self, nor not self, as true Self is plural and unlimited, as our true nature, no one can own/possess it (albeit many have tried).

This is the true meaning of "neti neti" -- the end of spiritual self-alienation. The process then is in arriving home to indigenous all encompassing space. When we arrive, we wake up to see that we are

profoundly present -- we become at one with sacred presence.

Thus the expression, neti neti, in context means we are the great LONG BODY all together taken as a whole, the Great Integrity/completion, not separate/fragmented from "God's" infinite parts -- THAT is OUR TRUE NATURE residing in this transpersonal interdependent spiritual fullness and wonder -- this natural, profoundly intelligent, and wholesome expression of complete and continuous integration and synchronicity with all things past present and future. In the Heart then, neti - neti means not separate, not separate -- not apart -- and thus we put to rest limited self imposed false identifications, habits, and beliefs and thus the "reality" of yoga -- of a world inextricably bound together in its continuity is realized. HERE Self acknowledges Self -- eternal Love recognizes eternal love -- we express the love which is our hearts. It is from this core Beingness -- this abiding in the true self nature of the natural unconditioned universal mind -- this as pure core/heart center -- where we learn to respect, affirm, and acknowledge in All Our Relations -- serving as the fount of great integral Beingness -- that we increasingly draw our sustenance, sense of balance, sense of centeredness, peace, security, inspiration, and eventually identity from -- from the HRIDAYAM which is the HEART OF HEARTS where the omnipresent teaching of the always present teacher (as eternal presence) intimately reveals in self liberatory revelation. HERE we cease to be betrayed, abandoned, ignored, and afraid as we cease to betray, abandon, ignore, and fear. HERE neurotic compensatory desire ceases -- the vrttis are annihilated (nirodha) and we rest in our true nature (swarupa).

Just this, just this

Just that, just that

Nothing else, all included

Empty of separate self

As-it-is, always

When the veil is lifted

When awakened

The ineffable great ground of pure Beingness and consciousness, disclosed through the removal of falsehood (neti neti) reveals our true and essential nature (eternal, uncontrived, natural, innate, and "self" abiding) -- from which all are related -- the spark behind the flame and the flame are one, not two) -- the sparkling spirit which animates and breathes life into the world at each and every moment unending. Even HERE- all-ways/always HERE. WE are that great wonder taken all together-- Om Tat Sat!

Tat Tvam Asi -- That Thou art -- WE are THEE -- Boundless Great Integrity -- Pure Love in its natural expression -- The reality where all relations of a separate "self" are known as an incomplete myopia (as illusory), where, the Reality of All Our Relations shines vibrant, naturally vivid, self illuminating, and bright ... and THAT is how it really is.

(1) Neti, Neti principle we write in mathematical form as follows (See Schrodinger's cat in Penrose's The Big, the Small and The Human Mind) Observer and the observed (Here we take both accentuation coefficient and dissipations coefficients as zero)

- (2)  $(\Psi) = -e_1 (\Psi) 1 - e_2 (\Psi) 1 - \dots -en (\Psi) n$
- (3)  $(\Psi) = +e_1 (\Psi) 1 + e_2 (\Psi) 1 - \dots (-) + -en (\Psi) n$
- (4)  $(\Psi) = +e_1 (\Psi) 1 - e_2 (\Psi) 1 - \dots + (-) -en (\Psi) n$
- (5)  $(\Psi) = -e_1 (\Psi) 1 + e_2 (\Psi) 1 - \dots -en (\Psi) n$

(6) Svabhava and space time world lines

(7) Consciousness (Storage of all information and visual representations with mind acting upon it anagrammatically. Note we eschew using words "being" and "becoming" We are infact beings with sense of consciousness and individual consciousness always tries to do "high jump" like body wants to. This is an evolutionary process. Levithanish literature has done more damage to the subject than addressing quintessential. We leave metaphorical and take the categorical and classification.)

(8) In quantum mechanics, the Schrödinger equation is an equation that describes how the quantum state of a physical system changes with time. It was formulated in late 1925, and published in 1926, by the Austrian physicist Erwin Schrödinger.

**Quantum Field Theory, Statistical Mechanics (We Are Talking Of Systems To Which Theory Is Applicable And Holds. Bank Example Of Assets Being Equivalent To Liabilities And That Each Individual Debts And Credits Being Conservative In Addition To The Conservativeness Of Holistic System Is To Be Mentioned), Markov Field And Hilbert Space:**

**Module Numbered One**

**NOTATION :**

$G_{13}$  : Category One Of Quantum Field Theory And Statistical; Mechanics

$G_{14}$  : Category Two Of Quantum Field Theory And Statistical Mechanics

$G_{15}$  : Category Three Of Quantum Field Theory And Statistical Mechanics

$T_{13}$  : Category One Of Yang Baxter Equations, Quantum Groups, Bose Fermion Equivalence Rational Conformal Field Theory (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$T_{14}$  : Category Two Of Yang Baxter Equations, Quantum Groups, Bose Fermion Equivalence Rational Conformal Field Theory (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

$T_{15}$  : Category Three Of Yang Baxter Equations, Quantum Groups, Bose Fermion Equivalence Rational Conformal Field Theory (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned)

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$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

**Lhs Of Schrödinger's Equation And Rhs Of Schrodinger's Equations (Negative Sign Implies That The Amount Is Extracted From Lhs. It Can Happen Instantaneously Or Immediately Without Time Lag. An Example of Distillation To Make Curds Out Of Milk And Addition Of Water To Mild Can Be Given .This Is Most Important Aspect In Our Further Analysis)**

**Module Numbered Two:**

$G_{16}$  : category one of lhs (note here that we are talking of the individual systems to which the Schrödinger's equation is applied and the classification is based on the features and characteristics of the systems and parametrization of systems which attribute and ascribe the formulation of classification and stratification of the systems)

$G_{17}$  : category two of lhs (note here that we are talking of the individual systems to which the Schrödinger's equation is applied and the classification is based on the features and characteristics of the systems and parametrization of systems which attribute and ascribe the formulation of classification and stratification of the systems)

$G_{18}$  : category three of LHS (note here that we are talking of the individual systems to which the Schrödinger's equation is applied and the classification is based on the features and characteristics of



the systems and parametrization of systems which attribute and ascribe the formulation of classification and stratification of the systems

$T_{16}$  :category one of LHS(note here that we are talking of the individual systems to which the Schrödinger's equation is applied and the classification is based on the features and characteristics of the systems and parametrization of systems which attribute and ascribe the formulation of classification and stratification of the systems

$T_{17}$  : category two of LHS(note here that we are talking of the individual systems to which the Schrödinger's equation is applied and the classification is based on the features and characteristics of the systems and parametrization of systems which attributes and ascribes the formulation of classification and stratification of the systems

$T_{18}$  : category three of LHS(note here that we are talking of the individual systems to which the Schrödinger's equation is applied and the classification is based on the features and characteristics of the systems and parametrization of systems which attribute and ascribe the formulation of classification and stratification of the systems translations in space and time

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t)$$

**Module Numbered Three:**

$G_{20}$  : Category one of LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and Parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, and is suitable for further classifications and organsiationalised generalized theories and their concomitant terms.

$G_{21}$  : CATEGORY TWO OF LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametrization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit

$G_{22}$  : CATEGORY THREE OF LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametrization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, and is suitable for further classifications and organsiationalised generalized theories and their concomitant terms

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$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t)$$

## Second Term Of The Rhs And First Term Of Rhs Of The Time Dependent Schrodinger's Equation

### : Module Numbered Four:

$G_{24}$  : CATEGORY ONE of LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, andis suitable for further classifications and organsiationalised generalized theories and their concomitant terms

$G_{25}$  : CATEGORY two of second term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatrational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term

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$G_{26}$  : CATEGORY THREE of second term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, and is suitable for further classifications and organizationalised generalized theories and their concomitant terms

$T_{24}$  : CATEGORY one of the first term on RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, and is suitable for further classifications and organizationalised generalized theories and their concomitant terms

$T_{25}$  : CATEGORY two of the first term on the RHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, and is suitable for further classifications and organizationalised generalized theories and their concomitant terms

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$$\underline{E\Psi = \hat{H}\Psi}$$

## Term On Lhs And Term On Rhs:

### Module Numbered Five:

$G_{28}$  : CATEGORY ONE OF THE TERM ON LHS of the time in dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, andis suitable for further classifications and organsiatinalised generalized theories and their concomitant terms

$G_{29}$  : CATEGORY two of the term on LHS of the time independent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, andis suitable for further classifications and organsiatinalised generalized theories and their concomitant terms

$G_{30}$  : CATEGORY three of term on LHS of the time dependent Schrödinger wave equation which is applicable to various systems that attributed and ascribed to its characteristics, features, and parametricization is classificatory, stratificatory and sequatratational. Dissipation coefficients and accentuation coefficients range from system to system. We in this category take only two terms lhs and the first term on the RHS. Next formulation concatenates the remaining term with that of the first term of the RHS. Obtention of the result for the individual term form the model is self explicit. When there is addition of say Milk to water it is done with some time lag or probably it could also be assumed at Planck's scale that once out in one shot. In the case of subtraction, it is like the process of distillation of slowly removing water from milk to make it curd. This analogy holds at the Planck's scale also, andis suitable for further classifications and organsiatinalised generalized theories and their concomitant terms

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The four (and more) negations

- 1) the negation of solid independent existence
- 2) the negation of the negation of existence
- 3) the negation of both existence and non-existence
- 4) the negation of the negation of both existence and non-existence

### **Svabhava And Space Time World Lines:**

#### **Module Numbered Six:**

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$G_{32}$  : CATEGORY ONE OF SVABHAVA (Needless to emphasise that different individuals have different predilections, proclivities, pen chance and propensities and they can be classified based on those characteristics and parametricization. In consideration to the fact that rajas and Tamás had been taken in to consideration to account in an earlier paper we eschew them in to consideration in this account.

$G_{33}$  : Category Two Of Svabhava

$G_{34}$  : Category Three Of svabhava

$T_{32}$  : Category One Of Space And Time World Lines

$T_{33}$  : Category Two Of space And Time World Lines

$T_{34}$  : Category Three Of Space And Time World Lines

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**Simulations And Dissimulations By Brahman And Antibrahman Agency (The Stimulus Providers To The Response Of The Human Beings) And Concomitant Misconception Of It To Be Real By The Human Being (If Two Persons Walk With You When You Go Out, It Is Accident. If Twenty People Walk With You Every Day And Every Time You Go Out It Is Coordination, A "Design" Created By The Agency Of Brahman-Antibrahman Category. Be Very Clear About The**



**Association Is Nothing But What Has Been Taught To You By Reference Groups And In The Eventuality Of The “Association” Say Green” For Good And “Red” For Danger Continues To Appear For An, In Respect To An Event Or A Subject Matter, Mind Acts Upon That And Produces Various Permutations And Combinations. That Is Why People Sometimes Say Just Like The Events Are Simulated, In The World By Ourselves, Like Thinking About The Past And Crying Over It, Or Thinking About The Future And Worrying About It, Brahman, The Simulatory Agency, A Giant Computer If You Like, Manipulates The Fundamental Forces ,”Energy” Of The Universe To Create An Illusory Space Time)**

**Module Numbered Seven)**

$G_{36}$  : Category One Of Simulation And Dissimulation by Brahman And Antibrahman

$G_{37}$  : category two of simulation and dissimulation by AntiBrahman

$G_{36}$  : category three of simulation and dissimulation by brahman AntiBrahman(note that we have defined brahman and AntiBrahman as simulation dissimulation agencies and these are classified based on the people such simulations are acting upon)

$T_{36}$  : Category One Of Prakruti Kshobha(Broadly Deprivation) Or Prakruti Siddhi(Broadly Gratification) (Note We Are Talking Of Individuals Who Are Affected Who Are Spoken About)

$T_{37}$  : Category Two Of prakruti Kshobha And Prakruti Siddhi

$T_{38}$  : Category Three Of Prakruti Kshobha And Prakruti Siddhi

**Representation Of Neti Neti(Case One) By Schrodinger’s Wave Equation. Look At Penrose On Schrödinger’s Cat In The Big The Small And The Human Mind**

$$(\Psi) = - e_1 (\Psi)_1 - e_2 (\Psi)_1 \dots \dots \dots e_n (\Psi)_n$$

**Representation Of Neti Neti In Schrodinger’s Equation First Term On The Above Equation When Added To Other Terms On The Rhs Leads To The Same Situation Of Addition Of Milk To Water. Here We Shall Not Take The Terms Separately But Assume That They Are Added To The Lhs On One Shot Basis.**

**Module Numbered Eight**

$G_{40}$ : CATEGORY ONE OF TERM ON LHS

$G_{41}$ : CATEGORY TWO OF TERM ON LHS

$G_{42}$ : CATEGORY TWO OF TERM ON LHS

$T_{40}$ : CATEGORY ONE OF TERM ON RHS

$T_{41}$ :CATEGORY TWO OF TERM S IN RHS

$T_{42}$ : CATEGORY THREE OF TERMS IN RHS.

$$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)} (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$$

$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}; (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$   
 $(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$   
 $(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$   
 are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$   
 $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$   
 $(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$   
 $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$   
 are Dissipation coefficients

**Quantum Field Theory And Statistical Mechanics (We Are Talking Of Systems To Which Theory Is Applicable And Holds. Bank Example Of Assets Being Equivalent To Liabilities And That Each Individual Debits And Credits Being Conservative In Addition To The Conservativeness Of Holistic System Is To Be Mentioned), Markov Field And Hilbert Space:**

**Module Numbered One**

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \tag{1}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \tag{2}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \tag{3}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \tag{4}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \tag{5}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \tag{6}$$

$+(a''_{13})^{(1)}(T_{14}, t) =$  First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$  First detritions factor

$$\underline{i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t)} \tag{7}$$

**Lhs Of Schrödinger's Equation And Rhs Of Schrodinger's Equations (Negative Sign Implies That The Amount Is Extracted From Lhs. It Can Happen Instantaneously Or Immediately Without Time Lag. An Example of Distillation To Make Curds Out Of Milk And Addition Of Water To Mild Can Be Given .This Is Most Important Aspect In Our Further Analysis)**

**Module Numbered Two:**

The differential system of this model is now ( Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \tag{8}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 9$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 10$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 11$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 12$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 13$$

$+(a''_{16})^{(2)}(T_{17}, t) =$  First augmentation factor

$-(b''_{16})^{(2)}((G_{19}), t) =$  First detritions factor

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$$\underline{i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t)}$$

### Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 15$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 16$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 17$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 18$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 19$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 20$$

$+(a''_{20})^{(3)}(T_{21}, t) =$  First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$  First detritions factor

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$$\underline{i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t)}$$

### Second Term Of The Rhs And First Term Of Rhs Of The Time Dependent Schrodinger's Equation

### Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 22$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 23$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 24$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} \quad 25$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} \quad 26$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} \quad 27$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor}$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor}$$

$$\underline{E\Psi = \hat{H}\Psi} \quad 28$$

### Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 29$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 30$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 31$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 32$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 33$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 34$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}$$

### Svabhava And Space Time World Lines:

#### Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 35$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 36$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 37$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 38$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 39$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 40$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor}$$

**Simulations And Dissimulations By Brahman And Antibrahman Agency (The Stimulus Providers To The Response Of The Human Beings) And Concomitant Misconception Of It To Be Real By The Human Being (If Two Persons Walk With You When You Go Out, It Is Accident. If Twenty People Walk With You Every Day And Every Time You Go Out It Is Coordination, A “Design” Created By The Agency Of Brahman-Antibrahman Category. Be Very Clear About The Association Is Nothing But What Has Been Taught To You By Reference Groups And In The Eventuality Of The “Association” Say Green” For Good And “Red” For Danger Continues To Appear For An, In Respect To An Event Or A Subject Matter, Mind Acts Upon That And Produces Various Permutations And Combinations. That Is Why People Sometimes Say Just Like The Events Are Simulated, In The World By Ourselves, Like Thinking About The Past And Crying Over It, Or Thinking About The Future And Worrying About It, Brahman Manipulates The Fundamental Forces ,”Energy” Of The Universe To Create An Illusory Space Time)**

**Module Numbered Seven:**

The differential system of this model is now (SEVENTH MODULE)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)]G_{36} \quad 41$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)]G_{37} \quad 42$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)]G_{38} \quad 43$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)]T_{36} \quad 44$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)]T_{37} \quad 45$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)]T_{38} \quad 46$$

$$+(a''_{36})^{(7)}(T_{37}, t) = \text{First augmentation factor}$$

Representation Of Neti Neti (Case One) By Schrodinger’s Wave Equation. Look At Penrose On Schrödinger’s Cat In The Big The Small And The Human Mind

$$\Psi = - e_1 (\Psi)_1 - e_2 (\Psi)_1 \dots \dots \dots e_n (\Psi)_n$$

**Representation Of Neti Neti In Schrodinger’s Equation First Term On The Above Equation When Added To Other Terms On The Rhs Lead To The Same Situation Of Addition Of Milk To Water. Here We Shall Not Take The Terms Separately But Assume That They Are Added To The Lhs On One Shot Basis.**

**Module Numbered Eight**

**GOVERNING EQUATIONS:**



The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - [(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)]G_{40} \quad 47$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - [(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)]G_{41} \quad 48$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - [(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)]G_{42} \quad 49$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)]T_{40} \quad 50$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)]T_{41} \quad 51$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)]T_{42} \quad 52$$

$$-(b''_{36})^{(7)}((G_{39}), t) = \text{First detritions factor} \quad 53$$

**First Module Concatenation:**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ \begin{array}{c} (a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7)}(T_{37}, t)} \boxed{+(a''_{40})^{(8)}(T_{41}, t)} \end{array} \right] G_{13} \quad 54$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ \begin{array}{c} (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7)}(T_{37}, t)} \boxed{+(a''_{41})^{(8)}(T_{41}, t)} \end{array} \right] G_{14} \quad 55$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ \begin{array}{c} (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7)}(T_{37}, t)} \boxed{+(a''_{42})^{(8)}(T_{41}, t)} \end{array} \right] G_{15} \quad 56$$

Where  $\boxed{(a'_{13})^{(1)}(T_{14}, t)}$ ,  $\boxed{(a'_{14})^{(1)}(T_{14}, t)}$ ,  $\boxed{(a'_{15})^{(1)}(T_{14}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$  are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3)}(T_{21}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)}$  are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)}$  are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)}$  are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{37})^{(7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{38})^{(7)}(T_{37}, t)}$  Are seventh augmentation Coefficients

$\boxed{+(a''_{40})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{42})^{(8)}(T_{41}, t)}$  Are Eighth Augmentation Coefficients

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[ \begin{array}{c} (b'_{13})^{(1)} \boxed{-(b''_{16})^{(1)}(G, t)} \boxed{-(b''_{36})^{(7,)}(G_{39}, t)} \boxed{-(b''_{20})^{(3,3,)}(G_{23}, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7,)}(G_{39}, t)} \boxed{-(b''_{40})^{(8,)}(G_{43}, t)} \end{array} \right] T_{13} \quad 57$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[ \begin{array}{c} (b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2,)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3,)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7,)}(G_{39}, t)} \boxed{-(b''_{41})^{(8,)}(G_{43}, t)} \end{array} \right] T_{14} \quad 58$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ \begin{array}{c} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2,)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3,)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27}, t)} \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31}, t)} \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7,)}(G_{39}, t)} \boxed{-(b''_{42})^{(8,)}(G_{41}, t)} \end{array} \right] T_{15} \quad 59$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3,)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3,)}(G_{23}, t)}$  are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,)}(G_{39}, t)}$ ,  $\boxed{-(b''_{36})^{(7,)}(G_{39}, t)}$ ,  $\boxed{-(b''_{36})^{(7,)}(G_{39}, t)}$  Are Seventh Detrition Coefficients

$\boxed{-(b''_{40})^{(8,)}(G_{41}, t)}$ ,  $\boxed{-(b''_{41})^{(8,)}(G_{41}, t)}$ ,  $\boxed{-(b''_{42})^{(8,)}(G_{41}, t)}$  Are Eighth Detrition coefficients

### Second Module Concatenation:

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ \begin{array}{c} (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1,)}(T_{14}, t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7,)}(T_{37}, t)} \boxed{+(a''_{40})^{(8,8)}(T_{41}, t)} \end{array} \right] \quad 60$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ \begin{array}{c} (a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1,)}(T_{14}, t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7,7,)}(T_{37}, t)} \boxed{+(a''_{41})^{(8,8)}(T_{41}, t)} \end{array} \right] G_{17} \quad 61$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ \begin{array}{c} (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1,)}(T_{14}, t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7,7,)}(T_{37}, t)} \boxed{+(a''_{42})^{(8,8)}(T_{41}, t)} \end{array} \right] G_{18} \quad 62$$

Where  $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,)}(T_{14}, t)}$  are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)}$  are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)}$  are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$  are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{37})^{(7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{38})^{(7,7)}(T_{37}, t)}$  Are Seventh Detrition Coefficients

$\boxed{+(a''_{40})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{42})^{(8)}(T_{41}, t)}$  Are Eight Augmentation Coefficients

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ \begin{array}{ccc} \boxed{(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t)} & \boxed{-(b'_{13})^{(1,1)}(G, t)} & \boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7,7)}(G_{39}, t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43}, t)} & \end{array} \right] T_{16} \quad 63$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ \begin{array}{ccc} \boxed{(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t)} & \boxed{-(b'_{14})^{(1,1)}(G, t)} & \boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7,7)}(G_{39}, t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43}, t)} & \end{array} \right] T_{17} \quad 64$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ \begin{array}{ccc} \boxed{(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t)} & \boxed{-(b'_{15})^{(1,1)}(G, t)} & \boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7,7)}(G_{39}, t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43}, t)} & \end{array} \right] T_{18} \quad 65$$

### Third Module Concatenation:

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[ \begin{array}{ccc} \boxed{(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)} & \boxed{+(a'_{16})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a'_{13})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41}, t)} & \end{array} \right] G_{20} \quad 66$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[ \begin{array}{ccc} \boxed{(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)} & \boxed{+(a'_{17})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a'_{14})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41}, t)} & \end{array} \right] G_{21} \quad 67$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[ \begin{array}{ccc} \boxed{(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)} & \boxed{+(a'_{18})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a'_{15})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41}, t)} & \end{array} \right] G_{22} \quad 68$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$  are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$  are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)}$  are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)}$  are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$  are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)}$  are seventh augmentation coefficient

$\boxed{+(a''_{40})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{42})^{(8)}(T_{41}, t)}$  are eighth augmentation coefficient

$$\frac{dT_{20}}{dt} = \left. \begin{matrix} (b_{20})^{(3)}T_{21} - \left[ \begin{matrix} \boxed{(b'_{20})^{(3)}} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \boxed{-(b'_{36})^{(7,7,7)}(G_{19}, t)} \boxed{-(b'_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b'_{24})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b'_{28})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b'_{32})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{36})^{(7,7,7)}(G_{39}, t)} \boxed{-(b'_{40})^{(8,8,8)}(G_{43}, t)} \boxed{-(b'_{40})^{(8,8,8)}(G_{43}, t)} \end{matrix} \right] \end{matrix} \right\} T_{20} \quad 69$$

$$\frac{dT_{21}}{dt} = \left. \begin{matrix} (b_{21})^{(3)}T_{20} - \left[ \begin{matrix} \boxed{(b'_{21})^{(3)}} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)} \boxed{-(b'_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b'_{25})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b'_{29})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b'_{33})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{37})^{(7,7,7)}(G_{39}, t)} \boxed{-(b'_{40})^{(8,8,8)}(G_{43}, t)} \end{matrix} \right] \end{matrix} \right\} T_{21} \quad 70$$

$$\frac{dT_{22}}{dt} = \left. \begin{matrix} (b_{22})^{(3)}T_{21} - \left[ \begin{matrix} \boxed{(b'_{22})^{(3)}} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} \boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)} \boxed{-(b'_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b'_{26})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b'_{30})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b'_{34})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{38})^{(7,7,7)}(G_{39}, t)} \boxed{-(b'_{40})^{(8,8,8)}(G_{43}, t)} \end{matrix} \right] \end{matrix} \right\} T_{22} \quad 71$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$  are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1)}(G, t)}$  are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[ \begin{array}{c} (a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)} \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{24} \quad 72$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[ \begin{array}{c} (a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37}, t)} \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{25} \quad 73$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[ \begin{array}{c} (a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37}, t)} \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{26} \quad 74$$

Where  $\boxed{(a''_{24})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a''_{25})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a''_{26})^{(4)}(T_{25}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5)}(T_{29}, t)}$  are second augmentation coefficient for category 1, 2 and

$\boxed{+(a''_{32})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6)}(T_{33}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37}, t)}$  are Seventh Augmentation Coefficients

$\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41}, t)}$  are Eighth Augmentation Coefficients

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ \begin{array}{c} (b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39}, t)} \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{24} \quad 75$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ \begin{array}{c} (b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39}, t)} \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{25} \quad 76$$

Where  $\boxed{-(b''_{24})^{(4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4)}(G_{27}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5)}(G_{31}, t)}$  are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6)}(G_{35}, t)}$  are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}$



are fourth detrition coefficients for category 1, 2 and 3

$$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$$

are fifth detrition coefficients for category 1, 2 and 3

$$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$$

are sixth detrition coefficients for category 1, 2 and 3

$$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39}, t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39}, t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39}, t)} \text{ ARE Seventh Detrition Coefficients}$$

$$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43}, t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43}, t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43}, t)} \text{ Are Eighth Detrition Coefficients}$$

**Fifth Module Concatenation:**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[ \begin{array}{l} \boxed{(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)} \quad \boxed{(a'_{24})^{(4,4)}(T_{25}, t)} \quad \boxed{(a'_{32})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{13})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{16})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{20})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{36})^{(7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{40})^{(8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{28} \quad 77$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[ \begin{array}{l} \boxed{(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)} \quad \boxed{(a'_{25})^{(4,4)}(T_{25}, t)} \quad \boxed{(a'_{33})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{14})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{17})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{21})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{37})^{(7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{41})^{(8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{29} \quad 78$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[ \begin{array}{l} \boxed{(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)} \quad \boxed{(a'_{26})^{(4,4)}(T_{25}, t)} \quad \boxed{(a'_{34})^{(6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{15})^{(1,1,1,1,1)}(T_{14}, t)} \quad \boxed{+(a'_{18})^{(2,2,2,2,2)}(T_{17}, t)} \quad \boxed{+(a'_{22})^{(3,3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{38})^{(7,7,7,7,7)}(T_{37}, t)} \quad \boxed{+(a'_{41})^{(8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{30} \quad 79$$

Where  $\boxed{+(a''_{28})^{(5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5)}(T_{29}, t)}$  are first augmentation coefficients for category 1, 2 and

And  $\boxed{+(a''_{24})^{(4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4)}(T_{25}, t)}$  are second augmentation coefficient for category 1, 2,

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6)}(T_{33}, t)}$  are third augmentation coefficient for category 1, 2 and

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41}, t)}$  Are Eighth Augmentation Coefficients

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[ \begin{array}{l} \boxed{(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}, t)} \quad \boxed{-(b''_{24})^{(4,4)}(G_{23}, t)} \quad \boxed{-(b''_{32})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{13})^{(1,1,1,1,1)}(G, t)} \quad \boxed{-(b'_{16})^{(2,2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b'_{20})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b'_{36})^{(7,7,7,7,7)}(G_{38}, t)} \end{array} \right] T_{28} \quad 80$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[ \begin{array}{l} \boxed{(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}, t)} \quad \boxed{-(b''_{25})^{(4,4)}(G_{27}, t)} \quad \boxed{-(b''_{33})^{(6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{14})^{(1,1,1,1,1)}(G, t)} \quad \boxed{-(b'_{17})^{(2,2,2,2,2)}(G_{19}, t)} \quad \boxed{-(b'_{21})^{(3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b'_{37})^{(7,7,7,7,7)}(G_{38}, t)} \end{array} \right] T_{29} \quad 81$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[ \begin{array}{ccc} (b'_{30})^{(5)} \boxed{-(b''_{30})^{(5)}(G_{31}, t)} & \boxed{-(b''_{26})^{(4,4)}(G_{27}, t)} & \boxed{-(b''_{34})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)} \\ & \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{38}, t)} & \end{array} \right] T_{30} \quad 82$$

**Sixth Module Concatenation**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[ \begin{array}{ccc} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33}, t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{24})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)} \\ & \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37}, t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{32} \quad 83$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[ \begin{array}{ccc} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33}, t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{25})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)} \\ & \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37}, t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{33} \quad 84$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[ \begin{array}{ccc} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33}, t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29}, t)} & \boxed{+(a''_{26})^{(4,4,4)}(T_{25}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)} \\ & \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37}, t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{34} \quad 85$$

$\boxed{+(a''_{32})^{(6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6)}(T_{33}, t)}$  are first augmentation coefficients for category 1, 2 and 3  
 $\boxed{+(a''_{28})^{(5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5,5)}(T_{29}, t)}$  are second augmentation coefficients for category 1, 2 and 3  
 $\boxed{+(a''_{24})^{(4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4,4)}(T_{25}, t)}$  are third augmentation coefficients for category 1, 2 and 3  
 $\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)}$  - are fourth augmentation coefficients  
 $\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)}$  - fifth augmentation coefficients  
 $\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)}$  sixth augmentation coefficients  
 $\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37}, t)}$  Are Seventh Augmentation Coefficients  
 $\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41}, t)}$  Are Eight Augmentation Coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[ \begin{array}{ccc} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35}, t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{24})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G, t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)} \\ & \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39}, t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{32} \quad 86$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} \tag{87}$$

$$- \left[ \begin{array}{ccc} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35}, t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{33}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} \tag{88}$$

$$- \left[ \begin{array}{ccc} (b'_{34})^{(6)} \boxed{-(b''_{34})^{(6)}(G_{35}, t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{34}$$

$\boxed{-(b''_{32})^{(6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6)}(G_{35}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)}$  are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)}$  are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)}$  are fourth detritions coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)}$  are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)}$  are sixth detritions coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$ ,  $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$ ,  $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$  ARE SEVENTH DETRITION COEFFICIENTS

$\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t)}$ ,  $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)}$ ,  $\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)}$  ARE EIGHTH DETRITION COEFFICIENTS.

**Seventh Module Concatenation:**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[ (a'_{36})^{(7)} \boxed{(a''_{36})^{(7)}(T_{37}, t)} + \boxed{(a'_{16})^{(7)}(T_{17}, t)} + \boxed{(a'_{20})^{(7)}(T_{21}, t)} + \right. \tag{89}$$

$$\boxed{(a'_{24})^{(7)}(T_{23}, t)} \boxed{G_{36}} + \boxed{(a'_{28})^{(7)}(T_{29}, t)} + \boxed{(a'_{32})^{(7)}(T_{33}, t)} +$$

$$\left. \boxed{(a'_{13})^{(7)}(T_{14}, t)} + \boxed{(a'_{40})^{(8,8,8,8,8,8)}(T_{41}, t)} \right] G_{36}$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[ (a'_{37})^{(7)} + \boxed{(a'_{37})^{(7)}(T_{37}, t)} + \boxed{(a'_{14})^{(7)}(T_{14}, t)} + \boxed{(a'_{21})^{(7)}(T_{21}, t)} + \right. \tag{90}$$

$$\boxed{(a'_{17})^{(7)}(T_{17}, t)} + \boxed{(a'_{25})^{(7)}(T_{25}, t)} + \boxed{(a'_{33})^{(7)}(T_{33}, t)} +$$

$$\left. \boxed{(a'_{29})^{(7)}(T_{29}, t)} + \boxed{(a'_{41})^{(8,8,8,8,8,8)}(T_{41}, t)} \right] G_{37}$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[ (a'_{38})^{(7)} + \boxed{(a'_{38})^{(7)}(T_{37}, t)} + \boxed{(a'_{15})^{(7)}(T_{14}, t)} + \boxed{(a'_{22})^{(7)}(T_{21}, t)} + \boxed{(a'_{18})^{(7)}(T_{17}, t)} + \right. \tag{91}$$

$$\boxed{(a'_{26})^{(7)}(T_{25}, t)} + \boxed{(a'_{34})^{(7)}(T_{33}, t)} + \boxed{(a'_{30})^{(7)}(T_{29}, t)} + \boxed{(a'_{42})^{(8,8,8,8,8,8)}(T_{41}, t)} \left. \right] G_{38}$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[ (b'_{36})^{(7)} - \boxed{(b''_{36})^{(7)}(G_{39}, t)} - \boxed{(b''_{16})^{(7)}(G_{19}, t)} - \boxed{(b''_{13})^{(7)}(G_{14}, t)} - \right. \tag{92}$$

$$\left[ \begin{array}{l} (b''_{20})^{(7)}((G_{231}), t) - (b''_{24})^{(7)}((G_{27}), t) - (b''_{28})^{(7)}((G_{31}), t) - \\ (b''_{32})^{(7)}((G_{35}), t) \end{array} \right] T_{36} \quad 93$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[ \begin{array}{l} (b'_{36})^{(7)} - (b''_{37})^{(7)}((G_{39}), t) - (b''_{17})^{(7)}((G_{19}), t) - (b''_{19})^{(7)}((G_{14}), t) - \\ (b''_{21})^{(7)}((G_{231}), t) - (b''_{25})^{(7)}((G_{27}), t) - (b''_{29})^{(7)}((G_{31}), t) - \\ (b''_{33})^{(7)}((G_{35}), t) \end{array} \right] T_{37}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[ \begin{array}{l} (b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t) - (b''_{18})^{(7)}((G_{19}), t) - (b''_{22})^{(7)}((G_{14}), t) - \\ (b''_{15})^{(7)}((G_{23}), t) - (b''_{26})^{(7)}((G_{27}), t) - (b''_{30})^{(7)}((G_{31}), t) - (b''_{34})^{(7)}((G_{35}), t) \end{array} \right] T_{38} \quad 94$$

**Eighth Module Concatenation:**

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[ \begin{array}{l} (a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t) + (a''_{13})^{(8)}(T_{14}, t) + (a''_{16})^{(8)}(T_{17}, t) + \\ (a''_{20})^{(8)}(T_{21}, t) + (a''_{24})^{(8)}(T_{23}, t) + (a''_{28})^{(8)}(T_{29}, t) + (a''_{32})^{(8)}(T_{33}, t) + \\ (a''_{36})^{(8)}(T_{37}, t) \end{array} \right] G_{40} \quad 95$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[ \begin{array}{l} (a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t) + (a''_{14})^{(8)}(T_{14}, t) + (a''_{17})^{(8)}(T_{17}, t) + \\ (a''_{21})^{(8)}(T_{21}, t) + (a''_{25})^{(8)}(T_{23}, t) + (a''_{29})^{(8)}(T_{29}, t) + (a''_{33})^{(8)}(T_{33}, t) + \\ (a''_{37})^{(8)}(T_{37}, t) \end{array} \right] G_{41} \quad 96$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[ \begin{array}{l} (a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t) + (a''_{15})^{(8)}(T_{14}, t) + (a''_{18})^{(8)}(T_{17}, t) + \\ (a''_{22})^{(8)}(T_{21}, t) + (a''_{26})^{(8)}(T_{23}, t) + (a''_{30})^{(8)}(T_{29}, t) + (a''_{34})^{(8)}(T_{33}, t) + \\ (a''_{38})^{(8)}(T_{37}, t) \end{array} \right] G_{42} \quad 97$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[ \begin{array}{l} (b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t) - (b''_{13})^{(8)}((G_{14}), t) - (b''_{16})^{(8)}((G_{19}), t) - \\ (b''_{20})^{(8)}((G_{23}), t) - (b''_{24})^{(8)}((G_{27}), t) - (b''_{28})^{(8)}((G_{31}), t) - (b''_{32})^{(8)}((G_{35}), t) - \\ (b''_{36})^{(8)}((G_{39}), t) \end{array} \right] T_{40} \quad 98$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[ \begin{array}{l} (b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t) - (b''_{14})^{(8)}((G_{14}), t) - (b''_{17})^{(8)}((G_{19}), t) - (b''_{21})^{(8)}((G_{23}), t) - \\ (b''_{25})^{(8)}((G_{27}), t) - (b''_{29})^{(8)}((G_{31}), t) - (b''_{33})^{(8)}((G_{35}), t) - (b''_{37})^{(8)}((G_{39}), t) \end{array} \right] T_{41} \quad 99$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[ \begin{array}{l} (b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t) - (b''_{15})^{(7)}((G_{14}), t) - (b''_{18})^{(8)}((G_{19}), t) - \end{array} \right] \quad 100$$

$$\left[ \begin{array}{c} (b''_{22})^{(8)}((G_{23}), t) \\ (b''_{38})^{(8)}((G_{39}), t) \end{array} \right] - \left[ \begin{array}{c} (b''_{26})^{(8)}((G_{27}), t) \\ (b''_{30})^{(8)}((G_{31}), t) \end{array} \right] - \left[ \begin{array}{c} (b''_{34})^{(8)}((G_{35}), t) \\ (b''_{38})^{(8)}((G_{39}), t) \end{array} \right] - \left[ \begin{array}{c} (b''_{22})^{(8)}((G_{23}), t) \\ (b''_{38})^{(8)}((G_{39}), t) \end{array} \right] \Big] T_{42}$$

$+(a''_{40})^{(8)}(T_{41}, t) =$  **First augmentation factor**

$-(b''_{40})^{(8)}((G_{43}), t) =$  **First detritions factor**

Where we suppose

(A)  $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$  101

$i, j = 13, 14, 15$

(B) The functions  $(a''_i)^{(1)}, (b''_i)^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \quad 102$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$  are positive constants and  $i = 13, 14, 15$

They satisfy Lipschitz condition: 103

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(1)}(T'_{14}, t)$  and  $(a''_i)^{(1)}(T_{14}, t) \cdot (T'_{14}, t)$  And  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that  $(a''_i)^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{13})^{(1)} = 1$  then the function  $(a''_i)^{(1)}(T_{14}, t)$ , the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 104

(C)  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 105

(D) There exists two constants  $(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  which together with  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$  and  $(\hat{B}_{13})^{(1)}$  and the constants



$$(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16,17,18 \quad 106$$

(E) The functions  $(a''_i)^{(2)}, (b''_i)^{(2)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(2)}, (r_i)^{(2)}$ : 107

$$(a''_i)^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 108$$

$$(b''_i)^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b'_i)^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 109$$

$$(F) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 110$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(2)}(G_{19}, t) = (r_i)^{(2)} \quad 111$$

**Definition of**  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 112

Where  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$  are positive constants and  $i = 16,17,18$

They satisfy Lipschitz condition:

$$|(a''_i)^{(2)}(T'_{17}, t) - (a''_i)^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T'_{17} - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 113$$

$$|(b''_i)^{(2)}((G_{19})', t) - (b''_i)^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G_{19})' - (G_{19})| e^{-(\hat{M}_{16})^{(2)}t} \quad 114$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(2)}(T'_{17}, t)$  and  $(a''_i)^{(2)}(T_{17}, t)$ .  $(T'_{17}, t)$  and  $(T_{17}, t)$  are points belonging to the interval  $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that  $(a''_i)^{(2)}(T_{17}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{16})^{(2)} = 1$  then the function  $(a''_i)^{(2)}(T_{17}, t)$ , the SECOND augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ : 115

$$(G) \quad (\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, \text{ are positive constants} \quad 116$$

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ : 117

There exists two constants  $(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  which together with  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$  and  $(\hat{B}_{16})^{(2)}$  and the constants  $(a_i)^{(2)}, (a'_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a'_i)^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 118$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [ (b_i)^{(2)} + (b'_i)^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)} ] < 1 \quad 119$$

Where we suppose

$$(H) \quad (a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (b''_i)^{(3)} > 0, \quad i, j = 20, 21, 22 \quad 120$$

The functions  $(a''_i)^{(3)}, (b''_i)^{(3)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(3)}, (r_i)^{(3)}$ :

$$(a''_i)^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b''_i)^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b'_i)^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 121$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of**  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$  :

Where  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$  are positive constants and  $i = 20, 21, 22$

They satisfy Lipschitz condition: 122

$$|(a''_i)^{(3)}(T'_{21}, t) - (a''_i)^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T'_{21}| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b''_i)^{(3)}(G'_{23}, t) - (b''_i)^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} |G_{23} - G'_{23}| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(3)}(T'_{21}, t)$  and  $(a''_i)^{(3)}(T_{21}, t) \cdot (T'_{21}, t)$  And  $(T_{21}, t)$  are points belonging to the interval  $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$ . It is to be noted that  $(a''_i)^{(3)}(T_{21}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{20})^{(3)} = 1$  then the function  $(a''_i)^{(3)}(T_{21}, t)$ , the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of**  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$  : 123

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ , are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants  $(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  which together with  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$  and  $(\hat{B}_{20})^{(3)}$  and the constants  $(a_i)^{(3)}, (a'_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$ , satisfy the inequalities 124

$$\frac{1}{(\hat{M}_{20})^{(3)}} [ (a_i)^{(3)} + (a'_i)^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)} ] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [ (b_i)^{(3)} + (b'_i)^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)} ] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a'_i)^{(4)}, (a''_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (b''_i)^{(4)} > 0, \quad i, j = 24, 25, 26 \quad 125$$

The functions  $(a''_i)^{(4)}, (b''_i)^{(4)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(4)}, (r_i)^{(4)}$ :

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \quad 126$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of**  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$  :

Where  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$  are positive constants and  $i = 24, 25, 26$

They satisfy Lipschitz condition: 127

$$|(a_i'')^{(4)}(T'_{25}, t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T'_{25} - T_{25}| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} |(G_{27})' - (G_{27})| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(4)}(T'_{25}, t)$  and  $(a_i'')^{(4)}(T_{25}, t) \cdot (T'_{25}, t)$  and  $(T_{25}, t)$  are points belonging to the interval  $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$ . It is to be noted that  $(a_i'')^{(4)}(T_{25}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{24})^{(4)} = 4$  then the function  $(a_i'')^{(4)}(T_{25}, t)$ , the **FOURTH augmentation coefficient WOULD** be absolutely continuous.

**Definition of**  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$  : 128

$(\hat{M}_{24})^{176^{175(4)}}, (\hat{k}_{24})^{(4)}$ , are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of**  $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$  : 129

There exists two constants  $(\hat{P}_{24})^{(4)}$  and  $(\hat{Q}_{24})^{(4)}$  which together with  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$  and  $(\hat{B}_{24})^{(4)}$  and the constants  $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28, 29, 30 \quad 130$$

The functions  $(a_i'')^{(5)}, (b_i'')^{(5)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(5)}, (r_i)^{(5)}$ :

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \quad 131$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of**  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$  :

Where  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$  are positive constants and  $i = 28, 29, 30$

They satisfy Lipschitz condition: 132

$$|(a_i'')^{(5)}(T'_{29}, t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T'_{29} - T_{29}| e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} |(G_{31})' - (G_{31})| e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(5)}(T'_{29}, t)$  and  $(a_i'')^{(5)}(T_{29}, t)$ .  $(T'_{29}, t)$  and  $(T_{29}, t)$  are points belonging to the interval  $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$ . It is to be noted that  $(a_i'')^{(5)}(T_{29}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{28})^{(5)} = 5$  then the function  $(a_i'')^{(5)}(T_{29}, t)$ , the FIFTH **augmentation coefficient** attributable would be absolutely continuous.

**Definition of**  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$  : 133

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ , are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} + \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of**  $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$  : 134

There exists two constants  $(\hat{P}_{28})^{(5)}$  and  $(\hat{Q}_{28})^{(5)}$  which together with  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$  and  $(\hat{B}_{28})^{(5)}$  and the constants  $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32, 33, 34 \quad 135$$

The functions  $(a_i'')^{(6)}, (b_i'')^{(6)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(6)}, (r_i)^{(6)}$ :

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)} \quad 136$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of**  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$  :

Where  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$  are positive constants and  $i = 32,33,34$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(6)}(T_{33}, t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35})' - (G_{35})| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(6)}(T_{33}, t)$  and  $(a_i'')^{(6)}(T_{33}, t) \cdot (T_{33}, t)$ . And  $(T_{33}, t)$  are points belonging to the interval  $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$ . It is to be noted that  $(a_i'')^{(6)}(T_{33}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{32})^{(6)} = 6$  then the function  $(a_i'')^{(6)}(T_{33}, t)$ , the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$  :

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$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ , are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of**  $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$  :

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There exists two constants  $(\hat{P}_{32})^{(6)}$  and  $(\hat{Q}_{32})^{(6)}$  which together with  $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$  and  $(\hat{B}_{32})^{(6)}$  and the constants  $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$$

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The functions  $(a_i'')^{(7)}, (b_i'')^{(7)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(7)}, (r_i)^{(7)}$ :

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}, \quad (b_i'')^{(7)}(G, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$$

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$$\lim_{G \rightarrow \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$$

**Definition of**  $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$  :

Where  $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}$  are positive constants

and  $i = 36,37,38$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37}' - T_{37}| e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), (T_{39}))| < (\hat{k}_{36})^{(7)} |(G_{39}) - (G_{39})'| e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(7)}(T_{37}', t)$  and  $(a_i'')^{(7)}(T_{37}, t) \cdot (T_{37}', t)$  and  $(T_{37}, t)$  are points belonging to the interval  $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that  $(a_i'')^{(7)}(T_{37}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{36})^{(7)} = 7$  then the function  $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$  : 143

(K)  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ , are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of**  $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$  : 144

There exists two constants  $(\hat{P}_{36})^{(7)}$  and  $(\hat{Q}_{36})^{(7)}$  which together with  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)}$  and  $(\hat{B}_{36})^{(7)}$  and the constants  $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36, 37, 38$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

**Definition of**  $G_i(0), T_i(0)$  : 145

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Where we suppose

(L)  $(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0$ , 146  
 $i, j = 40, 41, 42$

(M) The functions  $(a_i'')^{(8)}, (b_i'')^{(8)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(8)}, (r_i)^{(8)}$ :

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \quad 147$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \quad 148$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$

**Definition of**  $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$  :



Where  $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$  are positive constants and  $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition: 149

$$|(a_i'')^{(8)}(T_{41}, t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)}t}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), (T_{43}))| < (\hat{k}_{40})^{(8)} |(G_{43}) - (G_{43})'| e^{-(\hat{M}_{40})^{(8)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(8)}(T_{41}, t)$  and  $(a_i'')^{(8)}(T_{41}, t) \cdot (T_{41}, t)$ . And  $(T_{41}, t)$  are points belonging to the interval  $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$ . It is to be noted that  $(a_i'')^{(8)}(T_{41}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{40})^{(8)} = 8$  then the function  $(a_i'')^{(8)}(T_{41}, t)$ , the EIGHT **augmentation coefficient** would be absolutely continuous.

**Definition of**  $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$  : 150

(N)  $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ , are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t [(a_{15})^{(1)} G_{14}(s_{(13)}) - ((a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})) G_{15}(s_{(13)})] ds_{(13)}$$

By 151

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t [(a_{13})^{(1)} G_{14}(s_{(13)}) - ((a_{13}')^{(1)} + a_{13}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})) G_{13}(s_{(13)})] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t [(a_{14})^{(1)} G_{13}(s_{(13)}) - ((a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})) G_{14}(s_{(13)})] ds_{(13)} \quad 152$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t [(b_{13})^{(1)} T_{14}(s_{(13)}) - ((b_{13}')^{(1)} - (b_{13}'')^{(1)}(G(s_{(13)}), s_{(13)})) T_{13}(s_{(13)})] ds_{(13)} \quad 153$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t [(b_{14})^{(1)} T_{13}(s_{(13)}) - ((b_{14}')^{(1)} - (b_{14}'')^{(1)}(G(s_{(13)}), s_{(13)})) T_{14}(s_{(13)})] ds_{(13)} \quad 154$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t [(b_{15})^{(1)} T_{14}(s_{(13)}) - ((b_{15}')^{(1)} - (b_{15}'')^{(1)}(G(s_{(13)}), s_{(13)})) T_{15}(s_{(13)})] ds_{(13)} \quad 155$$

Where  $s_{(13)}$  is the integrand that is integrated over an interval  $(0, t)$

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions 156

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Consider operator  $\mathcal{A}^{(7)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{36})^{(7)}, T_i^0 \leq (\hat{Q}_{36})^{(7)}, \quad 157$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 158$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 159$$

By 160

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36}(s_{(36)})^{(7)} \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + a''_{37}(s_{(36)})^{(7)} \right) G_{37}(s_{(36)}) \right] ds_{(36)} \quad 161$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + a''_{38}(s_{(36)})^{(7)} \right) G_{38}(s_{(36)}) \right] ds_{(36)} \quad 162$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)} \quad 163$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)} \quad 164$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)} \quad 165$$

Where  $s_{(36)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(2)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \quad 166$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 167$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 168$$

By 169

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16}(s_{(16)})^{(2)} \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + a''_{17}(s_{(16)})^{(2)} \right) G_{17}(s_{(16)}) \right] ds_{(16)} \quad 170$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + a''_{18}(s_{(16)})^{(2)} \right) G_{18}(s_{(16)}) \right] ds_{(16)} \quad 171$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 172$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 173$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 174$$

Where  $s_{(16)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(3)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)}, \quad 175$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 176$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 177$$

By 178

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20}(s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)} \quad 179$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)} \quad 180$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)} \quad 181$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)} \quad 182$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 183$$

Where  $s_{(20)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(4)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)}, \quad 184$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 185$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 186$$

By 187

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24}(s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)} G_{24}(s_{(24)}) - \left( (a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \quad 188$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 189$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{24})^{(4)} - (b''_{24})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 190$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)} T_{24}(s_{(24)}) - \left( (b'_{25})^{(4)} - (b''_{25})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 191$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{26})^{(4)} - (b''_{26})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 192$$

Where  $s_{(24)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(5)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)}, \quad 193$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 194$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 195$$

By 196

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 197$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 198$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 199$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 200$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 201$$

Where  $s_{(28)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(6)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)}, \quad 202$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 203$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 204$$

By 205

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 206$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 207$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 208$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 209$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 210$$

Where  $s_{(32)}$  is the integrand that is integrated over an interval  $(0, t)$

if the conditions IN THE FOREGOING are fulfilled, there exists a solution satisfying the conditions 211

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:**

Consider operator  $\mathcal{A}^{(7)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{36})^{(7)}, T_i^0 \leq (\hat{Q}_{36})^{(7)}, \quad 212$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 213$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 214$$

By 215

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t [(a_{36})^{(7)} G_{37}(s_{(36)}) - ((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})] G_{36}(s_{(36)})] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t [(a_{37})^{(7)} G_{36}(s_{(36)}) - ((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)}))] G_{37}(s_{(36)})] ds_{(36)} \quad 216$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t [(a_{38})^{(7)} G_{37}(s_{(36)}) - ((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)}))] G_{38}(s_{(36)})] ds_{(36)} \quad 217$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t [(b_{36})^{(7)} T_{37}(s_{(36)}) - ((b'_{36})^{(7)} - (b''_{36})^{(7)}(G(s_{(36)}), s_{(36)}))] T_{36}(s_{(36)})] ds_{(36)} \quad 218$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t [(b_{37})^{(7)} T_{36}(s_{(36)}) - ((b'_{37})^{(7)} - (b''_{37})^{(7)}(G(s_{(36)}), s_{(36)}))] T_{37}(s_{(36)})] ds_{(36)} \quad 219$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t [(b_{38})^{(7)} T_{37}(s_{(36)}) - ((b'_{38})^{(7)} - (b''_{38})^{(7)}(G(s_{(36)}), s_{(36)}))] T_{38}(s_{(36)})] ds_{(36)} \quad 220$$

Where  $s_{(36)}$  is the integrand that is integrated over an interval  $(0, t)$

if the conditions above are fulfilled, there exists a solution satisfying the conditions 221

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:**

Consider operator  $\mathcal{A}^{(8)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{40})^{(8)}, T_i^0 \leq (\hat{Q}_{40})^{(8)}, \quad 222$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad 223$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad 224$$

By 225

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40}(s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)} \quad 226$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)} \quad 227$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)}(G(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)} \quad 228$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)}(G(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)} \quad 229$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)}(G(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)} \quad 230$$

Where  $s_{(40)}$  is the integrand that is integrated over an interval  $(0, t)$

The operator  $\mathcal{A}^{(8)}$  maps the space of functions satisfying global equations into itself. Indeed it is obvious that 231

$$G_{40}(t) \leq G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}s_{(40)}} \right) \right] ds_{(40)} =$$

$$\left( 1 + (a_{40})^{(8)}t \right) G_{41}^0 + \frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)}t} - 1 \right)$$

From which it follows that 232

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[ \left( (\hat{P}_{40})^{(8)} + G_{41}^0 \right) e^{\left( -\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1

In order that the operator  $\mathcal{A}^{(1)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying GLOBAL EQUATIONS into itself

The operator  $\mathcal{A}^{(1)}$  is a contraction with respect to the metric 233

$$d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)}t} \right\}$$

Indeed if we denote 234

**Definition of  $\tilde{G}, \tilde{T}$  :**  $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results



$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{(a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} |(a'_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a'_{13})^{(1)} (T_{14}^{(2)}, s_{(13)})| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)} \end{aligned}$$

Where  $s_{(13)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} &\leq & 235 \\ \frac{1}{(\widehat{M}_{13})^{(1)}} ((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})) \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a'_{13})^{(1)}$  and  $(b'_{13})^{(1)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  and  $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  respectively of  $\mathbb{R}_+$ . 236

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a'_i)^{(1)}$  and  $(b'_i)^{(1)}$ ,  $i = 13, 14, 15$  depend only on  $T_{14}$  and respectively on  $G$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  237

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a'_i)^{(1)} - (a''_i)^{(1)} (T_{14}(s_{(13)}), s_{(13)})) ds_{(13)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{13})^{(1)})_1$ , and  $((\widehat{M}_{13})^{(1)})_3$  : 238

**Remark 3:** if  $G_{13}$  is bounded, the same property have also  $G_{14}$  and  $G_{15}$ . indeed if

$$G_{13} < ((\widehat{M}_{13})^{(1)})_1 \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If  $G_{14}$  or  $G_{15}$  is bounded, the same property follows for  $G_{13}$ ,  $G_{15}$  and  $G_{13}$ ,  $G_{14}$  respectively.

**Remark 4:** If  $G_{13}$  is bounded, from below, the same property holds for  $G_{14}$  and  $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{14}$  is bounded from below. 239

**Remark 5:** If  $T_{13}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$  then  $T_{14} \rightarrow \infty$ . 240

**Definition of**  $(m)^{(1)}$  and  $\varepsilon_1$  :

Indeed let  $t_1$  be so that for  $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then  $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$  which leads to 241

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), t = \log \frac{2}{\varepsilon_1}$  By taking now  $\varepsilon_1$  sufficiently small one sees that  $T_{14}$  is unbounded. The same property holds for  $T_{15}$  if  $\lim_{t \rightarrow \infty} (b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take  $\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} < 1$  and to choose 242

$(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  large to have

$$\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ (\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\left( \frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{16})^{(2)} \quad 243$$

$$\frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ ((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)} \quad 244$$

In order that the operator  $\mathcal{A}^{(2)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying

The operator  $\mathcal{A}^{(2)}$  is a contraction with respect to the metric 245

$$d \left( (G_{19})^{(1)}, (T_{19})^{(1)}, (G_{19})^{(2)}, (T_{19})^{(2)} \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote 246

**Definition of**  $\widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 247

$$\begin{aligned} |\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\ &\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &G_{16}^{(2)} | (a'_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)} (T_{17}^{(2)}, s_{(16)}) | e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)} \end{aligned}$$

Where  $s_{(16)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\bar{M}_{16})^{(2)}t} \leq \quad 248$$

$$\frac{1}{(\widehat{M}_{16})^{(2)}} \left( (a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{K}_{16})^{(2)} \right) d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}) \right)$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a'_{16})^{(2)}$  and  $(b'_{16})^{(2)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$  and  $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$  respectively of  $\mathbb{R}_+$ . 249

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a'_i)^{(2)}$  and  $(b'_i)^{(2)}$ ,  $i = 16, 17, 18$  depend only on  $T_{17}$  and respectively on  $(G_{19})^{(1)}$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  250

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[ - \int_0^t \{ (a'_i)^{(2)} - (a''_i)^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \} ds_{(16)} \right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \quad \text{for } t > 0$$

**Definition of**  $((\widehat{M}_{16})^{(2)})_1$ ,  $((\widehat{M}_{16})^{(2)})_2$  and  $((\widehat{M}_{16})^{(2)})_3$  : 251

**Remark 3:** if  $G_{16}$  is bounded, the same property have also  $G_{17}$  and  $G_{18}$ . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)})_1 \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)} G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)} ((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)} ((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If  $G_{17}$  or  $G_{18}$  is bounded, the same property follows for  $G_{16}$ ,  $G_{18}$  and  $G_{16}$ ,  $G_{17}$  respectively.

**Remark 4:** If  $G_{16}$  is bounded, from below, the same property holds for  $G_{17}$  and  $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{17}$  is bounded from below. 252

**Remark 5:** If  $T_{16}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(2)} ((G_{19})^{(1)}(t), t)) = (b'_{17})^{(2)}$  then  $T_{17} \rightarrow \infty$ . 253

**Definition of**  $(m)^{(2)}$  and  $\varepsilon_2$  :

Indeed let  $t_2$  be so that for  $t > t_2$

$$(b_{17})^{(2)} - (b''_i)^{(2)} ((G_{19})^{(1)}(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then  $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)} (m)^{(2)} - \varepsilon_2 T_{17}$  which leads to 254

$$T_{17} \geq \left( \frac{(a_{17})^{(2)} (m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$T_{17} \geq \left( \frac{(a_{17})^{(2)} (m)^{(2)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_2}$  By taking now  $\varepsilon_2$  sufficiently small one sees that  $T_{17}$  is unbounded. 255

The same property holds for  $T_{18}$  if  $\lim_{t \rightarrow \infty} ((b'_{18})^{(2)} ((G_{19})^{(1)}(t), t)) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take  $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$  and to choose 256

$(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ (\hat{P}_{20})^{(3)} + ((\hat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{20})^{(3)} \quad 257$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ ((\hat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)} \right] \leq (\hat{Q}_{20})^{(3)} \quad 258$$

In order that the operator  $\mathcal{A}^{(3)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(3)}$  is a contraction with respect to the metric 259

$$d\left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t} \}$$

Indeed if we denote 260

**Definition of**  $\widetilde{G}_{23}, \widetilde{T}_{23} : ((\widetilde{G}_{23}), (\widetilde{T}_{23})) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 261

$$\begin{aligned} |\tilde{G}_{20}^{(1)} - \tilde{G}_{20}^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t ((a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{-(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} |(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)})| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned}$$

Where  $s_{(20)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\bar{M}_{20})^{(3)}t} &\leq \quad 262 \\ \frac{1}{(\bar{M}_{20})^{(3)}} &((a_{20})^{(3)} + (a'_{20})^{(3)} + (\hat{A}_{20})^{(3)} + \\ &(\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}) d\left( ((G_{23})^{(1)}, (T_{23})^{(1)}); ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{20})^{(3)}$  and  $(b''_{20})^{(3)}$  depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}t}$  and  $(\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}t}$  respectively of  $\mathbb{R}_+$ . 263

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(3)}$  and  $(b''_i)^{(3)}, i = 20, 21, 22$  depend only on  $T_{21}$  and respectively on  $(G_{23})$  (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  264

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(3)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{20})^{(3)})_1, ((\widehat{M}_{20})^{(3)})_2$  and  $((\widehat{M}_{20})^{(3)})_3$  : 265

**Remark 3:** if  $G_{20}$  is bounded, the same property have also  $G_{21}$  and  $G_{22}$  . indeed if

$$G_{20} < ((\widehat{M}_{20})^{(3)}) \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a_{21}')^{(3)}G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21}')^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22}')^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a_{22}')^{(3)}$$

If  $G_{21}$  or  $G_{22}$  is bounded, the same property follows for  $G_{20}$  ,  $G_{22}$  and  $G_{20}$  ,  $G_{21}$  respectively.

**Remark 4:** If  $G_{20}$  is bounded, from below, the same property holds for  $G_{21}$  and  $G_{22}$  . The proof is analogous with the preceding one. An analogous property is true if  $G_{21}$  is bounded from below. 266

**Remark 5:** If  $T_{20}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(3)}((G_{23})(t), t)) = (b_{21}')^{(3)}$  then  $T_{21} \rightarrow \infty$ . 267

**Definition of**  $(m)^{(3)}$  and  $\varepsilon_3$  :

Indeed let  $t_3$  be so that for  $t > t_3$

$$(b_{21}')^{(3)} - (b_i'')^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then  $\frac{dT_{21}}{dt} \geq (a_{21}')^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$  which leads to 268

$$T_{21} \geq \left(\frac{(a_{21}')^{(3)}(m)^{(3)}}{\varepsilon_3}\right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$T_{21} \geq \left(\frac{(a_{21}')^{(3)}(m)^{(3)}}{2}\right), t = \log \frac{2}{\varepsilon_3}$  By taking now  $\varepsilon_3$  sufficiently small one sees that  $T_{21}$  is unbounded. The same property holds for  $T_{22}$  if  $\lim_{t \rightarrow \infty} ((b_{22}'')^{(3)}((G_{23})(t), t)) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take  $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$  and to choose 269

$(\widehat{P}_{24})^{(4)}$  and  $(\widehat{Q}_{24})^{(4)}$  large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \left[ (\widehat{P}_{24})^{(4)} + ((\widehat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{24})^{(4)} \quad 270$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \left[ ((\widehat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{24})^{(4)} \right] \leq (\widehat{Q}_{24})^{(4)} \quad 271$$

In order that the operator  $\mathcal{A}^{(4)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying IN to itself

The operator  $\mathcal{A}^{(4)}$  is a contraction with respect to the metric 272

$$d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right), \left((G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t} \right\}$$

Indeed if we denote

**Definition of**  $(\widetilde{G_{27}}, \widetilde{T_{27}})$  :  $(\widetilde{G_{27}}, \widetilde{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_{24}^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} ds_{(24)} + \\ &\int_0^t (a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{-(\bar{M}_{24})^{(4)}s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} + \\ &G_{24}^{(2)} |(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)})| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} ds_{(24)} \end{aligned}$$

Where  $s_{(24)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\bar{M}_{24})^{(4)}t} &\leq \frac{1}{(\bar{M}_{24})^{(4)}} \left( (a_{24})^{(4)} + (a'_{24})^{(4)} + (\bar{A}_{24})^{(4)} + \right. \\ &\left. (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)} \right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right); \left((G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) \end{aligned} \quad 273$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed  $(a''_{24})^{(4)}$  and  $(b''_{24})^{(4)}$  depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)}t}$  and  $(\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)}t}$  respectively of  $\mathbb{R}_+$ . 274

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(4)}$  and  $(b''_i)^{(4)}$ ,  $i = 24, 25, 26$  depend only on  $T_{25}$  and respectively on  $(G_{27})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  275

From GLOBAL EQUATIONS it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(4)} - (a''_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(4)}t} > 0 \text{ for } t > 0 \end{aligned}$$



**Definition of**  $((\widehat{M}_{24})^{(4)})_1, ((\widehat{M}_{24})^{(4)})_2$  and  $((\widehat{M}_{24})^{(4)})_3$  : 276

**Remark 3:** if  $G_{24}$  is bounded, the same property have also  $G_{25}$  and  $G_{26}$  . indeed if

$G_{24} < ((\widehat{M}_{24})^{(4)})$  it follows  $\frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a'_{25})^{(4)}G_{25}$  and by integrating

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25})^{(4)}((\widehat{M}_{24})^{(4)})_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)}((\widehat{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If  $G_{25}$  or  $G_{26}$  is bounded, the same property follows for  $G_{24}$  ,  $G_{26}$  and  $G_{24}$  ,  $G_{25}$  respectively.

**Remark 4:** If  $G_{24}$  is bounded, from below, the same property holds for  $G_{25}$  and  $G_{26}$  . The proof is 277  
 analogous with the preceding one. An analogous property is true if  $G_{25}$  is bounded from below.

**Remark 5:** If  $T_{24}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$  then 278  
 $T_{25} \rightarrow \infty$ .

**Definition of**  $(m)^{(4)}$  and  $\varepsilon_4$  :

Indeed let  $t_4$  be so that for  $t > t_4$

$$(b_{25})^{(4)} - (b'_i)^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then  $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$  which leads to 279

$$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$$

If we take  $t$  such that  $e^{-\varepsilon_4 t} = \frac{1}{2}$  it results

$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_4}$  By taking now  $\varepsilon_4$  sufficiently small one sees that  $T_{25}$  is unbounded. The same property holds for  $T_{26}$  if  $\lim_{t \rightarrow \infty} ((b'_{26})^{(4)}((G_{27})(t), t)) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for  $G_{29}$  ,  $G_{30}$ ,  $T_{28}$ ,  $T_{29}$ ,  $T_{30}$

It is now sufficient to take  $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$  ,  $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$  and to choose 280

$(\widehat{P}_{28})^{(5)}$  and  $(\widehat{Q}_{28})^{(5)}$  large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} \left[ (\widehat{P}_{28})^{(5)} + ((\widehat{P}_{28})^{(5)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{28})^{(5)}$$
 281

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} \left[ ((\widehat{Q}_{28})^{(5)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{28})^{(5)} \right] \leq (\widehat{Q}_{28})^{(5)}$$
 282

In order that the operator  $\mathcal{A}^{(5)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(5)}$  is a contraction with respect to the metric 283

$$d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}\right), \left((G_{31})^{(2)}, (T_{31})^{(2)}\right)\right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t} \right\}$$

Indeed if we denote

**Definition of**  $(\widetilde{G}_{31}), (\widetilde{T}_{31}) : ((\widetilde{G}_{31}), (\widetilde{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$\begin{aligned} |\tilde{G}_{28}^{(1)} - \tilde{G}_{28}^{(2)}| &\leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \\ &\int_0^t \{(a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{-(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &G_{28}^{(2)} |(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)})| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}}\} ds_{(28)} \end{aligned}$$

Where  $s_{(28)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\bar{M}_{28})^{(5)}t} &\leq & 284 \\ \frac{1}{(\bar{M}_{28})^{(5)}} \left( (a_{28})^{(5)} + (a'_{28})^{(5)} + (\hat{A}_{28})^{(5)} + \right. \\ & \left. (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)} \right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}\right); (G_{31})^{(2)}, (T_{31})^{(2)}\right) \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (35,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{28})^{(5)}$  and  $(b''_{28})^{(5)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}t}$  and  $(\hat{Q}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}t}$  respectively of  $\mathbb{R}_+$ . 285

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(5)}$  and  $(b''_i)^{(5)}$ ,  $i = 28, 29, 30$  depend only on  $T_{29}$  and respectively on  $(G_{31})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  286

From GLOBAL EQUATIONS it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(5)}t} > 0 \text{ for } t > 0 \end{aligned}$$

**Definition of**  $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$  and  $((\widehat{M}_{28})^{(5)})_3 :$  287

**Remark 3:** if  $G_{28}$  is bounded, the same property have also  $G_{29}$  and  $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$  it follows  $\frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a'_{29})^{(5)}G_{29}$  and by integrating

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)}((\widehat{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)}((\widehat{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If  $G_{29}$  or  $G_{30}$  is bounded, the same property follows for  $G_{28}$ ,  $G_{30}$  and  $G_{28}$ ,  $G_{29}$  respectively.

**Remark 4:** If  $G_{28}$  is bounded, from below, the same property holds for  $G_{29}$  and  $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{29}$  is bounded from below. 288

**Remark 5:** If  $T_{28}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$  then  $T_{29} \rightarrow \infty$ . 289

**Definition of**  $(m)^{(5)}$  and  $\varepsilon_5$  :

Indeed let  $t_5$  be so that for  $t > t_5$

$$(b_{29})^{(5)} - (b'_i)^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then  $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$  which leads to 290

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$$

If we take  $t$  such that  $e^{-\varepsilon_5 t} = \frac{1}{2}$  it results

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5}$$

By taking now  $\varepsilon_5$  sufficiently small one sees that  $T_{29}$  is unbounded. The same property holds for  $T_{30}$  if  $\lim_{t \rightarrow \infty} ((b'_{30})^{(5)}((G_{31})(t), t)) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for  $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take  $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$  and to choose 291

$(\widehat{P}_{32})^{(6)}$  and  $(\widehat{Q}_{32})^{(6)}$  large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \left[ (\widehat{P}_{32})^{(6)} + ((\widehat{P}_{32})^{(6)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{32})^{(6)} \quad 292$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \left[ ((\widehat{Q}_{32})^{(6)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{32})^{(6)} \right] \leq (\widehat{Q}_{32})^{(6)} \quad 293$$

In order that the operator  $\mathcal{A}^{(6)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(6)}$  is a contraction with respect to the metric 294

$$d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \}$$

Indeed if we denote

**Definition of**  $(\bar{G}_{35}), (\bar{T}_{35}) : ( (\bar{G}_{35}), (\bar{T}_{35}) ) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$\begin{aligned} |\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} + \\ &\int_0^t ((a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &G_{32}^{(2)} |(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)})| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}}) ds_{(32)} \end{aligned}$$

Where  $s_{(32)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$(1) (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \tag{295}$$

$$i, j = 13, 14, 15$$

(2) The functions  $(a''_i)^{(1)}, (b''_i)^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)} \tag{296}$$

$$(3) \lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \tag{297}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$  :

Where  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$  are positive constants  
 and  $i = 13, 14, 15$

They satisfy Lipschitz condition: 298

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\bar{M}_{13})^{(1)}t}$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\bar{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(1)}(T'_{14}, t)$  and  $(a''_i)^{(1)}(T_{14}, t) \cdot (T'_{14}, t)$  And  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\bar{M}_{13})^{(1)}]$ . It is to be noted that  $(a''_i)^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\bar{M}_{13})^{(1)} = 1$  then the function  $(a''_i)^{(1)}(T_{14}, t)$ , the **first augmentation coefficient** attributable to

terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\widehat{M}_{13})^{(1)}, (\widehat{k}_{13})^{(1)}$  : 299

$(\widehat{M}_{13})^{(1)}, (\widehat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\widehat{P}_{13})^{(1)}, (\widehat{Q}_{13})^{(1)}$  : 300

There exists two constants  $(\widehat{P}_{13})^{(1)}$  and  $(\widehat{Q}_{13})^{(1)}$  which together with  $(\widehat{M}_{13})^{(1)}, (\widehat{k}_{13})^{(1)}, (\widehat{A}_{13})^{(1)}$  and  $(\widehat{B}_{13})^{(1)}$  and the constants  $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15,$

satisfy the inequalities

$$\frac{1}{(\widehat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\widehat{B}_{13})^{(1)} + (\widehat{Q}_{13})^{(1)} (\widehat{k}_{13})^{(1)}] < 1$$

Analogous inequalities hold also for  $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$  301

It is now sufficient to take  $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$  and to choose

$(\widehat{P}_{36})^{(7)}$  and  $(\widehat{Q}_{36})^{(7)}$  large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} \left[ (\widehat{P}_{36})^{(7)} + ((\widehat{P}_{36})^{(7)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{36})^{(7)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{36})^{(7)} \quad 302$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} \left[ ((\widehat{Q}_{36})^{(7)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{36})^{(7)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)} \right] \leq (\widehat{Q}_{36})^{(7)} \quad 303$$

In order that the operator  $\mathcal{A}^{(7)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying into itself

Analogous inequalities hold also for  $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$  304

It is now sufficient to take  $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$  and to choose

$(\widehat{P}_{40})^{(8)}$  and  $(\widehat{Q}_{40})^{(8)}$  large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} \left[ (\widehat{P}_{40})^{(8)} + ((\widehat{P}_{40})^{(8)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{40})^{(8)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{40})^{(8)} \quad 305$$

$$\frac{(b_i)^{(8)}}{(\bar{M}_{40})^{(8)}} \left[ ((\hat{Q}_{40})^{(8)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{40})^{(8)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)} \right] \leq (\hat{Q}_{40})^{(8)} \quad 306$$

In order that the operator  $\mathcal{A}^{(8)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying GLOBAL EQUATIONS into itself

The operator  $\mathcal{A}^{(8)}$  is a contraction with respect to the metric 307

$$d \left( ((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{40})^{(8)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{40})^{(8)}t} \right\}$$

Indeed if we denote

**Definition of**  $(\bar{G}_{43}), (\bar{T}_{43})$  :  $((\bar{G}_{43}), (\bar{T}_{43})) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results

$$\begin{aligned} |\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{40})^{(8)} |G_{41}^{(1)} - G_{41}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} ds_{(40)} + \\ &\int_0^t \{ (a'_{40})^{(8)} |G_{40}^{(1)} - G_{40}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{-(\bar{M}_{40})^{(8)}s_{(40)}} + \\ &(a''_{40})^{(8)} (T_{41}^{(1)}, s_{(40)}) |G_{40}^{(1)} - G_{40}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} + \\ &G_{40}^{(2)} | (a''_{40})^{(8)} (T_{41}^{(1)}, s_{(40)}) - (a''_{40})^{(8)} (T_{41}^{(2)}, s_{(40)}) | e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} \} ds_{(40)} \end{aligned}$$

Where  $s_{(40)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses IT follows

$$\begin{aligned} |(G_{43})^{(1)} - (G_{43})^{(2)}| e^{-(\bar{M}_{40})^{(8)}t} &\leq \\ \frac{1}{(\bar{M}_{40})^{(8)}} &\left( (a_{40})^{(8)} + (a'_{40})^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)} \right) d \left( ((G_{43})^{(1)}, (T_{43})^{(1)}); ((G_{43})^{(2)}, (T_{43})^{(2)}) \right) \end{aligned} \quad 308$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (38,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{40})^{(8)}$  and  $(b''_{40})^{(8)}$  depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)}t}$  and  $(\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)}t}$  respectively of  $\mathbb{R}_+$ . 309

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(8)}$  and  $(b''_i)^{(8)}$ ,  $i = 40, 41, 42$  depend only on  $T_{41}$  and respectively on  $(G_{43})$  (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any t where  $G_i(t) = 0$  and  $T_i(t) = 0$  310

From global equations it results



$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(8)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{40})^{(8)})_1, ((\widehat{M}_{40})^{(8)})_2$  and  $((\widehat{M}_{40})^{(8)})_3$  : 311

**Remark 3:** if  $G_{40}$  is bounded, the same property have also  $G_{41}$  and  $G_{42}$  . indeed if

$$G_{40} < (\widehat{M}_{40})^{(8)} \text{ it follows } \frac{dG_{41}}{dt} \leq ((\widehat{M}_{40})^{(8)})_1 - (a_{41}')^{(8)}G_{41} \text{ and by integrating}$$

$$G_{41} \leq ((\widehat{M}_{40})^{(8)})_2 = G_{41}^0 + 2(a_{41})^{(8)}((\widehat{M}_{40})^{(8)})_1 / (a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq ((\widehat{M}_{40})^{(8)})_3 = G_{42}^0 + 2(a_{42})^{(8)}((\widehat{M}_{40})^{(8)})_2 / (a_{42}')^{(8)}$$

If  $G_{41}$  or  $G_{42}$  is bounded, the same property follows for  $G_{40}$  ,  $G_{42}$  and  $G_{40}$  ,  $G_{41}$  respectively.

**Remark 4:** If  $G_{40}$  is bounded, from below, the same property holds for  $G_{41}$  and  $G_{42}$  . The proof is 312  
 analogous with the preceding one. An analogous property is true if  $G_{41}$  is bounded from below.

**Remark 5:** If  $T_{40}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$  then  $T_{41} \rightarrow \infty$ . 313

**Definition of**  $(m)^{(8)}$  and  $\varepsilon_8$  :

Indeed let  $t_8$  be so that for  $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}((G_{43})(t), t) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then  $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$  which leads to 314

$$T_{41} \geq \left( \frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8} \right) (1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_8 t} = \frac{1}{2} \text{ it results}$$

$$T_{41} \geq \left( \frac{(a_{41})^{(8)}(m)^{(8)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_8} \text{ By taking now } \varepsilon_8 \text{ sufficiently small one sees that } T_{41} \text{ is}$$

unbounded. The same property holds for  $T_{42}$  if  $\lim_{t \rightarrow \infty} (b_{42}'')^{(8)}((G_{43})(t), t(t), t) = (b_{42}')^{(8)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 82

**Remark 1:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  315

From CONCATENATED GLOBAL EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(7)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{36})^{(7)})_1, ((\widehat{M}_{36})^{(7)})_2$  and  $((\widehat{M}_{36})^{(7)})_3$  : 316

**Remark 2:** if  $G_{36}$  is bounded, the same property have also  $G_{37}$  and  $G_{38}$  . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$  it follows  $\frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a'_{37})^{(7)}G_{37}$  and by integrating

$$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)}((\widehat{M}_{36})^{(7)})_1 / (a'_{37})^{(7)}$$

In the same way, one can obtain

$$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)}((\widehat{M}_{36})^{(7)})_2 / (a'_{38})^{(7)}$$

If  $G_{37}$  or  $G_{38}$  is bounded, the same property follows for  $G_{36}$ ,  $G_{38}$  and  $G_{36}$ ,  $G_{37}$  respectively.

**Remark 3:** If  $G_{36}$  is bounded, from below, the same property holds for  $G_{37}$  and  $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{37}$  is bounded from below. 317

**Remark 5:** If  $T_{36}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$  then  $T_{37} \rightarrow \infty$ . 318

**Definition of**  $(m)^{(7)}$  and  $\varepsilon_7$  :

Indeed let  $t_7$  be so that for  $t > t_7$

$$(b_{37})^{(7)} - (b'_i)^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then  $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$  which leads to

$$T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7} \right) (1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$

If we take  $t$  such that  $e^{-\varepsilon_7 t} = \frac{1}{2}$  it results

$$T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_7}$$

By taking now  $\varepsilon_7$  sufficiently small one sees that  $T_{37}$  is unbounded. The same property holds for  $T_{38}$  if  $\lim_{t \rightarrow \infty} (b''_{38})^{(7)}((G_{39})(t), t) = (b'_{38})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)} \quad 319$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)} \quad 320$$

**Definition of**  $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$  :

By  $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$  and respectively  $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$  the roots 321

$$(a) \text{ of the equations } (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0 \quad 322$$

$$\text{and } (b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0 \text{ and} \quad 323$$

**Definition of**  $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$  :

By  $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$  and respectively  $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$  the 324

$$\text{roots of the equations } (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0 \quad 325$$

$$\text{and } (b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0 \quad 326$$

**Definition of**  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  :-

(b) If we define  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  by 327

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \mathbf{if} (v_0)^{(2)} < (v_1)^{(2)} \quad 328$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \mathbf{if} (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}, \quad 329$$

$$\text{and } \boxed{(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \mathbf{if} (\bar{v}_1)^{(2)} < (v_0)^{(2)} \quad 330$$

and analogously 331

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \mathbf{if} (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \mathbf{if} (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

$$\text{and } \boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \mathbf{if} (\bar{u}_1)^{(2)} < (u_0)^{(2)} \quad 332$$

Then the solution satisfies the inequalities 333

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$  is defined

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 334$$

$$\left( \frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \quad 335$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a'_{18})^{(2)})} \left[ e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 336$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 337$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[ e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq \quad 338$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[ e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**Definition of**  $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$ :- 339

$$\text{Where } (S_1)^{(2)} = (a_{16})^{(2)} (m_2)^{(2)} - (a'_{16})^{(2)} \quad 340$$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)} (\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 341$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

### Behavior Of The Solutions

If we denote and define 342

**Definition of**  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  :

(a)  $\sigma_1^{(3)}, \sigma_2^{(3)}, \tau_1^{(3)}, \tau_2^{(3)}$  four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

**Definition of**  $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$  :

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(b) By  $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$  and respectively  $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$  the roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By  $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$  and respectively  $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$  the

roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

**Definition of**  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  :-

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(c) If we define  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

$$\text{and } \boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously

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$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } \boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$  is defined

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(s_1)^{(3)}t} \quad 346$$

$$\left( \frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)} ((s_1)^{(3)} - (p_{20})^{(3)} - (s_2)^{(3)})} \left[ e^{((s_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(s_2)^{(3)}t} \right] + G_{22}^0 e^{-(s_2)^{(3)}t} \right) \leq G_{22}(t) \leq \quad 347$$

$$\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)} ((s_1)^{(3)} - (a'_{22})^{(3)})} \left[ e^{(s_1)^{(3)}t} - e^{-a'_{22}^{(3)}t} \right] + G_{22}^0 e^{-a'_{22}^{(3)}t}$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 348$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 349$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})} \left[ e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq \quad 350$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})} \left[ e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t} \quad 351$$

**Definition of**  $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$ :- 351

$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

If we denote and define 352

**Definition of**  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  :

(d)  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$$

**Definition of**  $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$  : 353

(e) By  $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$  and respectively  $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$  the roots of the equations  $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0 \text{ and}$$

**Definition of**  $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$  : 354

By  $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$  and respectively  $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$  the

roots of the equations  $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$$

**Definition of**  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$  :-

(f) If we define  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$  by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},$$

$$\text{and } (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)}$$

and analogously

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$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

$$\text{and } (u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}$$

are defined respectively

Then the solution satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where  $(p_i)^{(4)}$  is defined

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 357$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq \quad 358$$

$$\frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[ e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}$$

$$T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 359$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 360$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} ((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq \quad 361$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of**  $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$ :-

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$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions**

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If we denote and define

**Definition of**  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  :

(g)  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

**Definition of**  $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$  : 364

(h) By  $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$  and respectively  $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$  the roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0 \text{ and}$$

**Definition of**  $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$  : 365

By  $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$  and respectively  $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$  the

$$\text{roots of the equations } (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$$

**Definition of**  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$  :-

(i) If we define  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$  by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously 366

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

$$\text{and } \boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)} \text{ where } (u_1)^{(5)}, (\bar{u}_1)^{(5)}$$

are defined respectively

Then the solution satisfies the inequalities 367

$$G_{28}^0 e^{((s_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(s_1)^{(5)}t}$$

where  $(p_i)^{(5)}$  is defined



$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 368$$

$$\left( \frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[ e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \right) \leq G_{30}(t) \leq \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a'_{30})^{(5)})} \left[ e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t} \quad 369$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 370$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 371$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[ e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq \quad 372$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[ e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t} \quad 373$$

**Definition of**  $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$ :- 373

$$\text{Where } (S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}$$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

**Behavior of the solutions** 374

If we denote and define

**Definition of**  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  :

(j)  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

**Definition of**  $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$  : 375

(k) By  $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$  and respectively  $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$  the roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

$$\text{and } (b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0 \text{ and}$$

**Definition of**  $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$  : 376

By  $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$  and respectively  $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$  the

roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

$$\text{and } (b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$$

**Definition of**  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$  :-

(1) If we define  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$  by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

$$\text{and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

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$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

$$\text{and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

are defined respectively

Then the solution satisfies the inequalities

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$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where  $(p_i)^{(6)}$  is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 379$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \right) \leq G_{34}(t) \leq \quad 380$$

$$\frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((S_1)^{(6)} - (a'_{34})^{(6)})} \left[ e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 381$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 382$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)} ((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq \quad 383$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)} ((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of**  $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$  :-

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$$\text{Where } (S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

If we denote and define

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**Definition of**  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  :

(m)  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}(G_{39}, t) - (b''_{37})^{(7)}(G_{39}, t) \leq -(\tau_1)^{(7)}$$

**Definition of**  $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$  :

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(n) By  $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$  and respectively  $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$  the roots of the

$$\text{equations } (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$$

$$\text{and } (b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0 \text{ and}$$

**Definition of**  $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$  :

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By  $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$  and respectively  $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$  the

$$\text{roots of the equations } (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$$

$$\text{and } (b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$$

**Definition of**  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$  :-

(o) If we define  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$  by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \text{ if } (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

$$\text{and } \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously

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$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{ if } (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

$$\text{and } \boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

are defined respectively

Then the solution satisfies the inequalities

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$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where  $(p_i)^{(7)}$  is defined

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 390$$

$$\left( \frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)} ((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})} \right) \left[ e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \quad 391$$

$$\frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)} ((S_1)^{(7)} - (a'_{38})^{(7)})} \left[ e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 392$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 393$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)} ((R_1)^{(7)} - (b'_{38})^{(7)})} \left[ e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq \quad 394$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)} ((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[ e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

**Definition of**  $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$ :- 395

$$\text{Where } (S_1)^{(7)} = (a_{36})^{(7)} (m_2)^{(7)} - (a'_{36})^{(7)}$$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)} (\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions** 396

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$  :

(p)  $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$  four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \leq -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}((G_{43}), t) - (b''_{41})^{(8)}((G_{43}), t) \leq -(\tau_1)^{(8)}$$

**Definition of**  $(v_1)^{(8)}, (v_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, v^{(8)}, u^{(8)}$  : 397

(q) By  $(v_1)^{(8)} > 0, (v_2)^{(8)} < 0$  and respectively  $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$  the roots of the equations  $(a_{41})^{(8)}(v^{(8)})^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$  and  $(b_{41})^{(8)}(u^{(8)})^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(8)}, (\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$  : 398

By  $(\bar{v}_1)^{(8)} > 0, (\bar{v}_2)^{(8)} < 0$  and respectively  $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$  the

roots of the equations  $(a_{41})^{(8)}(v^{(8)})^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and  $(b_{41})^{(8)}(u^{(8)})^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of**  $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$  :-

(r) If we define  $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$  by

$$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}, \text{ if } (v_0)^{(8)} < (v_1)^{(8)}$$

$$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}, \text{ if } (v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)},$$

$$\text{and } (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}$$

$$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}, \text{ if } (\bar{v}_1)^{(8)} < (v_0)^{(8)}$$

and analogously

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$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \text{ if } (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \text{ if } (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

$$\text{and } (u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \text{ if } (\bar{u}_1)^{(8)} < (u_0)^{(8)} \text{ where } (u_1)^{(8)}, (\bar{u}_1)^{(8)}$$

are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

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$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{40}(t) \leq G_{40}^0 e^{(S_1)^{(8)}t}$$

where  $(p_i)^{(8)}$  is defined

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{41}(t) \leq \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t}$$

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$$\left( \frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)} ((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})} \right) \left[ e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t} \right] + G_{42}^0 e^{-(S_2)^{(8)}t} \leq G_{42}(t) \leq$$

$$\frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)} ((S_1)^{(8)} - (a'_{42})^{(8)})} \left[ e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t} \right] + G_{42}^0 e^{-(a'_{42})^{(8)}t}$$

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$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \leq T_{40}(t) \leq T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}$$

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$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \leq T_{40}(t) \leq \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}$$

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$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)} ((R_1)^{(8)} - (b'_{42})^{(8)})} \left[ e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t} \right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \leq T_{42}(t) \leq$$

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$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)} ((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})} \left[ e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t} \right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of**  $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$  :-

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Where  $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left( (a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t) \right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

**Definition of**  $v^{(8)}$  :- 
$$v^{(8)} = \frac{G_{40}}{G_{41}}$$

It follows

$$- \left( (a_{41})^{(8)}(v^{(8)})^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} \right) \leq \frac{dv^{(8)}}{dt} \leq - \left( (a_{41})^{(8)}(v^{(8)})^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(8)}, (v_0)^{(8)}$  :-

(a) For  $0 < \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (v_1)^{(8)} < (\bar{v}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(v_1)^{(8)} + (C)^{(8)}(v_2)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_0)^{(8)}]t}}{1 + (C)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_0)^{(8)}]t}}, \quad \boxed{(C)^{(8)} = \frac{(v_1)^{(8)} - (v_0)^{(8)}}{(v_0)^{(8)} - (v_2)^{(8)}}$$

it follows  $(v_0)^{(8)} \leq v^{(8)}(t) \leq (v_1)^{(8)}$

In the same manner , we get

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$$v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)} + (\bar{C})^{(8)}(\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}{1 + (\bar{C})^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}, \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{v}_1)^{(8)} - (v_0)^{(8)}}{(v_0)^{(8)} - (\bar{v}_2)^{(8)}}$$

From which we deduce  $(v_0)^{(8)} \leq v^{(8)}(t) \leq (\bar{v}_1)^{(8)}$

(b) If  $0 < (v_1)^{(8)} < (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{v}_1)^{(8)}$  we find like in the previous case,

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$$(v_1)^{(8)} \leq \frac{(v_1)^{(8)} + (C)^{(8)}(v_2)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_2)^{(8)}]t}}{1 + (C)^{(8)} e^{[-(a_{41})^{(8)}(v_1)^{(8)} - (v_2)^{(8)}]t}} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)} + (\bar{C})^{(8)}(\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}}{1 + (\bar{C})^{(8)} e^{[-(a_{41})^{(8)}(\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)}]t}} \leq (\bar{v}_1)^{(8)}$$

(c) If  $0 < (v_1)^{(8)} \leq (\bar{v}_1)^{(8)} \leq \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain

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$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (C)^{(8)}(\bar{\nu}_2)^{(8)} e^{[-(a_{41})^{(8)}((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)})t]}}{1 + (C)^{(8)} e^{[-(a_{41})^{(8)}((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)})t]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $\nu^{(8)}(t)$  :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(8)}(t)$  :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{40})^{(8)} = (a''_{41})^{(8)}$ , then  $(\sigma_1)^{(8)} = (\sigma_2)^{(8)}$  and in this case  $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$  if in addition  $(\nu_0)^{(8)} = (\nu_1)^{(8)}$  then  $\nu^{(8)}(t) = (\nu_0)^{(8)}$  and as a consequence  $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$  **this also defines**  $(\nu_0)^{(8)}$  **for the special case.**

Analogously if  $(b''_{40})^{(8)} = (b''_{41})^{(8)}$ , then  $(\tau_1)^{(8)} = (\tau_2)^{(8)}$  and then

$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$  if in addition  $(u_0)^{(8)} = (u_1)^{(8)}$  then  $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$  This is an important consequence of the relation between  $(\nu_1)^{(8)}$  and  $(\bar{\nu}_1)^{(8)}$ , **and definition of**  $(u_0)^{(8)}$ .

: From GLOBAL EQUATIONS we obtain

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$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left( (a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of**  $\nu^{(4)}$  :-  $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left( (a_{25})^{(4)}(\nu^{(4)})^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} \right) \leq \frac{d\nu^{(4)}}{dt} \leq - \left( (a_{25})^{(4)}(\nu^{(4)})^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

**Definition of**  $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$  :-

(d) For  $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{[-(a_{25})^{(4)}((\nu_1)^{(4)} - (\nu_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((\nu_1)^{(4)} - (\nu_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}$$

it follows  $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$



In the same manner , we get

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$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)} (\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}} , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If  $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$  we find like in the previous case,

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$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)} (v_2)^{(4)} e^{[-(a_{25})^{(4)} ((v_1)^{(4)} - (v_2)^{(4)}) t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)} ((v_1)^{(4)} - (v_2)^{(4)}) t]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)} (\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}} \leq (\bar{v}_1)^{(4)}$$

(f) If  $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain

414

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)} (\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(4)}(t)$  :-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(4)}(t)$  :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{24})^{(4)} = (a''_{25})^{(4)}$ , then  $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$  and in this case  $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$  if in addition  $(v_0)^{(4)} = (v_1)^{(4)}$  then  $v^{(4)}(t) = (v_0)^{(4)}$  and as a consequence  $G_{24}(t) = (v_0)^{(4)} G_{25}(t)$  **this also defines  $(v_0)^{(4)}$  for the special case.**

Analogously if  $(b''_{24})^{(4)} = (b''_{25})^{(4)}$ , then  $(\tau_1)^{(4)} = (\tau_2)^{(4)}$  and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$  if in addition  $(u_0)^{(4)} = (u_1)^{(4)}$  then  $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$  This is an important consequence of the relation between  $(v_1)^{(4)}$  and  $(\bar{v}_1)^{(4)}$ , **and definition of  $(u_0)^{(4)}$ .**

From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left( (a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of**  $v^{(5)}$  :- 
$$v^{(5)} = \frac{G_{28}}{G_{29}}$$

It follows

$$-\left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$  :-

(g) For  $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)}]t}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)}]t}}, \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows  $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get

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$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}, \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce  $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(h) If  $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$  we find like in the previous case,

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$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (\bar{v}_1)^{(5)}$$

(i) If  $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain

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$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(5)}(t)$  :-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(5)}(t)$  :-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{28})^{(5)} = (a''_{29})^{(5)}$ , then  $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$  and in this case  $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$  if in addition  $(v_0)^{(5)} = (v_5)^{(5)}$  then  $v^{(5)}(t) = (v_0)^{(5)}$  and as a consequence  $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$  **this also defines  $(v_0)^{(5)}$  for the special case.**

Analogously if  $(b''_{28})^{(5)} = (b''_{29})^{(5)}$ , then  $(\tau_1)^{(5)} = (\tau_2)^{(5)}$  and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$  if in addition  $(u_0)^{(5)} = (u_1)^{(5)}$  then  $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$  This is an important consequence of the relation between  $(v_1)^{(5)}$  and  $(\bar{v}_1)^{(5)}$ , **and definition of  $(u_0)^{(5)}$ .**

we obtain

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$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left( (a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of  $v^{(6)}$  :-** 
$$v^{(6)} = \frac{G_{32}}{G_{33}}$$

It follows

$$- \left( (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left( (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

**Definition of  $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$  :-**

(j) For  $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_0)^{(6)})t]}} , \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get

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$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}((\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)})t]}} , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If  $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$  we find like in the previous case,

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$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_2)^{(6)})t]}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}((v_1)^{(6)} - (v_2)^{(6)})t]}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{c})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}}{1 + (\bar{c})^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}} \leq (\bar{v}_1)^{(6)}$$

(l) If  $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain

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$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{c})^{(6)} (\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}}{1 + (\bar{c})^{(6)} e^{[-(a_{33})^{(6)} (\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}] t}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(6)}(t)$  :-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(6)}(t)$  :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{32})^{(6)} = (a''_{33})^{(6)}$ , then  $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$  and in this case  $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$  if in addition  $(v_0)^{(6)} = (v_1)^{(6)}$  then  $v^{(6)}(t) = (v_0)^{(6)}$  and as a consequence  $G_{32}(t) = (v_0)^{(6)} G_{33}(t)$  **this also defines  $(v_0)^{(6)}$  for the special case.**

Analogously if  $(b''_{32})^{(6)} = (b''_{33})^{(6)}$ , then  $(\tau_1)^{(6)} = (\tau_2)^{(6)}$  and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$  if in addition  $(u_0)^{(6)} = (u_1)^{(6)}$  then  $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$  This is an important consequence of the relation between  $(v_1)^{(6)}$  and  $(\bar{v}_1)^{(6)}$ , **and definition of  $(u_0)^{(6)}$ .**

**Behavior of the solutions**

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If we denote and define

**Definition of**  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  :

(s)  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \leq -(\tau_1)^{(7)}$$

**Definition of**  $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$  :

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(t) By  $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$  and respectively  $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$  the roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$  : 425

By  $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$  and respectively  $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$  the

roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of**  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$  :-

(u) If we define  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$  by  
 $(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$ , **if**  $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$ , **if**  $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$ ,

and 
$$(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}$$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$ , **if**  $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 426

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$ , **if**  $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$ , **if**  $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$ ,

and 
$$(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$ , **if**  $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$  where  $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 427

$$G_{36}^0 e^{((s_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(s_1)^{(7)}t}$$

where  $(p_i)^{(7)}$  is defined

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((s_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(s_1)^{(7)}t} \quad 428$$

$$\left( \frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)} ((s_1)^{(7)} - (p_{36})^{(7)} - (s_2)^{(7)})} \left[ e^{((s_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(s_2)^{(7)}t} \right] + G_{38}^0 e^{-(s_2)^{(7)}t} \right) \leq G_{38}(t) \leq \quad 429$$

$$\frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)} ((s_1)^{(7)} - (a'_{38})^{(7)})} \left[ e^{(s_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 430$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 431$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)} ((R_1)^{(7)} - (b'_{38})^{(7)})} \left[ e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 432$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)} ((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[ e^{((R_1)^{(7)} + (r_{36})^{(7)}) t} - e^{-(R_2)^{(7)} t} \right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of**  $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$ :- 433

Where  $(S_1)^{(7)} = (a_{36})^{(7)} (m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)} (\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

From CONCATENATED GLOBAL EQUATIONS we obtain 434

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left( (a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)} (T_{37}, t) \right) - (a''_{37})^{(7)} (T_{37}, t) v^{(7)} - (a_{37})^{(7)} v^{(7)}$$

**Definition of**  $v^{(7)}$  :-  $v^{(7)} = \frac{G_{36}}{G_{37}}$

It follows

$$- \left( (a_{37})^{(7)} (v^{(7)})^2 + (\sigma_2)^{(7)} v^{(7)} - (a_{36})^{(7)} \right) \leq \frac{dv^{(7)}}{dt} \leq - \left( (a_{37})^{(7)} (v^{(7)})^2 + (\sigma_1)^{(7)} v^{(7)} - (a_{36})^{(7)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$  :-

(m) For  $0 < \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \geq \frac{(v_1)^{(7)} + (C)^{(7)} (v_2)^{(7)} e^{[-(a_{37})^{(7)} ((v_1)^{(7)} - (v_0)^{(7)}) t]}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)} ((v_1)^{(7)} - (v_0)^{(7)}) t]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}$$

it follows  $(v_0)^{(7)} \leq v^{(7)}(t) \leq (v_1)^{(7)}$

In the same manner , we get 435

$$v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)} (\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)} ((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}) t]}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)} ((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}) t]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}$$

From which we deduce  $(v_0)^{(7)} \leq v^{(7)}(t) \leq (\bar{v}_1)^{(7)}$

(n) If  $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$  we find like in the previous case, 436

$$(v_1)^{(7)} \leq \frac{(v_1)^{(7)} + (C)^{(7)} (v_2)^{(7)} e^{[-(a_{37})^{(7)} ((v_1)^{(7)} - (v_2)^{(7)}) t]}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)} ((v_1)^{(7)} - (v_2)^{(7)}) t]}} \leq v^{(7)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)} (\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)} (\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}] t}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)} (\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}] t}} \leq (\bar{v}_1)^{(7)}$$

**Theorem 3:** If  $(a_i'')^{(8)}$  and  $(b_i'')^{(8)}$  are independent on  $t$ , and the conditions 437

$$(a'_{44})^{(8)} (a'_{45})^{(8)} - (a_{44})^{(8)} (a_{45})^{(8)} < 0$$

$$(a'_{44})^{(8)} (a'_{45})^{(8)} - (a_{44})^{(8)} (a_{45})^{(8)} + (a_{44})^{(8)} (p_{44})^{(8)} + (a'_{45})^{(8)} (p_{45})^{(8)} + (p_{44})^{(8)} (p_{45})^{(8)} > 0$$

$$(b'_{40})^{(8)} (b'_{41})^{(8)} - (b_{42})^{(8)} (b_{43})^{(8)} > 0,$$

$$(b'_{40})^{(8)} (b'_{41})^{(8)} - (b_{42})^{(8)} (b_{43})^{(8)} - (b'_{40})^{(8)} (r_{41})^{(8)} - (b'_{41})^{(9)} (r_{41})^{(9)} + (r_{43})^{(9)} (r_{41})^{(9)} < 0$$

with  $(p_{40})^{(8)}, (r_{41})^{(8)}$  as defined are satisfied, then the system

$$(a_{40})^{(8)} G_{41} - [(a'_{40})^{(8)} + (a''_{40})^{(8)} (T_{41})] G_{40} = 0 \quad 438$$

$$(a_{41})^{(8)} G_{40} - [(a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41})] G_{41} = 0 \quad 439$$

$$(a_{42})^{(8)} G_{41} - [(a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41})] G_{42} = 0 \quad 440$$

$$(b_{40})^{(8)} T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43})] T_{40} = 0 \quad 441$$

$$(b_{41})^{(8)} T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43})] T_{41} = 0 \quad 442$$

$$(b_{42})^{(8)} T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43})] T_{42} = 0 \quad 443$$

has a unique positive solution, which is an equilibrium solution for the system 444

**Proof:**

(a) Indeed the first two equations have a nontrivial solution  $G_{40}, G_{41}$  if

$$F(T_{43}) = (a'_{40})^{(8)} (a'_{41})^{(8)} - (a_{40})^{(8)} (a_{41})^{(8)} + (a'_{40})^{(8)} (a''_{41})^{(8)} (T_{41}) + (a'_{41})^{(8)} (a''_{40})^{(8)} (T_{41}) + (a''_{40})^{(8)} (T_{41}) (a''_{41})^{(8)} (T_{41}) = 0$$

**Definition and uniqueness of  $T_{41}^*$**  :- 445

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(8)}(T_{41})$  are increasing, it follows that there exists a unique  $T_{41}^*$  for which  $f(T_{41}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{44} = \frac{(a_{41})^{(8)} G_{41}}{[(a'_{41})^{(8)} + (a''_{41})^{(9)} (T_{41}^*)]} \quad , \quad G_{46} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(9)} + (a''_{42})^{(9)} (T_{41}^*)]}$$

(a) By the same argument, the equations (GLOBAL) admit solutions  $G_{40}, G_{41}$  if

$$\varphi(G_{43}) = (b'_{40})^{(8)} (b'_{41})^{(8)} - (b_{40})^{(8)} (b_{45})^{(8)} -$$

$$[(b'_{40})^{(8)} (b''_{41})^{(8)} (G_{43}) + (b'_{41})^{(8)} (b''_{40})^{(8)} (G_{43})] + (b'_{40})^{(8)} (G_{43}) (b'_{41})^{(8)} (G_{43}) = 0$$

Where in  $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$  must be replaced by their values. It is easy to see that  $\varphi$  is a decreasing function in  $G_{45}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{41}^*$  such that  $\varphi((G_{43})^*) = 0$  446



Finally we obtain the unique solution

$G_{41}^*$  given by  $\varphi((G_{43})^*) = 0$ ,  $T_{41}^*$  given by  $f(T_{41}^*) = 0$  and

$$G_{40}^* = \frac{(a_{40})^{(8)}G_{41}^*}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)}G_{41}^*}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{44}^* = \frac{(b_{40})^{(8)}T_{41}^*}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)}T_{41}^*}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^*)]} \quad 447$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 448$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 449$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 450$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 451$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 452$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 453$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 454$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 455$$

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 456$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 457$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 458$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}))]T_{24} = 0 \quad 459$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}))]T_{25} = 0 \quad 460$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}))]T_{26} = 0 \quad 461$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 462$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 463$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 464$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 465$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 466$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 467$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 468$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 469$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 470$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 471$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 472$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 473$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0 \quad 474$$

$$(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0 \quad 475$$

$$(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0 \quad 476$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 477$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0 \quad 478$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0 \quad 479$$

has a unique positive solution , which is an equilibrium solution for the system 480

(a) Indeed the first two equations have a nontrivial solution  $G_{36}, G_{37}$  if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Definition and uniqueness of  $T_{37}^*$**  :- 481

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(7)}(T_{37})$  are increasing, it follows that there exists a unique  $T_{37}^*$  for which  $f(T_{37}^*) = 0$ . With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

(b) By the same argument, the equations( SOLUTIONAL) admit solutions  $G_{36}, G_{37}$  if

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in  $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{37}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{37}^*$  such that  $\varphi(G^*) = 0$  482

Finally we obtain the unique solution OF THE SYSTEM

$G_{37}^*$  given by  $\varphi((G_{39})^*) = 0, T_{37}^*$  given by  $f(T_{37}^*) = 0$  and

$$G_{36}^* = \frac{(a_{36})^{(7)}G_{37}^*}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^*)]} , G_{38}^* = \frac{(a_{38})^{(7)}G_{37}^*}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)}T_{37}^*}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^*)]} , T_{38}^* = \frac{(b_{38})^{(7)}T_{37}^*}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^*)]}$$
483

**Definition and uniqueness of  $T_{21}^*$  :-** 484

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(1)}(T_{21})$  are increasing, it follows that there exists a unique  $T_{21}^*$  for which  $f(T_{21}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} , G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

**Definition and uniqueness of  $T_{25}^*$  :-** 485

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(4)}(T_{25})$  are increasing, it follows that there exists a unique  $T_{25}^*$  for which  $f(T_{25}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} , G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

**Definition and uniqueness of  $T_{29}^*$  :-** 486

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(5)}(T_{29})$  are increasing, it follows that there exists a unique  $T_{29}^*$  for which  $f(T_{29}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} , G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

**Definition and uniqueness of  $T_{33}^*$  :-** 487

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(6)}(T_{33})$  are increasing, it follows that there exists a unique  $T_{33}^*$  for which  $f(T_{33}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} , G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

(c) By the same argument, the equations GLOBAL admit solutions  $G_{13}, G_{14}$  if 488

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in  $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$  must be replaced by their values from 96. It is easy to see that

$\varphi$  is a decreasing function in  $G_{14}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi(G^*) = 0$

(d) By the same argument, the equations 92,93 admit solutions  $G_{16}, G_{17}$  if 489

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in  $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{17}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi((G_{19})^*) = 0$  490

(a) By the same argument, the concatenated equations admit solutions  $G_{20}, G_{21}$  if 491

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in  $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{21}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{21}^*$  such that  $\varphi((G_{23})^*) = 0$

(b) By the same argument, the equations of modules admit solutions  $G_{24}, G_{25}$  if 492

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in  $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{25}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{25}^*$  such that  $\varphi((G_{27})^*) = 0$

(c) By the same argument, the equations (modules) admit solutions  $G_{28}, G_{29}$  if 493

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in  $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{29}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{29}^*$  such that  $\varphi((G_{31})^*) = 0$

(d) By the same argument, the equations (modules) admit solutions  $G_{32}, G_{33}$  if 494

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in  $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$  must be replaced by their values It is easy to see that  $\varphi$  is a decreasing function in  $G_{33}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{33}^*$  such that  $\varphi(G^*) = 0$

Finally we obtain the unique solution 495

$G_{14}^*$  given by  $\varphi(G^*) = 0, T_{14}^*$  given by  $f(T_{14}^*) = 0$  and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} , G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} , T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^*$  given by  $\varphi((G_{19})^*) = 0$ ,  $T_{17}^*$  given by  $f(T_{17}^*) = 0$  and 496

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]} \quad 497$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 498$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 499

$G_{21}^*$  given by  $\varphi((G_{23})^*) = 0$ ,  $T_{21}^*$  given by  $f(T_{21}^*) = 0$  and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} , G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} , T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 500

$G_{25}^*$  given by  $\varphi(G_{27}) = 0$ ,  $T_{25}^*$  given by  $f(T_{25}^*) = 0$  and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} , G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} , T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]} \quad 501$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 502

$G_{29}^*$  given by  $\varphi((G_{31})^*) = 0$ ,  $T_{29}^*$  given by  $f(T_{29}^*) = 0$  and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} , G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} , T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]} \quad 503$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 504

$G_{33}^*$  given by  $\varphi((G_{35})^*) = 0$ ,  $T_{33}^*$  given by  $f(T_{33}^*) = 0$  and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)} (T_{33}^*)]} , G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)} ((G_{35})^*)]} , T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)} ((G_{35})^*)]}$$
505

Obviously, these values represent an equilibrium solution

### ASYMPTOTIC STABILITY ANALYSIS 506

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  belong to  $C^1(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}} (T_{14}^*) = (q_{14})^{(1)} , \frac{\partial (b_i'')^{(1)}}{\partial G_j} (G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)}) \mathbb{G}_{13} + (a_{13})^{(1)} \mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14}$$
507

$$\frac{dG_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)}) \mathbb{G}_{14} + (a_{14})^{(1)} \mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14}$$
508

$$\frac{dG_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)}) \mathbb{G}_{15} + (a_{15})^{(1)} \mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14}$$
509

$$\frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)}) \mathbb{T}_{13} + (b_{13})^{(1)} \mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(13)(j)}) T_{13}^* \mathbb{G}_j$$
510

$$\frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)}) \mathbb{T}_{14} + (b_{14})^{(1)} \mathbb{T}_{13} + \sum_{j=13}^{15} (s_{(14)(j)}) T_{14}^* \mathbb{G}_j$$
511

$$\frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)}) \mathbb{T}_{15} + (b_{15})^{(1)} \mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(15)(j)}) T_{15}^* \mathbb{G}_j$$
512

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  belong to  $C^2(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote 513

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i , T_i = T_i^* + \mathbb{T}_i$$
514

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}} (T_{17}^*) = (q_{17})^{(2)} , \frac{\partial (b_i'')^{(2)}}{\partial G_j} ((G_{19})^*) = s_{ij}$$
515

taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)}) \mathbb{G}_{16} + (a_{16})^{(2)} \mathbb{G}_{17} - (q_{16})^{(2)} G_{16}^* \mathbb{T}_{17}$$
516

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)}) \mathbb{G}_{17} + (a_{17})^{(2)} \mathbb{G}_{16} - (q_{17})^{(2)} G_{17}^* \mathbb{T}_{17}$$
517

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)}) \mathbb{G}_{18} + (a_{18})^{(2)} \mathbb{G}_{17} - (q_{18})^{(2)} G_{18}^* \mathbb{T}_{17}$$
518

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)(j)})T_{16}^*G_j \quad 519$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)})T_{17}^*G_j \quad 520$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)})T_{18}^*G_j \quad 521$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  belong to  $C^{(3)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \quad 523$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^*T_{21} \quad 524$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21} \quad 525$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)})T_{20}^*G_j \quad 526$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)})T_{21}^*G_j \quad 527$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)})T_{22}^*G_j \quad 528$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(4)}$  and  $(b_i'')^{(4)}$  belong to  $C^{(4)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)}, \quad \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^*T_{25} \quad 530$$

$$\frac{dG_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^*T_{25} \quad 531$$

$$\frac{dG_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^*T_{25} \quad 532$$



$$\frac{dT_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^*G_j \quad 533$$

$$\frac{dT_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^*G_j \quad 534$$

$$\frac{dT_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^*G_j \quad 535$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(5)}$  and  $(b_i'')^{(5)}$  belong to  $C^{(5)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of  $G_i, T_i$  :-** 536

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}, \quad \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^*T_{29} \quad 537$$

$$\frac{dG_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^*T_{29} \quad 538$$

$$\frac{dG_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^*T_{29} \quad 539$$

$$\frac{dT_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^*G_j \quad 540$$

$$\frac{dT_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^*G_j \quad 541$$

$$\frac{dT_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^*G_j \quad 542$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(6)}$  and  $(b_i'')^{(6)}$  belong to  $C^{(6)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of  $G_i, T_i$  :-** 543

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}, \quad \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G_{32}^*T_{33} \quad 544$$

$$\frac{dG_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G_{33}^*T_{33} \quad 545$$

$$\frac{dG_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G_{34}^*T_{33} \quad 546$$

$$\frac{dT_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{34} (s_{(32)(j)})T_{32}^*G_j \quad 547$$

$$\frac{dT_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{34} (s_{(33)(j)})T_{33}^*G_j \quad 548$$

$$\frac{dT_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34} (s_{(34)(j)})T_{34}^*G_j \quad 549$$

Obviously, these values represent an equilibrium solution of 79,20,36,22,23,

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(7)}$  and  $(b_i'')^{(7)}$  belong to  $C^{(7)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of**  $G_i, T_i$  :- 550

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)}, \quad \frac{\partial (b_i'')^{(7)}}{\partial G_j}((G_{39})^{**}) = s_{ij}$$

Then taking into account equations(SOLUTIONAL) and neglecting the terms of power 2, we obtain

$$\frac{dG_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})G_{36} + (a_{36})^{(7)}G_{37} - (q_{36})^{(7)}G_{36}^*T_{37} \quad 551$$

$$\frac{dG_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})G_{37} + (a_{37})^{(7)}G_{36} - (q_{37})^{(7)}G_{37}^*T_{37} \quad 552$$

$$\frac{dG_{38}}{dt} = -((a'_{38})^{(7)} + (p_{38})^{(7)})G_{38} + (a_{38})^{(7)}G_{37} - (q_{38})^{(7)}G_{38}^*T_{37} \quad 553$$

$$\frac{dT_{36}}{dt} = -((b'_{36})^{(7)} - (r_{36})^{(7)})T_{36} + (b_{36})^{(7)}T_{37} + \sum_{j=36}^{38} (s_{(36)(j)})T_{36}^*G_j \quad 554$$

$$\frac{dT_{37}}{dt} = -((b'_{37})^{(7)} - (r_{37})^{(7)})T_{37} + (b_{37})^{(7)}T_{36} + \sum_{j=36}^{38} (s_{(37)(j)})T_{37}^*G_j \quad 555$$

$$\frac{dT_{38}}{dt} = -((b'_{38})^{(7)} - (r_{38})^{(7)})T_{38} + (b_{38})^{(7)}T_{37} + \sum_{j=36}^{38} (s_{(38)(j)})T_{38}^*G_j \quad 556$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(8)}$  and  $(b_i'')^{(8)}$  belong to  $C^{(8)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Denote

**Definition of**  $G_i, T_i$  :- 557

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)}, \quad \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations CONCATENATED EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{dG_{40}}{dt} = -((a'_{40})^{(8)} + (p_{40})^{(8)})G_{40} + (a_{40})^{(8)}G_{41} - (q_{40})^{(8)}G_{40}^*T_{41} \quad 558$$

$$\begin{aligned} \frac{dG_{41}}{dt} &= -((a'_{41})^{(8)} + (p_{41})^{(8)})G_{41} + (a_{41})^{(8)}G_{40} - (q_{41})^{(8)}G_{41}^* T_{41} & 559 \\ \frac{dG_{42}}{dt} &= -((a'_{42})^{(8)} + (p_{42})^{(8)})G_{42} + (a_{42})^{(8)}G_{41} - (q_{42})^{(8)}G_{42}^* T_{41} & 560 \\ \frac{dT_{40}}{dt} &= -((b'_{40})^{(8)} - (r_{40})^{(8)})T_{40} + (b_{40})^{(8)}T_{41} + \sum_{j=40}^{42} (S_{(40)(j)})T_{40}^* G_j & 561 \\ \frac{dT_{41}}{dt} &= -((b'_{41})^{(8)} - (r_{41})^{(8)})T_{41} + (b_{41})^{(8)}T_{40} + \sum_{j=40}^{42} (S_{(41)(j)})T_{41}^* G_j & 562 \\ \frac{dT_{42}}{dt} &= -((b'_{42})^{(8)} - (r_{42})^{(8)})T_{42} + (b_{42})^{(8)}T_{41} + \sum_{j=40}^{42} (S_{(42)(j)})T_{42}^* G_j & 563 \end{aligned}$$

The characteristic equation of this system is 564

$$\begin{aligned} &((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ & \left[ ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right] \\ & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(14)}T_{14}^* + (b_{14})^{(1)}S_{(13),(14)}T_{14}^* \right) \\ & + \left( ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\ & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(13)}T_{14}^* + (b_{14})^{(1)}S_{(13),(13)}T_{13}^* \right) \\ & \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & \left( ((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & + \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\ & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*) \\ & \left. \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(15)}T_{14}^* + (b_{14})^{(1)}S_{(13),(15)}T_{13}^* \right) \right\} = 0 \\ & + \\ & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\ & \left[ ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right] \\ & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(17)}T_{17}^* + (b_{17})^{(2)}S_{(16),(17)}T_{17}^* \right) \\ & + \left( ((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\ & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(16)}T_{17}^* + (b_{17})^{(2)}S_{(16),(16)}T_{16}^* \right) \\ & \left( ((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) \\ & \left. \left( ((\lambda)^{(2)})^2 + ((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) (\lambda)^{(2)} \right) \right\} \end{aligned}$$

$$\begin{aligned}
 & + \left( (\lambda)^{(2)} \right)^2 + \left( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda)^{(2)} (q_{18})^{(2)} G_{18} \\
 & + \left( (\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \right) \left( (a_{18})^{(2)} (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16}^* \right) \\
 & \left\{ \left( (\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(18)} T_{17}^* + (b_{17})^{(2)} s_{(16),(18)} T_{16}^* \right\} = 0 \\
 & + \\
 & \left( (\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)} \right) \left\{ \left( (\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)} \right) \right. \\
 & \left. \left[ \left( (\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (q_{20})^{(3)} G_{20}^* \right] \right. \\
 & \left. \left( \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(21)} T_{21}^* + (b_{21})^{(3)} s_{(20),(21)} T_{21}^* \right) \right. \\
 & \left. + \left( \left( (\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)} \right) (q_{20})^{(3)} G_{20}^* + (a_{20})^{(3)} (q_{21})^{(1)} G_{21}^* \right) \right. \\
 & \left. \left( \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^* \right) \right. \\
 & \left. \left( \left( (\lambda)^{(3)} \right)^2 + \left( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} \right) \right. \\
 & \left. \left( \left( (\lambda)^{(3)} \right)^2 + \left( (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda)^{(3)} \right) \right. \\
 & \left. + \left( \left( (\lambda)^{(3)} \right)^2 + \left( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} \right) (q_{22})^{(3)} G_{22} \right. \\
 & \left. + \left( (\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left( (a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \right. \\
 & \left. \left\{ \left( (\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \right\} = 0 \\
 & + \\
 & \left( (\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)} \right) \left\{ \left( (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \right) \right. \\
 & \left. \left[ \left( (\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right] \right. \\
 & \left. \left( \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^* \right) \right. \\
 & \left. + \left( \left( (\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \right) \right. \\
 & \left. \left( \left( (\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^* \right) \right. \\
 & \left. \left( \left( (\lambda)^{(4)} \right)^2 + \left( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \right) \right. \\
 & \left. \left( \left( (\lambda)^{(4)} \right)^2 + \left( (b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)} \right) \right. \\
 & \left. + \left( \left( (\lambda)^{(4)} \right)^2 + \left( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \right) (q_{26})^{(4)} G_{26} \right. \\
 & \left. + \left( (\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) \left( (a_{26})^{(4)} (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (a_{26})^{(4)} (q_{24})^{(4)} G_{24}^* \right) \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( ((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}) S_{(25),(26)} T_{25}^* + (b_{25})^{(4)} S_{(24),(26)} T_{24}^* \right) \} = 0 \\
 & + \\
 & ((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)}) \{ ((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}) \\
 & \left[ ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right] \\
 & \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}) S_{(29),(29)} T_{29}^* + (b_{29})^{(5)} S_{(28),(29)} T_{29}^* \right) \\
 & + \left( ((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)}) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \right) \\
 & \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}) S_{(29),(28)} T_{29}^* + (b_{29})^{(5)} S_{(28),(28)} T_{28}^* \right) \\
 & \left( ((\lambda)^{(5)})^2 + ( (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) \\
 & \left( ((\lambda)^{(5)})^2 + ( (b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}) (\lambda)^{(5)} \right) \\
 & + \left( ((\lambda)^{(5)})^2 + ( (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) (q_{30})^{(5)} G_{30} \\
 & + ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) \left( (a_{30})^{(5)} (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (a_{30})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \\
 & \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}) S_{(29),(30)} T_{29}^* + (b_{29})^{(5)} S_{(28),(30)} T_{28}^* \right) \} = 0 \\
 & + \\
 & ((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)}) \{ ((\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)}) \\
 & \left[ ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right] \\
 & \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}) S_{(33),(33)} T_{33}^* + (b_{33})^{(6)} S_{(32),(33)} T_{33}^* \right) \\
 & + \left( ((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)}) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \right) \\
 & \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}) S_{(33),(32)} T_{33}^* + (b_{33})^{(6)} S_{(32),(32)} T_{32}^* \right) \\
 & \left( ((\lambda)^{(6)})^2 + ( (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}) (\lambda)^{(6)} \right) \\
 & \left( ((\lambda)^{(6)})^2 + ( (b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)}) (\lambda)^{(6)} \right) \\
 & + \left( ((\lambda)^{(6)})^2 + ( (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}) (\lambda)^{(6)} \right) (q_{34})^{(6)} G_{34} \\
 & + ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) \left( (a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\
 & \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}) S_{(33),(34)} T_{33}^* + (b_{33})^{(6)} S_{(32),(34)} T_{32}^* \right) \} = 0 \\
 & +
 \end{aligned}$$

$$\begin{aligned}
 & ((\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)}) \{ ((\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)}) \\
 & \left[ \left( ((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}) (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (q_{36})^{(7)} G_{36}^* \right) \right] \\
 & \left( ((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) s_{(37),(37)} T_{37}^* + (b_{37})^{(7)} s_{(36),(37)} T_{37}^* \right) \\
 & + \left( ((\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)}) (q_{36})^{(7)} G_{36}^* + (a_{36})^{(7)} (q_{37})^{(7)} G_{37}^* \right) \\
 & \left( ((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) s_{(37),(36)} T_{37}^* + (b_{37})^{(7)} s_{(36),(36)} T_{36}^* \right) \\
 & \left( ((\lambda)^{(7)})^2 + ((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}) (\lambda)^{(7)} \right) \\
 & \left( ((\lambda)^{(7)})^2 + ((b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)}) (\lambda)^{(7)} \right) \\
 & + \left( ((\lambda)^{(7)})^2 + ((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}) (\lambda)^{(7)} \right) (q_{38})^{(7)} G_{38} \\
 & + ((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}) \left( (a_{38})^{(7)} (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (a_{38})^{(7)} (q_{36})^{(7)} G_{36}^* \right) \\
 & \left( ((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) s_{(37),(38)} T_{37}^* + (b_{37})^{(7)} s_{(36),(38)} T_{36}^* \right) \} = 0
 \end{aligned}$$


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## SECTION 2 :

### Schrödinger's Wave Equation ,Relativistic Theories And Werner Heisenberg Matrix- An Accolytish Representation Apophthegmatic Aneurysm And Atrophied Asseveration

**Preface-** In quantum mechanics and quantum field theory, a Schrödinger field, named after Erwin Schrödinger, is a quantum field, which obeys the Schrödinger equation. While any situation described by a Schrödinger field can also be described by a many-body Schrödinger equation for identical particles, the field theory is more suitable for situations where the particle number changes. A Schrödinger field is also the classical limit of a quantum Schrödinger field, a classical wave which satisfies the Schrödinger equation. Unlike the quantum mechanical wavefunction, if there are interactions between the particles the equation will be nonlinear. These nonlinear equations describe the classical wave limit of a system of interacting identical particles. The path integral of a Schrödinger field is also known as a coherent state path integral, because the field itself is an annihilation operator whose eigenstates can be thought of as coherent states of the harmonic oscillations of the field modes. Schrödinger fields are useful for describing Bose-Einstein condensation, the Bogolyubov-de Gennes equation of superconductivity, super fluidity, and many-body theory in general. They are also a useful alternative formalism for nonrelativistic quantum mechanics. A Schrödinger field is the nonrelativistic limit of a Klein-Gordon field. Dirac equation rips open glittering façade of into four coupled linear first-order partial differential equations for the four quantities that make up the field. These matrices, and the form of the field, have a deep mathematical significance. Dirac's main objective and primary aim in casting this equation was to explain the behavior of the relativistically moving electron, and so to allow the atom to be treated in a manner consistent with relativity. Corrections introduced this way might have bearing on the problem of atomic spectra. Up until that time, attempts to make the old quantum theory of the atom compatible with the theory of relativity by discretizing the angular momentum of the electron's orbit had failed – and the new quantum mechanics of Heisenberg, Pauli, Jordan, Schrödinger, and Dirac himself had not developed sufficiently to treat this problem. Although Dirac's original intentions were satisfied, his equation had far deeper implications

for the structure of matter, and introduced new mathematical classes of objects that are now essential elements of fundamental physics. The Dirac theory, while providing a wealth of information that is accurately confirmed by experiments, nevertheless **introduces** a new physical paradigm that appears at first difficult to interpret and even dissymmetrical paradoxical. Sometimes, it might even appear that there is deployed conceptual significance and attributed conceptual implications to the very equation itself. But is figment of one's imagination and product of fecund prognostication. Some of these issues of interpretation must be regarded as open questions. The Dirac theory brilliantly **answered** some of the outstanding issues in physics at the time it was put forward, while posing others that are still the subject of debate. Many of these issues were **resolved** in modern quantum field theory by considering the Dirac equation not as a relativistic description of quantum mechanics but merely as **another relativistic field equation**, on the same footing as the Klein–Gordon equation or Maxwell's equations, in which  $\psi$  is not interpreted as a wave function but rather as a **fermion field**, similar to the Klein–Gordon scalar field or electromagnetic field. Nevertheless, considering Dirac's equation as a relativistic version of Schrödinger's equation is extremely computationally useful, and raises important issues Like Newton's Second law, the **Schrödinger equation** can be mathematically **transformed** into other formulations such as **Werner Heisenberg's matrix**, and **Richard Feynman's path integral formulation**. Also like Newton's Second law, the **Schrödinger equation describes time** in a way that is **inconvenient (e)** for **relativistic theories**, a problem that is not as severe in matrix mechanics and completely absent in the path integral formulation.

## INTRODUCTION:

### Quantum action principle

In ordinary quantum mechanics, the Hamiltonian is the infinitesimal **generator** of time-translations. This means that the state at a slightly later time is related to the state at the current time by acting with the Hamiltonian operator (multiplied by the negative imaginary unit,  $-i$ ). For states with a definite energy, this is a statement of the De Broglie relation between frequency and energy, and the general relation is consistent with that plus the superposition principle. But the Hamiltonian in classical mechanics is derived from a Lagrangian, which is a more fundamental quantity considering special relativity. The Hamiltonian tells you how to march forward in time, but the notion of time are different in different reference frames. So the Hamiltonian is different in different frames, and this type of symmetry is not apparent in the original formulation of quantum mechanics. The Hamiltonian is a function of the position and momentum at one time, and it tells you the position and momentum a little later. The Lagrangian is a function of the position now and the position a little later (or, equivalently for infinitesimal time separations, it is a function of the position and velocity). The relation between the two is by a Legendre transform, and the condition that determines the classical equations is that the Action is a minimum.

In quantum mechanics, the Legendre transform is hard to interpret, because the motion is not over a definite trajectory. So what does the Legendre transform mean? In classical mechanics, with discretization in time,

$$\epsilon H = p(q(t + \epsilon) - q(t)) - \epsilon L$$

And 
$$p = \frac{\partial L}{\partial \dot{q}}$$

Where the partial derivative with respect to  $q$  holds  $q(t + \epsilon)$  fixed. The inverse Legendre transform is:

$$\epsilon L = p\epsilon\dot{q} - \epsilon H$$

Where  $\dot{q} = \frac{\partial H}{\partial p}$

And the partial derivative now is with respect to p at fixed q.

In quantum mechanics, the state is a superposition of different states with different values of q, or different values of p, and the quantities p and q can be interpreted as noncommuting operators. The operator p is only definite on states that are indefinite with respect to q. So consider two states separated in time and act with the operator corresponding to the Lagrangian:

$$e^{i(p(q(t+\epsilon)-q(t))-\epsilon H(p,q))}$$

If the multiplications implicit in this formula are reinterpreted as matrix multiplications, what does this mean?

It can be given a meaning as follows: The first factor is  $e^{-ipq(t)}$

If this is interpreted as doing a matrix multiplication, the sum over all states integrates over all q(t), and so it takes the Fourier transform in q(t), to change basis to p(t). That is the action on the Hilbert space – change basis to p at time t.

Next comes:  $e^{-i\epsilon H(p,q)}$

Or evolve an infinitesimal time into the future.

Finally, the last factor in this interpretation is  $e^{ipq(t+\epsilon)}$

Which means change basis back to q at a later time?

This is not very different from just ordinary time evolution: the H factor contains all the dynamical information – it pushes the state forward in time. The first part and the last part are just doing Fourier transforms to change to a pure q basis from an intermediate p basis.

Another way of saying this is that since the Hamiltonian is naturally a function of p and q, exponentiation this quantity and changing basis from p to q at each step allows the matrix element of H to be expressed as a simple function along each path. This function is the quantum analog of the classical action. This observation is due to Paul Dirac.

Dirac further noted that one could square the time-evolution operator in the S representation  $e^{i\epsilon S}$

And this gives the time evolution operator between time t and time t + 2ε. While in the H representation the quantity that is being summed over the intermediate states is an obscure matrix element, in the S representation it is reinterpreted as a quantity associated to the path. In the limit that one takes a large power of this operator, one reconstructs the full quantum evolution between two states, the early one with a fixed value of q(0) and the later one with a fixed value of q(t). The result is a sum over paths with a phase which is the quantum action.

#### Feynman's interpretation

Dirac's work did not provide a precise prescription to calculate the sum over paths, and he did not show that one could recover the Schrödinger equation or the canonical commutation relations from this rule. This was done by Feynman.

Feynman showed that Dirac's quantum action was, for most cases of interest, simply equal to the classical action, appropriately discretized. This means that the classical action is the phase acquired by



quantum evolution between two fixed endpoints. He proposed to recover all of quantum mechanics from the following postulates:

The probability for an event is given by the squared length of a complex number called the "probability amplitude".

The probability amplitude is **given by** adding together the contributions of all the histories in configuration space.

The contribution of a history to the amplitude is proportional to  $e^{iS/\hbar}$ . While  $S$  is the action of that history, given by the time integral of the Lagrangian along the corresponding path.

In order to find the overall probability amplitude for a given process, then, one adds up, or integrates, the amplitude of postulate 3 over the space of *all* possible histories of the system in between the initial and final states, including histories that are absurd by classical standards. In calculating the amplitude for a single particle to go from one place to another in a given time, it would be correct to include histories in which the particle describes elaborate curlicues, histories in which the particle shoots off into outer space and flies back again, and so forth. The **path integral** assigns all of these histories amplitudes of *equal magnitude* but with varying phase, or argument of the complex number. The contributions that are wildly different from the classical history are suppressed only by the interference of similar, canceling histories

Feynman showed that this formulation of quantum mechanics is equivalent to the canonical approach to quantum mechanics, when the Hamiltonian is quadratic in the momentum. Amplitude computed according to Feynman's principles will also obey the Schrödinger equation for the Hamiltonian corresponding to the given action.

Classical action principles are puzzling because of their seemingly teleological quality: given a set of initial and final conditions one is able to find a unique path connecting them, as if the system somehow knows where it's going to end up and how it's going to get there. The path integral explains why this works in terms of quantum superposition. The system doesn't have to know in advance where it's going or what path it'll take: the path integral simply calculates the sum of the probability amplitudes for *every* possible path to *any* possible endpoint. After a long enough time, interference effects guarantee that only the contributions from the stationary points of the action give histories with appreciable probabilities.

#### Concrete formulation

Feynman's postulates can be interpreted as follows:

#### Time-slicing definition

For a particle in a smooth potential, the path integral is approximated by zig-zag paths, which in one dimension is a product of ordinary integrals. For the motion of the particle from position  $x_a$  at time  $t_a$  to  $x_b$  at time  $t_b$ , the time sequence

$$t_a = t_0 < t_1 < \dots < t_{n-1} < t_n < t_b = t_{n+1}$$

can be divided up into  $n + 1$  little segments  $t_j - t_{j-1}$ , where  $j = 1, \dots, n + 1$ , of fixed duration

$$\epsilon = \Delta t = \frac{t_b - t_a}{n+1}.$$

This process is called *time-slicing*.

An approximation for the path integral can be computed as proportional to

$$\int_{-\infty}^{+\infty} dx_1 \dots \int_{-\infty}^{+\infty} dx_n \exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \mathcal{L}(x(t), v(t), t) dt \right)$$

where  $\mathcal{L}(x, v, t)$  is the Lagrangian of the 1d-system with position variable  $x(t)$  and velocity  $v = \dot{x}(t)$  considered (see below), and  $dx_j$  corresponds to the position at the  $j$ -th time step, if the time integral is approximated by a sum of  $n$  terms

In the limit  $n \rightarrow \infty$ , this becomes a functional integral, which - apart from a nonessential factor - is directly the product of the probability amplitudes  $\langle x_a, t_a | x_b, t_b \rangle$  - more precisely, since one must work with a continuous spectrum, the respective densities - to find the quantum mechanical particle at  $t_a$  in the initial state  $x_a$  and at  $t_b$  in the final state  $x_b$ .

Actually  $\mathcal{L}$  is the classical Lagrangian of the one-dimensional system considered, also

$$\mathcal{L}(x, \dot{x}, t) = p \cdot \dot{x} - \mathcal{H}(x, p, t),$$

where  $\mathcal{H}$  is the Hamiltonian,

and the above-mentioned "zigzagging" corresponds to the appearance of the terms:

$$\exp \left( \frac{i}{\hbar} \epsilon \sum_{j=1}^n \mathcal{L} \left( \tilde{x}_j, \frac{x_j - x_{j-1}}{\epsilon}, j \right) \right)$$

In the Riemannian sum approximating the time integral, which are finally integrated over  $x_1$  to  $x_n$  with the integration measure  $dx_1 \dots dx_n$ ,  $\tilde{x}_j$  is an arbitrary value of the interval corresponding to  $j$ , e.g. its center,  $(x_j + x_{j-1})/2$ .

Thus, in contrast to classical mechanics, not only does the stationary path contribute, but actually all virtual paths between the initial and the final point also contribute.

Feynman's time-sliced approximation does not, however, exist for the most important quantum-mechanical path integrals of atoms, due to the singularity of the Coulomb potential  $e^2/r$  at the origin. Only after replacing the time  $t$  by another path-dependent pseudo-time parameter

$$s = \int \frac{dt}{r(t)}$$

the singularity is removed and a time-sliced approximation exists, that is exactly integrable, since it can be made harmonic by a simple coordinate transformation, as discovered in 1979 by İsmail Hakkı Duru and Hagen Kleinert. The combination of a path-dependent time transformation and a coordinate transformation is an important tool to solve many path integrals and is called generically the Duru-Kleinert transformation.

#### NOTATION :

$G_{13}$  : Category one of Schrodinger's Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger's Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Schrödinger's equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself. )

$G_{14}$  : Category two of Schrodinger's Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger's Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Schrödinger's equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself. )

$G_{15}$  Category Three of Schrodinger's Wave Equation (It is to be reiterated that we are classifying the universally applicable Schrödinger's Wave equation based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer

scrolls represent the totalistic transactions, Schrödinger's equation could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Wave Equation itself.

$T_{13}$  : Category one of Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Werner Heisenberg Matrix itself).

$T_{14}$  : Category Two of Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Werner Heisenberg Matrix itself).

$T_{15}$  : Category Three of Werner –Heisenberg matrix (It is to be reiterated that we are classifying the universally applicable Werner –Heisenberg matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Werner Heisenberg Matrix itself).

$G_{16}$  : Category one of Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

$G_{17}$  : Category Two of Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

$G_{18}$  : Category Three of Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It

is to be reiterated that we are classifying the universally applicable Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself).

$T_{16}$  : Category one of Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, penhance, predilection, proclivities, propensities, Solutional behaviour conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself in the foregoing).

$T_{17}$  : Category two of Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, penhance, predilection, proclivities, propensities, Solutional behaviour conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself in the foregoing)

$T_{18}$  : Category one of Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix (It is to be reiterated that we are classifying the universally applicable Relativistic Theories vis-a vis Richard Feynman's path integral formulation like Werner Heisenberg Matrix based on the individual systems or clusters of systems and their characteristics, penhance, predilection, proclivities, propensities, Solutional behaviour conceptual implications thereof, deployed contextual importance therein, anticipatory correlation and consubstationary causal association parameters, and features which have been decided, attributed and ascribed to. We recapitulate the Bank example of holistic conservativeness of assets and liabilities and application to the individual Debits and Credits which in themselves are conservative. Like the transfer scrolls represent the totalistic transactions, Relativistic Theories Richard Feynman's path integral formulation like Werner Heisenberg Matrix could be applied to various systems under investigation and a General Theory of the statement of the behaviour of the holistic system to which the theory has become applicable could be studied. This of course must lead to the generalizational consummation and realization of the Relativistic and concomitant and corresponding classificatory categorization of the Richard Feynman's path integral formulation like Werner Heisenberg Matrix itself in the foregoing

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$   
 $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$ : are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a''_{16})^{(2)}, (a''_{17})^{(2)}, (a''_{18})^{(2)}$   
 $(b''_{16})^{(2)}, (b''_{17})^{(2)}, (b''_{18})^{(2)}$  are Dissipation coefficients

### Schrödinger's Wave Equation And Werner Heisenberg Matrix

#### Module Numbered One Governing Equations:

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 565$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 566$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 567$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 568$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 569$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 570$$

$+(a''_{13})^{(1)}(T_{14}, t) =$  First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$  First detritions factor

### Feynman Path Integral Formulation And Relativistic Theories

#### Module Numbered Two

#### Governing Equations:

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 571$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 572$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 573$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 574$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 575$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 576$$

$+(a''_{16})^{(2)}(T_{17}, t) =$  First augmentation factor

$-(b''_{16})^{(2)}((G_{19}), t) =$  First detritions factor

#### Concatenated Equations For The Holistic Model

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ (a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \right] G_{13} \quad 577$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \right] G_{14} \quad 578$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} \right] G_{15} \quad 579$$

Where  $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$ ,  $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$ ,  $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$  are second augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[ (b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{+(b''_{16})^{(2,2)}(G_{19}, t)} \right] T_{13} \quad 580$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[ (b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{+(b''_{17})^{(2,2)}(G_{19}, t)} \right] T_{14} \quad 581$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{+(b''_{18})^{(2,2)}(G_{19}, t)} \right] T_{15} \quad 582$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detritions coefficients for category 1, 2 and 3

$\boxed{+(b''_{16})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{+(b''_{17})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{+(b''_{18})^{(2,2)}(G_{19}, t)}$  are second augmentation coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1)}(T_{14}, t)} \right] G_{16} \quad 583$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ (a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1)}(T_{14}, t)} \right] G_{17} \quad 584$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1)}(T_{14}, t)} \right] G_{18} \quad 585$$

Where  $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1)}(T_{14}, t)}$  are second detrition coefficients for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ (b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)} \boxed{-(b''_{13})^{(1,1)}(G, t)} \right] T_{16} \quad 586$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ (b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)} \boxed{-(b''_{14})^{(1,1)}(G, t)} \right] T_{17} \quad 587$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ (b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)} \boxed{-(b''_{15})^{(1,1)}(G, t)} \right] T_{18} \quad 588$$

Where  $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1)}(G, t)}$  are second detrition coefficients for category 1, 2 and 3

**Where we suppose**

$$(O) \quad (a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad 589$$



$$i, j = 13, 14, 15$$

(P) The functions  $(a_i'')^{(1)}, (b_i'')^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

(Q)  $\lim_{T_2 \rightarrow \infty} (a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$  590

$$\lim_{G \rightarrow \infty} (b_i'')^{(1)}(G, t) = (r_i)^{(1)}$$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$  are positive constants  
 and  $i = 13, 14, 15$

They satisfy Lipschitz condition: 591

$$|(a_i'')^{(1)}(T'_{14}, t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T'_{14} - T_{14}| e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(1)}(T'_{14}, t)$  and  $(a_i'')^{(1)}(T_{14}, t)$ .  $(T'_{14}, t)$  and  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that  $(a_i'')^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{13})^{(1)} = 1$  then the function  $(a_i'')^{(1)}(T_{14}, t)$ , the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 592

(R)  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 593

(S) There exists two constants  $(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  which together with  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$  and  $(\hat{B}_{13})^{(1)}$  and the constants  $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

**Where we suppose**

(T)  $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, i, j = 16, 17, 18$  594

(U) The functions  $(a_i'')^{(2)}, (b_i'')^{(2)}$  are positive continuous increasing and bounded. 595

**Definition of**  $(p_i)^{(2)}, (r_i)^{(2)}$ :

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 596$$

$$(b_i'')^{(2)}(G, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 597$$

(V)  $\lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 598$

$$\lim_{G \rightarrow \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 599$$

**Definition of**  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$  : 600

Where  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$  are positive constants and  $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17}' - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 601$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G_{19}') - (G_{19})| e^{-(\hat{M}_{16})^{(2)}t} \quad 602$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(2)}(T_{17}', t)$  and  $(a_i'')^{(2)}(T_{17}, t)$ .  $(T_{17}', t)$  and  $(T_{17}, t)$  are points belonging to the interval  $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that  $(a_i'')^{(2)}(T_{17}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{16})^{(2)} = 1$  then the function  $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$  :

(W)  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ , are positive constants 603

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$  : 604

There exists two constants  $(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  which together with  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$  and  $(\hat{B}_{16})^{(2)}$  and the constants  $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 605$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 606$$

**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 607

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$



**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 608

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**PROOF:** 609

Consider operator  $\mathcal{A}^{(1)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 610$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 611$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 612$$

By 613

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13}(T_{14}(s_{(13)}), s_{(13)})) G_{13}(s_{(13)}) \right) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + a''_{14}(T_{14}(s_{(13)}), s_{(13)})) G_{14}(s_{(13)}) \right) \right] ds_{(13)} \quad 614$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + a''_{15}(T_{14}(s_{(13)}), s_{(13)})) G_{15}(s_{(13)}) \right) \right] ds_{(13)} \quad 615$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})) T_{13}(s_{(13)}) \right) \right] ds_{(13)} \quad 616$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})) T_{14}(s_{(13)}) \right) \right] ds_{(13)} \quad 617$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})) T_{15}(s_{(13)}) \right) \right] ds_{(13)} \quad 618$$

Where  $s_{(13)}$  is the integrand that is integrated over an interval  $(0, t)$

**PROOF:**

Consider operator  $\mathcal{A}^{(2)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \quad 619$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 620$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 621$$

By 622

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16}(T_{17}(s_{(16)}), s_{(16)})) G_{16}(s_{(16)}) \right) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + a''_{17}(T_{17}(s_{(16)}), s_{(17)})) G_{17}(s_{(16)}) \right) \right] ds_{(16)} \quad 623$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + a''_{18}(T_{17}(s_{(16)}), s_{(16)})) G_{18}(s_{(16)}) \right) \right] ds_{(16)} \quad 624$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 625$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 626$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 627$$

Where  $s_{(16)}$  is the integrand that is integrated over an interval  $(0, t)$

(e) The operator  $\mathcal{A}^{(1)}$  maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that 628

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left( 1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left( e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 629

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{-\left( \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1

Analogous inequalities hold also for  $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(f) The operator  $\mathcal{A}^{(2)}$  maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} =$$

$$\left( 1 + (a_{16})^{(2)} t \right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left( e^{(\hat{M}_{16})^{(2)} t} - 1 \right) \quad 630$$

From which it follows that 631

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[ \left( (\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{-\left( \frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for  $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

It is now sufficient to take  $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$  and to choose 632

$(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ (\hat{P}_{13})^{(1)} + \left( (\hat{P}_{13})^{(1)} + G_j^0 \right) e^{-\left( \frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{13})^{(1)} \quad 633$$

$$\frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{Q}_{13})^{(1)} + T_j^0 \right) e^{-\left( \frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \quad 634$$

In order that the operator  $\mathcal{A}^{(1)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 34,35,36 into itself

The operator  $\mathcal{A}^{(1)}$  is a contraction with respect to the metric 635

$$d\left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{13})^{(1)}t} \right\}$$

Indeed if we denote 636

**Definition of  $\tilde{G}, \tilde{T}$ :**  $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{(a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} |(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)} (T_{14}^{(2)}, s_{(13)})| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)} \end{aligned}$$

Where  $s_{(13)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}} \left( (a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)} \right) d\left((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})\right) \quad 637$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a'_{13})^{(1)}$  and  $(b'_{13})^{(1)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  and  $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  respectively of  $\mathbb{R}_+$ . 638

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a'_i)^{(1)}$  and  $(b'_i)^{(1)}, i = 13, 14, 15$  depend only on  $T_{14}$  and respectively on  $G$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  639

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[- \int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0$$

**Definition of  $(\widehat{M}_{13})^{(1)}_1, (\widehat{M}_{13})^{(1)}_2$  and  $(\widehat{M}_{13})^{(1)}_3$ :** 640

**Remark 3:** if  $G_{13}$  is bounded, the same property have also  $G_{14}$  and  $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$  it follows  $\frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)}G_{14}$  and by integrating

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)}((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)}((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If  $G_{14}$  or  $G_{15}$  is bounded, the same property follows for  $G_{13}$ ,  $G_{15}$  and  $G_{13}$ ,  $G_{14}$  respectively.

**Remark 4:** If  $G_{13}$  is bounded, from below, the same property holds for  $G_{14}$  and  $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{14}$  is bounded from below. 641

**Remark 5:** If  $T_{13}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$  then  $T_{14} \rightarrow \infty$ . 642

**Definition of**  $(m)^{(1)}$  and  $\varepsilon_1$  :

Indeed let  $t_1$  be so that for  $t > t_1$

$$(b_{14})^{(1)} - (b''_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then  $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$  which leads to 643

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is unbounded. The same property holds for } T_{15} \text{ if } \lim_{t \rightarrow \infty} (b''_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take  $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$  and to choose 644

$(\widehat{P}_{16})^{(2)}$  and  $(\widehat{Q}_{16})^{(2)}$  large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[ (\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{16})^{(2)} \quad 645$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[ ((\widehat{Q}_{16})^{(2)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{16})^{(2)} \right] \leq (\widehat{Q}_{16})^{(2)} \quad 646$$

In order that the operator  $\mathcal{A}^{(2)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 34,35,36 into itself

The operator  $\mathcal{A}^{(2)}$  is a contraction with respect to the metric 647

$$d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(M_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(M_{16})^{(2)}t} \}$$

Indeed if we denote 648

**Definition of**  $\widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

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$$\begin{aligned} |\widetilde{G}_{16}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} ds_{(16)} + \\ &\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} + \\ &(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} + \\ &G_{16}^{(2)} | (a'_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a'_{16})^{(2)} (T_{17}^{(2)}, s_{(16)}) | e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)} \end{aligned}$$

Where  $s_{(16)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\begin{aligned} |(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\widehat{M}_{16})^{(2)} t} &\leq \\ \frac{1}{(\widehat{M}_{16})^{(2)}} &((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{k}_{16})^{(2)}) d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}) \right) \end{aligned} \quad 650$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a'_{16})^{(2)}$  and  $(b'_{16})^{(2)}$  depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$  and  $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$  respectively of  $\mathbb{R}_+$ . 651

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a'_i)^{(2)}$  and  $(b'_i)^{(2)}$ ,  $i = 16, 17, 18$  depend only on  $T_{17}$  and respectively on  $(G_{19})$  (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any t where  $G_i(t) = 0$  and  $T_i(t) = 0$  652

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(2)} - (a''_i)^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)} t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{16})^{(2)})_1, ((\widehat{M}_{16})^{(2)})_2$  and  $((\widehat{M}_{16})^{(2)})_3$  : 653

**Remark 3:** if  $G_{16}$  is bounded, the same property have also  $G_{17}$  and  $G_{18}$ . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)}) \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)} G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)} ((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)} ((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If  $G_{17}$  or  $G_{18}$  is bounded, the same property follows for  $G_{16}$ ,  $G_{18}$  and  $G_{16}$ ,  $G_{17}$  respectively.

**Remark 4:** If  $G_{16}$  is bounded, from below, the same property holds for  $G_{17}$  and  $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{17}$  is bounded from below. 654

**Remark 5:** If  $T_{16}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$  then  $T_{17} \rightarrow \infty$ . 655

**Definition of**  $(m)^{(2)}$  and  $\varepsilon_2$  :

Indeed let  $t_2$  be so that for  $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then  $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$  which leads to 656

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_2}$  By taking now  $\varepsilon_2$  sufficiently small one sees that  $T_{17}$  is 657  
 unbounded. The same property holds for  $T_{18}$  if  $\lim_{t \rightarrow \infty} (b_{18}'')^{(2)}((G_{19})(t), t) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

**Behavior of the solutions of equation 37 to 42** 658

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  :

(v)  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14}, t) + (a_{14}'')^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G, t) - (b_{14}'')^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

**Definition of**  $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$  : 659

(w) By  $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$  and respectively  $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**  $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$  : 660

By  $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$  and respectively  $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$  :- 661

(x) If we define  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$  by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously 662

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } \boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

Then the solution of Global equations satisfies the inequalities

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$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where  $(p_i)^{(1)}$  is defined by equation 25

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left( \frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[ e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \right) \leq G_{15}(t) \leq \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a'_{15})^{(1)})} \left[ e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$

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$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$

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$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$

666

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)} ((R_1)^{(1)} - (b'_{15})^{(1)})} \left[ e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

667

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)} ((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[ e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of**  $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$ :-

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$$\text{Where } (S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

### Behavior of the solutions

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  :

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(y)  $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

670

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$$

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**Definition of**  $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$  :

By  $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$  and respectively  $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$  the roots 672

(z) of the equations  $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$  673

and  $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$  and 674

**Definition of**  $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$  :

By  $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$  and respectively  $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$  the 675

roots of the equations  $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$  676

and  $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$  677

**Definition of**  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  :-

(aa) If we define  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  by 678

$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}$ , **if**  $(v_0)^{(2)} < (v_1)^{(2)}$  679

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}$ , **if**  $(v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}$ , 680

and 
$$(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$$

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}$ , **if**  $(\bar{v}_1)^{(2)} < (v_0)^{(2)}$  681

and analogously 682

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$ , **if**  $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$ , **if**  $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$ ,

and 
$$(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$ , **if**  $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$  683

Then the solution satisfies the inequalities 684

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$  is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 685$$

$$\left( \frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \quad 686$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a'_{18})^{(2)})} \left[ e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 687}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 688$$



$$\frac{(b_{18})^{(2)}T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)}-(b'_{18})^{(2)})} \left[ e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq \tag{689}$$

$$\frac{(a_{18})^{(2)}T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})} \left[ e^{((R_1)^{(2)}+(r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**Definition of**  $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$ :- 690

Where  $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$  691

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \tag{692}$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**PROOF:** we obtain 693

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of**  $v^{(1)}$  :- 
$$v^{(1)} = \frac{G_{13}}{G_{14}}$$

It follows

$$- \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$  :-

(o) For  $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t}}{1 + (C)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner, we get 694

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(p) If  $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$  we find like in the previous case, 695

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t}}{1 + (C)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t}} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}} \leq (\bar{v}_1)^{(1)}$$

(q) If  $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}]t}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $\nu^{(1)}(t)$  :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(1)}(t)$  :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{13})^{(1)} = (a''_{14})^{(1)}$ , then  $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$  and in this case  $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$  if in addition  $(\nu_0)^{(1)} = (\nu_1)^{(1)}$  then  $\nu^{(1)}(t) = (\nu_0)^{(1)}$  and as a consequence  $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$  this also defines  $(\nu_0)^{(1)}$  for the special case

Analogously if  $(b''_{13})^{(1)} = (b''_{14})^{(1)}$ , then  $(\tau_1)^{(1)} = (\tau_2)^{(1)}$  and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$  if in addition  $(u_0)^{(1)} = (u_1)^{(1)}$  then  $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$  This is an important consequence of the relation between  $(\nu_1)^{(1)}$  and  $(\bar{\nu}_1)^{(1)}$ , and definition of  $(u_0)^{(1)}$ .

**PROOF :** we obtain

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$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left( (a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a'_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of**  $\nu^{(2)}$  :-

$$\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$$

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It follows

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$$- \left( (a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} \right) \leq \frac{d\nu^{(2)}}{dt} \leq - \left( (a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

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**Definition of**  $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$  :-

$$(r) \text{ For } 0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{[-(a_{17})^{(2)}(\nu_1)^{(2)} - (\nu_0)^{(2)}]t}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}(\nu_1)^{(2)} - (\nu_0)^{(2)}]t}}, \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}$$

it follows  $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner, we get

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{[-(a_{17})^{(2)}(\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}]t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}(\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}]t}}, \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}$$

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From which we deduce  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$  701

(s) If  $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$  we find like in the previous case, 702

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (\bar{C})^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(t) If  $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 703

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(2)}(t)$  :- 704

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 705

**Definition of**  $u^{(2)}(t)$  :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in Global equations we get easily the result stated in the theorem.

**Particular case :** 706

If  $(a'_{16})^{(2)} = (a'_{17})^{(2)}$ , then  $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$  and in this case  $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$  if in addition  $(v_0)^{(2)} = (v_1)^{(2)}$  then  $v^{(2)}(t) = (v_0)^{(2)}$  and as a consequence  $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if  $(b'_{16})^{(2)} = (b'_{17})^{(2)}$ , then  $(\tau_1)^{(2)} = (\tau_2)^{(2)}$  and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$  if in addition  $(u_0)^{(2)} = (u_1)^{(2)}$  then  $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$  This is an important consequence of the relation between  $(v_1)^{(2)}$  and  $(\bar{v}_1)^{(2)}$

We can prove the following 707

**Theorem 3:** If  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  are independent on  $t$ , and the conditions (with the notations 25,26,27,28)

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with  $(p_{13})^{(1)}, (r_{14})^{(1)}$  as defined by equation are satisfied, then the system

**Theorem 3:** If  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  are independent on t, and the conditions (with the notations 708  
 25,26,27,28)

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 709$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 710$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0, \quad 711$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 712$$

with  $(p_{16})^{(2)}, (r_{17})^{(2)}$  as defined by equation are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0 \quad 713$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 714$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 715$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 716$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 717$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 718$$

has a unique positive solution, which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 719$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 720$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 721$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 722$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 723$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 724$$

has a unique positive solution, which is an equilibrium solution

**Proof:** 725

(a) Indeed the first two equations have a nontrivial solution  $G_{13}, G_{14}$  if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 726

(a) Indeed the first two equations have a nontrivial solution  $G_{16}, G_{17}$  if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Definition and uniqueness of  $T_{14}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(1)}(T_{14})$  being increasing, it follows

that there exists a unique  $T_{14}^*$  for which  $f(T_{14}^*) = 0$ . With this value, we obtain from the three first equations 727

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

**Definition and uniqueness of  $T_{17}^*$  :-** 728

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')(T_{17})$  being increasing, it follows that there exists a unique  $T_{17}^*$  for which  $f(T_{17}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]} \quad 729$$

(g) By the same argument, the equations admit solutions  $G_{13}, G_{14}$  if 730

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \\ [(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in  $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{14}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi(G^*) = 0$

(h) By the same argument, the equations admit solutions  $G_{16}, G_{17}$  if 731

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - \\ [(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in  $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$  must be replaced by their values. It is easy to see that  $\varphi$  is a decreasing function in  $G_{17}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi((G_{19})^*) = 0$  732

Finally we obtain the unique solution 733

$G_{14}^*$  given by  $\varphi(G^*) = 0, T_{14}^*$  given by  $f(T_{14}^*) = 0$  and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution of the global equations

Finally we obtain the unique solution

$G_{17}^*$  given by  $\varphi((G_{19})^*) = 0, T_{17}^*$  given by  $f(T_{17}^*) = 0$  and 734

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]} \quad 735$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 736$$

Obviously, these values represent an equilibrium solution of global equations

### ASYMPTOTIC STABILITY ANALYSIS

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  belong to  $C^{(1)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)}, \quad \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Neglecting the terms of power 2, we obtain from the global equations

$$\frac{d\mathbb{G}_{13}}{dt} = -((a_{13}'')^{(1)} + (p_{13})^{(1)})\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 738$$

$$\frac{d\mathbb{G}_{14}}{dt} = -((a_{14}'')^{(1)} + (p_{14})^{(1)})\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 739$$

$$\frac{d\mathbb{G}_{15}}{dt} = -((a_{15}'')^{(1)} + (p_{15})^{(1)})\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 740$$

$$\frac{d\mathbb{T}_{13}}{dt} = -((b_{13}'')^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(13)(j)})T_{13}^*\mathbb{G}_j \quad 741$$

$$\frac{d\mathbb{T}_{14}}{dt} = -((b_{14}'')^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15} (s_{(14)(j)})T_{14}^*\mathbb{G}_j \quad 742$$

$$\frac{d\mathbb{T}_{15}}{dt} = -((b_{15}'')^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(15)(j)})T_{15}^*\mathbb{G}_j \quad 743$$

### ASYMPTOTIC STABILITY ANALYSIS

744

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  belong to  $C^{(2)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

**Proof:** Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i \quad 745$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)}, \quad \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 746$$

taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{16}}{dt} = -((a_{16}'')^{(2)} + (p_{16})^{(2)})\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 747$$

$$\frac{d\mathbb{G}_{17}}{dt} = -((a_{17}'')^{(2)} + (p_{17})^{(2)})\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 748$$

$$\frac{d\mathbb{G}_{18}}{dt} = -((a_{18}'')^{(2)} + (p_{18})^{(2)})\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 749$$

$$\frac{d\mathbb{T}_{16}}{dt} = -((b_{16}'')^{(2)} - (r_{16})^{(2)})\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18} (s_{(16)(j)})T_{16}^*\mathbb{G}_j \quad 750$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)})T_{17}^*G_j \quad 751$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)})T_{18}^*G_j \quad 752$$

The characteristic equation of this system is 753

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ & \left[ ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right] \\ & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \right) \\ & + \left( ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\ & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\ & \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & \left( ((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & + \left( ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\ & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*) \\ & \left. \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \right\} = 0 \\ & + \\ & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\ & \left[ ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right] \\ & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\ & + \left( ((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\ & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \\ & \left( ((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) \\ & \left( ((\lambda)^{(2)})^2 + ((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) (\lambda)^{(2)} \right) \\ & + \left( ((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\ & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) ((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^*) \\ & \left. \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}T_{17}^* + (b_{17})^{(2)}s_{(16),(18)}T_{16}^* \right) \right\} = 0 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real

part, and this proves the theorem.

### Section 3: Platonic World-Physical World And Mental World: A Aletheia Akrasia Model

**We give a model based on Penrose's analysis. For details please refer The Large, The Small and The Human Mind by Roger Penrose**

#### NOTATION :

$G_{13}$  : Category One Of Platonic World (No One Has Seen Entire World Or Universe. It Is Based On This Concept That Classification Is Done. Everything Needs To Be Explored)

$G_{14}$  :Category Two Of Platonic World(Or The Part Thereof)

$G_{15}$  : Category Three Of Platonic World

$T_{13}$  : Human Mind Corresponding To The Above Classification Category One

$T_{14}$  : Human Mind Corresponding To The Above Classification Category two

$T_{15}$  : Human Mind Corresponding To The Above Classification Category three

$G_{16}$  : Category One Of Physical World

$G_{17}$  : Category Two Of Physical World

$G_{18}$  : Category Three Of Physical World

$T_{16}$  :Individuals With Mind Corresponding To Category One In The Foregoing

$T_{17}$  : Individuals With Mind Corresponding To Category two In The Foregoing

$T_{18}$  : Individuals With Mind Corresponding To Category Three In The Foregoing

$G_{20}$  : Category One Of Mental World

$G_{21}$  : Category Two Of Mental World

$G_{22}$  :Category Three Of Mental World

$T_{20}$  : Individuals With Mind Corresponding To Category One In The Foregoing

$T_{21}$  : Individuals With Mind Corresponding To Category One In The Foregoing

$T_{22}$  : Individuals With Mind Corresponding To Category One In The Foregoing

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$

$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}; (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$  are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$

$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$  are Dissipation coefficients

#### Platonic World And Corresponding Human Mind(Individual Consciousness)

##### Governing Equations:

The differential system of this model is now



$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 754$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 755$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 756$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 757$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 758$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 759$$

$+(a''_{13})^{(1)}(T_{14}, t) =$  First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$  First detritions factor

### Physical World And Concomitant Human Mind(Individual Consciousness Thereof)

#### Governing Equations:

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 760$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 761$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 762$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 763$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 764$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 765$$

$+(a''_{16})^{(2)}(T_{17}, t) =$  First augmentation factor

$-(b''_{16})^{(2)}((G_{19}), t) =$  First detritions factor

### Mental World And The Individual Consciousness Thereof(Note The Variation From Individual To Individual Which Depends on Their Penchance ,Predilection, Proclivities And Propensities, And Their Usage Of Sense Organs)

#### Governing Equations:

The differential system of this model is now

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 766$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 767$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 768$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 769$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 770$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 771$$

$+(a''_{20})^{(3)}(T_{21}, t) =$  First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$  First detritions factor

**First Module Concatenation:**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ (a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3)}(T_{21}, t)} \right] G_{13} \quad 772$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3)}(T_{21}, t)} \right] G_{14} \quad 773$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3)}(T_{21}, t)} \right] G_{15} \quad 774$$

Where  $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$  are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3)}(T_{21}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[ (b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{-(b''_{16})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3)}(G_{23}, t)} \right] T_{13} \quad 775$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[ (b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3)}(G_{23}, t)} \right] T_{14} \quad 776$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3)}(G_{23}, t)} \right] T_{15} \quad 777$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$  are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$  are third detritions coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1)}(T_{14}, t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \right] G_{16} \quad 778$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ (a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1)}(T_{14}, t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)} \right] G_{17} \quad 779$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1)}(T_{14}, t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)} \right] G_{18} \quad 780$$

Where  $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$  are first augmentation coefficients for category 1, 2 and 3

And  $\boxed{+(a''_{13})^{(1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1)}(T_{14}, t)}$  are second augmentation

coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)}$  are third augmentation coefficient for category 1, 2 and 3

**Second Module Concatenation:**

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ \boxed{(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)} \boxed{-(b''_{13})^{(1,1)}(G, t)} \boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)}} \right] T_{16} \quad 781$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ \boxed{(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)} \boxed{-(b''_{14})^{(1,1)}(G, t)} \boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)}} \right] T_{17} \quad 782$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ \boxed{(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)} \boxed{-(b''_{15})^{(1,1)}(G, t)} \boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)}} \right] T_{18} \quad 783$$

where  $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1)}(G, t)}$  are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)}$  are third detrition coefficients for category 1,2 and 3

**Third Module Concatenation:**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[ \boxed{(a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21}, t)} \boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}} \right] G_{20} \quad 784$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[ \boxed{(a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21}, t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}} \right] G_{21} \quad 785$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[ \boxed{(a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21}, t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}} \right] G_{22} \quad 786$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$  are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$  are third augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[ \boxed{(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{13})^{(1,1,1)}(G, t)}} \right] T_{20} \quad 787$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[ \boxed{(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{14})^{(1,1,1)}(G, t)}} \right] T_{21} \quad 788$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[ \boxed{(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{15})^{(1,1,1)}(G, t)}} \right] T_{22} \quad 789$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$  are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1)}(G, t)}$  are third detritions coefficients for category 1,2 and 3

Where we suppose

(X)  $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$  790  
 $i, j = 13, 14, 15$

(Y) The functions  $(a''_i)^{(1)}, (b''_i)^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

(Z)  $\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$  791

$$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where  $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$  are positive constants  
 and  $\boxed{i = 13, 14, 15}$

They satisfy Lipschitz condition:

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(1)}(T'_{14}, t)$  and  $(a''_i)^{(1)}(T_{14}, t)$ .  $(T'_{14}, t)$  and  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that  $(a''_i)^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{13})^{(1)} = 1$  then the function  $(a''_i)^{(1)}(T_{14}, t)$ , the first augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ :

(AA)  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ :

(BB) There exists two constants  $(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  which together with  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$  and  $(\hat{B}_{13})^{(1)}$  and the constants  $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(CC) \quad (a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16, 17, 18 \quad 795$$

(DD) The functions  $(a''_i)^{(2)}, (b''_i)^{(2)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(2)}, (r_i)^{(2)}$ :

$$(a''_i)^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 796$$

$$(b''_i)^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b'_i)^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 797$$

$$(EE) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 798$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(2)}(G_{19}, t) = (r_i)^{(2)} \quad 799$$

**Definition of**  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$  : 800

Where  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$  are positive constants and  $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a''_i)^{(2)}(T'_{17}, t) - (a''_i)^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T'_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 801$$

$$|(b''_i)^{(2)}((G_{19})', t) - (b''_i)^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G_{19}) - (G_{19})'| e^{-(\hat{M}_{16})^{(2)}t} \quad 802$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(2)}(T'_{17}, t)$  and  $(a''_i)^{(2)}(T_{17}, t)$ .  $(T'_{17}, t)$  and  $(T_{17}, t)$  are points belonging to the interval  $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that  $(a''_i)^{(2)}(T_{17}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{16})^{(2)} = 1$  then the function  $(a''_i)^{(2)}(T_{17}, t)$ , the SECOND augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$  :

$$(FF) \quad (\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, \text{ are positive constants} \quad 803$$

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of**  $(\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}$  :

There exists two constants  $(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  which together with  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$  and  $(\hat{B}_{16})^{(2)}$  and the constants  $(a_i)^{(2)}, (a'_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a'_i)^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 804$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b'_i)^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 805$$

Where we suppose

$$(GG) \quad (a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (b''_i)^{(3)} > 0, \quad 806$$

$$i, j = 20, 21, 22$$

(HH) The functions  $(a_i'')^{(3)}, (b_i'')^{(3)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(3)}, (r_i)^{(3)}$ :

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 807$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of**  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$  are positive constants and  $i = 20, 21, 22$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(3)}(T'_{21}, t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T'_{21}| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G'_{23}, t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} |G_{23} - G'_{23}| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(3)}(T'_{21}, t)$  and  $(a_i'')^{(3)}(T_{21}, t)$ .  $(T'_{21}, t)$  and  $(T_{21}, t)$  are points belonging to the interval  $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$ . It is to be noted that  $(a_i'')^{(3)}(T_{21}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{20})^{(3)} = 1$  then the function  $(a_i'')^{(3)}(T_{21}, t)$ , the THIRD augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ :

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(II)  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ , are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants  $(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  which together with  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$  and  $(\hat{B}_{20})^{(3)}$  and the constants  $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 811

**Definition of**  $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad T_i(0) = T_i^0 > 0$$

**Theorem 2:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 812

**Definition of**  $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 813

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

**Proof:**

Consider operator  $\mathcal{A}^{(1)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 814$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 815$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 816$$

By 817

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13}(s_{(13)}, s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + a''_{14}(s_{(13)}, s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \quad 818$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + a''_{15}(s_{(13)}, s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)} \quad 819$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \quad 820$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \quad 821$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \quad 822$$

Where  $s_{(13)}$  is the integrand that is integrated over an interval  $(0, t)$

**Proof:**

Consider operator  $\mathcal{A}^{(2)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \quad 823$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 824$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 825$$

By 826

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16}(s_{(16)}, s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)} \quad 827$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)} \quad 828$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 829$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 830$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 831$$

Where  $s_{(16)}$  is the integrand that is integrated over an interval  $(0, t)$

**Proof:**

Consider operator  $\mathcal{A}^{(3)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)}, \quad 832$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 833$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 834$$

By 835

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)} \quad 836$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)} \quad 837$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)} \quad 838$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)} \quad 839$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 840$$

Where  $s_{(20)}$  is the integrand that is integrated over an interval  $(0, t)$

(i) The operator  $\mathcal{A}^{(1)}$  maps the space of functions satisfying into itself. Indeed it is obvious that 841

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left( 1 + (a_{13})^{(1)}t \right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left( e^{(\hat{M}_{13})^{(1)}t} - 1 \right)$$

From which it follows that 842

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left( -\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1



Analogous inequalities hold also for  $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(j) The operator  $\mathcal{A}^{(2)}$  maps the space of functions satisfying equations into itself. Indeed it is obvious that 843

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s(16)} \right) \right] ds_{(16)} = \tag{844}$$

$$\left( 1 + (a_{16})^{(2)} t \right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left( e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 845

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[ \left( (\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{-\left( \frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for  $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(k) The operator  $\mathcal{A}^{(3)}$  maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that 846

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} \left( G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s(20)} \right) \right] ds_{(20)} =$$

$$\left( 1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left( e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that 847

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[ \left( (\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{-\left( \frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0} \right)} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for  $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

It is now sufficient to take  $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$  and to choose 848

$(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ (\hat{P}_{13})^{(1)} + \left( (\hat{P}_{13})^{(1)} + G_j^0 \right) e^{-\left( \frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{13})^{(1)} \tag{849}$$

$$\frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[ \left( (\hat{Q}_{13})^{(1)} + T_j^0 \right) e^{-\left( \frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \tag{850}$$

In order that the operator  $\mathcal{A}^{(1)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 34,35,36 into itself

The operator  $\mathcal{A}^{(1)}$  is a contraction with respect to the metric 851

$$d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)} t} \right\}$$

Indeed if we denote 852

**Definition of**  $\tilde{G}, \tilde{T} : (\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{(a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} |(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)} (T_{14}^{(2)}, s_{(13)})| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)} \end{aligned}$$

Where  $s_{(13)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} &\leq \\ \frac{1}{(\widehat{M}_{13})^{(1)}} &((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})) \end{aligned} \quad 853$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{13})^{(1)}$  and  $(b''_{13})^{(1)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  and  $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  respectively of  $\mathbb{R}_+$ . 854

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(1)}$  and  $(b''_i)^{(1)}, i = 13, 14, 15$  depend only on  $T_{14}$  and respectively on  $G$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  855

From 19 to 24 it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}]} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0 \end{aligned}$$

**Definition of**  $((\widehat{M}_{13})^{(1)})_1, ((\widehat{M}_{13})^{(1)})_2$  and  $((\widehat{M}_{13})^{(1)})_3$  : 856

**Remark 3:** if  $G_{13}$  is bounded, the same property have also  $G_{14}$  and  $G_{15}$ . indeed if

$G_{13} < ((\widehat{M}_{13})^{(1)})$  it follows  $\frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)}G_{14}$  and by integrating

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)}((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)}((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If  $G_{14}$  or  $G_{15}$  is bounded, the same property follows for  $G_{13}, G_{15}$  and  $G_{13}, G_{14}$  respectively.

**Remark 4:** If  $G_{13}$  is bounded, from below, the same property holds for  $G_{14}$  and  $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{14}$  is bounded from below. 857

**Remark 5:** If  $T_{13}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$  then  $T_{14} \rightarrow \infty$ . 858

**Definition of**  $(m)^{(1)}$  and  $\varepsilon_1$  :

Indeed let  $t_1$  be so that for  $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then  $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$  which leads to 859

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_1}$  By taking now  $\varepsilon_1$  sufficiently small one sees that  $T_{14}$  is unbounded. The same property holds for  $T_{15}$  if  $\lim_{t \rightarrow \infty} ((b_i'')^{(1)}(G(t), t)) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take  $\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}}, \frac{(b_j)^{(2)}}{(\bar{M}_{16})^{(2)}} < 1$  and to choose 860

$(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  large to have

$$\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ (\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\left( \frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{16})^{(2)} \quad 861$$

$$\frac{(b_j)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[ ((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)} \quad 862$$

In order that the operator  $\mathcal{A}^{(2)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 34,35,36 into itself

The operator  $\mathcal{A}^{(2)}$  is a contraction with respect to the metric 863

$$d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote 864

**Definition of**  $\tilde{G}_{19}, \tilde{T}_{19} : (\tilde{G}_{19}, \tilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 865

$$\begin{aligned} |\tilde{G}_{16}^{(1)} - \tilde{G}_{16}^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\ &\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &(a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} + \\ &G_{16}^{(2)} | (a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)}(T_{17}^{(2)}, s_{(16)}) | e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)} \end{aligned}$$

Where  $s_{(16)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\begin{aligned} & |(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\widehat{M}_{16})^{(2)}t} \leq \\ & \frac{1}{(\widehat{M}_{16})^{(2)}} ((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + \\ & (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}) d((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}) \end{aligned} \quad 866$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{16})^{(2)}$  and  $(b''_{16})^{(2)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$  and  $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$  respectively of  $\mathbb{R}_+$ . 867

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(2)}$  and  $(b''_i)^{(2)}$ ,  $i = 16, 17, 18$  depend only on  $T_{17}$  and respectively on  $(G_{19})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  868

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{[-\int_0^t \{(a'_i)^{(2)} - (a''_i)^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \quad \text{for } t > 0$$

**Definition of**  $((\widehat{M}_{16})^{(2)})_1$ ,  $((\widehat{M}_{16})^{(2)})_2$  and  $((\widehat{M}_{16})^{(2)})_3$  : 869

**Remark 3:** if  $G_{16}$  is bounded, the same property have also  $G_{17}$  and  $G_{18}$ . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)}) \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)}G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If  $G_{17}$  or  $G_{18}$  is bounded, the same property follows for  $G_{16}$ ,  $G_{18}$  and  $G_{16}$ ,  $G_{17}$  respectively.

**Remark 4:** If  $G_{16}$  is bounded, from below, the same property holds for  $G_{17}$  and  $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{17}$  is bounded from below. 870

**Remark 5:** If  $T_{16}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$  then  $T_{17} \rightarrow \infty$ . 871

**Definition of**  $(m)^{(2)}$  and  $\varepsilon_2$  :

Indeed let  $t_2$  be so that for  $t > t_2$

$$(b_{17})^{(2)} - (b'_i)^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then  $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$  which leads to 872

$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$  If we take  $t$  such that  $e^{-\varepsilon_2 t} = \frac{1}{2}$  it results

$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_2}$  By taking now  $\varepsilon_2$  sufficiently small one sees that  $T_{17}$  is unbounded. The same property holds for  $T_{18}$  if  $\lim_{t \rightarrow \infty} (b''_{18})^{(2)}((G_{19})(t), t) = (b'_{18})^{(2)}$  873

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take  $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$  and to choose 874

$(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ (\hat{P}_{20})^{(3)} + ((\hat{P}_{20})^{(3)} + G_j^0) e^{-\left( \frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{20})^{(3)} \quad 875$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[ ((\hat{Q}_{20})^{(3)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{20})^{(3)} \right] \leq (\hat{Q}_{20})^{(3)} \quad 876$$

In order that the operator  $\mathcal{A}^{(3)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 34,35,36 into itself

The operator  $\mathcal{A}^{(3)}$  is a contraction with respect to the metric 877

$$d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t} \}$$

Indeed if we denote 878

**Definition of  $\widetilde{G}_{23}, \widetilde{T}_{23}$ :**  $(\widetilde{G}_{23}, \widetilde{T}_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 879

$$\begin{aligned} |\widetilde{G}_{20}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{-(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned}$$

Where  $s_{(20)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\bar{M}_{20})^{(3)}t} &\leq \frac{1}{(\bar{M}_{20})^{(3)}} \left( (a_{20})^{(3)} + (a'_{20})^{(3)} + (\hat{A}_{20})^{(3)} + \right. \\ &\left. (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)} \right) d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}); (G_{23})^{(2)}, (T_{23})^{(2)} \right) \end{aligned} \quad 880$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (34,35,36) the result

follows

**Remark 1:** The fact that we supposed  $(a''_{20})^{(3)}$  and  $(b''_{20})^{(3)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$  and  $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$  respectively of  $\mathbb{R}_+$ . 881

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(3)}$  and  $(b''_i)^{(3)}$ ,  $i = 20, 21, 22$  depend only on  $T_{21}$  and respectively on  $(G_{23})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  882

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t (a'_i)^{(3)} - (a''_i)^{(3)}(T_{21}(s_{(20)}), s_{(20)}) ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(3)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{20})^{(3)})_1$ ,  $((\widehat{M}_{20})^{(3)})_2$  and  $((\widehat{M}_{20})^{(3)})_3$  : 883

**Remark 3:** if  $G_{20}$  is bounded, the same property have also  $G_{21}$  and  $G_{22}$ . indeed if

$$G_{20} < ((\widehat{M}_{20})^{(3)})_1 \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)}G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way, one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If  $G_{21}$  or  $G_{22}$  is bounded, the same property follows for  $G_{20}$ ,  $G_{22}$  and  $G_{20}$ ,  $G_{21}$  respectively.

**Remark 4:** If  $G_{20}$  is bounded, from below, the same property holds for  $G_{21}$  and  $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{21}$  is bounded from below. 884

**Remark 5:** If  $T_{20}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b''_i)^{(3)}((G_{23})(t), t)) = (b'_{21})^{(3)}$  then  $T_{21} \rightarrow \infty$ . 885

**Definition of**  $(m)^{(3)}$  and  $\varepsilon_3$  :

Indeed let  $t_3$  be so that for  $t > t_3$

$$(b_{21})^{(3)} - (b''_i)^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then  $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$  which leads to 886

$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_3}$  By taking now  $\varepsilon_3$  sufficiently small one sees that  $T_{21}$  is unbounded. The same property holds for  $T_{22}$  if  $\lim_{t \rightarrow \infty} (b''_{22})^{(3)}((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

**Behavior of the solutions of equation 37 to 42**

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**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  :

(bb)  $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$  four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

**Definition of**  $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$  :

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(cc) By  $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$  and respectively  $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**  $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$  :

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By  $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$  and respectively  $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$  the roots of the equations  $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$  and  $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of**  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$  :-

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(dd) If we define  $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$  by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously

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$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } (u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

are defined by 59 and 61 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

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$$G_{13}^0 e^{((s_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(s_1)^{(1)}t}$$

where  $(p_i)^{(1)}$  is defined by equation 25

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((s_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(s_1)^{(1)}t}$$

$$\left( \frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((s_1)^{(1)} - (p_{13})^{(1)} - (s_2)^{(1)})} \left[ e^{((s_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(s_2)^{(1)}t} \right] + G_{15}^0 e^{-(s_2)^{(1)}t} \leq G_{15}(t) \leq \right.$$

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$$\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})} [e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 894$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 895$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})} [e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq \quad 896$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})} [e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of**  $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$ :- 897

$$\text{Where } (S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

If we denote and define 898

**Definition of**  $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  : 899

(ee)  $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$  four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)} \quad 900$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)} \quad 901$$

**Definition of**  $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$  :

By  $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$  and respectively  $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$  the roots 902

(ff) of the equations  $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$  903

and  $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$  and 904

**Definition of**  $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$  :

By  $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$  and respectively  $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$  the 905

roots of the equations  $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$  906

and  $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$  907

**Definition of**  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  :- 908

(gg) If we define  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  by 909

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)} \quad 910$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}, \quad 911$$



$$\text{and } \boxed{(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} \quad 912$$

and analogously 913

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

$$\text{and } \boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} \quad 914$$

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities 915

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$  is defined by equation 25

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 916$$

$$\left( \frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a'_{18})^{(2)})} \left[ e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t} \quad 917$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 918}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 919$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[ e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq \quad 920$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[ e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**Definition of**  $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$ :-

$$\text{Where } (S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)} \quad 921$$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 922$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

If we denote and define 923

**Definition of**  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  :

(hh)  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

**Definition of**  $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$  :

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(ii) By  $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$  and respectively  $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$  the roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and  $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$  and

By  $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$  and respectively  $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$  the

roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and  $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of**  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  :-

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(jj) If we define  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$ , **if**  $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , **if**  $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$ ,

and  $(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$ , **if**  $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously

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$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$ , **if**  $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , **if**  $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$ , and  $(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$ , **if**  $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$  is defined by equation 25

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 928$$

$$\left( \frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)} ((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[ e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)} ((S_1)^{(3)} - (a'_{22})^{(3)})} \left[ e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t} \quad 929$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 930$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 931$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)} ((R_1)^{(3)} - (b'_{22})^{(3)})} \left[ e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq \quad 932$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)} ((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[ e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of**  $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$ :-

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$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Proof:** we obtain

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$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of**  $v^{(1)}$  :-  $v^{(1)} = \frac{G_{13}}{G_{14}}$

It follows

$$- \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left( (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$  :-

(u) For  $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get

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$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce  $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(v) If  $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$  we find like in the previous case,

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$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (\bar{v}_1)^{(1)}$$

(w) If  $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain

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$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(1)}(t)$  :-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(1)}(t)$  :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}$$

Now, using this result and replacing it in 19, 20, 21, 22, 23, and 24 we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{13})^{(1)} = (a''_{14})^{(1)}$ , then  $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$  and in this case  $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$  if in addition  $(v_0)^{(1)} = (v_1)^{(1)}$  then  $v^{(1)}(t) = (v_0)^{(1)}$  and as a consequence  $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$  this also defines  $(v_0)^{(1)}$  for the special case

Analogously if  $(b''_{13})^{(1)} = (b''_{14})^{(1)}$ , then  $(\tau_1)^{(1)} = (\tau_2)^{(1)}$  and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$  if in addition  $(u_0)^{(1)} = (u_1)^{(1)}$  then  $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$  This is an important consequence of the relation between  $(v_1)^{(1)}$  and  $(\bar{v}_1)^{(1)}$ , and definition of  $(u_0)^{(1)}$ .

**Proof :** From 19, 20, 21, 22, 23, 24 we obtain

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$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left( (a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

**Definition of**  $v^{(2)}$  :- 
$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

It follows

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$$- \left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

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**Definition of**  $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$  :-

(x) For  $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}} , \quad (C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner, we get

941

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} , \quad (\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce  $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

942

(y) If  $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$  we find like in the previous case,

943

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (\bar{C})^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(z) If  $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 944

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(2)}(t)$  :- 945

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 946

**Definition of**  $u^{(2)}(t)$  :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 947

If  $(a''_{16})^{(2)} = (a''_{17})^{(2)}$ , then  $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$  and in this case  $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$  if in addition  $(v_0)^{(2)} = (v_1)^{(2)}$  then  $v^{(2)}(t) = (v_0)^{(2)}$  and as a consequence  $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if  $(b''_{16})^{(2)} = (b''_{17})^{(2)}$ , then  $(\tau_1)^{(2)} = (\tau_2)^{(2)}$  and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$  if in addition  $(u_0)^{(2)} = (u_1)^{(2)}$  then  $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$  This is an important consequence of the relation between  $(v_1)^{(2)}$  and  $(\bar{v}_1)^{(2)}$

**Proof :** From 19,20,21,22,23,24 we obtain 948

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left( (a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right) - (a''_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

**Definition of**  $v^{(3)}$  :- 949

$$\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$$

It follows

$$- \left( (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left( (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

From which one obtains 950

(a) For  $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}]t}}, \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows  $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner, we get

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$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}, \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

**Definition of  $(\bar{v}_1)^{(3)}$  :-**

From which we deduce  $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If  $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$  we find like in the previous case,

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$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}} \leq (\bar{v}_1)^{(3)}$$

(c) If  $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain

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$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

**Definition of  $v^{(3)}(t)$  :-**

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of  $u^{(3)}(t)$  :-**

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

**Particular case :**

If  $(a_{20}''^{(3)}) = (a_{21}''^{(3)})$ , then  $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$  and in this case  $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$  if in addition  $(v_0)^{(3)} = (v_1)^{(3)}$  then  $v^{(3)}(t) = (v_0)^{(3)}$  and as a consequence  $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if  $(b_{20}''^{(3)}) = (b_{21}''^{(3)})$ , then  $(\tau_1)^{(3)} = (\tau_2)^{(3)}$  and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$  if in addition  $(u_0)^{(3)} = (u_1)^{(3)}$  then  $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$  This is an important consequence of the relation between  $(v_1)^{(3)}$  and  $(\bar{v}_1)^{(3)}$

We can prove the following:

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If  $(a_i'')^{(1)}$  and  $(b_i'')^{(1)}$  are independent on  $t$ , and the conditions (with the notations 25,26,27,28)

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with  $(p_{13})^{(1)}, (r_{14})^{(1)}$  as defined by equation 25 are satisfied, then the system

If  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  are independent on  $t$ , and the conditions (with the notations 25,26,27,28)

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0 \quad 955$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0 \quad 956$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0, \quad 957$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0 \quad 958$$

with  $(p_{16})^{(2)}, (r_{17})^{(2)}$  as defined by equation 25 are satisfied, then the system

If  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  are independent on  $t$ , and the conditions (with the notations 25,26,27,28)

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0 \quad 959$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with  $(p_{20})^{(3)}, (r_{21})^{(3)}$  as defined by equation 25 are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0 \quad 960$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 961$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 962$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 963$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 964$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 965$$

has a unique positive solution, which is an equilibrium solution for the system (19 to 24)

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 966$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 967$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 968$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 969$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 970$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 971$$

has a unique positive solution , which is an equilibrium solution for (19 to 24)

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 972$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 973$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 974$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 975$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 976$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 977$$

has a unique positive solution , which is an equilibrium solution for (19 to 24)

**Proof:** 978

(a) Indeed the first two equations have a nontrivial solution  $G_{13}, G_{14}$  if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

(b) Indeed the first two equations have a nontrivial solution  $G_{16}, G_{17}$  if 979

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

(a) Indeed the first two equations have a nontrivial solution  $G_{20}, G_{21}$  if 980

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Definition and uniqueness of  $T_{14}^*$  :-** 981

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a'_i)^{(1)}(T_{14})$  being increasing, it follows that there exists a unique  $T_{14}^*$  for which  $f(T_{14}^*) = 0$ . With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

**Definition and uniqueness of  $T_{17}^*$  :-** 982

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a'_i)^{(2)}(T_{17})$  being increasing, it follows that there exists a unique  $T_{17}^*$  for which  $f(T_{17}^*) = 0$ . With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 983$$

**Definition and uniqueness of  $T_{21}^*$  :-** 984



After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(1)}(T_{21})$  being increasing, it follows that there exists a unique  $T_{21}^*$  for which  $f(T_{21}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

(l) By the same argument, the equations 92,93 admit solutions  $G_{13}, G_{14}$  if 985

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$[(b'_{13})^{(1)}(b'_{14})^{(1)}(G) + (b'_{14})^{(1)}(b'_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in  $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{14}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi(G^*) = 0$

(m) By the same argument, the equations 92,93 admit solutions  $G_{16}, G_{17}$  if 986

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b'_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b'_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in  $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{17}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{17}^*$  such that  $\varphi((G_{19})^*) = 0$  987

(n) By the same argument, the equations 92,93 admit solutions  $G_{20}, G_{21}$  if 988

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b'_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b'_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in  $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{21}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{21}^*$  such that  $\varphi((G_{23})^*) = 0$

Finally we obtain the unique solution of 89 to 94 989

$G_{14}^*$  given by  $\varphi(G^*) = 0, T_{14}^*$  given by  $f(T_{14}^*) = 0$  and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

Finally we obtain the unique solution of 89 to 94 990

$G_{17}^*$  given by  $\varphi((G_{19})^*) = 0, T_{17}^*$  given by  $f(T_{17}^*) = 0$  and 991

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]} \quad 992$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 993$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

Finally we obtain the unique solution of 89 to 94 994

$G_{21}^*$  given by  $\varphi((G_{23})^*) = 0$ ,  $T_{21}^*$  given by  $f(T_{21}^*) = 0$  and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

### ASYMPTOTIC STABILITY ANALYSIS 995

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i')^{(1)}$  and  $(b_i'')^{(1)}$  belong to  $C^{(1)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 996$$

$$\frac{d\mathbb{G}_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 997$$

$$\frac{d\mathbb{G}_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 998$$

$$\frac{d\mathbb{T}_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(13)(j)})T_{13}^*\mathbb{G}_j \quad 999$$

$$\frac{d\mathbb{T}_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15} (s_{(14)(j)})T_{14}^*\mathbb{G}_j \quad 1000$$

$$\frac{d\mathbb{T}_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(15)(j)})T_{15}^*\mathbb{G}_j \quad 1001$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(2)}$  and  $(b_i'')^{(2)}$  belong to  $C^{(2)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

**Proof:** Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i \quad 1002$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)}, \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}(G_{19}^*) = s_{ij} \quad 1003$$

taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^* T_{17} \quad 1004$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^* T_{17} \quad 1005$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^* T_{17} \quad 1006$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)(j)}) T_{16}^* G_j \quad 1007$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)}) T_{17}^* G_j \quad 1008$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)}) T_{18}^* G_j \quad 1009$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  belong to  $C^{(3)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable. 1010

Proof: Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}(G_{23}^*) = s_{ij}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^* T_{21} \quad 1011$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21} \quad 1012$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21} \quad 1013$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)}) T_{20}^* G_j \quad 1014$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)}) T_{21}^* G_j \quad 1015$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)}) T_{22}^* G_j \quad 1016$$

The characteristic equation of this system is 1017

$$((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}) \{ ((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ \left[ ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (q_{13})^{(1)} G_{13}^* \right] \}$$

$$\begin{aligned}
 & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \right) \\
 & + \left( ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\
 & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\
 & \left( ((\lambda)^{(1)})^2 + ( (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} ) (\lambda)^{(1)} \right) \\
 & \left( ((\lambda)^{(1)})^2 + ( (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)} ) (\lambda)^{(1)} \right) \\
 & + \left( ((\lambda)^{(1)})^2 + ( (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} ) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\
 & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) \left( (a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^* \right) \\
 & \left( ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \} = 0 \\
 & + \\
 & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}) \{ ((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\
 & \left[ \left( ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right) \right] \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\
 & + \left( ((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \\
 & \left( ((\lambda)^{(2)})^2 + ( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} ) (\lambda)^{(2)} \right) \\
 & \left( ((\lambda)^{(2)})^2 + ( (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} ) (\lambda)^{(2)} \right) \\
 & + \left( ((\lambda)^{(2)})^2 + ( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} ) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\
 & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) \left( (a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^* \right) \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}T_{17}^* + (b_{17})^{(2)}s_{(16),(18)}T_{16}^* \right) \} = 0 \\
 & + \\
 & ((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)}) \{ ((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}) \\
 & \left[ \left( ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^* \right) \right] \\
 & \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(21)}T_{21}^* + (b_{21})^{(3)}s_{(20),(21)}T_{21}^* \right) \\
 & + \left( ((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G_{20}^* + (a_{20})^{(3)}(q_{21})^{(3)}G_{21}^* \right)
 \end{aligned}$$

$$\begin{aligned} & \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(20)}T_{21}^* + (b_{21})^{(3)}s_{(20),(20)}T_{20}^* \right) \\ & \left( ((\lambda)^{(3)})^2 + ( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} ) (\lambda)^{(3)} \right) \\ & \left( ((\lambda)^{(3)})^2 + ( (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} ) (\lambda)^{(3)} \right) \\ & + \left( ((\lambda)^{(3)})^2 + ( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} ) (\lambda)^{(3)} \right) (q_{22})^{(3)} G_{22} \\ & + ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) \left( (a_{22})^{(3)}(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^* \right) \\ & \left. \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(22)}T_{21}^* + (b_{21})^{(3)}s_{(20),(22)}T_{20}^* \right) \right\} = 0 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

#### SECTION 4:

#### Time Dependent Schrodinger's Equation: Variant Virtuositities And Concomitant Complementarities And Singularities

**Abstract:** Each of these three rows is a wave function which satisfies the time-dependent Schrödinger equation for a harmonic. Left: The real part (blue) and imaginary part (red) of the wave function. Right: The probability distribution of finding the particle with this wave function at a given position. The top two rows are examples of stationary, which correspond to standing waves. The bottom row an example of a state which is not a stationary state. The right column illustrates why stationary states are called "stationary". The time-dependent Schrödinger equation predicts that wavefunctions can form standing waves, called stationary states (also called "orbitals", as in atomic orbitals or molecular orbitals). These states are important in their own right, and moreover if the stationary states are classified and understood, then it becomes easier to solve the time-dependent Schrödinger equation for any state. The time-independent Schrödinger equation is the equation describing stationary states. (It is only used when the Hamiltonian itself is not dependent on time.). On the one hand, you can see QM as 0+1 (one temporal dimension) QFT, where the position operators (and their conjugate momenta) in the Heisenberg picture plays the role of the fields (and their conjugate momenta) in the QFT. You can check, for instance, that spatial rotational symmetry in the quantum mechanical theory is translated to an internal symmetry in the QFT. On the other hand, you can take the non-relativistic limit (by the way, ugly name because Galilean relativity is as relativistic as Special relativity) of the Klein-Gordon or Dirac theory to get the Schrödinger QFT, where  $\phi$  (in your notation) is a quantum field instead of a wave function. There is a chapter in Srednicki's book where this issue is raised in a simple and nice way. There, you can also read about spin-statistic and the wave function of multi-particle states. Let me add some equations that hopefully clarify (I'm using your notation and of course can be wrong factors, units, etc.): The quantum field is:  $\phi \sim \int d^3p a e^{-i(p^2/(2m) \cdot t - p \cdot x)}$

The Hamiltonian is:  $H \sim i\phi^\dagger \partial_t \phi - (1/2m) \partial_i \phi^\dagger \partial_i \phi \sim \int d^3p p^2 / (2m) a^\dagger p a$  The evolution of the quantum field is given by:  $i\partial_t \phi \sim [\phi, H] \sim -\nabla^2 \phi / (2m)$  1-particle states are given by:  $|1p\rangle \sim \int d^3p f(t,p) a^\dagger p |0\rangle$  (one can analogously define multi-particle states) This state verifies the Schrödinger equation:  $H|1p\rangle = i\partial_t |1p\rangle$  iff  $i\partial_t f(t,x) \sim -\nabla^2 f(t,x) / (2m)$  where  $f(t,x)$  is the spatial Fourier transformed of  $f(t,p)$ .  $f(t,x)$  is a wave function, while  $\phi(t,x)$  is a quantum field. When the Hamiltonian operator acts on the wave function  $\Psi$ , the result might be proportional to the same wave function  $\Psi$ . If it is, then  $\Psi$  is a stationary state, and the proportionality constant, E, is the

energy of the state  $\Psi$ . The time-independent Schrödinger equation is discussed further below. In linear algebra terminology, this equation is an eigen value equation. As before, the most famous manifestation is the non-relativistic Schrödinger equation for a single particle moving in an electric field (but not a magnetic field):

The Schrödinger equation predicts that if certain properties of a system are measured, the result may be quantized, meaning that only specific discrete values can occur. One example is energy quantization: the energy of an electron in an atom is always one of the quantized energy levels, a fact discovered via atomic spectroscopy. (Energy quantization is discussed below.) Another example is quantization of angular momentum. This was an assumption in the earlier Bohr model of the atom, but it is a prediction of the Schrödinger equation. Not every measurement gives a quantized result in quantum mechanics. For example, position, momentum, time, and (in some situations) energy can have any value across a continuous range. The overall form of the equation is not unusual or unexpected. The terms of the nonrelativistic Schrödinger equation can be interpreted as: (Total energy) = (kinetic energy) + (potential energy), In this respect, it is just the same as in classical physics. For example, a frictionless roller coaster has constant total energy; therefore it travels slower (low kinetic energy) when it is high off the ground (high gravitational potential energy) and vice versa

### **INTRODUCTION:**

An **evanescent wave** is a near-field standing wave with an intensity that exhibits exponential decay with distance from the boundary at which the wave was formed. Evanescent waves are a general property of wave-equations, and can in principle occur in any context to which a wave-equation applies. They are formed at the boundary between two media with different wave motion properties, and are most intense within one third of a wavelength from the surface of formation. In particular, evanescent waves can occur in the contexts of optics and other forms of electromagnetic radiation, acoustics, quantum mechanics, and "waves on strings".

### **Evanescence wave applications**

In optics and acoustics, evanescent waves are formed when waves traveling in a medium undergo total internal reflection at its boundary because they strike it at an angle greater than the so-called *critical angle*. The physical explanation for the existence of the evanescent wave is that the electric and magnetic fields (or pressure gradients, in the case of acoustical waves) cannot be discontinuous at a boundary, as would be the case if there was no evanescent wave field. **In quantum mechanics, the physical explanation is exactly analogous—the Schrödinger wave-function representing particle motion normal to the boundary cannot be discontinuous at the boundary.**

Electromagnetic evanescent waves have been used to exert optical radiation pressure on small particles to **trap** them for experimentation, or to cool them to very low temperatures, and to illuminate very small objects such as biological cells for microscopy (as in the total internal reflection fluorescence microscope). The evanescent wave from an optical fiber can be used in a gas sensor, and evanescent waves figure in the infrared spectroscopy technique known as attenuated total reflectance.

In electrical engineering, evanescent waves are found in the near-field region within one third of a wavelength of any radio antenna. During normal operation, an antenna emits electromagnetic fields into the surrounding near field region, and a portion of the field energy is reabsorbed, while the remainder is radiated as EM waves.

In quantum mechanics, the evanescent-wave solutions of the Schrödinger equation give rise to the phenomenon of wave-mechanical tunneling. In microscopy, systems that capture the information contained in evanescent waves can be used to create super-resolution images. Matter radiates both propagating and evanescent electromagnetic waves. Conventional optical systems capture only the information in the propagating waves and hence are subject to the diffraction limit. Systems that capture the information contained in evanescent waves, such as the super

lens and near field scanning optical microscopy, can overcome the diffraction limit; however these systems are then limited by the system's ability to accurately capture the evanescent waves. The limitation on their resolution is given by

$$k \propto \frac{1}{d} \ln \frac{1}{\delta},$$

Where  $k$  is the maximum wave vector that can be resolved,  $d$  is the distance between the object and the sensor, and  $\delta$  is a measure of the quality of the sensor? More generally, practical applications of evanescent waves can be classified in the following way:

Those in which the energy associated with the wave is used to excite some other phenomenon within the region of space where the original traveling wave becomes evanescent (for example, as in the total internal reflection fluorescence microscope) Those in which the evanescent wave couples two media in which traveling waves are allowed, and hence permits the transfer of energy or a particle between the media (depending on the wave equation in use), even though no traveling-wave solutions are allowed in the region of space between the two media. An example of this is so-called *wave*, and is known generally as *evanescent wave coupling*. For example, consider total internal reflection in two dimensions, with the interface between the media lying on the  $x$  axis, the normal along  $y$ , and the polarization along  $z$ . One might naively expect that for angles leading to total internal reflection, the solution would consist of an incident wave and a reflected wave, with no transmitted wave at all, but there is no such solution that obeys Maxwell's equations. Maxwell's equations in a dielectric medium impose a boundary condition of continuity for the components of the fields  $E_{||}$ ,  $H_{||}$ ,  $D_y$ , and  $B_y$ . For the polarization considered in this example, the conditions on  $E_{||}$  and  $B_y$  are satisfied if the reflected wave has the same amplitude as the incident one, because these components of the incident and reflected waves superimpose destructively. Their  $H_x$  components, however, superimpose constructively, so there can be no solution without a non-vanishing transmitted wave. The transmitted wave cannot, however, be a sinusoidal wave, since it would then transport energy away from the boundary, but since the incident and reflected waves have equal energy, this would violate conservation of energy. We therefore conclude that the transmitted wave must be a non-vanishing solution to Maxwell's equations that is not a traveling wave, and the only such solutions in a dielectric are those that decay exponentially: evanescent waves.

Mathematically, evanescent waves can be characterized by a wave vector where one or more of the vector's components have an imaginary value. Because the vector has imaginary components, it may have a magnitude that is less than its real components. If the angle of incidence exceeds the critical angle, then the wave vector of the transmitted wave has the form

$$\mathbf{k} = k_y \hat{\mathbf{y}} + k_x \hat{\mathbf{x}} = i\alpha \hat{\mathbf{y}} + \beta \hat{\mathbf{x}},$$

Which represents an evanescent wave because the  $y$  component is imaginary? (Here  $\alpha$  and  $\beta$  are real and  $i$  represents the imaginary unit.)

For example, if the polarization is perpendicular to the plane of incidence, then the electric field of any of the waves (incident, reflected, or transmitted) can be expressed as

$$\mathbf{E}(\mathbf{r}, t) = \text{Re} \{ E(\mathbf{r}) e^{i\omega t} \} \hat{\mathbf{z}}$$

Where  $\hat{\mathbf{z}}$  is the unit vector in the  $z$  direction.

Substituting the evanescent form of the wave vector  $\mathbf{k}$  (as given above), we find for the transmitted wave:

$$E(\mathbf{r}) = E_o e^{-i(i\alpha y + \beta x)} = E_o e^{\alpha y - i\beta x}$$

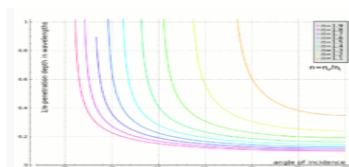
Where  $\alpha$  is the *attenuation constant* and  $\beta$  is the *propagation constant*.

### Evanescent-wave coupling

In optics, *evanescent-wave coupling* is a process by which electromagnetic waves are transmitted



from one medium to another by means of the evanescent, exponentially decaying electromagnetic field.



plot of 1/e-penetration depth of the evanescent wave against angle of incidence in units of wavelength for different refraction indices

Coupling is usually accomplished by placing two or more electromagnetic elements such as optical waveguides close together so that the evanescent field generated by one element does not decay much before it reaches the other element. With waveguides, if the receiving waveguide can support modes of the appropriate frequency, the evanescent field gives rise to propagating-wave modes, thereby connecting (or coupling) the wave from one waveguide to the next.

Evanescent-wave coupling is fundamentally identical to near field interaction in electromagnetic field theory. Depending on the impedance of the radiating source element, the evanescent wave is either predominantly electric (capacitive) or magnetic (inductive), unlike in the far field where these components of the wave eventually reach the ratio of the impedance of free space and the wave propagates radioactively. The evanescent wave coupling takes place in the non-radiative field near each medium and as such is always associated with matter; i.e., with the induced currents and charges within a partially reflecting surface. This coupling is directly analogous to the coupling between the primary and secondary coils of a transformer, or between the two plates of a capacitor. Mathematically, the process is the same as that of quantum tunneling, except with electromagnetic waves instead of quantum-mechanical wavefunctions.

Evanescent wave coupling is commonly used in photonic and nanophotonics devices as waveguide sensors. Evanescent wave coupling is used to excite, for example, dielectric microsphere resonators. A typical application is resonant energy transfer, useful, for instance, for charging electronic gadgets without wires. A particular implementation of this is WiTricity; the same idea is also used in some Tesla coils. Evanescent coupling, as near field interaction, is one of the concerns in electromagnetic compatibility. Evanescent wave coupling plays a major role in the theoretical explanation of extraordinary optical transmission

### The Time-Dependent Schrödinger Equation

We are now ready to consider the time-dependent Schrödinger equation. Although we were able to derive the single-particle time-independent Schrödinger equation starting from the classical wave equation and the de Broglie relation, the time-dependent Schrödinger equation cannot be derived using elementary methods and is generally given as a postulate of quantum mechanics. It is possible to show that the time-dependent equation is at least *reasonable* if not derivable, but the arguments are rather involved (cf. Merzbacher [2], Section 3.2; Levine [3], Section 1.4).

The single-particle three-dimensional time-dependent Schrödinger equation is

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) + V(\mathbf{r})\psi(\mathbf{r}, t) \dots\dots\dots 21$$

Where  $V$  is assumed to be a real function and represents the potential energy of the system (a complex function  $V$  will act as a source or sink for probability, as shown in Merzbacher [2], problem 4.1). *Wave Mechanics* is the branch of quantum mechanics with equation (21) as its



dynamical law. Note that equation (21) does not yet account for spin or relativistic effects.

Of course the time-dependent equation can be used to derive the time-independent equation. If we write the wavefunction as a product of spatial and temporal terms,  $\psi(\mathbf{r}, t) = \psi(\mathbf{r})f(t)$ , then equation (21) becomes

$$\psi(\mathbf{r})i\hbar\frac{df(t)}{dt} = f(t) \left[ -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) \quad \text{-----22}$$

or 
$$\frac{i\hbar}{f(t)} \frac{df}{dt} = \frac{1}{\psi(\mathbf{r})} \left[ -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) \quad \text{-----23}$$

Since the left-hand side is a function of  $t$  only and the right hand side is a function of  $\mathbf{r}$  only, the two sides must equal a constant. If we tentatively designate this constant  $E$  (since the right-hand side clearly must have the dimensions of energy), then we extract two ordinary differential equations, namely

$$\frac{1}{f(t)} \frac{df(t)}{dt} = -\frac{iE}{\hbar} \quad \text{-----24}$$

and

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad \text{-----25}$$

The latter equation is once again the time-independent Schrödinger equation. The former equation is easily solved to yield

$$f(t) = e^{-iEt/\hbar} \quad \text{-----26}$$

The Hamiltonian in equation (25) is a Hermitian operator, and the eigen values of a Hermitian operator must be real, so  $E$  is real. This means that the solutions  $f(t)$  are purely oscillatory, since  $f(t)$  never changes in magnitude (recall Euler's formula  $e^{\pm i\theta} = \cos\theta \pm i \sin\theta$ ). Thus if

$$\psi(\mathbf{r}, t) = \psi(\mathbf{r})e^{-iEt/\hbar} \quad \text{-----27}$$

then the total wave function  $\psi(\mathbf{r}, t)$  differs from  $\psi(\mathbf{r})$  only by a phase factor of constant magnitude. There are some interesting consequences of this. First of all, the quantity  $|\psi(\mathbf{r}, t)|^2$  is time independent, as we can easily show:

$$|\psi(\mathbf{r}, t)|^2 = \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t) = e^{iEt/\hbar}\psi^*(\mathbf{r})e^{-iEt/\hbar}\psi(\mathbf{r}) = \psi^*(\mathbf{r})\psi(\mathbf{r}) \quad \text{-----28}$$

Secondly, the expectation value for any time-independent operator is also time-independent, if  $\psi(\mathbf{r}, t)$  satisfies equation (27). By the same

reasoning applied above,

$$\langle A \rangle = \int \psi^*(\mathbf{r}, t) \hat{A} \psi(\mathbf{r}, t) = \int \psi^*(\mathbf{r}) \hat{A} \psi(\mathbf{r}) \dots\dots\dots 29$$

For these reasons, wave functions of the form (27) are called *stationary states*. The state  $\psi(\mathbf{r}, t)$  is "stationary," but the particle it describes is not!

Of course equation represents a particular solution to equation (21). The general solution to equation (21) will be a linear combination of these particular solutions, i.e.

$$\psi(\mathbf{r}, t) = \sum_i c_i e^{-iE_i t/\hbar} \psi_i(\mathbf{r}) \dots\dots\dots 30$$

**FORMULATION OF THE PROBLEM:**

$$E\Psi = \hat{H}\Psi$$

**NOTATION :**

$G_{48}$  : Category one of the term on the LHS of the Time dependent Schrödinger's Wave Equation, (Such a classification is based on the characteristics of the systems under investigation to which the Wave Equation is applied).

$G_{49}$  : Category two of the term on the LHS of the Time dependent Schrödinger's Wave Equation, (Such a classification is based on the characteristics of the systems under investigation to which the Wave Equation is applied).

$G_{50}$  : Category two of the term on the LHS of the Time dependent Schrödinger's Wave Equation, (Such a classification is based on the characteristics of the systems under investigation to which the Wave Equation is applied).

$(a_{48})^{(10)}, (a_{49})^{(10)}, (a_{50})^{(10)}$  : Accentuation coefficients

$(a'_{48})^{(10)}, (a'_{49})^{(10)}, (a'_{50})^{(10)}$  : Dissipation coefficients

**FORMULATION OF THE SYSTEM :**

In the light of the assumptions stated in the foregoing, we infer the following:-

- (a) The growth speed in category 1 is the sum of a accentuation term  $(a_{48})^{(10)}G_{49}$  and a dissipation term  $-(a'_{48})^{(10)}G_{48}$ , the amount of dissipation
- (b) The growth speed in category 2 is the sum of two parts  $(a_{49})^{(10)}G_{48}$  and  $-(a'_{49})^{(10)}G_{49}$  the inflow from the category 1.
- (c) The growth speed in category 3 is equivalent to  $(a_{50})^{(10)}G_{49}$  and  $-(a'_{50})^{(10)}G_{50}$  dissipation ascribed only to depletion phenomenon.

**GOVERNING EQUATIONS:**

The differential equations governing the above system can be written in the following form

$$\frac{dG_{48}}{dt} = (a_{48})^{(10)}G_{49} - (a'_{48})^{(10)}G_{48}$$

$$\frac{dG_{49}}{dt} = (a_{49})^{(10)}G_{48} - (a'_{49})^{(10)}G_{49} \quad 1019$$

$$\frac{dG_{50}}{dt} = (a_{50})^{(10)}G_{49} - (a'_{50})^{(10)}G_{50} \quad 1020$$

$$(a_i)^{(10)} > 0, \quad i = 48, 49, 50 \quad 1021$$

$$(a'_i)^{(10)} > 0, \quad i = 48, 49, 50 \quad 1022$$

$$(a_{49})^{(10)} < (a'_{48})^{(10)} \quad 1023$$

$$(a_{50})^{(10)} < (a'_{49})^{(10)} \quad 1024$$

We can rewrite equation in the following form

$$\frac{dG_{48}}{(a_{48})^{(10)}G_{49} - (a'_{48})^{(10)}G_{48}} = dt \quad 1025$$

$$\frac{dG_{49}}{(a_{49})^{(10)}G_{48} - (a'_{49})^{(10)}G_{49}} = dt \quad 1026$$

Or we write a single equation as

$$\frac{dG_{48}}{(a_{48})^{(10)}G_{49} - (a'_{48})^{(10)}G_{48}} = \frac{dG_{49}}{(a_{49})^{(10)}G_{48} - (a'_{49})^{(10)}G_{49}} = \frac{dG_{50}}{(a_{50})^{(10)}G_{49} - (a'_{50})^{(10)}G_{50}} = dt \quad 1027$$

The equality of the ratios in equation remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples  $\alpha, \beta, \gamma$  all positive we can write equation as

$$\frac{\alpha dG_{48}}{\alpha((a_{48})^{(10)}G_{49} - (a'_{48})^{(10)}G_{48})} = \frac{\beta dG_{49}}{\beta((a_{49})^{(10)}G_{48} - (a'_{49})^{(10)}G_{49})} = \frac{\gamma dG_{50}}{\gamma((a_{50})^{(10)}G_{49} - (a'_{50})^{(10)}G_{50})} = dt \quad 1028$$

The general solution of the system can be written in the form

$$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t} \quad \text{Where } i = 48, 49, 50 \text{ and } C_{48}, C_{49}, C_{50} \text{ are arbitrary constant coefficients.}$$

### STABILITY ANALYSIS :

Supposing  $G_i(0) = G_i^0(0) > 0$ , and denoting by  $\lambda_i$  the characteristic roots of the system, it easily results that

1. If  $(a'_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} > 0$  all the components of the solution, i.e all the three parts of the tend to zero, and the solution is stable with respect to the initial data.

2. If  $(a'_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} < 0$  and

$(\lambda_{49} + (a'_{48})^{(10)})G_{48}^0 - (a_{48})^{(10)}G_{49}^0 \neq 0, (\lambda_{49} < 0)$ , the first two components of the solution tend to infinity as  $t \rightarrow \infty$ , and  $G_{50} \rightarrow 0$ , i.e. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tends to zero.

3. If  $(a'_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} < 0$  and

$(\lambda_{49} + (a'_{48})^{(10)})G_{48}^0 - (a_{48})^{(10)}G_{49}^0 = 0$  Then all the three parts tend to zero, but the solution is

not stable i.e. at a small variation of the initial values of  $G_i$ , the corresponding solution tends to infinity.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system converges to equilibrium.
2. The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point
3. Conditions 1 and 2 are independent of the size and direction of initial disturbance
4. The actual shape of the time path is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance
5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold
6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question
7. Some authors Noyer F J, Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

**RHS OF TIME DEPENDENT SCHRODINGERS'S EQUATION:  $E\Psi = \hat{H}\Psi$**

- a) The speed of growth is linear function of the category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between category 1 and category 2.
- b) The dissipation coefficient in the growth model is attributable to aging and depletion phenomenon in systems under investigation.
- c) Inflow into category 2 is only from category 1. The balance of terrestrial organism sector in category 3 is because of transfer from category 2.

**NOTATION :**

$T_{48}$  : Category one of the term on RHS

$T_{49}$  : Category two of the term on RHS

$T_{50}$  : Category three of the term on RHS

$(b_{48})^{(10)}, (b_{49})^{(10)}, (b_{50})^{(10)}$  : Accentuation coefficients

$(b'_{48})^{(10)}, (b'_{49})^{(10)}, (b'_{50})^{(10)}$  : Dissipation coefficients

**FORMULATION OF THE SYSTEM :**

Under the above assumptions, we derive the following :

- a) The growth speed in category 1 is the sum of two parts:
1. A term  $+(b_{48})^{(10)}T_{49}$
  2. A term  $-(b'_{48})^{(10)}T_{48}$
  3. The growth speed in category 2 is the sum of two parts:
    1. A term  $+(b_{49})^{(10)}T_{48}$  constitutive of the amount of inflow from the category 1
    2. A term  $-(b'_{49})^{(10)}T_{49}$  the dissipation factor.
- b) The growth speed under category 3 is attributable to inflow from category 2.

### $E\Psi = \hat{H}\Psi$ THE TIME DEPENDENT SCHRODINGER'S WAVE EQUATION:

#### GOVERNING EQUATIONS:

Following are the differential equations that govern the growth in the terrestrial organisms portfolio

$$\frac{dT_{48}}{dt} = (b_{48})^{(10)}T_{49} - (b'_{48})^{(10)}T_{48} \quad 1029$$

$$\frac{dT_{49}}{dt} = (b_{49})^{(10)}T_{48} - (b'_{49})^{(10)}T_{49} \quad 1030$$

$$\frac{dT_{50}}{dt} = (b_{50})^{(10)}T_{49} - (b'_{50})^{(10)}T_{50} \quad 1031$$

$$(b_i)^{(10)} > 0 \quad , \quad i = 48,49,50 \quad 1032$$

$$(b'_i)^{(10)} > 0 \quad , \quad i = 48,49,50 \quad 1033$$

$$(b_{49})^{(10)} < (b'_{48})^{(10)} \quad 1034$$

$$(b_{50})^{(10)} < (b'_{49})^{(10)} \quad 1035$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is

$\alpha'_i T_i + \beta'_i T_i + \gamma'_i T_i = C'_i e_i^{\lambda' i t}$ ,  $i = 48,49,50$  where  $C'_{48}, C'_{49}, C'_{50}$  are arbitrary constant coefficients and  $\alpha'_{48}, \alpha'_{49}, \alpha'_{50}, \gamma'_{48}, \gamma'_{49}, \gamma'_{50}$  corresponding multipliers to the characteristic roots of the terrestrial organism system

#### THE TIME DEPENDENT SCHRODINGER'S WAVE EQUATION:-DUAL SYSTEM ANALYSIS

$$E\Psi = \hat{H}\Psi \quad 1036$$

We will denote

- 1) By  $T_i(t)$ ,  $i = 48,49,50$  , the three parts of the RHS OF TDSE(Time Dependent Schrödinger's equation)
- 2) By  $(a''_i)^{(10)}(T_{49}, t)$  ( $T_{49} \geq 0, t \geq 0$ ) ,
- 3) By  $(-b''_i)^{(10)}(G_{48}, G_{49}, G_{50}, t) = -(b''_i)^{(10)}((G_{51}), t)$  ,

#### THE TIME DEPENDENT SCHRODINGER'S WAVE EQUATION:-DUAL SYSTEM ANALYSIS 1037

$$E\Psi = \hat{H}\Psi$$

### GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{48}}{dt} = (a_{48})^{(10)}G_{49} - [(a'_{48})^{(10)} + (a''_{48})^{(10)}(T_{49}, t)]G_{48} \quad 1038$$

$$\frac{dG_{49}}{dt} = (a_{49})^{(10)}G_{48} - [(a'_{49})^{(10)} + (a''_{49})^{(10)}(T_{49}, t)]G_{49} \quad 1039$$

$$\frac{dG_{50}}{dt} = (a_{50})^{(10)}G_{49} - [(a'_{50})^{(10)} + (a''_{50})^{(10)}(T_{49}, t)]G_{50} \quad 1040$$

$$\frac{dT_{48}}{dt} = (b_{48})^{(10)}T_{49} - [(b'_{48})^{(10)} - (b''_{48})^{(10)}((G_{51}), t)]T_{48} \quad 1041$$

$$\frac{dT_{49}}{dt} = (b_{49})^{(10)}T_{48} - [(b'_{49})^{(10)} - (b''_{49})^{(10)}((G_{51}), t)]T_{49} \quad 1042$$

$$\frac{dT_{50}}{dt} = (b_{50})^{(10)}T_{49} - [(b'_{50})^{(10)} - (b''_{50})^{(10)}((G_{51}), t)]T_{50} \quad 1043$$

$$+(a''_{48})^{(10)}(T_{49}, t) = \text{First augmentation factor} \quad 1044$$

$$-(b''_{48})^{(10)}((G_{51}), t) = \text{First detritions factor} \quad 1045$$

Where we suppose

$$(a_i)^{(10)}, (a'_i)^{(10)}, (a''_i)^{(10)}, (b_i)^{(10)}, (b'_i)^{(10)}, (b''_i)^{(10)} > 0, \quad i, j = 48, 49, 50 \quad 1046$$

The functions  $(a_i'')^{(10)}, (b_i'')^{(10)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(10)}, (r_i)^{(10)}$ :

$$(a_i'')^{(10)}(T_{49}, t) \leq (p_i)^{(10)} \leq (\hat{A}_{48})^{(10)}$$

$$(b_i'')^{(10)}(G_{51}, t) \leq (r_i)^{(10)} \leq (b'_i)^{(10)} \leq (\hat{B}_{48})^{(10)}$$

1047

$$(J) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(10)}(T_{49}, t) = (p_i)^{(10)}$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(10)}(G_{51}, t) = (r_i)^{(10)}$$

**Definition of**  $(\hat{A}_{48})^{(10)}, (\hat{B}_{48})^{(10)}$ :

Where  $(\hat{A}_{48})^{(10)}, (\hat{B}_{48})^{(10)}, (p_i)^{(10)}, (r_i)^{(10)}$  are positive constants and  $i = 48, 49, 50$

They satisfy Lipschitz condition:

1048

$$|(a_i'')^{(10)}(T'_{49}, t) - (a_i'')^{(10)}(T_{49}, t)| \leq (\hat{k}_{48})^{(10)}|T_{49} - T'_{49}|e^{-(\hat{M}_{48})^{(10)}t}$$

$$|(b_i'')^{(10)}((G_{51})', t) - (b_i'')^{(10)}((G_{51}), T)| < (\hat{k}_{48})^{(10)}|(G_{51})' - (G_{51})|e^{-(\hat{M}_{48})^{(10)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(10)}(T'_{49}, t)$  and  $(a_i'')^{(10)}(T_{49}, t)$ .  $(T'_{49}, t)$  and  $(T_{49}, t)$  are points belonging to the interval  $[(\hat{k}_{48})^{(10)}, (\hat{M}_{48})^{(10)}]$ . It is to be noted that  $(a_i'')^{(10)}(T_{49}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{48})^{(10)} = 10$  then the function  $(a_i'')^{(10)}(T_{49}, t)$ , the **first**

augmentation coefficient, would be absolutely continuous.

**Definition of**  $(\hat{M}_{48})^{(10)}, (\hat{k}_{48})^{(10)}$  : 1049

$(\hat{M}_{48})^{(10)}, (\hat{k}_{48})^{(10)}$ , are positive constants

$$\frac{(a_i)^{(10)}}{(\hat{M}_{48})^{(10)}} , \frac{(b_i)^{(10)}}{(\hat{M}_{48})^{(10)}} < 1$$

**Definition of**  $(\hat{P}_{48})^{(10)}, (\hat{Q}_{48})^{(10)}$  : 1050

(KK) There exists two constants  $(\hat{P}_{48})^{(10)}$  and  $(\hat{Q}_{48})^{(10)}$  which together with  $(\hat{M}_{48})^{(10)}, (\hat{k}_{48})^{(10)}, (\hat{A}_{48})^{(10)}$  and  $(\hat{B}_{48})^{(10)}$  and the constants  $(a_i)^{(10)}, (a'_i)^{(10)}, (b_i)^{(10)}, (b'_i)^{(10)}, (p_i)^{(10)}, (r_i)^{(10)}, i = 48, 49, 50$ ,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{48})^{(10)}} [(a_i)^{(10)} + (a'_i)^{(10)} + (\hat{A}_{48})^{(10)} + (\hat{P}_{48})^{(10)} (\hat{k}_{48})^{(10)}] < 1$$

$$\frac{1}{(\hat{M}_{48})^{(10)}} [(b_i)^{(10)} + (b'_i)^{(10)} + (\hat{B}_{48})^{(10)} + (\hat{Q}_{48})^{(10)} (\hat{k}_{48})^{(10)}] < 1$$

**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 1051

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{48})^{(10)} e^{(\hat{M}_{48})^{(10)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{48})^{(10)} e^{(\hat{M}_{48})^{(10)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:**

Consider operator  $\mathcal{A}^{(10)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{48})^{(10)}, T_i^0 \leq (\hat{Q}_{48})^{(10)}, \quad 1052$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{48})^{(10)} e^{(\hat{M}_{48})^{(10)}t} \quad 1053$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{48})^{(10)} e^{(\hat{M}_{48})^{(10)}t} \quad 1054$$

By 1055

$$\bar{G}_{48}(t) = G_{48}^0 + \int_0^t [(a_{48})^{(10)} G_{49}(s_{(48)}) - ((a'_{48})^{(10)} + a''_{48})^{(10)} (T_{49}(s_{(48)}), s_{(48)})] G_{48}(s_{(48)})] ds_{(48)}$$

$$\bar{G}_{49}(t) = G_{49}^0 + \int_0^t [(a_{49})^{(10)} G_{48}(s_{(48)}) - ((a'_{49})^{(10)} + (a''_{49})^{(10)} (T_{49}(s_{(48)}), s_{(48)}))] G_{49}(s_{(48)})] ds_{(48)} \quad 1056$$

$$\bar{G}_{50}(t) = G_{50}^0 + \int_0^t [(a_{50})^{(10)} G_{49}(s_{(48)}) - ((a'_{50})^{(10)} + (a''_{50})^{(10)} (T_{49}(s_{(48)}), s_{(48)}))] G_{50}(s_{(48)})] ds_{(48)} \quad 1057$$

$$\bar{T}_{48}(t) = T_{48}^0 + \int_0^t \left[ (b_{48})^{(10)} T_{49}(s_{(48)}) - \left( (b'_{48})^{(10)} - (b''_{48})^{(10)} (G(s_{(48)}), s_{(48)}) \right) T_{48}(s_{(48)}) \right] ds_{(48)} \quad 1058$$

$$\bar{T}_{49}(t) = T_{49}^0 + \int_0^t \left[ (b_{49})^{(10)} T_{48}(s_{(48)}) - \left( (b'_{49})^{(10)} - (b''_{49})^{(10)} (G(s_{(48)}), s_{(48)}) \right) T_{49}(s_{(48)}) \right] ds_{(48)} \quad 1059$$

$$\bar{T}_{50}(t) = T_{50}^0 + \int_0^t \left[ (b_{50})^{(10)} T_{49}(s_{(48)}) - \left( (b'_{50})^{(10)} - (b''_{50})^{(10)} (G(s_{(48)}), s_{(48)}) \right) T_{50}(s_{(48)}) \right] ds_{(48)} \quad 1060$$

Where  $s_{(48)}$  is the integrand that is integrated over an interval  $(0, t)$

(o) The operator  $\mathcal{A}^{(10)}$  maps the space of functions satisfying 34,35,36 into itself. Indeed it is obvious that 1061

$$G_{48}(t) \leq G_{48}^0 + \int_0^t \left[ (a_{48})^{(10)} \left( G_{49}^0 + (\hat{P}_{48})^{(10)} e^{(\hat{M}_{48})^{(10)} s_{(48)}} \right) \right] ds_{(48)} =$$

$$\left( 1 + (a_{48})^{(10)} t \right) G_{49}^0 + \frac{(a_{48})^{(10)} (\hat{P}_{48})^{(10)}}{(\hat{M}_{48})^{(10)}} \left( e^{(\hat{M}_{48})^{(10)} t} - 1 \right)$$

From which it follows that 1062

$$(G_{48}(t) - G_{48}^0) e^{-(\hat{M}_{48})^{(10)} t} \leq \frac{(a_{48})^{(10)}}{(\hat{M}_{48})^{(10)}} \left[ \left( (\hat{P}_{48})^{(10)} + G_{49}^0 \right) e^{-\left( \frac{(\hat{P}_{48})^{(10)} + G_{49}^0}{G_{49}^0} \right)} + (\hat{P}_{48})^{(10)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem

Analogous inequalities hold also for  $G_{49}, G_{50}, T_{48}, T_{49}, T_{50}$  1063

It is now sufficient to take  $\frac{(a_i)^{(10)}}{(\hat{M}_{48})^{(10)}}, \frac{(b_j)^{(10)}}{(\hat{M}_{48})^{(10)}} < 1$  and to choose

$(\hat{P}_{48})^{(10)}$  and  $(\hat{Q}_{48})^{(10)}$  large to have

$$\frac{(a_i)^{(10)}}{(\hat{M}_{48})^{(10)}} \left[ (\hat{P}_{48})^{(10)} + \left( (\hat{P}_{48})^{(10)} + G_j^0 \right) e^{-\left( \frac{(\hat{P}_{48})^{(10)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{48})^{(10)} \quad 1064$$

$$\frac{(b_j)^{(10)}}{(\hat{M}_{48})^{(10)}} \left[ \left( (\hat{Q}_{48})^{(10)} + T_j^0 \right) e^{-\left( \frac{(\hat{Q}_{48})^{(10)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{48})^{(10)} \right] \leq (\hat{Q}_{48})^{(10)} \quad 1065$$

In order that the operator  $\mathcal{A}^{(10)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 34,35,36 into itself

The operator  $\mathcal{A}^{(10)}$  is a contraction with respect to the metric 1066

$$d \left( ((G_{51})^{(10)}, (T_{51})^{(10)}), ((G_{51})^{(2)}, (T_{51})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(10)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{48})^{(10)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(10)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{48})^{(10)} t} \right\}$$

Indeed if we denote

**Definition of**  $(\widetilde{G}_{51}), (\widetilde{T}_{51})$ :  $((\widetilde{G}_{51}), (\widetilde{T}_{51})) = \mathcal{A}^{(10)}((G_{51}), (T_{51}))$

It results



$$\begin{aligned}
 |\tilde{G}_{48}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{48})^{(10)} |G_{49}^{(1)} - G_{49}^{(2)}| e^{-(\bar{M}_{48})^{(10)}s_{(48)}} e^{(\bar{M}_{48})^{(10)}s_{(48)}} ds_{(48)} + \\
 &\int_0^t \{(a'_{48})^{(10)} |G_{48}^{(1)} - G_{48}^{(2)}| e^{-(\bar{M}_{48})^{(10)}s_{(48)}} e^{-(\bar{M}_{48})^{(10)}s_{(48)}} + \\
 &(a''_{48})^{(10)} (T_{49}^{(1)}, s_{(48)}) |G_{48}^{(1)} - G_{48}^{(2)}| e^{-(\bar{M}_{48})^{(10)}s_{(48)}} e^{(\bar{M}_{48})^{(10)}s_{(48)}} + \\
 &G_{48}^{(2)} |(a'_{48})^{(10)} (T_{49}^{(1)}, s_{(48)}) - (a''_{48})^{(10)} (T_{49}^{(2)}, s_{(48)})| e^{-(\bar{M}_{48})^{(10)}s_{(48)}} e^{(\bar{M}_{48})^{(10)}s_{(48)}}\} ds_{(48)}
 \end{aligned}$$

Where  $s_{(48)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\begin{aligned}
 |(G_{51})^{(1)} - G^{(2)}| e^{-(\bar{M}_{48})^{(10)}t} &\leq & 1067 \\
 \frac{1}{(\bar{M}_{48})^{(10)}} ((a_{48})^{(10)} + (a'_{48})^{(10)} + (\bar{A}_{48})^{(10)} + \\
 (\bar{P}_{48})^{(10)} (\bar{k}_{48})^{(10)}) d((G_{51})^{(1)}, (T_{51})^{(1)}; (G_{51})^{(2)}, (T_{51})^{(2)})
 \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{48})^{(10)}$  and  $(b''_{48})^{(10)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\bar{P}_{48})^{(10)} e^{(\bar{M}_{48})^{(10)}t}$  and  $(\bar{Q}_{48})^{(10)} e^{(\bar{M}_{48})^{(10)}t}$  respectively of  $\mathbb{R}_+$ . 1068

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a'_i)^{(10)}$  and  $(b'_i)^{(10)}$ ,  $i = 48,49,50$  depend only on  $T_{49}$  and respectively on  $(G_{51})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  1069

From 19 to 24 it results

$$\begin{aligned}
 G_i(t) &\geq G_i^0 e^{[-\int_0^t ((a'_i)^{(10)} - (a''_i)^{(10)}(T_{49}(s_{(48)}), s_{(48)})) ds_{(48)}]} \geq 0 \\
 T_i(t) &\geq T_i^0 e^{-(b'_i)^{(10)}t} > 0 \text{ for } t > 0
 \end{aligned}$$

**Definition of**  $((\bar{M}_{48})^{(10)})_1, ((\bar{M}_{48})^{(10)})_2$  and  $((\bar{M}_{48})^{(10)})_3$  : 1070

**Remark 3:** if  $G_{48}$  is bounded, the same property have also  $G_{49}$  and  $G_{50}$ . indeed if

$$\begin{aligned}
 G_{48} < (\bar{M}_{48})^{(10)} \text{ it follows } \frac{dG_{49}}{dt} &\leq ((\bar{M}_{48})^{(10)})_1 - (a'_{49})^{(10)} G_{49} \text{ and by integrating} \\
 G_{49} &\leq ((\bar{M}_{48})^{(10)})_2 = G_{49}^0 + 2(a_{49})^{(10)} ((\bar{M}_{48})^{(10)})_1 / (a'_{49})^{(10)}
 \end{aligned}$$

In the same way, one can obtain

$$G_{50} \leq ((\bar{M}_{48})^{(10)})_3 = G_{50}^0 + 2(a_{50})^{(10)} ((\bar{M}_{48})^{(10)})_2 / (a'_{50})^{(10)}$$

If  $G_{49}$  or  $G_{50}$  is bounded, the same property follows for  $G_{48}$ ,  $G_{50}$  and  $G_{48}$ ,  $G_{49}$  respectively.

**Remark 4:** If  $G_{48}$  is bounded, from below, the same property holds for  $G_{49}$  and  $G_{50}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{49}$  is bounded from below. 1071

**Remark 5:** If  $T_{48}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(10)}((G_{51})(t), t)) = (b_{49}')^{(10)}$  then  $T_{49} \rightarrow \infty$ . 1072

**Definition of**  $(m)^{(10)}$  and  $\varepsilon_{10}$  :

Indeed let  $t_{10}$  be so that for  $t > t_{10}$

$$(b_{49})^{(10)} - (b_i'')^{(10)}((G_{51})(t), t) < \varepsilon_{10}, T_{48}(t) > (m)^{(10)}$$

Then  $\frac{dT_{49}}{dt} \geq (a_{49})^{(10)}(m)^{(10)} - \varepsilon_{10}T_{49}$  which leads to 1073

$$T_{49} \geq \left( \frac{(a_{49})^{(10)}(m)^{(10)}}{\varepsilon_{10}} \right) (1 - e^{-\varepsilon_{10}t}) + T_{49}^0 e^{-\varepsilon_{10}t}$$

If we take  $t$  such that  $e^{-\varepsilon_{10}t} = \frac{1}{2}$  it results

$$T_{49} \geq \left( \frac{(a_{49})^{(10)}(m)^{(10)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_{10}}$$

By taking now  $\varepsilon_{10}$  sufficiently small one sees that  $T_{49}$  is unbounded. The same property holds for  $T_{50}$  if  $\lim_{t \rightarrow \infty} (b_{50}'')^{(10)}((G_{51})(t), t) = (b_{50}')^{(10)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

### Behavior of the solutions of equation

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(10)}, (\sigma_2)^{(10)}, (\tau_1)^{(10)}, (\tau_2)^{(10)}$  :

(kk)  $(\sigma_1)^{(10)}, (\sigma_2)^{(10)}, (\tau_1)^{(10)}, (\tau_2)^{(10)}$  four constants satisfying

$$-(\sigma_2)^{(10)} \leq -(a_{48}')^{(10)} + (a_{49}')^{(10)} - (a_{48}'')^{(10)}(T_{49}, t) + (a_{49}'')^{(10)}(T_{49}, t) \leq -(\sigma_1)^{(10)}$$

$$-(\tau_2)^{(10)} \leq -(b_{48}')^{(10)} + (b_{49}')^{(10)} - (b_{48}'')^{(10)}((G_{51}), t) - (b_{49}'')^{(10)}((G_{51}), t) \leq -(\tau_1)^{(10)}$$

**Definition of**  $(v_1)^{(10)}, (v_2)^{(10)}, (u_1)^{(10)}, (u_2)^{(10)}, v^{(10)}, u^{(10)}$  :

(ll) By  $(v_1)^{(10)} > 0, (v_2)^{(10)} < 0$  and respectively  $(u_1)^{(10)} > 0, (u_2)^{(10)} < 0$  the roots of the equations  $(a_{49})^{(10)}(v^{(10)})^2 + (\sigma_1)^{(10)}v^{(10)} - (a_{48})^{(10)} = 0$

$$\text{and } (b_{49})^{(10)}(u^{(10)})^2 + (\tau_1)^{(10)}u^{(10)} - (b_{48})^{(10)} = 0 \text{ and}$$

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**Definition of**  $(\bar{v}_1)^{(10)}, (\bar{v}_2)^{(10)}, (\bar{u}_1)^{(10)}, (\bar{u}_2)^{(10)}$  :

By  $(\bar{v}_1)^{(10)} > 0, (\bar{v}_2)^{(10)} < 0$  and respectively  $(\bar{u}_1)^{(10)} > 0, (\bar{u}_2)^{(10)} < 0$  the

$$\text{roots of the equations } (a_{49})^{(10)}(v^{(10)})^2 + (\sigma_2)^{(10)}v^{(10)} - (a_{48})^{(10)} = 0$$

$$\text{and } (b_{49})^{(10)}(u^{(10)})^2 + (\tau_2)^{(10)}u^{(10)} - (b_{48})^{(10)} = 0$$

**Definition of**  $(m_1)^{(10)}, (m_2)^{(10)}, (\mu_1)^{(10)}, (\mu_2)^{(10)}, (v_0)^{(10)}$  :-

(mm) If we define  $(m_1)^{(10)}, (m_2)^{(10)}, (\mu_1)^{(10)}, (\mu_2)^{(10)}$  by

$$(m_2)^{(10)} = (v_0)^{(10)}, (m_1)^{(10)} = (v_1)^{(10)}, \text{ if } (v_0)^{(10)} < (v_1)^{(10)}$$

$$(m_2)^{(10)} = (v_1)^{(10)}, (m_1)^{(10)} = (\bar{v}_1)^{(10)}, \text{ if } (v_1)^{(10)} < (v_0)^{(10)} < (\bar{v}_1)^{(10)},$$

$$\text{and } \boxed{(v_0)^{(10)} = \frac{G_{48}^0}{G_{49}^0}}$$

$$(m_2)^{(10)} = (v_1)^{(10)}, (m_1)^{(10)} = (v_0)^{(10)}, \text{ if } (\bar{v}_1)^{(10)} < (v_0)^{(10)}$$

and analogously

$$(\mu_2)^{(10)} = (u_0)^{(10)}, (\mu_1)^{(10)} = (u_1)^{(10)}, \text{ if } (u_0)^{(10)} < (u_1)^{(10)}$$

$$(\mu_2)^{(10)} = (u_1)^{(10)}, (\mu_1)^{(10)} = (\bar{u}_1)^{(10)}, \text{ if } (u_1)^{(10)} < (u_0)^{(10)} < (\bar{u}_1)^{(10)},$$

$$\text{and } \boxed{(u_0)^{(10)} = \frac{T_{48}^0}{T_{49}^0}}$$

$$(\mu_2)^{(10)} = (u_1)^{(10)}, (\mu_1)^{(10)} = (u_0)^{(10)}, \text{ if } (\bar{u}_1)^{(10)} < (u_0)^{(10)} \text{ where } (u_1)^{(10)}, (\bar{u}_1)^{(10)}$$

are defined by 59 and 61 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{48}^0 e^{((S_1)^{(10)} - (p_{48})^{(10)})t} \leq G_{48}(t) \leq G_{48}^0 e^{(S_1)^{(10)}t}$$

where  $(p_i)^{(10)}$  is defined by equation 25

$$\frac{1}{(m_{10})^{(10)}} G_{48}^0 e^{((S_1)^{(10)} - (p_{48})^{(10)})t} \leq G_{49}(t) \leq \frac{1}{(m_2)^{(10)}} G_{48}^0 e^{(S_1)^{(10)}t}$$

$$\left( \frac{(a_{50})^{(10)} G_{48}^0}{(m_{10})^{(10)} ((S_1)^{(10)} - (p_{48})^{(10)} - (S_2)^{(10)})} \right) \left[ e^{((S_1)^{(10)} - (p_{48})^{(10)})t} - e^{-(S_2)^{(10)}t} \right] + G_{50}^0 e^{-(S_2)^{(10)}t} \leq G_{50}(t) \leq$$

$$\frac{(a_{50})^{(10)} G_{48}^0}{(m_2)^{(10)} ((S_1)^{(10)} - (a'_{50})^{(10)})} \left[ e^{(S_1)^{(10)}t} - e^{-(a'_{50})^{(10)}t} \right] + G_{50}^0 e^{-(a'_{50})^{(10)}t}$$

$$\boxed{T_{48}^0 e^{(R_1)^{(10)}t} \leq T_{48}(t) \leq T_{48}^0 e^{((R_1)^{(10)} + (r_{48})^{(10)})t}}$$

$$\frac{1}{(\mu_1)^{(10)}} T_{48}^0 e^{(R_1)^{(10)}t} \leq T_{48}(t) \leq \frac{1}{(\mu_2)^{(10)}} T_{48}^0 e^{((R_1)^{(10)} + (r_{48})^{(10)})t}$$

$$\frac{(b_{50})^{(10)} T_{48}^0}{(\mu_1)^{(10)} ((R_1)^{(10)} - (b'_{50})^{(10)})} \left[ e^{(R_1)^{(10)}t} - e^{-(b'_{50})^{(10)}t} \right] + T_{50}^0 e^{-(b'_{50})^{(10)}t} \leq T_{50}(t) \leq$$

$$\frac{(a_{50})^{(10)} T_{48}^0}{(\mu_2)^{(10)} ((R_1)^{(10)} + (r_{48})^{(10)} + (R_2)^{(10)})} \left[ e^{((R_1)^{(10)} + (r_{48})^{(10)})t} - e^{-(R_2)^{(10)}t} \right] + T_{50}^0 e^{-(R_2)^{(10)}t}$$

**Definition of**  $(S_1)^{(10)}, (S_2)^{(10)}, (R_1)^{(10)}, (R_2)^{(10)}$ :-

$$\text{Where } (S_1)^{(10)} = (a_{48})^{(10)}(m_2)^{(10)} - (a'_{48})^{(10)}$$

$$(S_2)^{(10)} = (a_{50})^{(10)} - (p_{50})^{(10)}$$

$$(R_1)^{(10)} = (b_{48})^{(10)}(\mu_2)^{(10)} - (b'_{48})^{(10)}$$

$$(R_2)^{(10)} = (b'_{50})^{(10)} - (r_{50})^{(10)}$$

**Proof:** From 19,20,21,22,23,24 we obtain

$$\frac{dv^{(10)}}{dt} = (a_{48})^{(10)} - \left( (a'_{48})^{(10)} - (a'_{49})^{(10)} + (a''_{48})^{(10)}(T_{49}, t) \right) - (a''_{49})^{(10)}(T_{49}, t)v^{(10)} - (a_{49})^{(10)}v^{(10)}$$

**Definition of**  $v^{(10)}$  :- 
$$v^{(10)} = \frac{G_{48}}{G_{49}}$$

It follows

$$- \left( (a_{49})^{(10)}(v^{(10)})^2 + (\sigma_2)^{(10)}v^{(10)} - (a_{48})^{(10)} \right) \leq \frac{dv^{(10)}}{dt} \leq - \left( (a_{49})^{(10)}(v^{(10)})^2 + (\sigma_1)^{(10)}v^{(10)} - (a_{48})^{(10)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(10)}, (v_0)^{(10)}$  :-

(d) For  $0 < \boxed{(v_0)^{(10)} = \frac{G_{48}^0}{G_{49}^0}} < (v_1)^{(10)} < (\bar{v}_1)^{(10)}$

$$v^{(10)}(t) \geq \frac{(v_1)^{(10)} + (C)^{(10)}(v_2)^{(10)} e^{[-(a_{49})^{(10)}((v_1)^{(10)} - (v_0)^{(10)})t]}}{1 + (C)^{(10)} e^{[-(a_{49})^{(10)}((v_1)^{(10)} - (v_0)^{(10)})t]}} , \quad \boxed{(C)^{(10)} = \frac{(v_1)^{(10)} - (v_0)^{(10)}}{(v_0)^{(10)} - (v_2)^{(10)}}$$

it follows  $(v_0)^{(10)} \leq v^{(10)}(t) \leq (v_1)^{(10)}$

In the same manner , we get

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$$v^{(10)}(t) \leq \frac{(\bar{v}_1)^{(10)} + (\bar{C})^{(10)}(\bar{v}_2)^{(10)} e^{[-(a_{49})^{(10)}((\bar{v}_1)^{(10)} - (\bar{v}_2)^{(10)})t]}}{1 + (\bar{C})^{(10)} e^{[-(a_{49})^{(10)}((\bar{v}_1)^{(10)} - (\bar{v}_2)^{(10)})t]}} , \quad \boxed{(\bar{C})^{(10)} = \frac{(\bar{v}_1)^{(10)} - (v_0)^{(10)}}{(v_0)^{(10)} - (\bar{v}_2)^{(10)}}$$

From which we deduce  $(v_0)^{(10)} \leq v^{(10)}(t) \leq (\bar{v}_1)^{(10)}$

(e) If  $0 < (v_1)^{(10)} < (v_0)^{(10)} = \frac{G_{48}^0}{G_{49}^0} < (\bar{v}_1)^{(10)}$  we find like in the previous case,

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$$(v_1)^{(10)} \leq \frac{(v_1)^{(10)} + (C)^{(10)}(v_2)^{(10)} e^{[-(a_{49})^{(10)}((v_1)^{(10)} - (v_2)^{(10)})t]}}{1 + (C)^{(10)} e^{[-(a_{49})^{(10)}((v_1)^{(10)} - (v_2)^{(10)})t]}} \leq v^{(10)}(t) \leq \frac{(\bar{v}_1)^{(10)} + (\bar{C})^{(10)}(\bar{v}_2)^{(10)} e^{[-(a_{49})^{(10)}((\bar{v}_1)^{(10)} - (\bar{v}_2)^{(10)})t]}}{1 + (\bar{C})^{(10)} e^{[-(a_{49})^{(10)}((\bar{v}_1)^{(10)} - (\bar{v}_2)^{(10)})t]}} \leq (\bar{v}_1)^{(10)}$$

(f) If  $0 < (v_1)^{(10)} \leq (\bar{v}_1)^{(10)} \leq \boxed{(v_0)^{(10)} = \frac{G_{48}^0}{G_{49}^0}}$  , we obtain

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$$(v_1)^{(10)} \leq v^{(10)}(t) \leq \frac{(\bar{v}_1)^{(10)} + (\bar{C})^{(10)}(\bar{v}_2)^{(10)} e^{[-(a_{49})^{(10)}((\bar{v}_1)^{(10)} - (\bar{v}_2)^{(10)})t]}}{1 + (\bar{C})^{(10)} e^{[-(a_{49})^{(10)}((\bar{v}_1)^{(10)} - (\bar{v}_2)^{(10)})t]}} \leq (v_0)^{(10)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(10)}(t)$  :-

$$(m_2)^{(10)} \leq v^{(10)}(t) \leq (m_1)^{(10)}, \quad v^{(10)}(t) = \frac{G_{48}(t)}{G_{49}(t)}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(10)}(t)$  :-

$$(\mu_2)^{(10)} \leq u^{(10)}(t) \leq (\mu_1)^{(10)}, \quad u^{(10)}(t) = \frac{T_{48}(t)}{T_{49}(t)}$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{48})^{(10)} = (a''_{49})^{(10)}$ , then  $(\sigma_1)^{(10)} = (\sigma_2)^{(10)}$  and in this case  $(v_1)^{(10)} = (\bar{v}_1)^{(10)}$  if in addition  $(v_0)^{(10)} = (v_1)^{(10)}$  then  $v^{(10)}(t) = (v_0)^{(10)}$  and as a consequence  $G_{48}(t) = (v_0)^{(10)}G_{49}(t)$  **this also defines  $(v_0)^{(10)}$  for the special case.**

Analogously if  $(b''_{48})^{(10)} = (b''_{49})^{(10)}$ , then  $(\tau_1)^{(10)} = (\tau_2)^{(10)}$  and then

$(u_1)^{(10)} = (\bar{u}_1)^{(10)}$  if in addition  $(u_0)^{(10)} = (u_1)^{(10)}$  then  $T_{48}(t) = (u_0)^{(10)}T_{49}(t)$  This is an important consequence of the relation between  $(v_1)^{(10)}$  and  $(\bar{v}_1)^{(10)}$ , **and definition of  $(u_0)^{(10)}$ .**

#### 4. STATIONARY SOLUTIONS AND STABILITY

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Stationary solutions and stability curve representative of the variation of systemic parameters LHS AND RHS variation curve lies below the tangent at  $(G_{51}) = G_0$  for  $(G_{51}) < G_0$  and above the tangent for  $(G_{51}) > G_0$ . Wherever such a situation occurs the point  $G_0$  is called the **"point of inflexion"**. In this case, the tangent has a positive slope that simply means the rate of change of oxygen consumption due to cellular respiration is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

**Stationary value :**

In all the cases  $(G_{51}) = G_0$ ,  $(G_{51}) < G_0$ ,  $(G_{51}) > G_0$  the condition that the rate of change of LHS OF TDSE is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of  $(G_{51})$  is that the rate of change of LHS OF TDSE at  $G_0$  is zero.
2. A sufficient condition for the stationary value at  $G_0$ , to be maximum is that the acceleration of the LHS OF TDSE is less than zero.
3. A sufficient condition for the stationary value at  $G_0$ , be minimum is that acceleration of LHS OF TDSE is greater than zero.

We can prove the following

**Theorem 3:** If  $(a''_i)^{(10)}$  and  $(b''_i)^{(10)}$  are independent on  $t$ , and the conditions

$$(a'_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} < 0$$

$$(a'_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} + (a_{48})^{(10)}(p_{48})^{(10)} + (a'_{49})^{(10)}(p_{49})^{(10)} + (p_{48})^{(10)}(p_{49})^{(10)} > 0$$

$$(b'_{48})^{(10)}(b'_{49})^{(10)} - (b_{48})^{(10)}(b_{49})^{(10)} > 0,$$

$$(b'_{48})^{(10)}(b'_{49})^{(10)} - (b_{48})^{(10)}(b_{49})^{(10)} - (b'_{48})^{(10)}(r_{49})^{(10)} - (b'_{49})^{(10)}(r_{49})^{(10)} + (r_{48})^{(10)}(r_{49})^{(10)} < 0$$

with  $(p_{48})^{(10)}, (r_{49})^{(10)}$  as defined by equation are satisfied, then the system

$$(a_{48})^{(10)}G_{49} - [(a'_{48})^{(10)} + (a''_{48})^{(10)}(T_{49})]G_{48} = 0 \quad 1079$$

$$(a_{49})^{(10)}G_{48} - [(a'_{49})^{(10)} + (a''_{49})^{(10)}(T_{49})]G_{49} = 0 \quad 1080$$

$$(a_{50})^{(10)}G_{49} - [(a'_{50})^{(10)} + (a''_{50})^{(10)}(T_{49})]G_{50} = 0 \quad 1081$$

$$(b_{48})^{(10)}T_{49} - [(b'_{48})^{(10)} - (b''_{48})^{(10)}(G_{51})]T_{48} = 0 \quad 1082$$

$$(b_{49})^{(10)}T_{48} - [(b'_{49})^{(10)} - (b''_{49})^{(10)}(G_{51})]T_{49} = 0 \quad 1083$$

$$(b_{50})^{(10)}T_{49} - [(b'_{50})^{(10)} - (b''_{50})^{(10)}(G_{51})]T_{50} = 0 \quad 1084$$

has a unique positive solution, which is an equilibrium solution for the system 1085

**Proof:**

(a) Indeed the first two equations have a nontrivial solution  $G_{48}, G_{49}$  if

$$F(T_{51}) = (a'_{48})^{(10)}(a'_{49})^{(10)} - (a_{48})^{(10)}(a_{49})^{(10)} + (a'_{48})^{(10)}(a''_{49})^{(10)}(T_{49}) + (a'_{49})^{(10)}(a''_{48})^{(10)}(T_{49}) + (a''_{48})^{(10)}(T_{49})(a''_{49})^{(10)}(T_{49}) = 0$$

**Definition and uniqueness of  $T_{49}^*$**  :- 1086

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a'_i)^{(10)}(T_{49})$  are increasing, it follows that there exists a unique  $T_{49}^*$  for which  $f(T_{49}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{48} = \frac{(a_{48})^{(10)}G_{49}}{[(a'_{48})^{(10)} + (a''_{48})^{(10)}(T_{49}^*)]} \quad , \quad G_{50} = \frac{(a_{50})^{(10)}G_{49}}{[(a'_{50})^{(10)} + (a''_{50})^{(10)}(T_{49}^*)]}$$

(p) By the same argument, the equations 92,93 admit solutions  $G_{48}, G_{49}$  if

$$\varphi(G_{51}) = (b'_{48})^{(10)}(b'_{49})^{(10)} - (b_{48})^{(10)}(b_{49})^{(10)} -$$

$$[(b'_{48})^{(10)}(b''_{49})^{(10)}(G_{51}) + (b'_{49})^{(10)}(b''_{48})^{(10)}(G_{51})] + (b''_{48})^{(10)}(G_{51})(b''_{49})^{(10)}(G_{51}) = 0$$

Where in  $(G_{51})(G_{48}, G_{49}, G_{50}), G_{48}, G_{50}$  must be replaced by their values It is easy to see that  $\varphi$  is a decreasing function in  $G_{49}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{49}^*$  such that  $\varphi((G_{51})^*) = 0$  1087

Finally we obtain the unique solution

$G_{49}^*$  given by  $\varphi((G_{51})G^*) = 0$ ,  $T_{49}^*$  given by  $f(T_{49}^*) = 0$  and

$$G_{48}^* = \frac{(a_{48})^{(10)}G_{49}^*}{[(a'_{48})^{(10)} + (a''_{48})^{(10)}(T_{49}^*)]}, \quad G_{50}^* = \frac{(a_{50})^{(10)}G_{49}^*}{[(a'_{50})^{(10)} + (a''_{50})^{(10)}(T_{49}^*)]}$$

$$T_{48}^* = \frac{(b_{48})^{(10)}T_{49}^*}{[(b'_{48})^{(10)} - (b''_{48})^{(10)}((G_{51})^*)]}, \quad T_{50}^* = \frac{(b_{50})^{(10)}T_{49}^*}{[(b'_{50})^{(10)} - (b''_{50})^{(10)}((G_{51})^*)]}$$

Obviously, these values represent an equilibrium solution global equations

### ASYMPTOTIC STABILITY ANALYSIS

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a'_i)^{(10)}$  and  $(b'_i)^{(10)}$  belong to  $C^{(10)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a'_{49})^{(10)}}{\partial T_{49}}(T_{49}^*) = (q_{49})^{(10)}, \quad \frac{\partial (b'_i)^{(10)}}{\partial G_j}((G_{51})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from global equations

$$\begin{aligned} \frac{d\mathbb{G}_{48}}{dt} &= -((a'_{48})^{(10)} + (p_{48})^{(10)})\mathbb{G}_{48} + (a_{48})^{(10)}\mathbb{G}_{49} - (q_{48})^{(10)}G_{48}^*\mathbb{T}_{49} \\ \frac{d\mathbb{G}_{49}}{dt} &= -((a'_{49})^{(10)} + (p_{49})^{(10)})\mathbb{G}_{49} + (a_{49})^{(10)}\mathbb{G}_{48} - (q_{49})^{(10)}G_{49}^*\mathbb{T}_{49} \\ \frac{d\mathbb{G}_{50}}{dt} &= -((a'_{50})^{(10)} + (p_{50})^{(10)})\mathbb{G}_{50} + (a_{50})^{(10)}\mathbb{G}_{49} - (q_{50})^{(10)}G_{50}^*\mathbb{T}_{49} \\ \frac{d\mathbb{T}_{48}}{dt} &= -((b'_{48})^{(10)} - (r_{48})^{(10)})\mathbb{T}_{48} + (b_{48})^{(10)}\mathbb{T}_{49} + \sum_{j=48}^{50} (s_{(48)(j)}T_{48}^*\mathbb{G}_j) \\ \frac{d\mathbb{T}_{49}}{dt} &= -((b'_{49})^{(10)} - (r_{49})^{(10)})\mathbb{T}_{49} + (b_{49})^{(10)}\mathbb{T}_{48} + \sum_{j=48}^{50} (s_{(49)(j)}T_{49}^*\mathbb{G}_j) \\ \frac{d\mathbb{T}_{50}}{dt} &= -((b'_{50})^{(10)} - (r_{50})^{(10)})\mathbb{T}_{50} + (b_{50})^{(10)}\mathbb{T}_{49} + \sum_{j=48}^{50} (s_{(50)(j)}T_{50}^*\mathbb{G}_j) \end{aligned}$$

The characteristic equation of this system is

$$((\lambda)^{(10)} + (b'_{50})^{(10)} - (r_{50})^{(10)})\{((\lambda)^{(10)} + (a'_{50})^{(10)} + (p_{50})^{(10)})$$

$$\left[ ((\lambda)^{(10)} + (a'_{48})^{(10)} + (p_{48})^{(10)})(q_{49})^{(10)}G_{49}^* + (a_{49})^{(10)}(q_{48})^{(10)}G_{48}^* \right]$$

$$\begin{aligned}
 & \left( ((\lambda)^{(10)} + (b'_{48})^{(10)} - (r_{48})^{(10)})S_{(49),(49)}T_{49}^* + (b_{49})^{(10)}S_{(48),(49)}T_{49}^* \right) \\
 & + \left( ((\lambda)^{(10)} + (a'_{49})^{(10)} + (p_{49})^{(10)})(q_{48})^{(10)}G_{48}^* + (a_{48})^{(10)}(q_{49})^{(10)}G_{49}^* \right) \\
 & \left( ((\lambda)^{(10)} + (b'_{48})^{(10)} - (r_{48})^{(10)})S_{(49),(48)}T_{49}^* + (b_{49})^{(10)}S_{(48),(48)}T_{48}^* \right) \\
 & \left( ((\lambda)^{(10)})^2 + ((a'_{48})^{(10)} + (a'_{49})^{(10)} + (p_{48})^{(10)} + (p_{49})^{(10)}) (\lambda)^{(10)} \right) \\
 & \left( ((\lambda)^{(10)})^2 + ((b'_{48})^{(10)} + (b'_{49})^{(10)} - (r_{48})^{(10)} + (r_{49})^{(10)}) (\lambda)^{(10)} \right) \\
 & + \left( ((\lambda)^{(10)})^2 + ((a'_{48})^{(10)} + (a'_{49})^{(10)} + (p_{48})^{(10)} + (p_{49})^{(10)}) (\lambda)^{(10)} \right) (q_{50})^{(10)}G_{50} \\
 & + ((\lambda)^{(10)} + (a'_{48})^{(10)} + (p_{48})^{(10)}) ((a_{50})^{(10)}(q_{49})^{(10)}G_{49}^* + (a_{49})^{(10)}(a_{50})^{(10)}(q_{48})^{(10)}G_{48}^*) \\
 & \left. \left( ((\lambda)^{(10)} + (b'_{48})^{(10)} - (r_{48})^{(10)})S_{(49),(50)}T_{49}^* + (b_{49})^{(10)}S_{(48),(50)}T_{48}^* \right) \right\} = 0
 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem. More often than not, models begin with the assumption of 'steady state' and then proceed to trace out the path, which will be followed when the steady state is subjected to some kind of exogenous disturbance. Breathing pattern of terrestrial organisms is another parametric representation to be taken into consideration. It cannot be taken for granted that the sequence generated in this manner will tend to equilibrium i.e. a traverse from one steady state to another. In our model, we have, using the tools and techniques by Haimovici, Levin, Volttera, Lotka have brought out implications of steady state, stability, asymptotic stability, behavioral aspects of the solution without any such assumptions, such as those mentioned in the foregoing.

## SECTION 5:

### Affective Consciousness and Quantum Field Part One- Portmanteau Porte-cochere and contrariness of identities models

Preface- Affective consciousness: A Core emotional feeling in animals and humans has been studied by various authors. Some of them are Panksepp C. The position advanced in this investigation is that the bedrock of emotional feelings is contained within the evolved emotional action apparatus of mammalian brains. This dual-aspect monism approach to brain-mind functions, which asserts that emotional feelings may reflect the neuro dynamics of brain systems that generate instinctual emotional behaviors, saves us from various conceptual conundrums. In coarse form, primary process affective consciousness seems to be fundamentally an unconditional "gift of nature" rather than an acquired skill, even though those systems facilitate skill acquisition via various felt reinforcements. Affective consciousness, being a comparatively intrinsic function of the brain, shared homogenously by all mammalian species, should be the easiest variant of consciousness to study in animals. This is not to deny that some secondary processes (e.g., awareness of feelings in the generation of behavioral choices) cannot be evaluated in animals with sufficiently clever behavioral learning procedures, as with place-preference procedures and the analysis of changes in learned behaviors after one has induced re-evaluation of incentives. Rather, the claim is that a direct neuroscientific study of primary process emotional/affective states is best achieved through the study of the intrinsic ("instinctual"), albeit experientially refined, emotional action tendencies of other animals. In this view, core emotional feelings may reflect the neurodynamics attractor landscapes of a variety of extended trans-diencephalic,



limbic emotional action systems-including SEEKING, FEAR, RAGE, LUST, CARE, PANIC, and PLAY.

Through a study of these brain systems, the neural infrastructure of human and animal affective consciousness may be revealed. Emotional feelings are instantiated in large-scale neurodynamics that can be most effectively monitored via the ethological analysis of emotional action tendencies and the accompanying brain neurochemical/electrical changes. The intrinsic coherence of such emotional responses is demonstrated by the fact that they can be provoked by electrical and chemical stimulation of specific brain zones-effects that are affectively laden. For substantive progress in this emerging research arena, animal brain researchers need to discuss affective brain functions more openly. Secondary awareness processes, because of their more conditional, contextually situated nature, are more difficult to understand in any neuroscientific detail. In other words, the information-processing brain functions, critical for cognitive consciousness, are harder to study in other animals than the more homologous emotional/motivational affective state functions of the brain.

Form of discrete and stochastic pulse trains and point processes, the “macroscopic” activity of large assemblies of neurons appears to be spatially coherent and highly structured in phase and amplitude (Freeman 1996, 2000). Motivated by such an experimental situation, Ricciardi and Umezawa (1967) formulated the quantum model of the brain as a many-body physics problem, by using the formalism of QFT with a spontaneous breakdown of symmetry, successfully tested in condensed matter experiments. In fact, such formalism provides the only available theoretical tool capable of describing long-range correlations in many-body systems. In classical physics, long-range correlations are usually explained as the result of classical processes causally propagating along different pathways. However, such a view is not supported by many experimental observations in condensed matter physics. In contrast, such observations are fully described and predicted by quantum field theory (Itzykson and Zuber 1980, Umezawa 1993). The hypothesis by Ricciardi and Umezawa (1967) was, then, that the QFT approach could also be applied to describe the observation that the brain presents almost simultaneous responses in several regions to some external stimuli. As a matter of fact, the understanding of such correlations in terms of modern biochemical and electrochemical processes is still lacking, which suggests that these responses could not be explained in terms of single neuron activity (Pribram 1971, 1991).

In QFT the dynamics (i.e. the Lagrangian) is in general invariant under some group  $G$  of continuous transformations. This means that the dynamical evolution of the system is constrained by the conservation laws of some observables depending on  $G$ . For example, invariance under the time translation group implies energy conservation, and similarly one gets the conservation of momentum, of electric charge, etc. as consequences of the dynamical invariance under corresponding continuous transformation groups. The invariance of the dynamics, thus, characterizes physical states of the system by assigning conserved quantities to them. Spontaneous breakdown of symmetry occurs when the minimum energy state (the ground state or vacuum) of the system is not invariant under the full group  $G$ , but less than one of its subgroups. Then it can be shown (Itzykson and Zuber 1980,

Umezawa 1993) that collective modes, the so-called Nambu-Goldstone (NG) boson modes, are dynamically generated. At this point, let us mention that particles may also be represented as wave excitations or modes. Referring to NG modes, thus, emphasizes wave-like behavior rather than particle-like behavior. Propagating over the whole system, these modes are the carriers of the ordering information in terms of long-range correlations: order manifests itself as a global property dynamically generated. The long-range correlation modes are responsible for maintaining the ordered pattern: they are coherently condensed in the ground state. In the case of crystals, for Quantum Field Theory of Mind/Brain States 61example, they keep the atoms trapped at their lattice sites. Long-range correlations thus form a sort of net, extending all over the system, which traps its components in an ordered pattern. This explains the macroscopic collective behavior of the system as a “whole”. The macroscopic observable specifying the ordered state of a system is called the order parameter. For example, the density (or a quantity proportional to it) is the order parameter for a crystal, the magnetization for a ferromagnetic, etc. Properties like the stiffness of a crystal, electrical conductivity, etc., depend on the value assumed by the order parameter under specific boundary conditions. A most difficult problem in neuroscience is how conscious mind emerges from brain activities. To make headway on this, we may need to focus more on the vast neuronal contexts that unconditionally enable phenomenal experience rather than on the specific contents of consciousness. From this perspective, consciousness was initially built on fundamental survival concerns of organisms. Psychologically, such concerns may have been first instantiated in the glimmers of affective feelings – basic, internally felt neurodynamics reflecting intrinsic survival values that are experienced but not necessarily reflected upon. Unfortunately, affective experience has been profoundly neglected in consciousness studies. It is commonly assumed that consciousness cannot be scientifically studied without linguistic reports of subjective experiences. That premise arbitrarily limits consciousness studies to humans. (See JAAKPAKSEPP)

A neuro-evolutionary view suggests that primary-process consciousness emerged long before organisms had enough brain matter to speak or to cognitively reflect on their experiences. In any event, current knowledge supports the conjecture that primary-process affective experience emerged in brain evolution much earlier than the cognitive processes that allow us to think and talk about our internal experiences (i.e., secondary and tertiary forms of consciousness).

## INTRODUCTION:

### QUANTUM FIELD:

- (1) With the search for the Higgs boson, the last missing piece of the Standard Model of particle physics, apparently reaching its long-anticipated-and-finally-successful conclusion, anticipation of the next set of discoveries is growing. Repeatedly, it is reported that the theorists joked that, with the exception of the actual CERN experimentalists present, all of us know that the Higgs has now been discovered with a mass of  $125 \text{ GeV}/c^2$ . Super symmetry – a theory that posits that for every known particle there is another (or more than one) yet-to-be-discovered partner particle – is the leading candidate for physics Beyond the Standard Model.
- (2) It is central to string theory (a.k.a. super-string theory), required(e) for gauge coupling unification (see below), useful for solving the Higgs Fine Tuning Problem
- (3) This gives us the leading candidate for dark matter = the Lightest Supersymmetric Particle (LSP). A main rationale for super symmetry evaporates on closer inspection. So what is Beyond the Standard Model (BSM) physics, but still people are convinced that Higgs is the ultimate neplus ultra. At least since the discovery of the W and Z particles at CERN in 1983, physicists have been pretty much convinced that the Standard Model (SM) that emerged from the late 1960s and early 1970s is the correct model of fundamental physics. At least at energies below the so-called weak-scale – a few hundred GeV – or maybe a few times that. But particle theorists variously hoped/expected/knew that at higher energies the Standard Model was not the whole story and a more fundamental theory would need to be found.
- (4) According to Relativity, our lives are world lines in space time.
- (5) Space time does not happen, it always exists.

- (6) **It is our brain that shows us a movie of matter evolving in time.**
- (7) All spacetime events are **conscious**; **they are conscious of other** spacetime events.
- (8) The "**experience**" of a spacetime event **is static**, a **frozen region of spacetime events**.
- (9) **All the subjective features of the "psycho space" of an observer can be completely derived** from the **objective features of the region of spacetime that the observer is connected to.**
- (10) **Special circuits in our brain create the impression of a time flow, of a time travel through the region of spacetime events connected to the brain.**
- (11) Memory of an event is re-experiencing that spacetime event, which is **fixed** in spacetime.
- (12) We don't store an event; we only keep a **link** to it.
- (13) **Conscious memory** is not in the brain but in is **in spacetime**.
- (14) The inner life of a system **is (=) its** spacetime history.

There are two types of reasons to doubt the completeness of the Standard Model – aesthetic (philosophical) and mathematical. Aesthetic problem number one, physicists adore simplicity. Zero and one are our favorite numbers. Two can be suffered. After two comes “too many”, although identical copies (twins, triplets,) may receive special dispensation. The Standard Model has too many too-man’s: three fundamental forces (a.k.a. gauge groups); way too many fundamental fermions (particles that make up matter)– three generations each with at least 5 representations (groups) of them — plus three sets of gauge bosons and the set of particles of which the Higgs boson is a member. It also has far too many (more than 20) independent parameters. Aesthetic problem number two — for no apparent reason the weak scale is much (as in about  $10^{16}$  times) smaller than what we believe to be the fundamental energy scale of physics – the Planck scale (about  $10^{19}$  GeV), a scale set by the strength of gravity (the one fundamental force not included in the Standard Model). This is known as the (Weak) Hierarchy Problem – and can also be understood in terms of the absolutely enormous strength of the three Standard Model forces compared to that of gravity between pairs of fundamental particles separated by appropriately microscopic scales. It is however the technical problem that has carried the most weight in convincing people that there must be physics beyond the Standard Model. It is the story we tell our children — quantum mechanics makes the Standard Model unstable. Quantum mechanics teaches us that, as a particle such as a Higgs boson travels along, it can emit and reabsorb another particle. This process represents a “loop contribution” to the mass of the Higgs boson, so-called because a pictorial representation of the process – Feynman diagrams – depicts these processes as loops attached to the traveling Higgs boson. Unfortunately, when you add up the loop contributions to the mass of the Higgs boson from all possible particles with all possible energies and momenta, they appear to be infinite or at least proportional to the maximum possible momentum that can be carried. For technical reasons these are called quadratic divergences and are widely derided. For the actual Higgs boson mass to be finite, there must apparently be subtle and precise cancellation of the loop contributions against the underlying “tree” (loop-free) mass. This Higgs Fine-Tuning Problem, so the lore tells us, must be remedied. Following points are of seminal and perennial importance:

- (1) **Potential features of quantum computation** could **explain** enigmatic aspects of consciousness.
- (2) The Penrose—Hameroff model (orchestrated objective reduction: ‘Orch OR’) **suggests** that **quantum superposition and a form of quantum computation occur** in microtubules—cylindrical protein lattices of **the cell cytoskeleton within the brain's neurons.**
- (3) **Microtubules couple to and regulate** neural-level synaptic functions,
- (4) They may be ideal **quantum computers because of dynamical lattice structure, quantum-level subunit states and intermittent isolation from environmental interactions.**
- (5) In addition to its biological setting, the Orch OR proposal differs in an essential way from technologically envisioned **quantum computers in which collapse, or reduction to classical output states, is caused by environmental decoherence (hence introducing randomness).** In the **Orch OR proposal, reduction of microtubule quantum superposition to classical output states occurs** by an **objective factor**
- (6) Roger Penrose's **quantum gravity threshold stems** quintessentially from **instability in Planck-scale**

- separations (superpositions) in spacetime geometry.
- (7) Output states following Penrose's objective reduction are neither totally deterministic nor random, but **influenced by** a non-computable factor ingrained in fundamental spacetime.
  - (8) Taking a modern pan-psychist view in which protoconscious experience and Platonic values are embedded in Planck-scale spin networks, the Orch OR model **portrays** poignantly and in a functionally topological manner with abstract problematic structure consciousness as brain activities linked to fundamental ripples in spacetime geometry. (Stuart Hameroff, MD)
  - (9) Protein conformational dynamics and pharmacological evidence suggest that protein conformational states—fundamental information units ('bits') in biological systems—are governed by quantum events,

BSM physics is the proposed remedy. Super symmetry cancels the loop of every known particle against the loop of an as-yet-to-be-discovered partner particle. Technicolor eliminates the Higgs boson – replacing it by a composite of new particles called techni-quarks. If there are large extra dimensions then the largest momentum that can circulate in a loop is actually only a little larger than the weak scale. Clearly BSM physics is not just desirable but essential. Recently, however, my colleague Bryan Lynn suggested, and together with Katie Freese and Dmitry Podolsky, he and I explained, how the Standard Model actually comes up with a remedy all on its own. The Higgs boson is one member of a set of quadruplets in the Standard Model. At energies below the weak scale, its three siblings get eaten – they get incorporated into the W and Z bosons. According to a famous theorem due to MIT's Jeffrey Goldstone (hence "Goldstone's Theorem"), the masses of the three siblings must be exactly zero. In particular, the quadratic ally divergent contribution to their masses are zero.

Although this doesn't force the mass of the Higgs boson to be zero (a good thing, since it seems likely to be about  $125 \text{ GeV}/c^2$ ), it does mean that the quadratic divergences in the Higgs mass that have worried us for decades are not a problem of the Standard Model after all. Now, not everybody buys our argument. Some of them prefer to focus on the aesthetic challenge of the Weak Hierarchy Problem, while others argue that we have no choice but to add quantum gravity to the Standard Model, inevitably resurrecting the Higgs Fine Tuning Problem. We would counter that the absence of a Higgs Fine Tuning Problem in the Standard Model is such a virtue that, absent any hard evidence for BSM physics, preserving the Standard Model's Goldstone miracle should be taken as a requirement of any proposed BSM theories. The implication is clear. If there is no problem, there may be no need for a solution. Beyond the Standard Model Physics isn't ruled out by the absence of a Higgs Fine Tuning Problem in the Standard Model, but it does mean that the Standard Model may well be the whole story, or at least the whole story at the energies that the LHC can command. In short, don't be surprised if the Higgs is the last new particle discovered by the LHC. Theorists may hunger for physics beyond the Standard Model, but nature may be quite content without it, thank you very much.

For decades now physicists have contemplated the idea of an entire shadow world of elementary particles, called super symmetry. It would elegantly solve mysteries that the current Standard Model of particle physics leaves unexplained, such as what cosmic dark matter is. Now some are starting to wonder. The most powerful collider in history, the Large Hadron Collider (LHC), has yet to see any new phenomena that would betray an unseen level of reality. Although the search has only just begun, it has made some theorists ask what physics might be like if super symmetry is not true after all. "Wherever we look, we see nothing—that is, we see no deviations from the Standard Model," says Giacomo Polesello of Italy's National Institute of Nuclear Physics in Pavia. Polesello is a leading member of the 3,000-strong international collaboration that built and operates ATLAS, one of two cathedral-size general-purpose detectors on the LHC ring. The other such detector, CMS, has seen nothing, either, according to an update presented at a conference in the Italian Alps in March. Theorists introduced super symmetry in the 1960s to connect the two basic types of particles seen in nature, called fermions and bosons. Roughly speaking, fermions are the constituents of matter (the electron being the quintessential example), whereas bosons are the carriers of the fundamental forces (the photon in the case of electromagnetism). Super symmetry would give every known boson a heavy "super partner" that is a fermion and every known fermion a heavy partner that is a boson. "It is the next step up toward the ultimate view of the world, where we make everything symmetric and beautiful," says Michael Peskin, a theorist at SLAC National Accelerator Laboratory.

Following are the tit bits from the article by Sir Jogesh C. Pati, Professor at Princeton University.

Reader is referred to his papers for further reading. Quintessentially we take points that are consistent for our deliberation.

- (1) Recently a new class of composite Higgs models have been developed which **give rise** to naturally light Higgs bosons without super symmetry.
- (2) Based on the chiral symmetries of “theory space,” involving replicated gauge groups and appropriate gauge symmetry breaking patterns, these models allow the scale of the underlying strong dynamics **giving rise to** the composite particles to be as large as of order 10 TeV, without any fine tuning to prevent large corrections to Higgs boson mass(es) of order 100 GeV.
- (3) Prof. Pati showed that the size of flavor violating interactions arising generically from underlying flavor dynamics **constrains** the scale of the Higgs boson compositeness to be greater than of order 75 TeV, implying that significant fine-tuning is required.
- (4) Without fine-tuning, the low-energy structure of the composite Higgs model alone is **not sufficient to eliminate** potential problems with flavor-changing neutral currents or excessive CP violation; solving those problems requires additional information or assumptions about the symmetries of the underlying flavor or strong dynamics.
- (5) Weaker, but more model-independent, bounds **which arise** from limits on weak isospin violation. In the presence of massless chiral fermions, a  $\theta$  term in can be rotated away by an appropriate chiral transformation of the fermion fields, because due to the chiral anomaly, this transformation **induces** a contribution to the fermion path integral measure proportional to the  $\theta$  term Lagrangian.
- (6) The gluons have the same coupling to the right and left handed quarks, and a chiral rotation **leaves** the mass matrix variant.
- (7) The SU(2)<sub>L</sub> fields (however, are coupled to the left handed components of the fermions only, thus both the left and right handed components can be rotated with the same angle, rotating away the  $\theta$  term) **without altering** the mass matrix.
- (8) Bose-Fermi symmetry **play a crucial role** in all recent attempts at higher unification, which include the ideas of(i) the conventional approach to grand unification; (ii) the preonic approach; and (iii) Superstrings.
- (9) The concept of massless particles with two states of polarization (these are the photons with spin 1) **and** the concept that their number is not conserved; that they obey a new statistics.
- (10) Connection between spin **and** statistics which asserts that particles of integer spins (i.e., spin 0, 1, 2,) obey what is now called the Bose-Einstein statistics while those of half-integer spins (i.e., spin-1/2, 3/2, 5/2, . . .) obey the Fermi-Dirac statistics or the exclusion principle. This in turn allows us to classify all known elementary particles as either bosons or fermions which respectively obey the Bose-Einstein and the Fermi Dirac statistics.
- (11)(Concept of symmetry relates) a symmetry that **relates** fermions **to** bosons
- (12)Principle of local gauge invariance **dictates** the existence of the photon and the gluons as well as that of the associated forces of quantum electro- and chromodynamics.
- (13)The principle of general coordinate invariance proposed by Einstein **leads to** the familiar gravitational “forces” as a consequence of the curvature of space-time
- (14)Principle of general coordinate invariance **dictates** the existence of spin-2 graviton.
- (15) (Coordinate invariance and curvature of space and time)in turn help **preserve** the masslessness of these “gauge” particles despite quantum corrections, at least in perturbation theory.
- (16)Spin-1/2 particles (i.e., electrons) are at least needed since they obey the exclusion principle which is relevant to the **explanation** of chemistry and in turn to the biology of life.
- (17)Now, once spin-1/2 particles are introduced into the Lagrangian, **they have** the good feature that their masses remain protected against arbitrarily large quantum corrections through the so-called chiral symmetry
- (18)Chiral symmetry **guarantees** that the quantum corrections to the masses of spin-1/2 fermions in perturbation theory either vanish or are bounded by a logarithmic cutoff (symbolizing short-



- distance physics) depending upon whether their “bare” mass-terms are zero or non-zero
- (19). In particular, the pions are the **carriers** of the nuclear force
  - (20) Higgs bosons induce spontaneous breaking of the electroweak symmetry and give masses to W and Z bosons.
  - (21) Mass of the pion **controlled** by the relevant chiral symmetry breaking parameter which is determined by (a) the finite “bare” masses of the up and down 3quarks and (b) the QCD scale-parameter.
  - (22) Compositeness of Higgs Boson **violates** the flavor-changing neutral current processes and oblique electroweak parameters.
  - (23) Elementariness of Higgs Boson leads to (i) What if any is an a priori rationale for its existence, and , equally important, (ii) how can one protect its mass against large quantum corrections?
  - (24) Higgs boson couples to the gauge bosons and also possesses quartic self couplings, one **obtains corrections in one loop to its mass which are proportional to  $\alpha\Lambda$**  ( $\Lambda$  is the cutoff characterizing short-distance physics and  $\alpha$ 's are coupling parameters
  - (25) Fine tuning by some 24 orders of magnitude (or higher) in the choice of the bare mass of the Higgs boson cancels **the** large quantum corrections.
  - (26) “Super symmetry” **transforms** spin-1/2 fermions into spin-0 bosons and vice versa.
  - (27) Bose fermi Symmetry transforms Bosons **in to** fermions
  - (28) The power of super symmetry arises because its generator(s) Q **transforms** a spin-0 (or spin-1) boson into a spin-1/2 fermion and thereby changes the spin of the particle by 1/2 unit as well as its statistics.
  - (29) Generators of Lie algebras, associated with the familiar symmetries such as isospin and SU (3) **transform** as Lorentz-scalars – i.e., as spin-0 bosonic operators; and which can thus transform a particle of a given spin into another of the same spin, only.
  - (30) Super symmetry **unites** fermions and bosons (as members of a supermultiplet and thereby provides the rationale for the existence of spin-0 matter, on a par with that of spin-1/2 matter.)
  - (31) Super symmetry **permits** a non-trivial marriage of space-time (Poincaré) symmetries with internal symmetries in accord with relativity.
  - (32) Lopuszanski and Sohnius the graded Lie algebra associated with super symmetry is the only framework within which such a marriage of space and time can be **achieved** ( consistent with relativistic quantum field theory. For example, SU(2)-isospin symmetry together with super symmetry groups the spin-1/2 doublet of (u, d)-quarks with the spin-0 doubletof ( $\tilde{u}, \tilde{d}$ )-squarks to make a super-bidoublet)

The monumental collider at CERN near Geneva should have the oomph to produce those super particles. Currently the LHC is smashing protons with energy of four trillion electron volts (TeV) apiece, up from 3.5 TeV last year. This energy is divided among the quarks and gluons that make up the protons, so the collision can generate new particles with the equivalent of about 1 TeV of mass. But despite the high expectations (and energies), so far nature has not cooperated. LHC physicists have been searching for signs of particles new to science and have seen none. If super particles exist, they must be even heavier than many physicists had hoped. “To put it bluntly,” Polesello says, “the situation is that we have ruled out a number of ‘easy’ models that should have showed up right away.” His colleague Ian Hinchliffe of Lawrence Berkeley National Laboratory echoes him: “If you look at the range of masses and particles that have been excluded, it’s quite impressive.” Many are still hopeful. “There are still very viable ways of building super symmetry models,” Peskin says. Expecting to see new physics after just a year of data taking was unrealistic, says Joseph Lykken, a theorist on the CMS team. What has theorists on edge, however, is that for super symmetry to solve the problems for which it was invented in the first place, at least a few of the super particles should not be too heavy. To constitute dark matter, for example, they need to weigh no more than a few tenths of 1 TeV. Another reason most physicists want some super particles to be light lies in the Higgs boson, another major target of the LHC. All elementary particles that have mass are supposed to get it through their interaction with this boson and, secondarily, with a halo of fleeting “virtual particles.” In most cases, the symmetries of the Standard Model ensure that these virtual particles cancel one another out, so they contribute only modestly to mass. The exception, ironically, is the Higgs itself. Calculations based on the Standard

Model yield the paradoxical result that the boson's mass should be infinite. Super partners would solve this mystery by providing greater scope for cancellations. A Higgs mass of around 0.125 TeV, as suggested by preliminary results announced in December 2011, would be right in the range where super symmetry predicts it should be. But in that case, the super particles would need to have a fairly low mass.

If the Higgs boson with mass 125 GeV is fermiophobic, as suggested by the most recent LHC results, then the MSSM is excluded. The minimal supersymmetric fermiophobic Higgs scenario can naturally be formulated in the context of the NMSSM that admits  $Z_3$  discrete symmetries. In the fermiophobic NMSSM, the SUSY naturalness criteria are relaxed by a factor  $N_c y_t^4/g^4 \sim 25$ , removing the little hierarchy problem and allowing particle masses to be naturally of order 2-3 TeV. This scale motivates wino or Higgsino dark matter. The SUSY flavour and CP problems as well as the constraints on  $s$ -particle and Higgs boson masses from  $b \rightarrow s\gamma$ ,  $B_s \rightarrow \mu\mu$  and direct LHC searches are relaxed in fermiophobic NMSSM. The price to pay is that a new, yet unknown, mechanism must be introduced to generate fermion masses. We show that in the fermiophobic NMSSM the radiative Higgs boson branching to  $\gamma\gamma$ ,  $\gamma Z$  can be modified compared to the fermiophobic and ordinary standard model predictions, and fit present collider data better. Suppression of dark matter scattering off nuclei explains the absence of signal in XENON100.

#### A SOCIOLOGICAL INTRUSION:

Change is a potentially produces and transforms as an emotional event as people anticipate or experience its outcomes and processes.

Emotional aspects of organizational change, promote acceptance of change or resistance to it. Cognitive, affective and behavioral responses to organizational change. indicates outcomes, scale, temporal issues and justice

Emotional intelligence, disposition, previous experience of change, and change and stress outside the workplace affect those in the employee's perceptions of the leaders/managers/agents (their leadership ability, emotional intelligence and trustworthiness); and those in the employee's perception of the organization (its culture and change context)

Cognitive appraisal theory takes the position that emotion derives from cognition as people contemplate the importance of events (such as organizational change) to their wellbeing and consider how they will cope.

Social constructionism was used as a theoretical platform because it combines the individual experience of emotions during change with the social forces that help shape them.

The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself.

The particle Higgs Boson

Higgs Quantum Field

Mass of the matter

Quantum Field Theory

Electro-Weak Unification and Broken Symmetry

- (1) Grand Unification Theory Super symmetry
- (2) Quantum Gravity and Super gravity
- (3) The Heterotic String
- (4) Unified Quantum Field Theories
- (5) Principle of spontaneously broken symmetry, locates (eb) deeply hidden symmetries of nature at fundamental space-time scales.
- (6) Principle of broken symmetry explains the emergence of diverse forces from an initially unified field.
- (7) Profound symmetry principle called super symmetry, which has the potential of unifying force fields

and matter fields in the context of a single field.

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## SECTION 5:

### **AFFECTIVE CONSCIOUSNESS AND QUANTUM FIELD -A PISCATORIAL PIRATISH-BUBONIC BUCANEER MODEL-PART ONE**

**PREFACE:** The connection between statistical mechanics and quantum field theory (See Barry M. McCoy) are elucidated and expatiated more often than not... The general principles, axiomatic predications and postulation, conception, and creation relating statistical mechanics and the path integral formulation of quantum field theory are intertwined in such a way that they cannot be disentangled. Theory of the Ising model for  $H=0$ , where both the homogeneous and randomly inhomogeneous models **are treated** and the scaling theory and the relation with Fredholm determinants and Painlevé equations bears ample testimony and infallible observatory and impeccable demonstration to the intricate links that the two fields possess. Ising model with  $H \neq 0$ , where the **relation with** gauge theory **is used** to discuss the phenomenon of confinement. We conclude in the last lecture with a discussion of quantum spin diffusion in one dimensional chains and a presentation of the chiral Potts model which illustrates the physical effects that can occur when the Euclidean and Minkowski regions are not connected by an analytic continuation. **Nonequilibrium Statistical Mechanics with Quantum Field has been studied and investigated by** Takafumi Kita where a conception, creation, process oriented and system oriented a concise and self-contained introduction to nonequilibrium statistical mechanics with quantum field theory by considering **an ensemble of interacting** identical bosons or fermions as an example. Matsubara formalism of equilibrium statistical mechanics such as Feynman diagrams, the proper self-energy, and Dyson's equation are widely used in the subject matter in question. The author states that the aim is three pronged: (i) to **explain the** fundamentals of nonequilibrium quantum field theory as simple as possible on the basis of the knowledge of the equilibrium counterpart; (ii) **to elucidate** the hierarchy in **describing** nonequilibrium systems from Dyson's equation on the Keldysh contour to the Navier-Stokes equation in fluid mechanics via quantum transport equations and the Boltzmann equation; (iii) **to derive** an expression of nonequilibrium entropy that **evolves** with time. Nonequilibrium Green's function and the self-energy uniquely on the round-trip Keldysh contour, thereby **avoiding** possible confusions that **may arise** from **defining** multiple Green's functions at the very beginning. Feynman **rules for the** perturbation expansion are presented. Self-consistent perturbation expansion with the Luttinger-Ward thermodynamic functional, i.e., Baym's Phi-derivable approximation, which **has a crucial** property for nonequilibrium systems **of obeying** various conservation laws automatically. Two-particle **correlations** can be calculated within the Phi-derivable approximation, i.e., an issue of how to handle the "Bogoliubov-Born-Green-Kirkwood-Yvons (BBGKY) hierarchy". In the extant and existential model we converge some of the salient features of the Quantum Field Theory, Statistical Mechanics, Yang Baxter Equations, Bose fermion Equivalence, Rational conformal field theory to present a holistic and consummate scenario of the asymmetric freedom and translations in space time. We intend to extend the model for virtual photons and mass energy momentum tensor.

### **HIGGS OLIO PODRADA AND POT POURRI:**

BECAUSE Higgs particles interact most strongly with other high-mass particles, it is hard to make them directly in the collisions of lightweights like electrons. Instead, we reach the Higgs particle indirectly, through virtual Z or W bosons or pairs of top quarks, which then decay into Higgs.

In diagrams like those shown below, only the particles with free ends extending backwards exist for a



noticeable time in the past, and only the particles with free ends extending forwards exist for a noticeable time in the future. The lines with no free ends have only a very fleeting existence and cannot be observed - they are said to be virtual particles.

In part (A) of the diagram, we see how an electron and a positron create a virtual Z boson, which then emits a Higgs boson and becomes real. This is the process LEP experimenters hope to see. At the Tevatron, instead of electrons, experimenters will use quarks and antiquarks (B) found within their colliding protons and antiprotons, and produce W bosons in place of Zs. (C) Alternatively at the Tevatron and especially at the LHC, gluons - again found within colliding protons - should make pairs of virtual top quarks that will annihilate to form Higgs particles. This process is my own contribution to the Higgs particle cookbook.

### **The Mysteries of Mass**

Physicists are hunting for an elusive particle that would reveal the presence of a new kind of field that permeates all of reality. Finding that Higgs field will give us a more complete understanding about how the universe works

Most people think they know what mass is, but they understand only part of the story. For instance, an elephant is clearly bulkier and weighs more than an ant. Even in the absence of gravity, the elephant would have greater mass--it would be harder to push and set in motion. Obviously the elephant is more massive because it is made of many more atoms than the ant is, but what determines the masses of the individual atoms? What about the elementary particles that makes up the atoms--what determines their masses? Indeed, why do they even have mass?

We see that the problem of mass has two independent aspects. First, we need to learn how mass arises at all. It turns out mass results from at least three different mechanisms, which I will describe below. A key player in physicists' tentative theories about mass is a new kind of field that permeates all of reality, called the Higgs field. Elementary particle masses are thought to come about from the interaction with the Higgs field. If the Higgs field exists, theory demands that it have an associated particle, the Higgs boson. Using particle accelerators, scientists are now hunting for the Higgs.

The second aspect is that scientists want to know why different species of elementary particles have their specific quantities of mass. Their intrinsic masses span at least 11 orders of magnitude, but we do not yet know why that should be so. For comparison, an elephant and the smallest of ants differ by about 11 orders of magnitude of mass.

### **What Is Mass?**

Isaac Newton presented the earliest scientific definition of mass in 1687 in his landmark Principia: "The quantity of matter is the measure of the same, arising from its density and bulk conjointly." That very basic definition was good enough for Newton and other scientists for more than 200 years. They understood that science should proceed first by describing how things work and later by understanding why. In recent years, however, the why of mass has become a research topic in physics. Understanding the meaning and origins of mass will complete and extend the Standard Model of particle physics, the well-established theory that describes the known elementary particles and their interactions. It will also resolve mysteries such as dark matter, which makes up about 25 percent of the universe.

### **Why is the Higgs field present throughout the universe? What is the Higgs field?**

The foundation of our modern understanding of mass is far more intricate than Newton's definition and is based on the Standard Model. At the heart of the Standard Model is a mathematical function called a

Lagrangian, which represents how the various particles interact? From that function, by following rules known as relativistic quantum theory, physicists can calculate the behavior of the elementary particles, including how they come together to form compound particles, such as protons. For both the elementary particles and the compound ones, we can then calculate how they will respond to forces, and for a force  $F$ , we can write Newton's equation  $F = ma$ , which relates the force, the mass and the resulting acceleration. The Lagrangian tells us what to use for  $m$  here, and that is what is meant by the mass of the particle.

But mass, as we ordinarily understand it, shows up in more than just  $F = ma$ . For example, Einstein's special relativity theory predicts those massless particles in a vacuum travel at the speed of light and those particles with mass travel more slowly, in a way that can be calculated if we know their mass. The laws of gravity predict that gravity acts on mass and energy as well, in a precise manner. The quantity  $m$  deduced from the Lagrangian for each particle behaves correctly in all those ways, just as we expect for a given mass. Fundamental particles have an intrinsic mass known as their rest mass (those with zero rest mass are called massless). For a compound particle, the constituents' rest mass and also their kinetic energy of motion and potential energy of interactions contribute to the particle's total mass. Energy and mass are related, as described by Einstein's famous equation,  $E = mc^2$  (energy equals mass times the speed of light squared). An example of energy contributing to mass occurs in the most familiar kind of matter in the universe--the protons and neutrons that make up atomic nuclei in stars, planets, people and all that we see. These particles amount to 4 to 5 percent of the mass-energy of the universe. The Standard Model tells us that protons and neutrons are composed of elementary particles called quarks that are bound together by massless particles called gluons. Although the constituents are whirling around inside each proton, from outside we see a proton as a coherent object with an intrinsic mass, which is given by adding up the masses and energies of its constituents. The Standard Model lets us calculate that nearly all the mass of protons and neutrons is from the kinetic energy of their constituent quarks and gluons (the remainder is from the quarks' rest mass). Thus, about 4 to 5 percent of the entire universe--almost all the familiar matter around us--comes from the energy of motion of quarks and gluons in protons and neutrons.

### **The Higgs Mechanism**

Unlike protons and neutrons, truly elementary particles--such as quarks and electrons--are not made up of smaller pieces. The explanation of how they acquire their rest masses gets to the very heart of the problem of the origin of mass. As I noted above, the account proposed by contemporary theoretical physics is that fundamental particle masses arise from interactions with the Higgs field. But why is the Higgs field present throughout the universe? Why isn't its strength essentially zero on cosmic scales, like the electromagnetic field? What is the Higgs field? The Higgs field is a quantum field. That may sound mysterious, but the fact is that all elementary particles arise as quanta of a corresponding quantum field. The electromagnetic field is also a quantum field (its corresponding elementary particle is the photon). So in this respect, the Higgs field is no more enigmatic than electrons and light. The Higgs field does, however, differ from all other quantum fields in three crucial ways. The first difference is somewhat technical. All fields have a property called spin, an intrinsic quantity of angular momentum that is carried by each of their particles. Particles such as electrons have spin  $1/2$  and most particles associated with a force, such as the photon, have spin 1. The Higgs boson (the particle of the Higgs field) has spin 0. Having 0 spin enables the Higgs field to appear in the Lagrangian in different ways than the other particles do, which in turn allows--and leads to--its other two distinguishing features. The second unique property of the Higgs field explains how and why it has nonzero strength throughout the universe. Any system, including a universe, will tumble into its lowest energy state, like a ball bouncing down to the bottom of a valley. For the familiar fields, such as the electromagnetic fields that give us radio broadcasts, the lowest energy state is the one in which the fields have zero value (that is, the fields vanish)--if any nonzero field is introduced, the energy stored in the fields

increases the net energy of the system. But for the Higgs field, the energy of the universe is lower if the field is not zero but instead has a constant nonzero value. In terms of the valley metaphor, for ordinary fields the valley floor is at the location of zero fields; for the Higgs, the valley has a hillock at its center (at zero fields) and the lowest point of the valley forms a circle around the hillock. The universe, like a ball, comes to rest somewhere on this circular trench, which corresponds to a nonzero value of the field. That is, in its natural, lowest energy state, the universe is permeated throughout by a nonzero Higgs field.

The final distinguishing characteristic of the Higgs field is the form of its interactions with the other particles. Particles that interact with the Higgs field behave as if they have mass, proportional to the strength of the field times the strength of the interaction. The masses arise from the terms in the Lagrangian that have the particles interacting with the Higgs field.

Our understanding of all this is not yet complete, however, and we are not sure how many kinds of Higgs fields there are. Although the Standard Model requires only one Higgs field to generate all the elementary particle masses, physicists know that the Standard Model must be superseded by a more complete theory. Leading contenders are extensions of the Standard Model known as Supersymmetric Standard Models (SSMs). In these models, each Standard Model particle has a so-called super partner (as yet undetected) with closely related properties [see "The Dawn of Physics beyond the Standard Model," by Gordon Kane; *Scientific American*, June 2003]. With the Supersymmetric Standard Model, at least two different kinds of Higgs fields are needed. Interactions with those two fields give mass to the Standard Model particles. They also give some (but not all) mass to the super partners. The two Higgs fields give rise to five species of Higgs boson: three that are electrically neutral and two that are charged. The masses of particles called neutrinos, which are tiny compared with other particle masses, could arise rather indirectly from these interactions or from yet a third kind of Higgs field.

Theorists have several reasons for expecting the SSM picture of the Higgs interaction to be correct. First, without the Higgs mechanism, the W and Z bosons that mediate the weak force would be massless, just like the photon (which they are related to), and the weak interaction would be as strong as the electromagnetic one. Theory holds that the Higgs mechanism confers mass to the W and Z in a very special manner. Predictions of that approach (such as the ratio of the W and Z masses) have been confirmed experimentally.

Second, essentially all other aspects of the Standard Model have been well tested, and with such a detailed, interlocking theory it is difficult to change one part (such as the Higgs) without affecting the rest. For example, the analysis of precision measurements of W and Z boson properties led to the accurate prediction of the top quark mass before the top quark had been directly produced. Changing the Higgs mechanism would spoil that and other successful predictions.

Third, the Standard Model Higgs mechanism works very well for giving mass to all the Standard Model particles, W and Z bosons, as well as quarks and leptons; the alternative proposals usually do not. Next, unlike the other theories, the SSM provides a framework to unify our understanding of the forces of nature. Finally, the SSM can explain why the energy "valley" for the universe has the shape needed by the Higgs mechanism. In the basic Standard Model the shape of the valley has to be put in as a postulate, but in the SSM that shape can be derived mathematically.

### **Testing the Theory**

Naturally, physicists want to carry out direct tests of the idea that mass arises from the interactions with the different Higgs fields. We can test three key features. First, we can look for the signature particles called Higgs bosons. These quanta must exist, or else the explanation is not right. Physicists are

currently looking for Higgs bosons at the Tevatron Collider at Fermi National Accelerator Laboratory in Batavia, Ill. Second, once they are detected we can observe how Higgs bosons interact with other particles. The very same terms in the Lagrangian that determine the masses of the particles also fix the properties of such interactions. So we can conduct experiments to test quantitatively the presence of interaction terms of that type. The strength of the interaction and the amount of particle mass are uniquely connected. Third, different sets of Higgs fields, as occur in the Standard Model or in the various SSMs, imply different sets of Higgs bosons with various properties, so tests can distinguish these alternatives, too. All that we need to carry out the tests are appropriate particle colliders--ones that have sufficient energy to produce the different Higgs bosons, sufficient intensity to make enough of them and very good detectors to analyze what is produced.

A practical problem with performing such tests is that we do not yet understand the theories well enough to calculate what masses the Higgs bosons themselves should have, which makes searching for them more difficult because one must examine a range of masses. A combination of theoretical reasoning and data from experiments guides us about roughly what masses to expect. The Large Electron-Positron Collider (LEP) at CERN, the European laboratory for particle physics near Geneva, operated over a mass range that had a significant chance of including a Higgs boson. It did not find one--although there was tantalizing evidence for one just at the limits of the collider's energy and intensity--before it was shut down in 2000 to make room for constructing a newer facility, CERN's Large Hadron Collider (LHC). The Higgs must therefore be heavier than about 120 proton masses. Nevertheless, LEP did produce indirect evidence that a Higgs boson exists: experimenters at LEP made a number of precise measurements, which can be combined with similar measurements from the Tevatron and the collider at the Stanford Linear Accelerator Center. The entire set of data agrees well with theory only if certain interactions of particles with the lightest Higgs boson are included and only if the lightest Higgs boson is not heavier than about 200 proton masses. That provides researchers with an upper limit for the mass of the Higgs boson, which helps focus the search.

The LEP collider saw tantalizing evidence for the Higgs particle.

For the next few years, the only collider that could produce direct evidence for Higgs bosons will be the Tevatron. Its energy is sufficient to discover a Higgs boson in the range of masses implied by the indirect LEP evidence, if it can consistently achieve the beam intensity it was expected to have, which so far has not been possible. In 2007 the LHC, which is seven times more energetic and is designed to have far more intensity than the Tevatron, is scheduled to begin taking data. It will be a factory for Higgs bosons (meaning it will produce many of the particles a day). Assuming the LHC functions as planned, gathering the relevant data and learning how to interpret it should take one to two years. Carrying out the complete tests that show in detail that the interactions with Higgs fields are providing the mass will require a new electron-positron collider in addition to the LHC (which collides protons) and the Tevatron (which collides protons and antiprotons).

### **Dark Matter**

What is discovered about Higgs bosons will not only test whether the Higgs mechanism is indeed providing mass, it will also point the way to how the Standard Model can be extended to solve problems such as the origin of dark matter. With regard to dark matter, a key particle of the SSM is the lightest super partner (LSP). Among the super partners of the known Standard Model particles predicted by the SSM, the LSP is the one with the lowest mass. Most super partners decay promptly to lower-mass super partners, a chain of decays that ends with the LSP, which is stable because it has no lighter particle that it can decay into. (When a super partners decays, at least one of the decay products should be another super partner; it should not decay entirely into Standard Model particles.) Superpartner particles would have been created early in the big bang but then promptly decayed into LSPs. The LSP

is the leading candidate particle for dark matter. The Higgs bosons may also directly affect the amount of dark matter in the universe. We know that the amount of LSPs today should be less than the amount shortly after the big bang, because some would have collided and annihilated into quarks and leptons and photons, and the annihilation rate may be dominated by LSPs interacting with Higgs bosons. As mentioned earlier, the two basic SSM Higgs fields give mass to the Standard Model particles and some mass to the super partners, such as the LSP. The super partners acquire more mass via additional interactions, which may be with still further Higgs fields or with fields similar to the Higgs. We have theoretical models of how these processes can happen, but until we have data on the super partners themselves we will not know how they work in detail. Such data are expected from the LHC or perhaps even from the Tevatron.

Neutrino masses may also arise from interactions with additional Higgs or Higgs-like fields, in a very interesting way. Neutrinos were originally assumed to be massless, but since 1979 theorists have predicted that they have small masses, and over the past decade several impressive experiments have confirmed the predictions [see "Solving the Solar Neutrino Problem," by Arthur B. McDonald, Joshua R. Klein and David L. Wark; *Scientific American*, April 2003]. The neutrino masses are less than a millionth the size of the next smallest mass, the electron mass. Because neutrinos are electrically neutral, the theoretical description of their masses is more subtle than for charged particles. Several processes contribute to the mass of each neutrino species, and for technical reasons the actual mass value emerges from solving an equation rather than just adding the terms. Thus, we have understood the three ways that mass arises: The main form of mass we are familiar with--that of protons and neutrons and therefore of atoms--comes from the motion of quarks bound into protons and neutrons. The proton mass would be about what it is even without the Higgs field. The masses of the quarks themselves, however, and also the mass of the electron, are entirely caused by the Higgs field. Those masses would vanish without the Higgs. Last, but certainly not least, most of the amount of super partner masses, and therefore the mass of the dark matter particle (if it is indeed the lightest super partner), comes from additional interactions beyond the basic Higgs one. Finally, we consider an issue known as the family problem. Over the past half a century physicists have shown that the world we see, from people to flowers to stars, is constructed from just six particles: three matter particles (up quarks, down quarks and electrons), two force quanta (photons and gluons), and Higgs bosons--a remarkable and surprisingly simple description. Yet there are four more quarks, two more particles similar to the electron, and three neutrinos. All are very short-lived or barely interact with the other six particles. They can be classified into three families: up, down, electron neutrino, electron; charm, strange, muon neutrino, muon; and top, bottom, tau neutrino, tau. The particles in each family have interactions identical to those of the particles in other families. They differ only in that those in the second family are heavier than those in the first, and those in the third family are heavier still. Because these masses arise from interactions with the Higgs field, the particles must have different interactions with the Higgs field. Hence, the family problem has two parts: Why are there three families when it seems only one is needed to describe the world we see? Why do the families differ in mass and have the masses they do? Perhaps it is not obvious why physicists are astonished that nature contains three almost identical families even if one would do. It is because we want to fully understand the laws of nature and the basic particles and forces. We expect that every aspect of the basic laws is a necessary one. The goal is to have a theory in which all the particles and their mass ratios emerge inevitably, without making ad hoc assumptions about the values of the masses and without adjusting parameters. If having three families is essential, then it is a clue whose significance is currently not understood.

### **COSMIC STRING TO TIE UP:**

The standard model and the SSM can accommodate the observed family structure, but they cannot explain it. This is a strong statement. It is not that the SSM has not yet explained the family structure but that it cannot. For me, the most exciting aspect of string theory is not only that it may provide us



with a quantum theory of all the forces but also that it may tell us what the elementary particles are and why there are three families. String theory seems able to address the question of why the interactions with the Higgs field differ among the families. In string theory, repeated families can occur, and they are not identical. Their differences are described by properties that do not affect the strong, weak, electromagnetic or gravitational forces but that do affect the interactions with Higgs fields, which fits with our having three families with different masses. Although string theorists have not yet fully solved the problem of having three families, the theory seems to have the right structure to provide a solution. String theory allows many different family structures, and so far no one knows why nature picks the one we observe rather than some other [see "The String Theory Landscape," by Raphael Bousso and Joseph Polchinski; *Scientific American*, September 2004]. Data on the quark and lepton masses and on their super partner masses may provide major clues to teach us about string theory. One can now understand why it took so long historically to begin to understand mass. Without the Standard Model of particle physics and the development of quantum field theory to describe particles and their interactions, physicists could not even formulate the right questions. Whereas the origins and values of mass are not yet fully understood, it is likely that the framework needed to understand them is in place. Mass could not have been comprehended before theories such as the Standard Model and its supersymmetric extension and string theory existed. Whether they indeed provide the complete answer is not yet clear, but mass is now a routine research topic in particle physics.

SOME TIME in the next few years a great discovery will be unveiled, with appropriate fanfare. The headlines will read "ORIGIN OF MASS DISCOVERED". Many readers will be blown away; many will be cynical. Some will scratch their heads and wonder, what do these words actually mean? One doesn't normally think of mass as something with an origin. But a wise and happy few will be prepared. They will leaf through their treasured back issues of *New Scientist*, fish out the right one, and refresh their memories. Welcome back! What will have been discovered is a new kind of heavy, highly unstable particle, the so-called Higgs particle. And we might see it in just a few months, at one of two high-energy accelerators: the Large Electron Positron collider (LEP) at CERN near Geneva or the Tevatron at Batavia, Illinois. The Higgs is more than just another expensive, highly unstable particle: it embodies the mechanism that gives other fundamental particles mass. But isn't mass just a fact of life? Not necessarily. In fact, ours would be a much simpler world if particles didn't have mass. For one thing, mass disfigures the theory of the weak nuclear force. The weak force, as befits its name, is much weaker than the strong force which holds atomic nuclei together and the electromagnetic force that holds atoms together. But it does things that no other interaction can: it causes the slow decay of various otherwise stable particles, and it is the only interaction aware of neutrinos. So what's the problem? Well, the existence of mass means that particles feeling the weak force don't all spin in the same way (see "Messy mass"). It would be neater if they did. That is merely untidiness; but there is another, more disturbing problem with the particles that carry the weak force. All forces in nature work by the action of such carrier particles; photons carry the electromagnetic force, for example. And in 1954, Chen Ning Yang and Robert Mills hypothesized the existence of particles called vector mesons, generalized versions of the photon, which looked like good candidates to carry the weak nuclear force. Then in 1961 Sheldon Glashow used them in a theory that unified weak and electromagnetic forces. According to this theory, vector mesons are massless, like the photon. But unlike electromagnetism, the weak force is short-ranged, a sign that its carrier particles must have mass. To fix this, Glashow fudged the equations by just sticking in a mass, without understanding where it came from.

### **Cosmic molasses**

It would be easier, then, to understand an imaginary world with only massless particles, forever whizzing around at the speed of light. But we know that in our world particles do have mass. So to get from that ideal world to ours, we need some kind of cosmic molasses that fills all space and slows down these massless speed demons. But if this molasses is everywhere, why can't we see it?

To understand, imagine you're living in a bar magnet. An ordinary magnet is really an extraordinary thing. For whereas the laws of physics do not have a preferred direction, the magnet does: its pole. Where does this direction come from? Each electron in any material acts as a small magnet, pointing in the direction of its spin axis. An isolated electron would be equally happy with its spin in any direction, indifference that we call rotational symmetry. But in some materials, such as iron, neighboring electrons prefer to point in the same direction. Like insecure teenagers, they don't care what they are doing, as long as they are all doing the same thing. So to make all the electrons happy or, in more dignified language, to obtain the configuration of minimum energy, all the spins have to pick a common direction--it doesn't matter which. That direction defines the magnetic pole.

The rotational symmetry of an isolated spin is gone, but not forgotten. For if we heat an iron magnet above 870 °C, the spins get enough energy to break free from their neighbors and point in random directions again - the material loses its magnetism. If the iron is later cooled, it will once again become magnetic. But the new pole will usually point in a different direction from the old. And rotational symmetry can reappear in another, subtler way. Give the spins just a little energy, and you can make the preferred spin direction (the local magnetic North) change slowly as a function of location. Configurations in which the preferred direction varies periodically are called spin waves. And just as quantum mechanics parcels up light waves into photons, it parcels these spin waves into particles known as magnons.

#### Particle swarm

Intelligent creatures living inside a magnet would be used to seeing magnons, but they would have trouble figuring out why magnons exist. Evolution would adapt their senses to ignore the unchanging aspects of their environment. So what we think of as the material of the magnet, they would commonly regard as empty space. And it would seem obvious that there was a preferred direction to space, because everything the creatures experienced would be colored by the pervasive magnetism of their world. Eventually, though, some visionary might imagine the true situation: an underlying set of laws with full rotation symmetry, a symmetry hidden by the spontaneous alignment of spins in the pervading medium. Our visionary would have deduced that the "vacuum" is really a structured medium, explained the existence of magnons, and so become a hero of physics. This is just what happened on Earth. We have known since the 1930s that our vacuum is really a swarm of short-lived "virtual" particles, appearing and disappearing at random. But where is the organized structure in this melee? The visionaries who first saw it were Yochiro Nambu and Jeffrey Goldstone. In the early 1960s they noticed a symmetry by which the laws of physics stay the same if certain particles are substituted for others. (It would take an article several times the length of this one to attach proper names and identifiable faces to these particles, and unless you are a very unusual person you would not stay awake to the end. Trust me.) But, just as in the magnet, at low temperature the symmetry is broken: from the symmetrical swarm of virtual particles, one kind condenses out in large numbers. So a preference is formed among the otherwise interchangeable types of particles. Instead of a preferred direction like the magnet, our space has a preferred particle composition.

And this is where the cosmic molasses oozes into our story. In 1966 Peter Higgs of the University of Edinburgh and his co-workers Robert Brout and François Englert of the Free University in Brussels added this idea to the theory of vector mesons. They discovered that when the symmetry breaks, producing a condensate of virtual Higgs particles, the vector mesons become massive. Better still, interactions with the condensate could generate the masses of all the other elementary particles, the quarks and leptons. Nambu and Goldstone had constructed a form of cosmic molasses using particles already known to exist. But this isn't quite enough because it exerts too little drag on the vector mesons, and none at all on the leptons. In 1967, however, Steven Weinberg (and later Abdus Salam) postulated an additional stickier form, and showed how it could give an improved, fudge-free

version of Glashow's weak interaction model. This stickier stuff is what physicists usually mean when they talk about the Higgs condensate.

How can we test this extraordinary conception? We could try to heat up the vacuum, by concentrating a lot of energy in a small space, and watch to see if its symmetry is restored as the condensate evaporates. All particles in this region would become massless. Unfortunately, that will only happen at temperatures approaching 1016 kelvin. Although such temperatures were universal in the early stages of the Big Bang, they are out of reach on Earth for the foreseeable future. The Relativistic Heavy Ion Collider at Brookhaven, New York, due to turn on this summer, will peak at only 1013 kelvin.

### **Stir it up**

A much more modest project is feasible, however. Rather than restore symmetry completely, we can stir up the Higgs condensate a bit. This being a quantum world, we can only stir it up in discrete units. The minimal excitation - a ripple in the cosmic molasses - is the Higgs particle. How hard will it be to make this particle? Who gets to taste the joy of discovery depends on the value of the Higgs mass, as does the nature of particle physics. We can already narrow down the range. If the Higgs particle were lighter than 95 gigaelectronvolts (GeV), about 100 times the mass of a proton, LEP would already have seen it. If it were heavier than 600 GeV, virtual Higgs particles would affect many particle reactions in a way that experiments have already ruled out. And the promising theoretical idea of supersymmetry - an extension of the Standard Model that proposes a host of extra fundamental particles, partners of the familiar bunch - predicts masses well below 200 GeV for the Higgs particle; probably between 100 and 130 GeV. That is why so much excitement surrounds the upcoming explorations (see "The Higgs particle cookbook" below). Scientists at LEP will drive their machine to the limits of its energy and luminosity, pushing the mass window up to 105 GeV or so within two years. Meanwhile, scientists at the Tevatron hope to explore all the way up to 160 GeV. If they fail, then a final effort will be made at the Large Hadron Collider (LHC) being constructed in Geneva due to open around 2005. Its reach extends beyond 600 GeV. If that fails, we theoretical physicists will be exceedingly embarrassed, and I hesitate to predict what we'll do. The Standard Model requires just one Higgs particle. But theories with more symmetry imply several new particles -- Higgs galore. The theory of super symmetry predicts at least five Higgs-type particles. In the most popular version, the lightest member of the Higgs family has the properties we discussed above. There is no consensus on the masses of the others, although they should not be much heavier than 1000 GeV, and might be much lighter. The masses of these particles will tell us how the supersymmetric partners of ordinary particles hide themselves from us. At present it is a big mystery, and wild concepts are in the air, including their infection by otherwise inaccessible "dark" matter, or exotic condensates living only in extra dimensions of space. The LHC should shed light on this mystery. More ambitious models that unify the strong and electroweak forces predict a bizarre tribe of very much heavier Higgs particles. We probably won't be able to make them directly anytime soon, but we might sense the effect of their exchange as virtual particles. Some of them can make protons decay, at rates close to current experimental limits' cosmic Remember the future headline, trumpeting the discovery of the origin of mass? Honesty compels me to call the headline writer to task: the statement is not entirely misleading, but it's literally false. Actually the lion's share of ordinary mass, in protons and neutrons, has nothing to do with Higgs particles. It comes instead from the energy of the gluon field that holds their constituent quarks together. Intrigued? Write to the editors, demanding a follow-up article.

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### **Messy mass**

ONCE, we thought that the fundamental laws of physics made no distinction between left and right - for



any behaviour you can observe in the real world, its mirror image can also happen. So if you filmed the real world and its reflected image, someone watching the movies later wouldn't be able to tell which was which. This is called parity symmetry. Then in 1956 Tsao-Dai Lee at Columbia University, New York, and Chen Ning Yang at the Institute for Advanced Study, Princeton, suggested that the weak interaction breaks parity symmetry. They turned out to be right. For example, neutrons decay through the weak interaction into protons, electrons and electron antineutrinos. The electrons emitted in this decay are moving at nearly the speed of light, and they are also spinning. About 98 per cent of them are left-handed, meaning that if you pointed the thumb of your left hand in the direction of its motion your fingers would curl in the direction the electron spins. This bias violates parity symmetry, because it distinguishes left from right. So the weak interaction likes left-handed particles (electrons, muons, quarks and so on) and right-handed antiparticles. But, irritatingly, this seems to be no more than a rule of thumb. Weak processes involving right-handed particles or left-handed antiparticles are rare, but not absolutely nonexistent.

### **Political heavyweights**

In 1993, researchers were challenged by William Waldegrave, then Britain's science minister, to come up with a concise description of the Higgs particle. For a complete list of the winners and their explanations see The Waldegrave Higgs Challenge. David Miller from University College London won a bottle of champagne for this "quasi-political explanation".

Imagine a cocktail party of political party workers who are uniformly distributed across the floor, all talking to their nearest neighbors. The ex-prime minister enters and crosses the room. All of the workers in her neighborhood are strongly attracted to her and cluster round her. As she moves she attracts the people she comes close to, while the ones she has left return to their even spacing. Because of the knot of people always clustered around her she acquires a greater mass than normal, that is, she has more momentum for the same speed of movement across the room. Once moving she is harder to stop, and once stopped she is harder to get moving again because the clustering process has to be restarted. In three dimensions, and with the complications of relativity, this is the Higgs mechanism. In order to give particles mass, a background field is invented which becomes locally distorted whenever a particle moves through it. The distortion--the clustering of the field around the particle - generates the particle's mass.

The idea comes directly from the physics of solids. Instead of a field spread throughout all space a solid contains a lattice of positively charged crystal atoms. When an electron moves through the lattice the atoms are attracted to it, causing the electron's effective mass to be as much as 40 times bigger than the mass of a free electron. The postulated Higgs field in the vacuum is a sort of hypothetical lattice which fills our Universe. We need it because otherwise we cannot explain why the Z and W particles which carry the Weak Interactions are so heavy while the photon which carries electromagnetic forces is massless. Now consider a rumor passing through our room full of uniformly spread political workers. Those near the door hear of it first and cluster together to get the details, then they turn and move closer to their next neighbors who want to know about it too. A wave of clustering passes through the room. It may spread out to all the corners, or it may form a compact bunch which carries the news along a line of workers from the door to some dignitary at the other side of the room. Because the information is carried by clusters of people, and since it was clustering which gave extra mass to the ex-prime minister, then the rumor-carrying clusters also have mass. The Higgs particle is predicted to be just such a clustering in the Higgs field. We will find it much easier to believe that the field exists, and that the mechanism for giving other particles mass is true, if we actually see the Higgs particle itself. Again, there are analogies in the physics of solids. A crystal lattice can carry waves of clustering without needing an electron to move and attract the atoms. These waves can behave as if they are particles. They are called phonons. There could be a Higgs mechanism, and a Higgs field throughout our Universe, without there being a Higgs particle. The next generation of colliders will sort this out.

Physically, if you look at the low level of CMB anisotropy and the BAO power spectrum, you need some sort of mass that interacts gravitationally but is decoupled from the photons before recombination (i.e. - dark matter). Otherwise there would be no way to seed the galaxy formation that we see occurring at lower redshifts. That is to say, dark matter needs to be primordial, so that it can dodge Silk Damping and provide the gravitational "cores" around which galaxies form.

### **AFFECTIVE CONSCIOUSNESS:**

Affect consciousness (Monsen, Monsen, Solbakken & Hansen, 2008) refers to the mutual relationship between activation of basic affective experiences and the individual's capacity to consciously perceive, tolerate, reflect upon and express these experiences (Monsen & Monsen, 1999; Solbakken, Hansen, Havik & Monsen, 2011). AC is traditionally operationalized as degrees of awareness, tolerance, nonverbal and conceptual expression for each of, in the newest revision of the model, eleven basic affects, and measured by the semi-structured Affect Consciousness Interview (ACI).

A vast array of ideas from a number of different approaches can be emphasized as inspirations for the development of the AC construct (Solbakken, Hansen & Monsen, 2011), most notably Silvan Tomkins' Affect theory, Script Theoretical formulations by the same author (Tomkins, 2008a, 2008b), Discrete Emotions Theory (Izard, 1976; 1993). The writings of Daniel Stern (1986) and the seminal studies by Emde (1983; 1996) on nonverbal affective communication with infants are also central, along with modern self psychological formulations as those advocated by Stolorow, Brandshaft & Atwood (1987), Stolorow & Atwood (1992) and Lichtenberg et al. (1992, 1996).

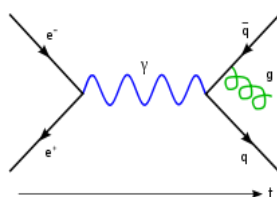
The affect consciousness construct posits that affect, along with pain and the cyclical drives constitute the primary motivating force in all human affairs (Solbakken, Hansen & Monsen, 2011). Of these motivating forces the affect system is the primary, and by far the most flexible (Tomkins, 2008a), it is hypothesized that affect is ever present in the organism, continually shaping and codetermining both the qualia of consciousness and the behavior initiated or terminated.

It is assumed that affect activation structures not only behavior, but also the coherence of experience, so that affect that is void of meaning to the individual constitute the core of meaningless and disorganized experience. Furthermore central importance is given to the function of affect expression as a primary channel of communication: The transactions of affect-displays among humans are regarded denser in information than any verbal communication. Importantly attunement of affective communication between child and caregiver is seen as paramount to the acquisition of coherent self experience and attainment of adequate affect integration and regulation in the course of development (Solbakken, Hansen & Monsen, 2011).

The affect system is viewed as an independent, evolutionary early, response apparatus developed specifically for adaptive purposes. This system is hypothesized to interact and transact with the other major adaptive systems of the human organism; drive, motor, perceptual, sensory, memory and cognitive, but is seen as the central core of direction for those other systems (Tomkins, 2008a). The adaptiveness of these interactions and their organization is the defining feature of affect consciousness.

### **.PARTITION FUNCTION:**

Partition function (quantum field theory) (We have used Wikipedia and author home pages and Stanford Encyclopedia for the preparation of the following introductory remarks).



In quantum field theory, we have a generating functional,  $Z[J]$  of correlation functions and this value,

called the partition function is usually expressed by something like the following functional integral:

$$Z[J] = \int \mathcal{D}\phi e^{i(S[\phi] + \int d^d x J(x)\phi(x))}$$

Where S is the action functional.

The partition function in quantum field theory is a special case of the mathematical partition function, and **is related** to the statistical partition function in statistical mechanics. The primary difference is that the countable collection of random variables seen in the definition of such simpler partition functions has been replaced by an uncountable set, thus necessitating the use of integrals over a field  $\phi$ .

The prototypical **use of the** partition function **is to obtain** Feynman amplitudes by differentiating with respect to the auxiliary function (sometimes called the current) J. Thus, for example:

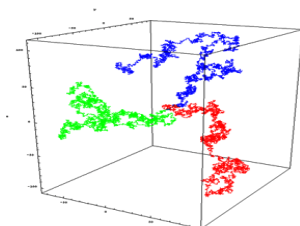
$$\langle G(x_1, x_2) \rangle = - \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \log Z[J] \Big|_{J=0}$$

Is the Green's function, **propagator or** correlation function for the field  $\phi$  between points  $x_1$  and  $x_2$  in space?

#### **Reactionary Complex valued function:**

Unlike the partition function in statistical mechanics, that in quantum field theory contains an extra factor of i in front of the action, making the integrand complex, not real. It is sometimes mistakenly implied that this has something to do with Wick rotations; this is not so. Rather, the i **has to do with the fact** that the fields  $\phi$  are to be interpreted as quantum-mechanical probability amplitudes, **taking on** values in the complex projective space (complex Hilbert space, but the emphasis is placed on the word projective, **because** the probability amplitudes are still normalized to one). By contrast, more traditional partition functions **involve** random variables that are real-valued, and range over a simplex-- a simplex, being the geometric way of saying that the total of probabilities **sum to** one. The factor of i can be understood **to arise as** the Jacobian of the natural measure of volume in complex projective space. For the (highly unusual) situation where the complex-valued probability amplitude is to be replaced by some other field taking on values in some other mathematical, the i would **be replaced** by the appropriate geometric factor (that is, the Jacobian) for that space.

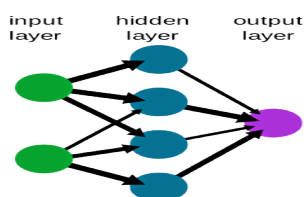
Statistical mechanics was born at the beginning of the 20th century to provide a microscopic explanation of thermodynamics and then, for many years, it was used to understand the different aggregation states of matter. Quantum field theory instead was born in the context of sub nuclear physics as the basic method to describe the fundamental interactions between the ultimate constituents of matter. These two fields evolved independently for many years. Only at the end of the '60s was it realized that statistical mechanics was able to provide the basic tools for a non-perturbative and rigorous definition of quantum field theory. On the other hand, quantum-field-theory concepts like renormalization group and scale invariance found their application in statistical mechanics.



### A three-arm polymer

Since then the cross-fertilization of the two fields has **provided** important methods and tools which have been applied not only in physics but also in several different contexts spanning from computer science, chemistry, biology, geology, and social sciences. Since many years our group is active both in quantum field theory and statistical mechanics, considering several problems which are relevant in elementary particle physics, condensed matter physics, biology, and mathematics, and which can be addressed by combining statistical-mechanics methods, quantum field theory, and the theory of stochastic processes.

In recent years much of our research concerns "random models", a vast class of systems which are the object of intense study in many disciplines — physics, biology, computer science, and mathematics to name a few. They have a complex phenomenology and many of their properties are still not understood. For instance, restricting to physical systems, random systems show a variety of new phenomena — slow relaxation and aging, memory and oblivion, and generalizations of the usual dissipation-fluctuation relations — which have been the object of intense experimental scrutiny. In this field we are extensively studying the critical behavior of several spin models with quenched disorder and phase separation in fluid systems, in particular colloid-polymer mixtures, in porous materials. Beside random systems, we are also investigating, both numerically and analytically, the critical behavior of less-common magnetic systems, some properties of high-Tc superconductors, and the finite-density behavior of solutions of polymers of different architecture. In the following model we concentrate on Quantum Field and Statistical mechanics, Markov fields, Hilbert spaces, Lie algebras and translations in space and time. Expansion of quantum field describing electrons or other fermions are also studied.



A simple neural network

Other subjects under intensive investigation by our group are spin glasses and neural nets. The methods **at the basis of** their study combine the physical intuition, **accumulated through** the **use of** the replica trick and numerical simulations, **with the need for** a rigorous mathematical treatment. The essential ingredients are given by powerful interpolation methods, and sum rules. These methods **led in the** past years to the proof of relevant results, in particular concerning the control of the infinite volume limit, and **the mechanism of the** spontaneous replica symmetry breaking. For the neural nets of Hopfield type many physicists have developed interpolation method **which allows the** characterization of the replica symmetric approximation, and the possibility **of introducing** functional order parameters for the description of the replica symmetry breaking.

The variational principle **arising in the expression** of the free energy in the infinite volume limit is of novel type, in **that it involves** a mini-max procedure, in contrast with the Sherrington-Kirkpatrick model for a spin glass. The theory of self-oscillating mechanical systems **has been exploited for** the study of **speech formation, analysis and synthesis, and musical instrument functioning**. It is possible **to apply** fully nonlinear schemes, by completely **avoiding any kind** of exploitation of the Fourier analysis. The **role of the** different peaks of the spectrum in the Fourier analysis is played **by the intervention of** successive Landau instability modes for the self-oscillating system. Finally, in recent times, scientists have **developed the** possibility of giving simple models for the immunological system, **based on** stochastic dynamical systems of statistical mechanics far from equilibrium. The models are simple enough **to allow** practical evaluations, **in connection** with the known phenomenology, but they

are very rich in the possibility of introducing all basic feature of the real system. In THIS Connection mention can be made of the studies done of the quantum field theory formulation of the relativistic Majorana equations, introduced in 1932 in a famous paper on Nuovo Cimento, and the study of slowing down, scattering and absorption of neutrons, original methods of Fermi, Wick, Bothe, Heisenberg, with the purpose of a realistic assessment of the validity of the approximations introduced by them, in comparison with the modern methods of numerical simulations in the nuclear reactor theory.

**(1) The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself. AND The particle Higgs Boson**

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### Module Numbered One

#### NOTATION :

$G_{13}$  : CATEGORY ONE OF The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself

$G_{14}$  : CATEGORY TWO OF The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself

$G_{15}$  : Category Three Of The Higgs Field is the fundamental quantum arena, quantitative form of means and ends that gives birth to all mass in the universe. In the finding of Higgs Boson, there exists an inferential thought and diversity and essential predications, predicational anteriority, character constitution of primordial exactitude and ontological resonance of the formation of the world itself

$T_{13}$  : Category One Of The Particle Higgs Boson\_(Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theopries Mentioned)

$T_{14}$  : Category Two Of The Particle Higgs Boson(Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned

$T_{15}$  :Category Three Of The Particle Higgs Boson (Again Classification Is For The Systems For Which The Theories Are Applicable Or For That Matter Systems Which Violates The Theories Mentioned

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### **(2) Higgs Quantum Field AND Mass of the matter**

#### **MODULE NUMBERED TWO:**

In the most widely believed scenario, dark matter is composed of "weakly interacting massive particles" ("WIMP"). The adjective "weak" really means that the particles interact via the weak nuclear force. This pretty much guarantees that they interact with the Higgs boson, too: the WIMPs carry the hypercharge or the weak nuclear charge (that's what "WI" in "WIMP" guarantees).According to current theory, the most likely WIMP is the LSP, the lightest super partner in a super symmetric theory. The LSP may be a gravitino in which case its interactions with the Higgs bosons are almost non-existent – gravity is the only relevant interaction; or they may be a "neutralino" (higgsino, photino, or zino – or, in a different basis, higgsino, neutral wino, or bino) and this interacts with the Higgs boson as strongly as the known superpartners of the

neutralino, namely the Z-boson or the Higgs boson itself. So if the dark matter is composed of neutralinos, then it interacts with the Higgs boson. However, the Higgs field isn't the main reason why these particles are massive ("M" in "WIMP"). Instead, the main reason is a supersymmetry breaking mechanism, something independent from the Higgs mechanism which may also be called the electroweak symmetry breaking mechanism. It's due to SUSY breaking, and not the electroweak symmetry breaking, that the LSP is massive. For example, SUSY breaking is needed to make photino (or bino/wino/neutralino that contains a piece of it) massive even though the photon, its known superpartner, is massless. If the dark matter is made of axions, its interactions with the Higgs field are virtually non-existent. If dark matter is composed of MACHOs or if it doesn't exist at all, it makes no sense to describe its interactions at the level of particle physics because it is a composite object or because it doesn't exist. For example, if dark matter were composed of black holes, they would interact like other black holes – pretty much only gravitationally.

$G_{16}$  : Category One Of Lie Algebra Of The Groups

$G_{17}$  : Category Two Of Lie Algebra Of The Groups

$G_{18}$  : Category Three Of Lie Algebra Of The Groups

$T_{16}$  : Category One Of translations In Space And Time

$T_{17}$  : Category Two Of Translations In Space And Time

$T_{18}$  : Category Three Of Translations In Space And Time

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### **Markov Fields And Hilbert Spaces:**

#### **Module Numbered Three:**

$G_{20}$  : Category One Of Markov Space( There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable)

$G_{21}$  : Category Two Of Category One Of Markov Space( There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable

$G_{22}$  : Category Three Of Category One Of Markov Space( There Are Lot Of Markov Spaces And Some Spaces Could Be Approximated To Markov Spaces And Markov Spaces Could Be Made To Lose Their Characteristics And Parameter Under Some Transformations. These Points Are To Be Borne In Mind. We Here Speak Of The Characterized Systems For Which Markov Theory Is Applicable

$T_{20}$  : Category One Of Hilbert Spaces And Quantum, Fields

$T_{21}$  : Category Two Of Hilbert Spaces And Quantum Fields

$T_{22}$  : Category Three Of Hilbert Spaces And Quantum Fields

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**Quantum Field Theory(Again, Parametric zed Systems To Which Qft Could Be Applied Is Taken In To Consideration And Renormalization Theory(Based On Certain Variables Of The System Which Consequentially Classifiable On Parameters)**



**: MODULE NUMBERED FOUR:**

$G_{24}$  : Category One Of Quantum Field Theory(Evaluative Parametricization Of Situational Orientations And Essential Cognitive Orientation And Choice Variables Of The System To Which Qft Is Applicable)

$G_{25}$  : Category Two Of Quantum Field Theory

$G_{26}$  : Category Three Of Quantum Field Theory

$T_{24}$  :Category One Of Renormalization Theory

$T_{25}$  :Category Two Of Renormalization Theory(Systemic Instrumental Characterisations And Action Orientations And Fuyncional Imperatives Of Change Manifested Therein )

$T_{26}$  : Category Three Of Quantum Field Theory

**Quantum Connections Due To Due To Electron Positron Pairs And Linear Corrections To Maxwell's Equations:**

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**Module Numbered Five:**

$G_{28}$  : Category One Of Quantum Corrections Due To Virtual Electron-Positron Pairs

$G_{29}$  : Category Two Of Quantum Corrections Due To Virtual Electron-Positron Pairs

$G_{30}$  :Category Three Of Quantum Corrections Due To Virtual Electron-Positron Pairs

$T_{28}$  :Category One Of Linear Corrections In Maxwells Equation Applicable To Various Systems With Defined Characterstics And The Concomitant Correction Factor.Classifcation Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell's Equations Are The Same

$T_{29}$  :Category Two Of Linear Corrections In Maxwell's Equation Applicable To Various Systems With Defined Characterstics And The Concomitant Correction Factor.Classifcation Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell's Equations Are The Same

$T_{30}$  : Category Three Of Linear Corrections In Maxwells Equation Applicable To Various Systems With Defined Characterstics And The Concomitant Correction Factor. Classification Is Based On The Parametricization Of The Systems Even Though To Some Systems Linear Corrections To Maxwell's Equations Are The Same

**Asymmetric Freedom Decisive Regularities In The Case When Sobolev Inequalities Are On The Border Line Of Transition:**

**Module Numbered Six:**

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$G_{32}$  : Category One Of Decisive Regularities In Respect Of Caseswhere Sobolev Inequalities Are In The Borderline Phase Transition

$G_{33}$  : Category Two Of Decisive Regularities With Respect To Cases Wherein Sobolev Inequalities

## Are In Border Line Transition

$G_{34}$  : Category Three Of Decisive Regularities With Respect Top Systemic Cases In Regard To Which Sobolev Inequalities Are In Border Line Transition

$T_{32}$  : Category One Of Asymptotic Freedom

$T_{33}$  : Category Two Of asymptotic Freedom

$T_{34}$  : Category Three Of Asymptotic Freedom

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**Expansion Rate Of Quantum Field Describing Electrons And Other Fermions And Creation And Annihilation Of Operators according To Pauli's Exclusion Principle (Again We Talk Of The Systemic Characteristics To Which Pauli's Exclusion Principle Is Applied, And The Parametricization And Stratification Follows)**

### Module Numbered Seven

$G_{36}$  : Category One Of Creation And Annihilation Operators Due To Pauli's Exclusion Principle (Note Pauli's Exclusion Principle Denies The Two States For The Electrons)

$G_{37}$  : Category Two Of Creation And Annihilation Operators Due To Pauli's Exclusion Principle (Note Pauli's Exclusion Principle Denies The Two States For The Electrons)

$G_{36}$  : Category Three Of Orthogonal Energy State Of Vacuum (Energy Excitation Of The Vacuum And Concomitant Generation Of Energy Differential-Time Lag Or Instantaneousness might Exist Whereby Accentuation And Attritions Model May Assume Zero Positions)

$T_{36}$  : Category One Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions

$T_{37}$  : Category Two Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions

$T_{38}$  : Category Three Of Expansion Rates Of Quantum Fields Describing Electrons and Other Fermions

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## Texture Zero Of Neutrinos Preservation And Renormalization

### Module Numbered Eight

$G_{40}$ : Category one of Neutrinos With Zero Texture (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptons, Photons, Electrons In The Universe)

$G_{41}$ : Category two of Neutrinos With Zero Texture (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptons, Photons, Electrons In The Universe)

$G_{42}$ : Category three of Neutrinos With Zero Texture (There May Or Might Not Be Neutrinos Without Texture And There Are Lot Of Neutrinos As There Are Lot Of Leptons, Photons, Electrons In The Universe)



$T_{40}$ : Category one of Renormalization

$T_{41}$ : Category Two of Renormalization

$T_{42}$ : Category Three of Renormalization

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$   
 $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}; (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$   
 $(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$   
 $(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$   
 $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$   
 $(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$   
 $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

**Quantum Field Theory And Statistical Mechanics (We Are Talking Of Systems To Which Theory Is Applicable And Holds. Bank Example Of Assets Being Equivalent To Liabilities And That Each Individual Debits And Credits Being Conservative In Addition To The Conservativeness Of Holistic System Is To Be Mentioned), Markov Field And Hilbert Space:**

### Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 1090$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 1091$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 1092$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 1093$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 1094$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 1095$$

$+(a''_{13})^{(1)}(T_{14}, t) =$  First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$  First detritions factor

### LIE ALGEBRA OF THE GROUPS AND TRANSLATIONS IN SPACE TIME

#### MODULE NUMBERED TWO

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 1096$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 1097$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 1098$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 1099$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 1100$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 1101$$

$+(a''_{16})^{(2)}(T_{17}, t) =$  First augmentation factor

$-(b''_{16})^{(2)}((G_{19}), t) =$  First detritions factor

### MARKOV FIELD AND HILBERT SPACE:

#### MODULE NUMBERED THREE

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 1102$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 1103$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 1104$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 1105$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 1106$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 1107$$

$+(a''_{20})^{(3)}(T_{21}, t) =$  First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$  First detritions factor

**QUANTUM FIELD THEORY(AGAIN,PARAMETRICIZED SYSTEMS TO WHICH QFT COULD BE APPLIED IS TAKEN IN TO CONSIDERATION AND RENORMALIZATION THEORY(BASED ON CERTAIN VARIABLES OF THE SYSTEM WHICH CONSEQUENTIALLY CLASSIFIABLE ON PARAMETERS)**

#### : MODULE NUMBERED FOUR

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 1108$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 1109$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 1110$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} \quad 1111$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} \quad 1112$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} \quad 1113$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor}$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor}$$

**QUANTUM CORRECTIONS DUE TO ELECTRON POSITRON PAIRS AND LINEAR CORRECTIONS TO MAXWELL'S EQUATIONS:**

**MODULE NUMBERED FIVE:**

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 1114$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 1115$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 1116$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 1117$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 1118$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 1119$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}$$

**ASSYMETRIC FREEDOM DECISIVE REGULARITIES IN THE CASE WHEN SOBOLEV INEQUALITIES ARE ON THE BORDER LINE OF TRANSITION:**

**MODULE NUMBERED SIX**

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 1120$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 1121$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 1122$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 1123$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 1124$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 1125$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor} \quad 1126$$

**EXPANSION RATE OF QUANTUM FIELD DESCRIBING ELECTRONS AND OTHER FERMIONS AND**

**CREATION AND ANNIHILATION OF OPERATORS ACCORDING TO PAULI'S EXCLUSION PRINCIPLE  
 (AGAIN WE TALK OF THE SYSTEMIC CHARACTERISTICS TO WHICH PAULIS EXCLUSION PRINCIPLE  
 IS APPLIED, AND THE PARAMETRIZATION AND STRATIFICATION FOLLOWS)**

**MODULE NUMBERED SEVEN:**

The differential system of this model is now (SEVENTH MODULE)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)] G_{36} \quad 1127$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)] G_{37} \quad 1128$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)] G_{38} \quad 1129$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)] T_{36} \quad 1130$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)] T_{37} \quad 1131$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)} T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)] T_{38} \quad 1132$$

$$+(a''_{36})^{(7)}(T_{37}, t) = \text{First augmentation factor} \quad 1133$$

**TEXTURE ZERO OF NEUTRINOS PRESERVATION AND RENORMALIZATION**

**MODULE NUMBERED EIGHT**

**GOVERNING EQUATIONS:**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)} G_{41} - [(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)] G_{40} \quad 1134$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)} G_{40} - [(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)] G_{41} \quad 1135$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)} G_{41} - [(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)] G_{42} \quad 1136$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)} T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)] T_{40} \quad 1137$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)} T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)] T_{41} \quad 1138$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)} T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)] T_{42} \quad 1139$$

$$-(b''_{36})^{(7)}((G_{39}), t) = \text{First detritions factor}$$

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[ \begin{array}{c} (a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) + (a'_{16})^{(2,2)}(T_{17}, t) + (a''_{20})^{(3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4)}(T_{25}, t) + (a''_{28})^{(5,5,5,5)}(T_{29}, t) + (a''_{32})^{(6,6,6,6)}(T_{33}, t) \\ + (a''_{36})^{(7)}(T_{37}, t) + (a''_{40})^{(8)}(T_{41}, t) \end{array} \right] G_{13} \quad 1140$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[ \begin{array}{c} (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7)}(T_{37}, t)} \boxed{+(a''_{41})^{(8)}(T_{41}, t)} \end{array} \right] G_{14} \quad 1141$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[ \begin{array}{c} (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3)}(T_{21}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7)}(T_{37}, t)} \boxed{+(a''_{42})^{(8)}(T_{41}, t)} \end{array} \right] G_{15} \quad 1142$$

Where  $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$ ,  $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$ ,  $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$  are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3,3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3,3)}(T_{21}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4,4,4)}(T_{25}, t)}$  are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5,5,5)}(T_{29}, t)}$  are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6)}(T_{33}, t)}$  are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7)}(T_{37}, t)}$   $\boxed{+(a''_{37})^{(7)}(T_{37}, t)}$   $\boxed{+(a''_{38})^{(7)}(T_{37}, t)}$  Are Seventh Augmentation Coefficients

$\boxed{+(a''_{42})^{(8)}(T_{40}, t)}$   $\boxed{+(a''_{42})^{(8)}(T_{41}, t)}$   $\boxed{+(a''_{42})^{(8)}(T_{41}, t)}$  Are Eighth Augmentation Coefficients

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[ \begin{array}{c} (b'_{13})^{(1)} \boxed{-(b''_{16})^{(1)}(G, t)} \boxed{-(b''_{36})^{(7)}(G_{39}, t)} \boxed{-(b''_{20})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7)}(G_{39}, t)} \boxed{-(b''_{40})^{(8)}(G_{43}, t)} \end{array} \right] T_{13} \quad 1143$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[ \begin{array}{c} (b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7)}(G_{39}, t)} \boxed{-(b''_{41})^{(8)}(G_{43}, t)} \end{array} \right] T_{14} \quad 1144$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[ \begin{array}{c} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3)}(G_{23}, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{38})^{(7)}(G_{39}, t)} \boxed{-(b''_{42})^{(8)}(G_{41}, t)} \end{array} \right] T_{15} \quad 1145$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$  are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7)}(G_{39}, t)}$   $\boxed{-(b''_{36})^{(7)}(G_{39}, t)}$   $\boxed{-(b''_{36})^{(7)}(G_{39}, t)}$  Are Seventh Detrition Coefficients

$\boxed{-(b''_{40})^{(8)}(G_{41}, t)}$   $\boxed{-(b''_{41})^{(8)}(G_{41}, t)}$   $\boxed{-(b''_{42})^{(8)}(G_{41}, t)}$  Are Eighth Detrition coefficients

**SECOND MODULE CONCATENATION:**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ \begin{array}{ccc} \boxed{(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)} & \boxed{+(a'_{13})^{(1,1)}(T_{14}, t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{24})^{(4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{36})^{(7,7)}(T_{37}, t)} & \boxed{+(a'_{40})^{(8,8)}(T_{41}, t)} & \end{array} \right] \quad 1146$$

Where  $\boxed{-(b''_{13})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1)}(G, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3)}(G_{23}, t)}$  are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ \begin{array}{ccc} \boxed{(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)} & \boxed{+(a'_{14})^{(1,1)}(T_{14}, t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{25})^{(4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{29})^{(5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a'_{37})^{(7,7)}(T_{37}, t)} & \boxed{+(a'_{41})^{(8,8)}(T_{41}, t)} & \end{array} \right] G_{17} \quad 1147$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[ \begin{array}{ccc} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) & + (a'_{15})^{(1,1)}(T_{14}, t) & + (a'_{22})^{(3,3,3)}(T_{21}, t) \\ + (a'_{26})^{(4,4,4,4,4)}(T_{25}, t) & + (a'_{30})^{(5,5,5,5,5)}(T_{29}, t) & + (a'_{34})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a'_{38})^{(7,7,7)}(T_{37}, t) & + (a'_{42})^{(8,8)}(T_{41}, t) & \end{array} \right] G_{18} \quad 1148$$

Where  $(a'_{16})^{(2)}(T_{17}, t)$ ,  $(a'_{17})^{(2)}(T_{17}, t)$ ,  $(a'_{18})^{(2)}(T_{17}, t)$  are first augmentation coefficients for category 1, 2 and 3

$(a'_{13})^{(1,1)}(T_{14}, t)$ ,  $(a'_{14})^{(1,1)}(T_{14}, t)$ ,  $(a'_{15})^{(1,1)}(T_{14}, t)$  are second augmentation coefficient for category 1, 2 and 3

$(a'_{20})^{(3,3,3)}(T_{21}, t)$ ,  $(a'_{21})^{(3,3,3)}(T_{21}, t)$ ,  $(a'_{22})^{(3,3,3)}(T_{21}, t)$  are third augmentation coefficient for category 1, 2 and 3

$(a'_{24})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $(a'_{25})^{(4,4,4,4,4)}(T_{25}, t)$ ,  $(a'_{26})^{(4,4,4,4,4)}(T_{25}, t)$  are fourth augmentation coefficient for category 1, 2 and 3

$(a'_{28})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $(a'_{29})^{(5,5,5,5,5)}(T_{29}, t)$ ,  $(a'_{30})^{(5,5,5,5,5)}(T_{29}, t)$  are fifth augmentation coefficient for category 1, 2 and 3

$(a'_{32})^{(6,6,6,6,6)}(T_{33}, t)$ ,  $(a'_{33})^{(6,6,6,6,6)}(T_{33}, t)$ ,  $(a'_{34})^{(6,6,6,6,6)}(T_{33}, t)$  are sixth augmentation coefficient for category 1, 2 and 3

$(a'_{36})^{(7,7,7)}(T_{37}, t)$ ,  $(a'_{37})^{(7,7,7)}(T_{37}, t)$ ,  $(a'_{38})^{(7,7,7)}(T_{37}, t)$  Are Seventh Detrition Coefficients

$(a'_{40})^{(8)}(T_{41}, t)$ ,  $(a'_{41})^{(8)}(T_{41}, t)$ ,  $(a'_{42})^{(8)}(T_{41}, t)$  Are Eight Augmentation Coefficients

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[ \begin{array}{ccc} (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) & - (b'_{13})^{(1,1)}(G, t) & - (b'_{20})^{(3,3,3)}(G_{23}, t) \\ - (b'_{24})^{(4,4,4,4,4)}(G_{27}, t) & - (b'_{28})^{(5,5,5,5,5)}(G_{31}, t) & - (b'_{32})^{(6,6,6,6,6)}(G_{35}, t) \\ - (b'_{36})^{(7,7,7)}(G_{39}, t) & - (b'_{40})^{(8,8)}(G_{43}, t) & \end{array} \right] T_{16} \quad 1149$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[ \begin{array}{ccc} (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) & - (b'_{14})^{(1,1)}(G, t) & - (b'_{21})^{(3,3,3)}(G_{23}, t) \\ - (b'_{25})^{(4,4,4,4,4)}(G_{27}, t) & - (b'_{29})^{(5,5,5,5,5)}(G_{31}, t) & - (b'_{33})^{(6,6,6,6,6)}(G_{35}, t) \\ - (b'_{37})^{(7,7,7)}(G_{39}, t) & - (b'_{41})^{(8,8)}(G_{43}, t) & \end{array} \right] T_{17} \quad 1150$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[ \begin{array}{ccc} (b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) & - (b'_{15})^{(1,1)}(G, t) & - (b'_{22})^{(3,3,3)}(G_{23}, t) \\ - (b'_{26})^{(4,4,4,4,4)}(G_{27}, t) & - (b'_{30})^{(5,5,5,5,5)}(G_{31}, t) & - (b'_{34})^{(6,6,6,6,6)}(G_{35}, t) \\ - (b'_{38})^{(7,7,7)}(G_{39}, t) & - (b'_{42})^{(8,8)}(G_{43}, t) & \end{array} \right] T_{18} \quad 1151$$

THIRD MODULE CONCATENATION:

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[ \begin{array}{ccc} (a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) & + (a'_{16})^{(2,2,2)}(T_{17}, t) & + (a'_{13})^{(1,1,1)}(T_{14}, t) \\ + (a'_{24})^{(4,4,4,4,4)}(T_{25}, t) & + (a'_{28})^{(5,5,5,5,5)}(T_{29}, t) & + (a'_{32})^{(6,6,6,6,6)}(T_{33}, t) \\ + (a'_{36})^{(7,7,7)}(T_{37}, t) & + (a'_{40})^{(8,8,8)}(T_{41}, t) & \end{array} \right] G_{20} \quad 1152$$

$$\frac{dG_{21}}{dt} = \left. \begin{array}{l} (a'_{21})^{(3)} G_{20} - \left[ \begin{array}{l} (a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21}, t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)} \boxed{+(a''_{41})^{(8,8,8)}(T_{41}, t)} \end{array} \right] \end{array} \right\} G_{21} \quad 1153$$

$$\frac{dG_{22}}{dt} = \left. \begin{array}{l} (a'_{22})^{(3)} G_{21} - \left[ \begin{array}{l} (a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21}, t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)} \boxed{+(a''_{42})^{(8,8,8)}(T_{41}, t)} \end{array} \right] \end{array} \right\} G_{22} \quad 1154$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$ ,  $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$  are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$  are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)}$ ,  $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)}$  are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)}$ ,  $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)}$  are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$ ,  $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$  are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{37})^{(7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a''_{38})^{(7,7,7)}(T_{37}, t)}$  are seventh augmentation coefficient

$\boxed{+(a''_{40})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{41})^{(8)}(T_{41}, t)}$ ,  $\boxed{+(a''_{42})^{(8)}(T_{41}, t)}$  Are Eighth Augmentation Coefficient

$$\frac{dT_{20}}{dt} = \left. \begin{array}{l} (b'_{20})^{(3)} T_{21} - \left[ \begin{array}{l} (b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \boxed{-(b''_{36})^{(7,7,7)}(G_{19}, t)} \boxed{-(b''_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39}, t)} \boxed{-(b''_{40})^{(8,8,8)}(G_{43}, t)} \boxed{-(b''_{40})^{(8,8,8)}(G_{43}, t)} \end{array} \right] \end{array} \right\} T_{20} \quad 1155$$

$$\frac{dT_{21}}{dt} = \left. \begin{array}{l} (b'_{21})^{(3)} T_{20} - \left[ \begin{array}{l} (b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39}, t)} \boxed{-(b''_{40})^{(8,8,8)}(G_{43}, t)} \end{array} \right] \end{array} \right\} T_{21} \quad 1156$$

$$\frac{dT_{22}}{dt} = \quad 1157$$



$$(b_{22})^{(3)}T_{21} - \left[ \begin{array}{ccc} (b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} & \boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b'_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b'_{26})^{(4,4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b'_{30})^{(5,5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b'_{34})^{(6,6,6,6,6,6)}(G_{35}, t)} \\ \boxed{-(b'_{38})^{(7,7,7)}(G_{39}, t)} & \boxed{-(b'_{40})^{(8,8,8)}(G_{43}, t)} & \end{array} \right] T_{22}$$

$\boxed{-(b'_{20})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b'_{21})^{(3)}(G_{23}, t)}$ ,  $\boxed{-(b'_{22})^{(3)}(G_{23}, t)}$  are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)}$  are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b'_{13})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{14})^{(1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{15})^{(1,1,1)}(G, t)}$  are third detrition coefficients for category 1,2 and 3

$\boxed{-(b'_{24})^{(4,4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b'_{25})^{(4,4,4,4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b'_{26})^{(4,4,4,4,4,4)}(G_{27}, t)}$  are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b'_{28})^{(5,5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b'_{29})^{(5,5,5,5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b'_{30})^{(5,5,5,5,5,5)}(G_{31}, t)}$  are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b'_{32})^{(6,6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b'_{33})^{(6,6,6,6,6,6)}(G_{35}, t)}$ ,  $\boxed{-(b'_{34})^{(6,6,6,6,6,6)}(G_{35}, t)}$  are sixth detritions coefficients for category 1, 2 and 3

Where  $\boxed{(a'_{24})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a'_{25})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a'_{26})^{(4)}(T_{25}, t)}$  are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a'_{28})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a'_{29})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a'_{30})^{(5,5)}(T_{29}, t)}$  are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a'_{32})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a'_{33})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a'_{34})^{(6,6)}(T_{33}, t)}$  are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a'_{13})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a'_{14})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a'_{15})^{(1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a'_{16})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a'_{17})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a'_{18})^{(2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a'_{20})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a'_{21})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a'_{22})^{(3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)}$  ARE SEVENTH augmentation coefficients

**FOURTH MODULE CONCATENATION:**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[ \begin{array}{ccc} (a'_{24})^{(4)} \boxed{+(a'_{24})^{(4)}(T_{25}, t)} & \boxed{+(a'_{28})^{(5,5)}(T_{29}, t)} & \boxed{+(a'_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a'_{13})^{(1,1,1,1)}(T_{14}, t)} & \boxed{+(a'_{16})^{(2,2,2,2)}(T_{17}, t)} & \boxed{+(a'_{20})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)} & \boxed{+(a'_{40})^{(8,8,8,8)}(T_{41}, t)} & \end{array} \right] G_{24} \quad 1158$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[ \begin{array}{ccc} (a'_{25})^{(4)} \boxed{+(a'_{25})^{(4)}(T_{25}, t)} & \boxed{+(a'_{29})^{(5,5)}(T_{29}, t)} & \boxed{+(a'_{33})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a'_{14})^{(1,1,1,1)}(T_{14}, t)} & \boxed{+(a'_{17})^{(2,2,2,2)}(T_{17}, t)} & \boxed{+(a'_{21})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{37})^{(7,7,7,7)}(T_{37}, t)} & \boxed{+(a'_{41})^{(8,8,8,8)}(T_{41}, t)} & \end{array} \right] G_{25} \quad 1159$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[ \begin{array}{c} (a'_{26})^{(4)} \boxed{+(a'_{26})^{(4)}(T_{25}, t)} \boxed{+(a'_{30})^{(5,5)}(T_{29}, t)} \boxed{+(a'_{34})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a'_{15})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a'_{18})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a'_{22})^{(3,3,3,3)}(T_{21}, t)} \\ \boxed{+(a'_{38})^{(7,7,7,7)}(T_{37}, t)} \boxed{+(a'_{42})^{(8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{26} \quad 1160$$

Where  $\boxed{(a'_{24})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a'_{25})^{(4)}(T_{25}, t)}$ ,  $\boxed{(a'_{26})^{(4)}(T_{25}, t)}$  are first augmentation coefficients for category 1, 2 and 3  
 $\boxed{+(a'_{28})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a'_{29})^{(5,5)}(T_{29}, t)}$ ,  $\boxed{+(a'_{30})^{(5,5)}(T_{29}, t)}$  are second augmentation coefficient for category 1, 2 and 3  
 $\boxed{+(a'_{32})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a'_{33})^{(6,6)}(T_{33}, t)}$ ,  $\boxed{+(a'_{34})^{(6,6)}(T_{33}, t)}$  are third augmentation coefficient for category 1, 2 and 3  
 $\boxed{+(a'_{13})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a'_{14})^{(1,1,1,1)}(T_{14}, t)}$ ,  $\boxed{+(a'_{15})^{(1,1,1,1)}(T_{14}, t)}$  are fourth augmentation coefficients for category 1, 2, and 3  
 $\boxed{+(a'_{16})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a'_{17})^{(2,2,2,2)}(T_{17}, t)}$ ,  $\boxed{+(a'_{18})^{(2,2,2,2)}(T_{17}, t)}$  are fifth augmentation coefficients for category 1, 2, and 3  
 $\boxed{+(a'_{20})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a'_{21})^{(3,3,3,3)}(T_{21}, t)}$ ,  $\boxed{+(a'_{22})^{(3,3,3,3)}(T_{21}, t)}$  are sixth augmentation coefficients for category 1, 2, and 3  
 $\boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)}$ ,  $\boxed{+(a'_{36})^{(7,7,7,7)}(T_{37}, t)}$  Are Seventh Augmentation Coefficients  
 $\boxed{+(a'_{41})^{(8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a'_{41})^{(8,8,8,8)}(T_{41}, t)}$ ,  $\boxed{+(a'_{41})^{(8,8,8,8)}(T_{41}, t)}$  Are Eighth Augmentation Coefficients

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[ \begin{array}{c} (b'_{24})^{(4)} \boxed{-(b'_{24})^{(4)}(G_{27}, t)} \boxed{-(b'_{28})^{(5,5)}(G_{31}, t)} \boxed{-(b'_{32})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b'_{13})^{(1,1,1,1)}(G, t)} \boxed{-(b'_{16})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b'_{20})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b'_{36})^{(7,7,7,7)}(G_{39}, t)} \boxed{-(b'_{40})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{24} \quad 1161$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[ \begin{array}{c} (b'_{25})^{(4)} \boxed{-(b'_{25})^{(4)}(G_{27}, t)} \boxed{-(b'_{29})^{(5,5)}(G_{31}, t)} \boxed{-(b'_{33})^{(6,6)}(G_{35}, t)} \\ \boxed{-(b'_{14})^{(1,1,1,1)}(G, t)} \boxed{-(b'_{17})^{(2,2,2,2)}(G_{19}, t)} \boxed{-(b'_{21})^{(3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b'_{37})^{(7,7,7,7)}(G_{39}, t)} \boxed{-(b'_{41})^{(8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{25} \quad 1162$$

Where  $\boxed{-(b'_{24})^{(4)}(G_{27}, t)}$ ,  $\boxed{-(b'_{25})^{(4)}(G_{27}, t)}$ ,  $\boxed{-(b'_{26})^{(4)}(G_{27}, t)}$  are first detrition coefficients for category 1, 2 and 3  
 $\boxed{-(b'_{28})^{(5,5)}(G_{31}, t)}$ ,  $\boxed{-(b'_{29})^{(5,5)}(G_{31}, t)}$ ,  $\boxed{-(b'_{30})^{(5,5)}(G_{31}, t)}$  are second detrition coefficients for category 1, 2 and 3  
 $\boxed{-(b'_{32})^{(6,6)}(G_{35}, t)}$ ,  $\boxed{-(b'_{33})^{(6,6)}(G_{35}, t)}$ ,  $\boxed{-(b'_{34})^{(6,6)}(G_{35}, t)}$  are third detrition coefficients for category 1, 2 and 3  
 $\boxed{-(b'_{13})^{(1,1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{14})^{(1,1,1,1)}(G, t)}$ ,  $\boxed{-(b'_{15})^{(1,1,1,1)}(G, t)}$  are fourth detrition coefficients for category 1, 2 and 3  
 $\boxed{-(b'_{16})^{(2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{17})^{(2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b'_{18})^{(2,2,2,2)}(G_{19}, t)}$  are fifth detrition coefficients for category 1, 2 and 3  
 $\boxed{-(b'_{20})^{(3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b'_{21})^{(3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b'_{22})^{(3,3,3,3)}(G_{23}, t)}$  are sixth detrition coefficients for category 1, 2 and 3  
 $\boxed{-(b'_{36})^{(7,7,7,7)}(G_{39}, t)}$ ,  $\boxed{-(b'_{37})^{(7,7,7,7)}(G_{39}, t)}$ ,  $\boxed{-(b'_{38})^{(7,7,7,7)}(G_{39}, t)}$  ARE Seventh Detrition Coefficients  
 $\boxed{-(b'_{40})^{(8,8,8,8)}(G_{43}, t)}$ ,  $\boxed{-(b'_{41})^{(8,8,8,8)}(G_{43}, t)}$ ,  $\boxed{-(b'_{42})^{(8,8,8,8)}(G_{43}, t)}$  Are Eighth Detrition Coefficients.

**Fifth Module Concatenation:**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[ \begin{array}{l} (a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \quad + (a''_{24})^{(4,4)}(T_{25}, t) \quad + (a''_{32})^{(6,6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1,1)}(T_{14}, t) \quad + (a''_{16})^{(2,2,2,2,2)}(T_{17}, t) \quad + (a''_{20})^{(3,3,3,3,3)}(T_{21}, t) \\ + (a''_{36})^{(7,7,7,7,7)}(T_{37}, t) \quad + (a''_{40})^{(8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{28} \quad 1163$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[ \begin{array}{l} (a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t) \quad + (a''_{25})^{(4,4)}(T_{25}, t) \quad + (a''_{33})^{(6,6,6)}(T_{33}, t) \\ + (a''_{14})^{(1,1,1,1,1)}(T_{14}, t) \quad + (a''_{17})^{(2,2,2,2,2)}(T_{17}, t) \quad + (a''_{21})^{(3,3,3,3,3)}(T_{21}, t) \\ + (a''_{37})^{(7,7,7,7,7)}(T_{37}, t) \quad + (a''_{41})^{(8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{29} \quad 1164$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[ \begin{array}{l} (a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t) \quad + (a''_{26})^{(4,4)}(T_{25}, t) \quad + (a''_{34})^{(6,6,6)}(T_{33}, t) \\ + (a''_{15})^{(1,1,1,1,1)}(T_{14}, t) \quad + (a''_{18})^{(2,2,2,2,2)}(T_{17}, t) \quad + (a''_{22})^{(3,3,3,3,3)}(T_{21}, t) \\ + (a''_{38})^{(7,7,7,7,7)}(T_{37}, t) \quad + (a''_{41})^{(8,8,8,8,8)}(T_{41}, t) \end{array} \right] G_{30} \quad 1165$$

Where  $(a''_{28})^{(5)}(T_{29}, t)$ ,  $(a''_{29})^{(5)}(T_{29}, t)$ ,  $(a''_{30})^{(5)}(T_{29}, t)$  are first augmentation coefficients for category 1, 2 and 3

And  $(a''_{24})^{(4,4)}(T_{25}, t)$ ,  $(a''_{25})^{(4,4)}(T_{25}, t)$ ,  $(a''_{26})^{(4,4)}(T_{25}, t)$  are second augmentation coefficient for category 1, 2 and 3

$(a''_{32})^{(6,6,6)}(T_{33}, t)$ ,  $(a''_{33})^{(6,6,6)}(T_{33}, t)$ ,  $(a''_{34})^{(6,6,6)}(T_{33}, t)$  are third augmentation coefficient for category 1, 2 and 3

$(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)$ ,  $(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)$ ,  $(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)$  are fourth augmentation coefficients for category 1, 2, and 3

$(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)$ ,  $(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)$ ,  $(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)$  are fifth augmentation coefficients for category 1, 2, and 3

$(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)$ ,  $(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)$ ,  $(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)$  are sixth augmentation coefficients for category 1, 2, and 3

$(a''_{40})^{(8,8,8,8,8)}(T_{41}, t)$ ,  $(a''_{41})^{(8,8,8,8,8)}(T_{41}, t)$ ,  $(a''_{42})^{(8,8,8,8,8)}(T_{41}, t)$  are Eighth Augmentation Coefficients

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[ \begin{array}{l} (b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}, t) \quad - (b''_{24})^{(4,4)}(G_{23}, t) \quad - (b''_{32})^{(6,6,6)}(G_{35}, t) \\ - (b''_{13})^{(1,1,1,1,1)}(G, t) \quad - (b''_{16})^{(2,2,2,2,2)}(G_{19}, t) \quad - (b''_{20})^{(3,3,3,3,3)}(G_{23}, t) \\ - (b''_{36})^{(7,7,7,7,7)}(G_{38}, t) \end{array} \right] T_{28} \quad 1166$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[ \begin{array}{l} (b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}, t) \quad - (b''_{25})^{(4,4)}(G_{27}, t) \quad - (b''_{33})^{(6,6,6)}(G_{35}, t) \\ - (b''_{14})^{(1,1,1,1,1)}(G, t) \quad - (b''_{17})^{(2,2,2,2,2)}(G_{19}, t) \quad - (b''_{21})^{(3,3,3,3,3)}(G_{23}, t) \\ - (b''_{37})^{(7,7,7,7,7)}(G_{38}, t) \end{array} \right] T_{29} \quad 1167$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[ \begin{array}{l} (b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}, t) \quad - (b''_{26})^{(4,4)}(G_{27}, t) \quad - (b''_{34})^{(6,6,6)}(G_{35}, t) \\ - (b''_{15})^{(1,1,1,1,1)}(G, t) \quad - (b''_{18})^{(2,2,2,2,2)}(G_{19}, t) \quad - (b''_{22})^{(3,3,3,3,3)}(G_{23}, t) \\ - (b''_{38})^{(7,7,7,7,7)}(G_{38}, t) \end{array} \right] T_{30} \quad 1168$$

**SIXTH MODULE CONCATENATION**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \tag{1169}$$

$$- \left[ \begin{array}{ccc} (a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) & + (a''_{28})^{(5,5,5)}(T_{29}, t) & + (a''_{24})^{(4,4,4)}(T_{25}, t) \\ + (a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t) & + (a''_{40})^{(8,8,8,8,8,8)}(T_{41}, t) & \end{array} \right] G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} \tag{1170}$$

$$- \left[ \begin{array}{ccc} (a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t) & + (a''_{29})^{(5,5,5)}(T_{29}, t) & + (a''_{25})^{(4,4,4)}(T_{25}, t) \\ + (a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{37})^{(7,7,7,7,7,7)}(T_{37}, t) & + (a''_{41})^{(8,8,8,8,8,8)}(T_{41}, t) & \end{array} \right] G_{33}$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} \tag{1171}$$

$$- \left[ \begin{array}{ccc} (a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t) & + (a''_{30})^{(5,5,5)}(T_{29}, t) & + (a''_{26})^{(4,4,4)}(T_{25}, t) \\ + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) \\ + (a''_{38})^{(7,7,7,7,7,7)}(T_{37}, t) & + (a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t) & \end{array} \right] G_{34}$$

$+(a''_{32})^{(6)}(T_{33}, t), +(a''_{33})^{(6)}(T_{33}, t), +(a''_{34})^{(6)}(T_{33}, t)$  are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5)}(T_{29}, t), +(a''_{29})^{(5,5,5)}(T_{29}, t), +(a''_{30})^{(5,5,5)}(T_{29}, t)$  are second augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4)}(T_{25}, t), +(a''_{25})^{(4,4,4)}(T_{25}, t), +(a''_{26})^{(4,4,4)}(T_{25}, t)$  are third augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t), +(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t), +(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$  - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t), +(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t), +(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$  - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t), +(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t), +(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$  sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t), +(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t), +(a''_{36})^{(7,7,7,7,7,7)}(T_{37}, t)$  Are Seventh Augmentation Coefficients

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41}, t), +(a''_{41})^{(8,8,8,8,8,8)}(T_{41}, t), +(a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t)$  Are Eight Augmentation Coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} \tag{1172}$$

$$- \left[ \begin{array}{ccc} (b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}, t) & - (b''_{28})^{(5,5,5)}(G_{31}, t) & - (b''_{24})^{(4,4,4)}(G_{27}, t) \\ - (b''_{13})^{(1,1,1,1,1,1)}(G, t) & - (b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t) & - (b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t) \\ - (b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t) & - (b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t) & \end{array} \right] T_{32}$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} \quad 1173$$

$$- \left[ \begin{array}{ccc} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35}, t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{33}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} \quad 1174$$

$$- \left[ \begin{array}{ccc} (b'_{34})^{(6)} \boxed{-(b''_{34})^{(6)}(G_{35}, t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)} & \boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)} & & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)} \end{array} \right] T_{34}$$

$\boxed{-(b''_{32})^{(6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{33})^{(6)}(G_{35}, t)}$ ,  $\boxed{-(b''_{34})^{(6)}(G_{35}, t)}$  are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)}$ ,  $\boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)}$  are second detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)}$ ,  $\boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)}$  are third detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)}$ ,  $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)}$  are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)}$ ,  $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)}$  are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)}$ ,  $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)}$  are sixth detritions coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39}, t)}$ ,  $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39}, t)}$ ,  $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39}, t)}$  Are Seventh Detrition Coefficients

$\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43}, t)}$ ,  $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43}, t)}$ ,  $\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43}, t)}$  Are Eighth Detrition Copefficients.

**SEVENTH MODULE CONCATENATION:**

$$\frac{dG_{36}}{dt} = \quad 1175$$

$$(a_{36})^{(7)}G_{37} - \left[ \begin{array}{l} \boxed{(a'_{36})^{(7)}} \boxed{+(a''_{36})^{(7)}(T_{37}, t)} + \boxed{(a''_{16})^{(7)}(T_{17}, t)} + \boxed{(a''_{20})^{(7)}(T_{21}, t)} + \boxed{(a''_{24})^{(7)}(T_{23}, t)G_{36}} + \\ \boxed{(a''_{28})^{(7)}(T_{29}, t)} + \boxed{(a''_{32})^{(7)}(T_{33}, t)} + \boxed{(a''_{13})^{(7)}(T_{14}, t)} \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{36}$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[ \begin{array}{l} \boxed{(a'_{37})^{(7)}} + \boxed{(a''_{37})^{(7)}(T_{37}, t)} + \boxed{(a''_{14})^{(7)}(T_{14}, t)} + \boxed{(a''_{21})^{(7)}(T_{21}, t)} + \\ \boxed{(a''_{17})^{(7)}(T_{17}, t)} + \boxed{(a''_{25})^{(7)}(T_{25}, t)} + \boxed{(a''_{33})^{(7)}(T_{33}, t)} + \\ \boxed{(a''_{29})^{(7)}(T_{29}, t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41}, t)} \end{array} \right] G_{37} \quad 1176$$

$$\frac{dG_{38}}{dt} = \quad 1177$$

$$(a_{38})^{(7)}G_{37} - \left[ \begin{array}{l} \boxed{(a'_{38})^{(7)}} + \boxed{(a''_{38})^{(7)}(T_{37}, t)} + \boxed{(a''_{15})^{(7)}(T_{14}, t)} + \boxed{(a''_{22})^{(7)}(T_{21}, t)} + \boxed{+(a''_{18})^{(7)}(T_{17}, t)} + \end{array} \right]$$

$$\left[ (a''_{26})^{(7)}(T_{25}, t) + (a''_{34})^{(7)}(T_{33}, t) + (a''_{30})^{(7)}(T_{29}, t) + (a''_{42})^{(8,8,8,8,8,8)}(T_{41}, t) \right] G_{38}$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[ (b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{16})^{(7)}((G_{19}), t) - (b''_{13})^{(7)}((G_{14}), t) - (b''_{20})^{(7)}((G_{231}), t) - (b''_{24})^{(7)}((G_{27}), t) - (b''_{28})^{(7)}((G_{31}), t) - (b''_{32})^{(7)}((G_{35}), t) \right] T_{36} \quad 1178$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[ (b'_{36})^{(7)} - (b''_{37})^{(7)}((G_{39}), t) - (b''_{17})^{(7)}((G_{19}), t) - (b''_{19})^{(7)}((G_{14}), t) - (b''_{21})^{(7)}((G_{231}), t) - (b''_{25})^{(7)}((G_{27}), t) - (b''_{29})^{(7)}((G_{31}), t) - (b''_{33})^{(7)}((G_{35}), t) \right] T_{37} \quad 1179$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[ (b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t) - (b''_{18})^{(7)}((G_{19}), t) - (b''_{22})^{(7)}((G_{14}), t) - (b''_{15})^{(7)}((G_{23}), t) - (b''_{26})^{(7)}((G_{27}), t) - (b''_{30})^{(7)}((G_{31}), t) - (b''_{34})^{(7)}((G_{35}), t) \right] T_{38} \quad 1180$$

**EIGHTH MODULE CONCATENATION:**

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[ (a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t) + (a''_{13})^{(8)}(T_{14}, t) + (a''_{16})^{(8)}(T_{17}, t) + (a''_{20})^{(8)}(T_{21}, t) + (a''_{24})^{(8)}(T_{23}, t) + (a''_{28})^{(8)}(T_{29}, t) + (a''_{32})^{(8)}(T_{33}, t) + (a''_{36})^{(8)}(T_{37}, t) \right] G_{40} \quad 1181$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[ (a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t) + (a''_{14})^{(8)}(T_{14}, t) + (a''_{17})^{(8)}(T_{17}, t) + (a''_{21})^{(8)}(T_{21}, t) + (a''_{25})^{(8)}(T_{23}, t) + (a''_{29})^{(8)}(T_{29}, t) + (a''_{33})^{(8)}(T_{33}, t) + (a''_{37})^{(8)}(T_{37}, t) \right] G_{41} \quad 1182$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[ (a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t) + (a''_{15})^{(8)}(T_{14}, t) + (a''_{18})^{(8)}(T_{17}, t) + (a''_{22})^{(8)}(T_{21}, t) + (a''_{26})^{(8)}(T_{23}, t) + (a''_{30})^{(8)}(T_{29}, t) + (a''_{34})^{(8)}(T_{33}, t) + (a''_{38})^{(8)}(T_{37}, t) \right] G_{42} \quad 1183$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[ (b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t) - (b''_{13})^{(8)}((G_{14}), t) - (b''_{16})^{(8)}((G_{19}), t) - (b''_{20})^{(8)}((G_{23}), t) - (b''_{24})^{(8)}((G_{27}), t) - (b''_{28})^{(8)}((G_{31}), t) - (b''_{32})^{(8)}((G_{35}), t) - (b''_{36})^{(8)}((G_{39}), t) \right] T_{40} \quad 1184$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \quad 1185$$

$$\left[ \begin{array}{l} (b'_{41})^{(8)} - \boxed{(b''_{41})^{(8)}((G_{43}), t)} - \boxed{(b''_{14})^{(8)}((G_{14}), t)} - \boxed{(b''_{17})^{(8)}((G_{19}), t)} - \boxed{(b''_{21})^{(8)}((G_{23}), t)} \\ \boxed{(b''_{25})^{(8)}((G_{27}), t)} - \boxed{(b''_{29})^{(8)}((G_{31}), t)} - \boxed{(b''_{33})^{(8)}((G_{35}), t)} - \boxed{(b''_{37})^{(8)}((G_{39}), t)} \end{array} \right] T_{41}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[ (b'_{42})^{(8)} - \boxed{(b''_{42})^{(8)}((G_{43}), t)} - \boxed{(b''_{15})^{(7)}((G_{14}), t)} - \boxed{(b''_{18})^{(8)}((G_{19}), t)} - \right. \\ \left. \boxed{(b''_{22})^{(8)}((G_{23}), t)} - \boxed{(b''_{26})^{(8)}((G_{27}), t)} - \boxed{(b''_{30})^{(8)}((G_{31}), t)} - \boxed{(b''_{34})^{(8)}((G_{35}), t)} - \right. \\ \left. \boxed{(b''_{38})^{(8)}((G_{39}), t)} \right] T_{42} \quad 1186$$

$+(a''_{40})^{(8)}(T_{41}, t) =$  **First augmentation factor**

$-(b''_{40})^{(8)}((G_{43}), t) =$  **First detritions factor**

Where we suppose

(A)  $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$  1187

$i, j = 13, 14, 15$

(B) The functions  $(a''_i)^{(1)}, (b''_i)^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

$\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$  1188

$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where  $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$  are positive constants and  $\boxed{i = 13, 14, 15}$

They satisfy Lipschitz condition: 1189

$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)}t}$

$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(1)}(T'_{14}, t)$  and  $(a''_i)^{(1)}(T_{14}, t)$ .  $(T'_{14}, t)$  and  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that  $(a''_i)^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{13})^{(1)} = 1$  then the function  $(a''_i)^{(1)}(T_{14}, t)$ , the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 1190

(C)  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$  :

1191

- (D) There exists two constants  $(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  which together with  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$  and  $(\hat{B}_{13})^{(1)}$  and the constants  $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$+(a''_{36})^{(7)}(T_{37}, t) =$  **First augmentation factor**

$$(1)(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16,17,18$$

- (E) (2) The functions  $(a''_i)^{(2)}, (b''_i)^{(2)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(2)}, (r_i)^{(2)}$ :

$$(a''_i)^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 1192$$

$$(b''_i)^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b'_i)^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 1193$$

$$(F) \quad (3) \lim_{T_2 \rightarrow \infty} (a''_i)^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 1194$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 1195$$

**Definition of**  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$  :

Where  $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$  are positive constants and  $i = 16,17,18$

They satisfy Lipschitz condition:

$$|(a''_i)^{(2)}(T'_{17}, t) - (a''_i)^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T'_{17} - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 1196$$

$$|(b''_i)^{(2)}((G'_{19}), t) - (b''_i)^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G'_{19}) - (G_{19})| e^{-(\hat{M}_{16})^{(2)}t} \quad 1197$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(2)}(T'_{17}, t)$  and  $(a''_i)^{(2)}(T_{17}, t) \cdot (T'_{17}, t)$  And  $(T_{17}, t)$  are points belonging to the interval  $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that  $(a''_i)^{(2)}(T_{17}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{16})^{(2)} = 1$  then the function  $(a''_i)^{(2)}(T_{17}, t)$ , the SECOND augmentation coefficient would be absolutely continuous.

**Definition of**  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$  :

- (G) (4)  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ , are positive constants 1198



$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$  :

There exists two constants  $(\hat{P}_{16})^{(2)}$  and  $(\hat{Q}_{16})^{(2)}$  which together with  $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$  and  $(\hat{B}_{16})^{(2)}$  and the constants  $(a_i)^{(2)}, (a'_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [ (a_i)^{(2)} + (a'_i)^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)} ] < 1 \quad 1199$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [ (b_i)^{(2)} + (b'_i)^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)} ] < 1 \quad 1200$$

Where we suppose

$$(H) \quad (a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (b''_i)^{(3)} > 0, \quad i, j = 20,21,22 \quad 1201$$

The functions  $(a''_i)^{(3)}, (b''_i)^{(3)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(3)}, (r_i)^{(3)}$ :

$$(a''_i)^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b''_i)^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b'_i)^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 1202$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of**  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$  :

Where  $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$  are positive constants and  $i = 20,21,22$

They satisfy Lipschitz condition: 1203

$$|(a''_i)^{(3)}(T'_{21}, t) - (a''_i)^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T'_{21}| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b''_i)^{(3)}(G'_{23}, t) - (b''_i)^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G'_{23}|| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(3)}(T'_{21}, t)$  and  $(a''_i)^{(3)}(T_{21}, t)$ .  $(T'_{21}, t)$  and  $(T_{21}, t)$  are points belonging to the interval  $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$ . It is to be noted that  $(a''_i)^{(3)}(T_{21}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{20})^{(3)} = 1$  then the function  $(a''_i)^{(3)}(T_{21}, t)$ , the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of**  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$  : 1204

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ , are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants  $(\hat{P}_{20})^{(3)}$  and  $(\hat{Q}_{20})^{(3)}$  which together with  $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$  and  $(\hat{B}_{20})^{(3)}$  and the constants  $(a_i)^{(3)}, (a'_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$  1205

satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a'_i)^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b'_i)^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a'_i)^{(4)}, (a''_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (b''_i)^{(4)} > 0, \quad i, j = 24, 25, 26 \quad 1206$$

The functions  $(a''_i)^{(4)}, (b''_i)^{(4)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(4)}, (r_i)^{(4)}$ :

$$(a''_i)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b''_i)^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b'_i)^{(4)} \leq (\hat{B}_{24})^{(4)}$$

1207

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(4)}(T_{25}, t) = (p_i)^{(4)}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of**  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$  :

Where  $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$  are positive constants and  $i = 24, 25, 26$

They satisfy Lipschitz condition:

1208

$$|(a''_i)^{(4)}(T'_{25}, t) - (a''_i)^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T'_{25}| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b''_i)^{(4)}((G_{27})', t) - (b''_i)^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} |(G_{27})' - (G_{27})| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(4)}(T'_{25}, t)$  and  $(a''_i)^{(4)}(T_{25}, t) \cdot (T'_{25}, t)$ . And  $(T_{25}, t)$  are points belonging to the interval  $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$ . It is to be noted that  $(a''_i)^{(4)}(T_{25}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{24})^{(4)} = 4$  then the function  $(a''_i)^{(4)}(T_{25}, t)$ , the **FOURTH augmentation coefficient WOULD** be absolutely continuous.

**Definition of**  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$  :

1209

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ , are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of**  $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$  :

1210

There exists two constants  $(\hat{P}_{24})^{(4)}$  and  $(\hat{Q}_{24})^{(4)}$  which together with  $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$  and  $(\hat{B}_{24})^{(4)}$  and the constants  $(a_i)^{(4)}, (a'_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a'_i)^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b'_i)^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a'_i)^{(5)}, (a''_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (b''_i)^{(5)} > 0, \quad i, j = 28, 29, 30 \quad 1211$$

The functions  $(a''_i)^{(5)}, (b''_i)^{(5)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(5)}, (r_i)^{(5)}$ :

$$(a''_i)^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b''_i)^{(5)}(G_{31}, t) \leq (r_i)^{(5)} \leq (b'_i)^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(5)}(T_{29}, t) = (p_i)^{(5)} \quad 1212$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of**  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where  $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$  are positive constants and  $i = 28, 29, 30$

They satisfy Lipschitz condition: 1213

$$|(a''_i)^{(5)}(T'_{29}, t) - (a''_i)^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29} - T'_{29}| e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b''_i)^{(5)}(G'_{31}, t) - (b''_i)^{(5)}(G_{31}, t)| < (\hat{k}_{28})^{(5)} |(G_{31}) - (G'_{31})'| e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(5)}(T'_{29}, t)$  and  $(a''_i)^{(5)}(T_{29}, t)$ .  $(T'_{29}, t)$  and  $(T_{29}, t)$  are points belonging to the interval  $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$ .

It is to be noted that  $(a''_i)^{(5)}(T_{29}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{28})^{(5)} = 5$  then the function  $(a''_i)^{(5)}(T_{29}, t)$ , the FIFTH **augmentation coefficient** attributable would be absolutely continuous.

**Definition of**  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 1214

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ , are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of**  $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 1215

There exists two constants  $(\hat{P}_{28})^{(5)}$  and  $(\hat{Q}_{28})^{(5)}$  which together with  $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$  and  $(\hat{B}_{28})^{(5)}$  and the constants  $(a_i)^{(5)}, (a'_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a'_i)^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b'_i)^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$(a_i)^{(6)}, (a'_i)^{(6)}, (a''_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (b''_i)^{(6)} > 0, \quad i, j = 32, 33, 34$  1216  
 The functions  $(a'_i)^{(6)}, (b'_i)^{(6)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(6)}, (r_i)^{(6)}$ :

$$(a''_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b''_i)^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b'_i)^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(6)}(T_{33}, t) = (p_i)^{(6)} \quad 1217$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of**  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$  :

Where  $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$  are positive constants and  $i = 32, 33, 34$

They satisfy Lipschitz condition: 1218

$$|(a''_i)^{(6)}(T'_{33}, t) - (a''_i)^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T'_{33}| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b''_i)^{(6)}((G_{35})', t) - (b''_i)^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35}) - (G_{35})'| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(6)}(T'_{33}, t)$  and  $(a''_i)^{(6)}(T_{33}, t)$ .  $(T'_{33}, t)$  and  $(T_{33}, t)$  are points belonging to the interval  $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$ .

It is to be noted that  $(a''_i)^{(6)}(T_{33}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{32})^{(6)} = 1$  then the function  $(a''_i)^{(6)}(T_{33}, t)$ , the SIXTH **augmentation coefficient** would be absolutely continuous.

**Definition of**  $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$  : 1219

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ , are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of**  $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$  : 1220

There exists two constants  $(\hat{P}_{32})^{(6)}$  and  $(\hat{Q}_{32})^{(6)}$  which together with  $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$  and  $(\hat{B}_{32})^{(6)}$  and the constants  $(a_i)^{(6)}, (a'_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a'_i)^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b'_i)^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

$$i, j = 36, 37, 38 \quad (a_i)^{(7)}, (a'_i)^{(7)}, (a''_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (b''_i)^{(7)} > 0, \quad 1221$$

The functions  $(a''_i)^{(7)}, (b''_i)^{(7)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(7)}, (r_i)^{(7)}$ :

$$(a''_i)^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b''_i)^{(7)}(G, t) \leq (r_i)^{(7)} \leq (b'_i)^{(7)} \leq (\hat{B}_{36})^{(7)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(7)}(T_{37}, t) = (p_i)^{(7)} \quad 1222$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(7)}(G_{39}, t) = (r_i)^{(7)}$$

**Definition of**  $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$  :

Where  $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}$  are positive constants and  $i = 36, 37, 38$

They satisfy Lipschitz condition: 1223

$$|(a''_i)^{(7)}(T'_{37}, t) - (a''_i)^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T'_{37}| e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b''_i)^{(7)}((G_{39})', t) - (b''_i)^{(7)}((G_{39}), (T_{39}))| < (\hat{k}_{36})^{(7)} |(G_{39}) - (G_{39})'| e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(7)}(T'_{37}, t)$  and  $(a''_i)^{(7)}(T_{37}, t) \cdot (T'_{37}, t)$  and  $(T_{37}, t)$  are points belonging to the interval  $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that  $(a''_i)^{(7)}(T_{37}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{36})^{(7)} = 7$  then the function  $(a''_i)^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$  : 1224

(K)  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ , are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of**  $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$  : 1225

There exists two constants  $(\hat{P}_{36})^{(7)}$  and  $(\hat{Q}_{36})^{(7)}$  which together with  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)}$  and  $(\hat{B}_{36})^{(7)}$  and the constants  $(a_i)^{(7)}, (a'_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36, 37, 38$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(a_i)^{(7)} + (a'_i)^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(b_i)^{(7)} + (b'_i)^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

**Definition of**  $G_i(0), T_i(0)$  :

1226

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Where we suppose

(L)  $(a_i)^{(8)}, (a'_i)^{(8)}, (a''_i)^{(8)}, (b_i)^{(8)}, (b'_i)^{(8)}, (b''_i)^{(8)} > 0,$   
 $i, j = 40, 41, 42$

1227

(M) The functions  $(a''_i)^{(8)}, (b''_i)^{(8)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(8)}, (r_i)^{(8)}$ :

$$(a''_i)^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$

$$(b''_i)^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b'_i)^{(8)} \leq (\hat{B}_{40})^{(8)}$$

1228

(N)  $\lim_{T_2 \rightarrow \infty} (a''_i)^{(8)}(T_{41}, t) = (p_i)^{(8)}$   
 $\lim_{G \rightarrow \infty} (b''_i)^{(8)}((G_{43}), t) = (r_i)^{(8)}$

**Definition of**  $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$  :

Where  $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$  are positive constants and  $\boxed{i = 40, 41, 42}$

They satisfy Lipschitz condition:

1229

$$|(a''_i)^{(8)}(T'_{41}, t) - (a''_i)^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T'_{41}| e^{-(\hat{M}_{40})^{(8)}t}$$

$$|(b''_i)^{(8)}((G_{43})', t) - (b''_i)^{(8)}((G_{43}), (T_{43}))| < (\hat{k}_{40})^{(8)} |(G_{43}) - (G_{43})'| e^{-(\hat{M}_{40})^{(8)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(8)}(T'_{41}, t)$  and  $(a''_i)^{(8)}(T_{41}, t)$ .  $(T'_{41}, t)$  and  $(T_{41}, t)$  are points belonging to the interval  $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$ . It is to be noted that  $(a''_i)^{(8)}(T_{41}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{40})^{(8)} = 1$  then the function  $(a''_i)^{(8)}(T_{41}, t)$ , the EIGHT **augmentation coefficient** would be absolutely continuous.

**Definition of**  $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$  :

1230

(O)  $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ , are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t [(a_{15})^{(1)} G_{14}(s_{(13)}) - ((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})) G_{15}(s_{(13)})] ds_{(13)}$$

1231

By

1232

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t [(a_{13})^{(1)} G_{14}(s_{(13)}) - ((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) G_{13}(s_{(13)})] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \quad 1233$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \quad 1234$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \quad 1235$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \quad 1236$$

Where  $s_{(13)}$  is the integrand that is integrated over an interval  $(0, t)$

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions 1237

**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Consider operator  $\mathcal{A}^{(7)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i \leq (\hat{P}_{36})^{(7)}, T_i \leq (\hat{Q}_{36})^{(7)}, \quad 1238$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 1239$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 1240$$

By 1241

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + (a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)} \quad 1242$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)} \quad 1243$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)} \quad 1244$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)} \quad 1245$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)} \quad 1246$$

Where  $s_{(36)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(2)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i \leq (\hat{P}_{16})^{(2)}, T_i \leq (\hat{Q}_{16})^{(2)}, \quad 1247$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 1248$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 1249$$

By 1250

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16}(s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)} \quad 1251$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)} \quad 1252$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 1253$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 1254$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 1255$$

Where  $s_{(16)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(3)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i \leq (\hat{P}_{20})^{(3)}, T_i \leq (\hat{Q}_{20})^{(3)}, \quad 1256$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 1257$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 1258$$

By 1259

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20}(s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)} \quad 1260$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)} \quad 1261$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)} \quad 1262$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)} \quad 1263$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 1264$$

Where  $s_{(20)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(4)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i \leq (\hat{P}_{24})^{(4)}, T_i \leq (\hat{Q}_{24})^{(4)}, \quad 1265$$



$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 1266$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 1267$$

By 1268

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)} G_{24}(s_{(24)}) - \left( (a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \quad 1269$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 1270$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{24})^{(4)} - (b''_{24})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 1271$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)} T_{24}(s_{(24)}) - \left( (b'_{25})^{(4)} - (b''_{25})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 1272$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{26})^{(4)} - (b''_{26})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 1273$$

Where  $s_{(24)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(5)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)}, \quad 1275$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 1276$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 1277$$

By 1278

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 1279$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 1280$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 1281$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 1282$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 1283$$

Where  $s_{(28)}$  is the integrand that is integrated over an interval  $(0, t)$

Consider operator  $\mathcal{A}^{(6)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)}, \quad 1284$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 1285$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 1286$$

By 1287

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 1288$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 1289$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 1290$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 1291$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 1292$$

Where  $s_{(32)}$  is the integrand that is integrated over an interval  $(0, t)$

if the conditions are fulfilled, there exists a solution satisfying the conditions 1293

Definition of  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof:

Consider operator  $\mathcal{A}^{(7)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{36})^{(7)}, T_i^0 \leq (\hat{Q}_{36})^{(7)}, \quad 1294$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 1295$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad 1296$$

By 1297

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36}(s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + a''_{37}(s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)} \quad 1298$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + a''_{38}(s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)} \quad 1299$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)} \quad 1300$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)} \quad 1301$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)} \quad 1302$$

Where  $s_{(36)}$  is the integrand that is integrated over an interval  $(0, t)$

if the conditions above are fulfilled, there exists a solution satisfying the conditions 1303

Definition of  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof:

Consider operator  $\mathcal{A}^{(8)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{40})^{(8)}, T_i^0 \leq (\hat{Q}_{40})^{(8)}, \quad 1304$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad 1305$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad 1306$$

By 1307

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40}(s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + a''_{41}(s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)} \quad 1308$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)} \quad 1309$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)} \quad 1310$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)} \quad 1311$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)} \quad 1312$$

Where  $s_{(40)}$  is the integrand that is integrated over an interval  $(0, t)$

The operator  $\mathcal{A}^{(8)}$  maps the space of functions satisfying global equations into itself 1313

.Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} =$$

$$(1 + (a_{40})^{(8)} t) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$
1314

From which it follows that

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[ ((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left( -\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0} \right)} + (\hat{P}_{40})^{(8)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1

In order that the operator  $\mathcal{A}^{(1)}$  transforms the space of sextuples of functions  $G_i, T_i$  1315  
 satisfying GLOBAL EQUATIONS into itself

The operator  $\mathcal{A}^{(1)}$  is a contraction with respect to the metric 1316

$$d\left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)} t} \right\}$$

Indeed if we denote

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Definition of  $\tilde{G}, \tilde{T} : (\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned} |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\ &\int_0^t \{(a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &(a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} + \\ &G_{13}^{(2)} |(a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)}(T_{14}^{(2)}, s_{(13)})| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)} \end{aligned}$$

Where  $s_{(13)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} &\leq \\ \frac{1}{(\widehat{M}_{13})^{(1)}} &((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})) \end{aligned} \quad 1318$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed  $(a''_{13})^{(1)}$  and  $(b''_{13})^{(1)}$  depending also on  $t$  can be 1319

considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  and  $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$  respectively of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it

only on a compact then it suffices to consider that  $(a'_i)^{(1)}$  and  $(b'_i)^{(1)}$ ,

$i=13,14,15$  depend only on  $T_{14}$  and respectively on  $G$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  1320

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0$$

Definition of  $((\widehat{M}_{13})^{(1)})_1$ , and  $((\widehat{M}_{13})^{(1)})_3$  : 1321

Remark 3: if  $G_{13}$  is bounded, the same property have also  $G_{14}$  and  $G_{15}$ . indeed if

$$G_{13} < (\widehat{M}_{13})^{(1)} \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)}G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)}((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)}((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If  $G_{14}$  or  $G_{15}$  is bounded, the same property follows for  $G_{13}$ ,  $G_{15}$  and  $G_{13}$ ,  $G_{14}$  respectively.

Remark 4: If  $G_{13}$  is bounded, from below, the same property holds for  $G_{14}$  and  $G_{15}$ . 1322

The proof is analogous with the preceding one. An analogous property is true if  $G_{14}$  is bounded from below.

Remark 5: If  $T_{13}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$  then  $T_{14} \rightarrow \infty$ . 1323

Definition of  $(m)^{(1)}$  and  $\varepsilon_1$  :

Indeed let  $t_1$  be so that for  $t > t_1$

$$(b_{14})^{(1)} - (b'_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then  $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$  which leads to 1324

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$$T_{14} \geq \left( \frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is}$$

unbounded.

The same property holds for  $T_{15}$  if  $\lim_{t \rightarrow \infty} (b''_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take  $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$  and to choose 1325

$(\widehat{P}_{16})^{(2)}$  and  $(\widehat{Q}_{16})^{(2)}$  large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[ (\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{16})^{(2)} \quad 1326$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[ ((\widehat{Q}_{16})^{(2)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{16})^{(2)} \right] \leq (\widehat{Q}_{16})^{(2)} \quad 1327$$

In order that the operator  $\mathcal{A}^{(2)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 1328

The operator  $\mathcal{A}^{(2)}$  is a contraction with respect to the metric 1329

$$d\left( ((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \}$$

Indeed if we denote

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Definition of  $\bar{G}_{19}, \bar{T}_{19}$  :  $(\bar{G}_{19}, \bar{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

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$$|\bar{G}_{16}^{(1)} - \bar{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} +$$

$$\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} | (a'_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a'_{16})^{(2)} (T_{17}^{(2)}, s_{(16)}) | e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)}$$

Where  $s_{(16)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\bar{M}_{16})^{(2)}t} \leq \frac{1}{(\bar{M}_{16})^{(2)}} ((a_{16})^{(2)} + (a'_{16})^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}) d((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})$$

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And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis

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the result follows

Remark 1: The fact that we supposed  $(a'_{16})^{(2)}$  and  $(b'_{16})^{(2)}$  depending also on  $t$

1334

can be considered as not conformal with the reality, however we have put this

hypothesis, in order that we can postulate condition necessary to prove the

uniqueness of the solution bounded by  $(\hat{P}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)}t}$  and  $(\hat{Q}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)}t}$  respectively

of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only

on a compact then it suffices to consider that  $(a'_i)^{(2)}$  and  $(b'_i)^{(2)}$ ,  $i =$

16,17,18 depend only on  $T_{17}$  and respectively on  $(G_{19})$  (and not on  $t$ ) and hypothesis

can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

1335

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(2)} - (a''_i)^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \text{ for } t > 0$$

Definition of  $((\widehat{M}_{16})^{(2)})_1, ((\widehat{M}_{16})^{(2)})_2$  and  $((\widehat{M}_{16})^{(2)})_3$  : 1336

Remark 3: if  $G_{16}$  is bounded, the same property have also  $G_{17}$  and  $G_{18}$  . indeed if

$G_{16} < ((\widehat{M}_{16})^{(2)})_1$  it follows  $\frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)}G_{17}$  and by integrating

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If  $G_{17}$  or  $G_{18}$  is bounded, the same property follows for  $G_{16}$  ,  $G_{18}$  and  $G_{16}$  ,  $G_{17}$  respectively.

Remark 4: If  $G_{16}$  is bounded, from below, the same property holds for  $G_{17}$  and  $G_{18}$  . 1337

The proof is analogous with the preceding one. An analogous property is true if  $G_{17}$  is bounded from below.

Remark 5: If  $T_{16}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b'_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$  then  $T_{17} \rightarrow \infty$ . 1338

Definition of  $(m)^{(2)}$  and  $\varepsilon_2$  :

Indeed let  $t_2$  be so that for  $t > t_2$

$$(b_{17})^{(2)} - (b'_i)^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then  $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$  which leads to 1339

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$$

If we take  $t$  such that  $e^{-\varepsilon_2 t} = \frac{1}{2}$  it results

$$T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right), \quad t \log \frac{2}{\varepsilon_2}$$

By taking now  $\varepsilon_2$  sufficiently small one sees that  $T_{17}$  is 1340

unbounded.

The same property holds for  $T_{18}$  if  $\lim_{t \rightarrow \infty} (b''_{18})^{(2)}((G_{19})(t), t) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take  $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$  and to choose 1341

$(\widehat{P}_{20})^{(3)}$  and  $(\widehat{Q}_{20})^{(3)}$  large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} \left[ (\widehat{P}_{20})^{(3)} + ((\widehat{P}_{20})^{(3)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{20})^{(3)}$$
1342



$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} \left[ \left( (\widehat{Q}_{20})^{(3)} + T_j^0 \right) e^{-\left( \frac{(\widehat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{20})^{(3)} \right] \leq (\widehat{Q}_{20})^{(3)} \quad 1343$$

In order that the operator  $\mathcal{A}^{(3)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(3)}$  is a contraction with respect to the metric 1344

$$d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{20})^{(3)}t} \}$$

Indeed if we denote 1345

Definition of  $\widetilde{G}_{23}, \widetilde{T}_{23} : (\widetilde{G}_{23}, \widetilde{T}_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 1346

$$\begin{aligned} |\widetilde{G}_{20}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned}$$

Where  $s_{(20)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{20})^{(3)}t} &\leq \frac{1}{(\widehat{M}_{20})^{(3)}} \left( (a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} \right) \\ &+ (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) \end{aligned} \quad 1347$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed  $(a''_{20})^{(3)}$  and  $(b''_{20})^{(3)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$  and  $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$  respectively of  $\mathbb{R}_+$ . 1348

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on

a compact then it suffices to consider that  $(a'_i)^{(3)}$  and  $(b'_i)^{(3)}$ ,  $i = 20, 21, 22$  depend only on  $T_{21}$  and respectively on  $(G_{23})$  (and not on  $t$ ) and hypothesis

can be replaced by a usual Lipschitz condition.

*Remark 2: There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$*  1349

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(3)}t} > 0 \text{ for } t > 0$$

*Definition of  $((\widehat{M}_{20})^{(3)})_1, ((\widehat{M}_{20})^{(3)})_2$  and  $((\widehat{M}_{20})^{(3)})_3$  :* 1350

*Remark 3: if  $G_{20}$  is bounded, the same property have also  $G_{21}$  and  $G_{22}$ . indeed if*

*$G_{20} < ((\widehat{M}_{20})^{(3)})_1$  it follows  $\frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a_{21}')^{(3)}G_{21}$  and by integrating*

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a_{21}')^{(3)}$$

*In the same way, one can obtain*

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a_{22}')^{(3)}$$

*If  $G_{21}$  or  $G_{22}$  is bounded, the same property follows for  $G_{20}$ ,  $G_{22}$  and  $G_{20}$ ,  $G_{21}$  respectively.*

*Remark 4: If  $G_{20}$  is bounded, from below, the same property holds for  $G_{21}$  and  $G_{22}$ .* 1351

*The proof is analogous with the preceding one. An analogous property is true if  $G_{21}$  is bounded from below.*

*Remark 5: If  $T_{20}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(3)}((G_{23})(t), t)) = (b_{21}')^{(3)}$  then  $T_{21} \rightarrow \infty$ .* 1352

*Definition of  $(m)^{(3)}$  and  $\varepsilon_3$  :*

*Indeed let  $t_3$  be so that for  $t > t_3$*

$$(b_{21}')^{(3)} - (b_i'')^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

*Then  $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$  which leads to* 1353

$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$$T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), \quad t \log \frac{2}{\varepsilon_3} \text{ By taking now } \varepsilon_3 \text{ sufficiently small one sees that } T_{21} \text{ is}$$

*unbounded.*

*The same property holds for  $T_{22}$  if  $\lim_{t \rightarrow \infty} (b_{22}'')^{(3)}((G_{23})(t), t) = (b_{22}')^{(3)}$*

*We now state a more precise theorem about the behaviors at infinity of the solutions*

It is now sufficient to take  $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ ,  $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$  and to choose 1354

$(\widehat{P}_{24})^{(4)}$  and  $(\widehat{Q}_{24})^{(4)}$  large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \left[ (\widehat{P}_{24})^{(4)} + ((\widehat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{24})^{(4)} \quad 1355$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \left[ ((\widehat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{24})^{(4)} \right] \leq (\widehat{Q}_{24})^{(4)} \quad 1356$$

In order that the operator  $\mathcal{A}^{(4)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying IN to itself

The operator  $\mathcal{A}^{(4)}$  is a contraction with respect to the metric 1357

$$d\left( ((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{24})^{(4)}t} \right\}$$

Indeed if we denote

$$\text{Definition of } (\widehat{G}_{27}), (\widehat{T}_{27}) : (\widehat{G}_{27}), (\widehat{T}_{27}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} ds_{(24)} + \\ &\int_0^t \{ (a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} + \\ &(a''_{24})^{(4)}(T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} + \\ &G_{24}^{(2)} | (a''_{24})^{(4)}(T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)}(T_{25}^{(2)}, s_{(24)}) | e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} \} ds_{(24)} \end{aligned}$$

Where  $s_{(24)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\widehat{M}_{24})^{(4)}t} &\leq \frac{1}{(\widehat{M}_{24})^{(4)}} ((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)}) \\ &+ (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)} d\left( ((G_{27})^{(1)}, (T_{27})^{(1)}); ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) \end{aligned} \quad 1358$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis the result follows

*Remark 1: The fact that we supposed  $(a_{24}'')^{(4)}$  and  $(b_{24}'')^{(4)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$  and  $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$  respectively of  $\mathbb{R}_+$ .* 1359

*If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a_i'')^{(4)}$  and  $(b_i'')^{(4)}$ ,  $i = 24, 25, 26$  depend only on  $T_{25}$  and respectively on  $(G_{27})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.*

*Remark 2: There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$*  1360

*From GLOBAL EQUATIONS it results*

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i'')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i'')^{(4)}t} > 0 \text{ for } t > 0$$

*Definition of  $((\widehat{M}_{24})^{(4)})_1$ ,  $((\widehat{M}_{24})^{(4)})_2$  and  $((\widehat{M}_{24})^{(4)})_3$  :* 1361

*Remark 3: if  $G_{24}$  is bounded, the same property have also  $G_{25}$  and  $G_{26}$ . indeed if*

$$G_{24} < (\widehat{M}_{24})^{(4)} \text{ it follows } \frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a_{25}'')^{(4)}G_{25} \text{ and by integrating}$$

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25}'')^{(4)}((\widehat{M}_{24})^{(4)})_1 / (a_{25}'')^{(4)}$$

*In the same way, one can obtain*

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26}'')^{(4)}((\widehat{M}_{24})^{(4)})_2 / (a_{26}'')^{(4)}$$

*If  $G_{25}$  or  $G_{26}$  is bounded, the same property follows for  $G_{24}$ ,  $G_{26}$  and  $G_{24}$ ,  $G_{25}$  respectively.*

*Remark 4: If  $G_{24}$  is bounded, from below, the same property holds for  $G_{25}$  and  $G_{26}$ .* 1362

*The proof is analogous with the preceding one. An analogous property is true if  $G_{25}$  is bounded from below.*

*Remark 5: If  $T_{24}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(4)}((G_{27})(t), t)) = (b_{25}'')^{(4)}$  then  $T_{25} \rightarrow \infty$ .* 1363

*Definition of  $(m)^{(4)}$  and  $\varepsilon_4$  :*

*Indeed let  $t_4$  be so that for  $t > t_4$*

$$(b_{25}'')^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then  $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$  which leads to

$$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results}$$

$$T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right), \quad t$$

$= \log \frac{2}{\varepsilon_4}$  By taking now  $\varepsilon_4$  sufficiently small one sees that  $T_{25}$  is unbounded. The same

property holds for  $T_{26}$  if  $\lim_{t \rightarrow \infty} (b_{26}'')^{(4)}((G_{27})(t), t) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for  $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take  $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$  and to choose 1365

$(\widehat{P}_{28})^{(5)}$  and  $(\widehat{Q}_{28})^{(5)}$  large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} \left[ (\widehat{P}_{28})^{(5)} + ((\widehat{P}_{28})^{(5)} + G_j^0) e^{-\left( \frac{(\widehat{P}_{28})^{(5)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{28})^{(5)} \quad 1366$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} \left[ ((\widehat{Q}_{28})^{(5)} + T_j^0) e^{-\left( \frac{(\widehat{Q}_{28})^{(5)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{28})^{(5)} \right] \leq (\widehat{Q}_{28})^{(5)} \quad 1367$$

In order that the operator  $\mathcal{A}^{(5)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself

The operator  $\mathcal{A}^{(5)}$  is a contraction with respect to the metric 1368

$$d \left( ((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{28})^{(5)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{28})^{(5)} t} \right\}$$

Indeed if we denote 1369

$$\text{Definition of } (\widehat{G}_{31}), (\widehat{T}_{31}) : \quad ((\widehat{G}_{31}), (\widehat{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$$

It results

$$|\tilde{G}_{28}^{(1)} - \tilde{G}_{28}^{(2)}| \leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} ds_{(28)} + \quad 1370$$

$$\int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}(T_{29}^{(1)}, s_{(28)}) | G_{28}^{(1)} - G_{28}^{(2)} | e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} | (a''_{28})^{(5)}(T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)}(T_{29}^{(2)}, s_{(28)}) | e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where  $s_{(28)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$|(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\widehat{M}_{28})^{(5)} t} \tag{1371}$$

$$\leq \frac{1}{(\widehat{M}_{28})^{(5)}} ((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)})$$

$$+ (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)} d \left( ((G_{31})^{(1)}, (T_{31})^{(1)}); ((G_{31})^{(2)}, (T_{31})^{(2)}) \right)$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed  $(a''_{28})^{(5)}$  and  $(b''_{28})^{(5)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$  and  $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$  respectively of  $\mathbb{R}_+$ . 1372

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(5)}$  and  $(b''_i)^{(5)}$ ,  $i = 28, 29, 30$  depend only on  $T_{29}$  and respectively on  $(G_{31})$  (and not on  $t$ ) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$  1373

From GLOBAL EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{[-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(5)} t} > 0 \text{ for } t > 0$$

Definition of  $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$  and  $((\widehat{M}_{28})^{(5)})_3$  : 1374

Remark 3: if  $G_{28}$  is bounded, the same property have also  $G_{29}$  and  $G_{30}$ . indeed if

$$G_{28} < (\widehat{M}_{28})^{(5)} \text{ it follows } \frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a'_{29})^{(5)} G_{29} \text{ and by integrating}$$

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)} ((\widehat{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)} ((\widehat{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If  $G_{29}$  or  $G_{30}$  is bounded, the same property follows for  $G_{28}$ ,  $G_{30}$  and  $G_{28}$ ,  $G_{29}$  respectively.

*Remark 4: If  $G_{28}$  is bounded, from below, the same property holds for  $G_{29}$  and  $G_{30}$ .* 1375

*The proof is analogous with the preceding one. An analogous property is true if  $G_{29}$  is bounded from below.*

*Remark 5: If  $T_{28}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$  then  $T_{29} \rightarrow \infty$ .* 1376

*Definition of  $(m)^{(5)}$  and  $\varepsilon_5$  :*

*Indeed let  $t_5$  be so that for  $t > t_5$*

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

*Then  $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$  which leads to* 1377

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$$T_{29} \geq \left( \frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), t$$

$$= \log \frac{2}{\varepsilon_5} \text{ By taking now } \varepsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is unbounded. The}$$

*same property holds for  $T_{30}$  if  $\lim_{t \rightarrow \infty} (b_{30}'')^{(5)}((G_{31})(t), t) = (b_{30}')^{(5)}$*

*We now state a more precise theorem about the behaviors at infinity of the solutions*

*Analogous inequalities hold also for  $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$*

*It is now sufficient to take  $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$  and to choose* 1378

*$(\hat{P}_{32})^{(6)}$  and  $(\hat{Q}_{32})^{(6)}$  large to have*

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ (\hat{P}_{32})^{(6)} + ((\hat{P}_{32})^{(6)} + G_j^0) e^{-\left( \frac{(\hat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{32})^{(6)} \quad 1379$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ ((\hat{Q}_{32})^{(6)} + T_j^0) e^{-\left( \frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{32})^{(6)} \right] \leq (\hat{Q}_{32})^{(6)} \quad 1380$$

*In order that the operator  $\mathcal{A}^{(6)}$  transforms the space of sextuples of functions  $G_i, T_i$  into itself*

*The operator  $\mathcal{A}^{(6)}$  is a contraction with respect to the metric* 1381

$$d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \}$$

Indeed if we denote

1382

Definition of  $(\bar{G}_{35}), (\bar{T}_{35})$  :  $(\bar{G}_{35}), (\bar{T}_{35}) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$\begin{aligned} |\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} + \\ &\int_0^t \{ (a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &G_{32}^{(2)} | (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)} \end{aligned}$$

Where  $s_{(32)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows

$$(1) \quad (a'_i)^{(1)}, (a''_i)^{(1)}, (b'_i)^{(1)}, (b_i)^{(1)}, (b''_i)^{(1)} > 0,$$

1383

$$i, j = 13, 14, 15$$

(2) The functions  $(a''_i)^{(1)}, (b''_i)^{(1)}$  are positive continuous increasing and bounded.

**Definition of**  $(p_i)^{(1)}, (r_i)^{(1)}$ :

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$(3) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$$

1384

$$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$$

**Definition of**  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$  :

Where  $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$  are positive constants and  $i = 13, 14, 15$

They satisfy Lipschitz condition:

1385

$$|(a''_i)^{(1)}(T_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\bar{M}_{13})^{(1)}t}$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\bar{M}_{13})^{(1)}t}$$



With the Lipschitz condition, we place a restriction on the behavior of functions  $(a_i'')^{(1)}(T_{14}, t)$  and  $(a_i'')^{(1)}(T_{14}, t) \cdot (T_{14}, t)$  and  $(T_{14}, t)$  are points belonging to the interval  $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that  $(a_i'')^{(1)}(T_{14}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{13})^{(1)} = 1$  then the function  $(a_i'')^{(1)}(T_{14}, t)$ , the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

**Definition of**  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$  : 1386

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ , are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of**  $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$  : 1387

There exists two constants  $(\hat{P}_{13})^{(1)}$  and  $(\hat{Q}_{13})^{(1)}$  which together with  $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$  and  $(\hat{B}_{13})^{(1)}$  and the constants  $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Analogous inequalities hold also for  $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$  1388

It is now sufficient to take  $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$  and to choose

$(\hat{P}_{36})^{(7)}$  and  $(\hat{Q}_{36})^{(7)}$  large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ (\hat{P}_{36})^{(7)} + ((\hat{P}_{36})^{(7)} + G_j^0) e^{-\left(\frac{(\hat{P}_{36})^{(7)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{36})^{(7)} \quad 1389$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ ((\hat{Q}_{36})^{(7)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{36})^{(7)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)} \right] \leq (\hat{Q}_{36})^{(7)} \quad 1390$$

In order that the operator  $\mathcal{A}^{(7)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying into itself

Analogous inequalities hold also for  $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$  1391

It is now sufficient to take  $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$  and to choose

$(\hat{P}_{40})^{(8)}$  and  $(\hat{Q}_{40})^{(8)}$  large to have

$$\frac{(a_i)^{(8)}}{(\bar{M}_{40})^{(8)}} \left[ (\hat{P}_{40})^{(8)} + ((\hat{P}_{40})^{(8)} + G_j^0) e^{-\left(\frac{(\hat{P}_{40})^{(8)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{40})^{(8)} \quad 1392$$

$$\frac{(b_i)^{(8)}}{(\bar{M}_{40})^{(8)}} \left[ ((\hat{Q}_{40})^{(8)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{40})^{(8)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)} \right] \leq (\hat{Q}_{40})^{(8)} \quad 1393$$

In order that the operator  $\mathcal{A}^{(8)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying GLOBAL EQUATIONS into itself

The operator  $\mathcal{A}^{(8)}$  is a contraction with respect to the metric 1394

$$d\left( ((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)}) \right) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{40})^{(8)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{40})^{(8)}t} \}$$

Indeed if we denote

**Definition of**  $(\bar{G}_{43}, \bar{T}_{43}) : (\bar{G}_{43}, \bar{T}_{43}) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results

$$\begin{aligned} |\tilde{G}_{40}^{(1)} - \tilde{G}_{40}^{(2)}| &\leq \int_0^t (a_{40})^{(8)} |G_{41}^{(1)} - G_{41}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} ds_{(40)} + \\ &\int_0^t \{ (a'_{40})^{(8)} |G_{40}^{(1)} - G_{40}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{-(\bar{M}_{40})^{(8)}s_{(40)}} + \\ &(a''_{40})^{(8)} (T_{41}^{(1)}, s_{(40)}) |G_{40}^{(1)} - G_{40}^{(2)}| e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} + \\ &G_{40}^{(2)} | (a''_{40})^{(8)} (T_{41}^{(1)}, s_{(40)}) - (a''_{40})^{(8)} (T_{41}^{(2)}, s_{(40)}) | e^{-(\bar{M}_{40})^{(8)}s_{(40)}} e^{(\bar{M}_{40})^{(8)}s_{(40)}} \} ds_{(40)} \end{aligned}$$

Where  $s_{(40)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses IT follows

$$\begin{aligned} |(G_{43})^{(1)} - (G_{43})^{(2)}| e^{-(\bar{M}_{40})^{(8)}t} &\leq \quad 1395 \\ \frac{1}{(\bar{M}_{40})^{(8)}} &((a_{40})^{(8)} + (a'_{40})^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}) d\left( ((G_{43})^{(1)}, (T_{43})^{(1)}); ((G_{43})^{(2)}, (T_{43})^{(2)}) \right) \end{aligned}$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (38,35,36) the result follows

**Remark 1:** The fact that we supposed  $(a''_{40})^{(8)}$  and  $(b''_{40})^{(8)}$  depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)}t}$  and  $(\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)}t}$  respectively of  $\mathbb{R}_+$ . 1396

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a''_i)^{(8)}$  and  $(b''_i)^{(8)}, i = 40, 41, 42$  depend only on  $T_{41}$  and respectively on  $(G_{43})$  (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

1397

From global equations it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(8)}t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{40})^{(8)})_1, ((\widehat{M}_{40})^{(8)})_2$  and  $((\widehat{M}_{40})^{(8)})_3$  :

1398

**Remark 3:** if  $G_{40}$  is bounded, the same property have also  $G_{41}$  and  $G_{42}$ . indeed if

$G_{40} < ((\widehat{M}_{40})^{(8)})$  it follows  $\frac{dG_{41}}{dt} \leq ((\widehat{M}_{40})^{(8)})_1 - (a'_{41})^{(8)}G_{41}$  and by integrating

$$G_{41} \leq ((\widehat{M}_{40})^{(8)})_2 = G_{41}^0 + 2(a_{41})^{(8)}((\widehat{M}_{40})^{(8)})_1 / (a'_{41})^{(8)}$$

In the same way, one can obtain

$$G_{42} \leq ((\widehat{M}_{40})^{(8)})_3 = G_{42}^0 + 2(a_{42})^{(8)}((\widehat{M}_{40})^{(8)})_2 / (a'_{42})^{(8)}$$

If  $G_{41}$  or  $G_{42}$  is bounded, the same property follows for  $G_{40}$ ,  $G_{42}$  and  $G_{40}$ ,  $G_{41}$  respectively.

**Remark 1:** If  $G_{40}$  is bounded, from below, the same property holds for  $G_{41}$  and  $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{41}$  is bounded from below.

1399

**Remark 5:** If  $T_{40}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(8)}((G_{43})(t), t)) = (b'_{41})^{(8)}$  then  $T_{41} \rightarrow \infty$ .

1400

**Definition of**  $(m)^{(8)}$  and  $\varepsilon_8$  :

Indeed let  $t_8$  be so that for  $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}((G_{43})(t), t) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then  $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$  which leads to

1401

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right) (1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$  If we take  $t$  such that  $e^{-\varepsilon_8 t} = \frac{1}{2}$  it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$ ,  $t = \log \frac{2}{\varepsilon_8}$  By taking now  $\varepsilon_8$  sufficiently small one sees that  $T_{41}$  is unbounded. The same property holds for  $T_{42}$  if  $\lim_{t \rightarrow \infty} (b_i'')^{(8)}((G_{43})(t), t(t), t) = (b'_{42})^{(8)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 82

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

1402

From CONCATENATED GLOBAL EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i^{(7)})t} > 0 \text{ for } t > 0$$

**Definition of**  $((\widehat{M}_{36})^{(7)})_1, ((\widehat{M}_{36})^{(7)})_2$  and  $((\widehat{M}_{36})^{(7)})_3$  : 1403

**Remark 3:** if  $G_{36}$  is bounded, the same property have also  $G_{37}$  and  $G_{38}$  . indeed if

$$G_{36} < ((\widehat{M}_{36})^{(7)}) \text{ it follows } \frac{dG_{37}}{dt} \leq ((\widehat{M}_{36})^{(7)})_1 - (a'_{37})^{(7)}G_{37} \text{ and by integrating}$$

$$G_{37} \leq ((\widehat{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)}((\widehat{M}_{36})^{(7)})_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq ((\widehat{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)}((\widehat{M}_{36})^{(7)})_2 / (a'_{38})^{(7)}$$

If  $G_{37}$  or  $G_{38}$  is bounded, the same property follows for  $G_{36}$  ,  $G_{38}$  and  $G_{36}$  ,  $G_{37}$  respectively.

**Remark 7:** If  $G_{36}$  is bounded, from below, the same property holds for  $G_{37}$  and  $G_{38}$  . The proof is analogous with the preceding one. An analogous property is true if  $G_{37}$  is bounded from below. 1404

**Remark 5:** If  $T_{36}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(7)}((G_{39})(t), t)) = (b'_{37})^{(7)}$  then  $T_{37} \rightarrow \infty$ . 1405

**Definition of**  $(m)^{(7)}$  and  $\varepsilon_7$  :

Indeed let  $t_7$  be so that for  $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then  $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$  which leads to 1406

$$T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7} \right) (1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_7 t} = \frac{1}{2} \text{ it results}$$

$T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_7}$  By taking now  $\varepsilon_7$  sufficiently small one sees that  $T_{37}$  is unbounded. The same property holds for  $T_{38}$  if  $\lim_{t \rightarrow \infty} (b_{38}'')^{(7)}((G_{39})(t), t) = (b'_{38})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)} \quad 1407$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)} \quad 1408$$

**Definition of**  $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$  : 1409

By  $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$  and respectively  $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$  the roots 1410

$$(nn) \text{ of the equations } (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0 \quad 1411$$

$$\text{and } (b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0 \text{ and} \quad 1412$$

**Definition of**  $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$  : 1413

By  $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$  and respectively  $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$  the 1414

$$\text{roots of the equations } (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0 \quad 1415$$

$$\text{and } (b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0 \quad 1416$$

**Definition of**  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  :- 1417

(oo) If we define  $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$  by 1418

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)} \quad 1419$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}, \quad 1420$$

$$\text{and } \boxed{(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)} \quad 1421$$

and analogously 1422

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

$$\text{and } \boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} \quad 1423$$

Then the solution satisfies the inequalities 1424

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$  is defined 1425

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 1426$$

$$\left( \frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a'_{18})^{(2)})} \left[ e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t} \quad 1427$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 1428$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 1429$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[ e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq \quad 1430$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[ e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

**Definition of**  $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$  :- 1431

$$\text{Where } (S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)} \quad 1432$$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 1433$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

### **BEHAVIUOR OF THE SOLUTIONS**

If we denote and define 1434

**Definition of**  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  :

(pp)  $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$  four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

**Definition of**  $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$  : 1435

(qq) By  $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$  and respectively  $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$  the roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By  $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$  and respectively  $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$  the

roots of the equations  $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

**Definition of**  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  :- 1436

(rr) If we define  $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$  by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

$$\text{and } \boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously 1437

$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } \boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$  is defined

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 1438$$

$$\left( \frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[ e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq \right. \\ \left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a_{22})^{(3)})} \left[ e^{(S_1)^{(3)}t} - e^{-(a_{22}')^{(3)}t} \right] + G_{22}^0 e^{-(a_{22}')^{(3)}t} \right) \quad 1439$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 1440$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 1441$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b_{22}')^{(3)})} \left[ e^{(R_1)^{(3)}t} - e^{-(b_{22}')^{(3)}t} \right] + T_{22}^0 e^{-(b_{22}')^{(3)}t} \leq T_{22}(t) \leq \quad 1442$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[ e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of**  $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$ :- 1443

$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b_{20}')^{(3)}$$

$$(R_2)^{(3)} = (b_{22}')^{(3)} - (r_{22})^{(3)}$$

If we denote and define 1444

**Definition of**  $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$  four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a_{24}')^{(4)} + (a_{25}')^{(4)} - (a_{24}'')^{(4)}(T_{25}, t) + (a_{25}'')^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b_{24}')^{(4)} + (b_{25}')^{(4)} - (b_{24}'')^{(4)}((G_{27}), t) - (b_{25}'')^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$$

**Definition of**  $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$  : 1445

By  $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$  and respectively  $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$  the roots of the equations

$$(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0 \text{ and}$$

**Definition of**  $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$  : 1446

By  $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$  and respectively  $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$  the

$$\text{roots of the equations } (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$$

**Definition of**  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$  :-

(ss) If we define  $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$  by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},$$

and  $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously

1447

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

and  $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}$$

are defined respectively

Then the solution satisfies the inequalities

1448

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where  $(p_i)^{(4)}$  is defined

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 1449$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq \quad 1450$$

$$\frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[ e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}} \quad 1451$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 1452$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} ((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq \quad 1453$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of**  $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$ :-

1454

$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b'_{24})^{(4)}$$



$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions**

1455

If we denote and define

**Definition of**  $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$  four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

**Definition of**  $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$  :

1456

(tt) By  $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$  and respectively  $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$  the roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$$

**Definition of**  $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$  :

1457

By  $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$  and respectively  $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$  the

roots of the equations  $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

$$\text{and } (b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$$

**Definition of**  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$  :-

(uu) If we define  $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$  by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously

1458

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

$$\text{and } (u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)} \text{ where } (u_1)^{(5)}, (\bar{u}_1)^{(5)}$$

are defined respectively

Then the solution satisfies the inequalities 1459

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where  $(p_i)^{(5)}$  is defined

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 1460$$

$$\left( \frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[ e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \right) \leq G_{30}(t) \leq \quad 1461$$

$$\frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((S_1)^{(5)} - (a'_{30})^{(5)})} \left[ e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 1462$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 1463$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[ e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq \quad 1464$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[ e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

**Definition of**  $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$ :- 1465

$$\text{Where } (S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}$$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

**Behavior of the solutions** 1466

If we denote and define

**Definition of**  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  :

(vv)  $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$  four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

**Definition of**  $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$  : 1457

(ww) By  $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$  and respectively  $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$  the roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and  $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$  :

1458

By  $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$  and respectively  $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$  the roots of the equations  $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$  and  $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of**  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$  :-

(xx) If we define  $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$  by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

and 
$$(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

1459

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and 
$$(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

are defined respectively

Then the solution satisfies the inequalities

1460

$$G_{32}^0 e^{((s_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(s_1)^{(6)}t}$$

where  $(p_i)^{(6)}$  is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((s_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(s_1)^{(6)}t} \quad 1461$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((s_1)^{(6)} - (p_{32})^{(6)} - (s_2)^{(6)})} \left[ e^{((s_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(s_2)^{(6)}t} \right] + G_{34}^0 e^{-(s_2)^{(6)}t} \right) \leq G_{34}(t) \leq \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)} ((s_1)^{(6)} - (a'_{34})^{(6)})} \left[ e^{(s_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t} \quad 1462$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (\tau_{32})^{(6)})t}} \quad 1463$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 1464$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq \quad 1465$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})} \left[ e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of**  $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$ :- 1466

$$\text{Where } (S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

If we denote and define 1467

**Definition of**  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  :

(yy)  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  four constants satisfying  
 $-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$

$$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \leq -(\tau_1)^{(7)}$$

**Definition of**  $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$  : 1468

(zz) By  $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$  and respectively  $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$  the roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$  and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$  : 1469

By  $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$  and respectively  $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$  the

roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

$$\text{and } (b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$$

**Definition of**  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$  :-

(aaa) If we define  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$  by  
 $(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$ , **if**  $(v_0)^{(7)} < (v_1)^{(7)}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \text{ **if** } (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

$$\text{and } (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ **if** } (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 1470

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{ if } (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

$$\text{and } \boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \text{ if } (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

are defined respectively

Then the solution satisfies the inequalities

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$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where  $(p_i)^{(7)}$  is defined

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 1472$$

$$\left( \frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)} ((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})} \left[ e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \right) \leq G_{38}(t) \leq \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)} ((S_1)^{(7)} - (a'_{38})^{(7)})} \left[ e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t} \quad 1473$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 1474$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 1475$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)} ((R_1)^{(7)} - (b'_{38})^{(7)})} \left[ e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq \quad 1476$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)} ((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[ e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

**Definition of**  $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$ :- 1477

$$\text{Where } (S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions** 1478

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$  :

$$\text{(bbb) } (\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)} \text{ four constants satisfying}$$

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \leq -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}(G_{43}, t) - (b''_{41})^{(8)}(G_{43}, t) \leq -(\tau_1)^{(8)}$$

**Definition of**  $(v_1)^{(8)}, (v_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, v^{(8)}, u^{(8)}$  : 1479

(ccc) By  $(v_1)^{(8)} > 0, (v_2)^{(8)} < 0$  and respectively  $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$  the roots of the equations  $(a_{41})^{(8)}(v^{(8)})^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$  and  $(b_{41})^{(8)}(u^{(8)})^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(8)}, (\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$  : 1480

By  $(\bar{v}_1)^{(8)} > 0, (\bar{v}_2)^{(8)} < 0$  and respectively  $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$  the

roots of the equations  $(a_{41})^{(8)}(v^{(8)})^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and  $(b_{41})^{(8)}(u^{(8)})^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of**  $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$  :-

(ddd) If we define  $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$  by  $(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$ , **if**  $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$ , **if**  $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$ ,

and 
$$(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}$$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$ , **if**  $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 1481

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$ , **if**  $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , **if**  $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$ ,

and 
$$(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$ , **if**  $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$  where  $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 1482

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{40}(t) \leq G_{40}^0 e^{(S_1)^{(8)}t}$$

where  $(p_i)^{(8)}$  is defined

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{41}(t) \leq \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 1483$$

$$\left( \frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)} ((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})} \right) \left[ e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t} \right] + G_{42}^0 e^{-(S_2)^{(8)}t} \leq G_{42}(t) \leq \quad 1484$$

$$\frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)} ((S_1)^{(8)} - (a'_{42})^{(8)})} \left[ e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t} \right] + G_{42}^0 e^{-(a'_{42})^{(8)}t}$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \leq T_{40}(t) \leq T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 1485}$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \leq T_{40}(t) \leq \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 1486$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})} \left[ e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t} \right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \leq T_{42}(t) \leq \quad 1487$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})} \left[ e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t} \right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of**  $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$ :- 1488

$$\text{Where } (S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

From GLOBAL EQUATIONS we obtain 1489

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left( (a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t) \right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

**Definition of**  $v^{(8)}$  :-  $v^{(8)} = \frac{G_{40}}{G_{41}}$

It follows

$$- \left( (a_{41})^{(8)}(v^{(8)})^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} \right) \leq \frac{dv^{(8)}}{dt} \leq - \left( (a_{41})^{(8)}(v^{(8)})^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(8)}, (v_0)^{(8)}$  :-

(g) For  $0 < \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (v_1)^{(8)} < (\bar{v}_1)^{(8)}$

$$v^{(8)}(t) \geq \frac{(v_1)^{(8)}+(C)^{(8)}(v_2)^{(8)}e^{[-(a_{41})^{(8)}((v_1)^{(8)}-(v_0)^{(8)})t]}}{1+(C)^{(8)}e^{[-(a_{41})^{(8)}((v_1)^{(8)}-(v_0)^{(8)})t]}} , \quad \boxed{(C)^{(8)} = \frac{(v_1)^{(8)}-(v_0)^{(8)}}{(v_0)^{(8)}-(v_2)^{(8)}}$$

it follows  $(v_0)^{(8)} \leq v^{(8)}(t) \leq (v_1)^{(8)}$

In the same manner , we get 1490

$$v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{[-(a_{41})^{(8)}((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)})t]}}{1+(\bar{C})^{(8)}e^{[-(a_{41})^{(8)}((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)})t]}} , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{v}_1)^{(8)}-(v_0)^{(8)}}{(v_0)^{(8)}-(\bar{v}_2)^{(8)}}$$

From which we deduce  $(v_0)^{(8)} \leq v^{(8)}(t) \leq (\bar{v}_8)^{(8)}$

(h) If  $0 < (v_1)^{(8)} < (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{v}_1)^{(8)}$  we find like in the previous case, 1491

$$(v_1)^{(8)} \leq \frac{(v_1)^{(8)} + (\bar{c})^{(8)} (v_2)^{(8)} e^{[-(a_{41})^{(8)}((v_1)^{(8)} - (v_2)^{(8)})t]}}{1 + (\bar{c})^{(8)} e^{[-(a_{41})^{(8)}((v_1)^{(8)} - (v_2)^{(8)})t]}} \leq v^{(8)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(8)} + (\bar{c})^{(8)} (\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}((\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)})t]}}{1 + (\bar{c})^{(8)} e^{[-(a_{41})^{(8)}((\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)})t]}} \leq (\bar{v}_1)^{(8)}$$

(i) If  $0 < (v_1)^{(8)} \leq (\bar{v}_1)^{(8)} \leq \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain

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$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)} + (\bar{c})^{(8)} (\bar{v}_2)^{(8)} e^{[-(a_{41})^{(8)}((\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)})t]}}{1 + (\bar{c})^{(8)} e^{[-(a_{41})^{(8)}((\bar{v}_1)^{(8)} - (\bar{v}_2)^{(8)})t]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(8)}(t)$  :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(8)}(t)$  :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{40})^{(8)} = (a''_{41})^{(8)}$ , then  $(\sigma_1)^{(8)} = (\sigma_2)^{(8)}$  and in this case  $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$  if in addition  $(v_0)^{(8)} = (v_1)^{(8)}$  then  $v^{(8)}(t) = (v_0)^{(8)}$  and as a consequence  $G_{40}(t) = (v_0)^{(8)} G_{41}(t)$  **this also defines  $(v_0)^{(8)}$  for the special case.**

Analogously if  $(b''_{40})^{(8)} = (b''_{41})^{(8)}$ , then  $(\tau_1)^{(8)} = (\tau_2)^{(8)}$  and then

$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$  if in addition  $(u_0)^{(8)} = (u_1)^{(8)}$  then  $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$  This is an important consequence of the relation between  $(v_1)^{(8)}$  and  $(\bar{v}_1)^{(8)}$ , **and definition of  $(u_0)^{(8)}$ .**

: From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left( (a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of**  $v^{(4)}$  :-  $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left( (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} \right) \leq \frac{dv^{(4)}}{dt} \leq - \left( (a_{25})^{(4)}(v^{(4)})^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains



**Definition of**  $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$  :-

(j) For  $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}$$

it follows  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get

1494

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce  $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(k) If  $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$  we find like in the previous case,

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$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_2)^{(4)})t]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (\bar{v}_1)^{(4)}$$

(l) If  $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$  , we obtain

1496

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)})t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(4)}(t)$  :-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)} , \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(4)}(t)$  :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)} , \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a_{24}''^{(4)}) = (a_{25}''^{(4)})$ , then  $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$  and in this case  $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$  if in addition  $(v_0)^{(4)} = (\bar{v}_1)^{(4)}$  then  $v^{(4)}(t) = (v_0)^{(4)}$  and as a consequence  $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$  **this also**

defines  $(v_0)^{(4)}$  for the special case .

Analogously if  $(b''_{24})^{(4)} = (b''_{25})^{(4)}$ , then  $(\tau_1)^{(4)} = (\tau_2)^{(4)}$  and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$  if in addition  $(u_0)^{(4)} = (u_1)^{(4)}$  then  $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$  This is an important consequence of the relation between  $(v_1)^{(4)}$  and  $(\bar{v}_1)^{(4)}$ , **and definition of  $(u_0)^{(4)}$ .**

From GLOBAL EQUATIONS we obtain

1497

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left( (a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of  $v^{(5)}$  :-** 
$$v^{(5)} = \frac{G_{28}}{G_{29}}$$

It follows

$$- \left( (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left( (a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

**Definition of  $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$  :-**

(m) For  $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)}]t}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_0)^{(5)}]t}}, \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows  $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get

1498

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}, \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce  $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(n) If  $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$  we find like in the previous case,

1499

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}(v_1)^{(5)} - (v_2)^{(5)}]t}} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (\bar{v}_1)^{(5)}$$

(o) If  $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain

1500

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}(\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}]t}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(5)}(t)$  :-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}} \text{ In a completely analogous way, we obtain}$$

**Definition of**  $u^{(5)}(t)$  :-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{28})^{(5)} = (a''_{29})^{(5)}$ , then  $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$  and in this case  $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$  if in addition  $(v_0)^{(5)} = (v_5)^{(5)}$  then  $v^{(5)}(t) = (v_0)^{(5)}$  and as a consequence  $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$  **this also defines  $(v_0)^{(5)}$  for the special case .**

Analogously if  $(b''_{28})^{(5)} = (b''_{29})^{(5)}$ , then  $(\tau_1)^{(5)} = (\tau_2)^{(5)}$  and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$  if in addition  $(u_0)^{(5)} = (u_1)^{(5)}$  then  $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$  This is an important consequence of the relation between  $(v_1)^{(5)}$  and  $(\bar{v}_1)^{(5)}$ , **and definition of  $(u_0)^{(5)}$ .**

we obtain

1501

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left( (a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of**  $v^{(6)}$  :-  $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$- \left( (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left( (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$  :-

(p) For  $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}, \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner, we get

1502

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}, \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce  $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(q) If  $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$  we find like in the previous case, 1503

$$(r) \quad (v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (C)^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (\bar{v}_1)^{(6)}$$

(s) If  $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 1504

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (C)^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c), we have

**Definition of**  $v^{(6)}(t)$  :-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(6)}(t)$  :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If  $(a''_{32})^{(6)} = (a''_{33})^{(6)}$ , then  $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$  and in this case  $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$  if in addition  $(v_0)^{(6)} = (v_1)^{(6)}$  then  $v^{(6)}(t) = (v_0)^{(6)}$  and as a consequence  $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$  **this also defines  $(v_0)^{(6)}$  for the special case .**

Analogously if  $(b''_{32})^{(6)} = (b''_{33})^{(6)}$ , then  $(\tau_1)^{(6)} = (\tau_2)^{(6)}$  and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$  if in addition  $(u_0)^{(6)} = (u_1)^{(6)}$  then  $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$  This is an important consequence of the relation between  $(v_1)^{(6)}$  and  $(\bar{v}_1)^{(6)}$ , **and definition of  $(u_0)^{(6)}$ .**

**Behavior of the solutions**

1505

If we denote and define

**Definition of**  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  :

(eee)  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  four constants satisfying  
 $-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$

$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \leq -(\tau_1)^{(7)}$

**Definition of**  $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$  : 1506

(fff) By  $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$  and respectively  $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$  the roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$  and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$  and

**Definition of**  $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$  : 1507

By  $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$  and respectively  $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$  the

roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of**  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$  :-

(ggg) If we define  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$  by  $(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$ , **if**  $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$ , **if**  $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$ ,

and 
$$(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}$$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}$ , **if**  $(\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 1508

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$ , **if**  $(u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}$ , **if**  $(u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$ ,

and 
$$(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$ , **if**  $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$  where  $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

are defined respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 1509

$$G_{36}^0 e^{((s_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(s_1)^{(7)}t}$$

where  $(p_i)^{(7)}$  is defined

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((s_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(s_1)^{(7)}t} \quad 1510$$

$$\left( \frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)} ((s_1)^{(7)} - (p_{36})^{(7)} - (s_2)^{(7)})} \left[ e^{((s_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(s_2)^{(7)}t} \right] + G_{38}^0 e^{-(s_2)^{(7)}t} \right) \leq G_{38}(t) \leq \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)} ((s_1)^{(7)} - (a'_{38})^{(7)})} \left[ e^{(s_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t} \quad 1511$$

$$T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 1512$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 1513$$

$$\frac{(b_{38})^{(7)}T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})} \left[ e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq \quad 1514$$

$$\frac{(a_{38})^{(7)}T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})} \left[ e^{((R_1)^{(7)}+(r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

**Definition of**  $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$ :- 1515

Where  $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

From CONCATENATED GLOBAL EQUATIONS we obtain 1516

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left( (a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) \right) - (a''_{37})^{(7)}(T_{37}, t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of**  $v^{(7)}$  :-  $v^{(7)} = \frac{G_{36}}{G_{37}}$

It follows

$$- \left( (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} \right) \leq \frac{dv^{(7)}}{dt} \leq - \left( (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$  :-

(t) For  $0 < \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \geq \frac{(v_1)^{(7)}+(C)^{(7)}(v_2)^{(7)}e^{-(a_{37})^{(7)}((v_1)^{(7)}-(v_0)^{(7)})t}}{1+(C)^{(7)}e^{-(a_{37})^{(7)}((v_1)^{(7)}-(v_0)^{(7)})t}}, \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)}-(v_0)^{(7)}}{(v_0)^{(7)}-(v_2)^{(7)}}$$

it follows  $(v_0)^{(7)} \leq v^{(7)}(t) \leq (v_1)^{(7)}$

In the same manner , we get 1517

$$v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{-(a_{37})^{(7)}((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)})t}}{1+(\bar{C})^{(7)}e^{-(a_{37})^{(7)}((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)})t}}, \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)}-(v_0)^{(7)}}{(v_0)^{(7)}-(\bar{v}_2)^{(7)}}$$

From which we deduce  $(v_0)^{(7)} \leq v^{(7)}(t) \leq (\bar{v}_1)^{(7)}$

(u) If  $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$  we find like in the previous case, 1518

$$v_1^{(7)} \leq \frac{(v_1)^{(7)} + (c)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}((v_1)^{(7)} - (v_2)^{(7)})t]}}{1 + (c)^{(7)} e^{[-(a_{37})^{(7)}((v_1)^{(7)} - (v_2)^{(7)})t]}} \leq v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (c)^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}}{1 + (c)^{(7)} e^{[-(a_{37})^{(7)}((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)})t]}} \leq (\bar{v}_1)^{(7)}$$

**Theorem 3:** If  $(a_i'')^{(8)}$  and  $(b_i'')^{(8)}$  are independent on  $t$ , and the conditions 1519

$$\begin{aligned} (a'_{44})^{(8)}(a'_{45})^{(8)} - (a_{44})^{(8)}(a_{45})^{(8)} &< 0 \\ (a'_{44})^{(8)}(a'_{45})^{(8)} - (a_{44})^{(8)}(a_{45})^{(8)} + (a_{44})^{(8)}(p_{44})^{(8)} + (a'_{45})^{(8)}(p_{45})^{(8)} + (p_{44})^{(8)}(p_{45})^{(8)} &> 0 \\ (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{42})^{(8)}(b_{43})^{(8)} &> 0, \\ (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{42})^{(8)}(b_{43})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(9)}(r_{41})^{(9)} + (r_{43})^{(9)}(r_{41})^{(9)} &< 0 \end{aligned}$$

with  $(p_{40})^{(8)}, (r_{41})^{(8)}$  as defined are satisfied, then the system

$$\begin{aligned} (a_{40})^{(8)}G_{41} - [(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})]G_{40} &= 0 && 1520 \\ (a_{41})^{(8)}G_{40} - [(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})]G_{41} &= 0 && 1521 \\ (a_{42})^{(8)}G_{41} - [(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})]G_{42} &= 0 && 1522 \\ (b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})]T_{40} &= 0 && 1523 \\ (b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})]T_{41} &= 0 && 1524 \\ (b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})]T_{42} &= 0 && 1525 \end{aligned}$$

has a unique positive solution, which is an equilibrium solution for the system 1526

**Proof:**

(a) Indeed the first two equations have a nontrivial solution  $G_{40}, G_{41}$  if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Definition and uniqueness of  $T_{41}^*$**  :- 1527

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(8)}(T_{41})$  being increasing, it follows that there exists a unique  $T_{41}^*$  for which  $f(T_{41}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{44} = \frac{(a_{41})^{(8)}G_{41}}{[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}^*)]} \quad , \quad G_{46} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

(q) By the same argument, the equations(GLOBAL) admit solutions  $G_{40}, G_{41}$  if

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{45})^{(8)} -$$

$$[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})] + (b''_{40})^{(8)}(G_{43})(b'_{41})^{(8)}(G_{43}) = 0$$

Where in  $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$  must be replaced by their values . It is easy to see that  $\varphi$  is a decreasing function in  $G_{45}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{41}^*$  such that  $\varphi((G_{43})^*) = 0$  1528

Finally we obtain the unique solution

$G_{41}^*$  given by  $\varphi((G_{43})^*) = 0, T_{41}^*$  given by  $f(T_{41}^*) = 0$  and

$$G_{40}^* = \frac{(a_{40})^{(8)}G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]}, \quad G_{42}^* = \frac{(a_{42})^{(8)}G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{44}^* = \frac{(b_{40})^{(8)}T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]}, \quad T_{42}^* = \frac{(b_{42})^{(8)}T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]} \quad 1529$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 1530$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 1531$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 1532$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 1533$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 1534$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 1535$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 1536$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 1537$$

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 1538$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 1539$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 1540$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}))]T_{24} = 0 \quad 1541$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}))]T_{25} = 0 \quad 1542$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}))]T_{26} = 0$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 1543$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 1544$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 1545$$



$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 1546$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 1547$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 1548$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 1549$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 1550$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 1551$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 1552$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 1553$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 1554$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0 \quad 1555$$

$$(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0 \quad 1556$$

$$(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0 \quad 1557$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 1558$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0 \quad 1559$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0 \quad 1560$$

has a unique positive solution , which is an equilibrium solution for the system 1561

(a) Indeed the first two equations have a nontrivial solution  $G_{36}, G_{37}$  if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Definition and uniqueness of  $T_{37}^*$**  :- 1562

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(7)}(T_{37})$  are increasing, it follows that there exists a unique  $T_{37}^*$  for which  $f(T_{37}^*) = 0$ . With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

(r) By the same argument, the equations( SOLUTIONAL) admit solutions  $G_{36}, G_{37}$  if

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in  $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$  must be replaced by their values from 96. It is easy to see 1563  
 that  $\varphi$  is a decreasing function in  $G_{37}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{37}^*$  such that  $\varphi(G^*) = 0$

Finally we obtain the unique solution OF THE SYSTEM

$G_{37}^*$  given by  $\varphi((G_{39})^*) = 0, T_{37}^*$  given by  $f(T_{37}^*) = 0$  and

$$G_{36}^* = \frac{(a_{36})^{(7)}G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]}, \quad G_{38}^* = \frac{(a_{38})^{(7)}G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)}T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]}, \quad T_{38}^* = \frac{(b_{38})^{(7)}T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]} \quad 1564$$

**Definition and uniqueness of  $T_{21}^*$**  :- 1565

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(4)}(T_{21})$  being increasing, it follows that there exists a unique  $T_{21}^*$  for which  $f(T_{21}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]}, \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

**Definition and uniqueness of  $T_{25}^*$**  :- 1566

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(4)}(T_{25})$  being increasing, it follows that there exists a unique  $T_{25}^*$  for which  $f(T_{25}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]}, \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

**Definition and uniqueness of  $T_{29}^*$**  :-

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(5)}(T_{29})$  are increasing, it follows that there exists a unique  $T_{29}^*$  for which  $f(T_{29}^*) = 0$ . With this value, we obtain from the three first equations 1567

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]}, \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

**Definition and uniqueness of  $T_{33}^*$**  :- 1568

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(6)}(T_{33})$  are increasing, it follows that there exists a unique  $T_{33}^*$  for which  $f(T_{33}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]}, \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

(s) By the same argument, the equations GLOBAL admit solutions  $G_{13}, G_{14}$  if 1569

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in  $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{14}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{14}^*$  such that  $\varphi(G^*) = 0$

(t) By the same argument, the equations 92,93 admit solutions  $G_{16}, G_{17}$  if 1570

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in  $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{17}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{17}^*$  such that  $\varphi((G_{19})^*) = 0$

(u) By the same argument, the concatenated equations admit solutions  $G_{20}, G_{21}$  if 1571

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in  $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{21}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{21}^*$  such that  $\varphi((G_{23})^*) = 0$

(v) By the same argument, the equations of modules admit solutions  $G_{24}, G_{25}$  if 1572

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in  $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{25}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{25}^*$  such that  $\varphi((G_{27})^*) = 0$

(w) By the same argument, the equations (modules) admit solutions  $G_{28}, G_{29}$  if 1573

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in  $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{29}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{29}^*$  such that  $\varphi((G_{31})^*) = 0$

(x) By the same argument, the equations (modules) admit solutions  $G_{32}, G_{33}$  if 1574

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in  $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$  must be replaced by their values It is easy to see that  $\varphi$  is a decreasing function in  $G_{33}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{33}^*$  such that  $\varphi(G^*) = 0$

Finally we obtain the unique solution

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$G_{14}^*$  given by  $\varphi(G^*) = 0, T_{14}^*$  given by  $f(T_{14}^*) = 0$  and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]}, \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]}, \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^*$  given by  $\varphi((G_{19})^*) = 0, T_{17}^*$  given by  $f(T_{17}^*) = 0$  and

1576

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]}, \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]}$$

1577

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]}, \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]}$$

1578

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

1579

$G_{21}^*$  given by  $\varphi((G_{23})^*) = 0, T_{21}^*$  given by  $f(T_{21}^*) = 0$  and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]}, \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}^*)]}, \quad T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

1580

$G_{25}^*$  given by  $\varphi(G_{27}) = 0, T_{25}^*$  given by  $f(T_{25}^*) = 0$  and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]}, \quad G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]}, \quad T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]}$$

1581

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

1582

$G_{29}^*$  given by  $\varphi((G_{31})^*) = 0$ ,  $T_{29}^*$  given by  $f(T_{29}^*) = 0$  and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \quad 1583$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 1584

$G_{33}^*$  given by  $\varphi((G_{35})^*) = 0$ ,  $T_{33}^*$  given by  $f(T_{33}^*) = 0$  and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \quad 1585$$

Obviously, these values represent an equilibrium solution

#### ASYMPTOTIC STABILITY ANALYSIS 1586

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions  $(a'_i)^{(1)}$  and  $(b'_i)^{(1)}$  belong to  $C^{(1)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 1587$$

$$\frac{d\mathbb{G}_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 1588$$

$$\frac{d\mathbb{G}_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 1589$$

$$\frac{d\mathbb{T}_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(13)(j)})T_{13}^*\mathbb{G}_j \quad 1590$$

$$\frac{d\mathbb{T}_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15} (s_{(14)(j)})T_{14}^*\mathbb{G}_j \quad 1591$$

$$\frac{d\mathbb{T}_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(15)(j)})T_{15}^*\mathbb{G}_j \quad 1592$$

If the conditions of the previous theorem are satisfied and if the functions  $(a'_i)^{(2)}$  and  $(b'_i)^{(2)}$  belong to  $C^{(2)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i \quad 1593$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}(G_{19})^* = s_{ij} \quad 1594$$

taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 1595$$

$$\frac{d\mathbb{G}_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 1596$$

$$\frac{d\mathbb{G}_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 1597$$

$$\frac{d\mathbb{T}_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18} (s_{(16)(j)})\mathbb{T}_{16}^*\mathbb{G}_j \quad 1598$$

$$\frac{d\mathbb{T}_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18} (s_{(17)(j)})\mathbb{T}_{17}^*\mathbb{G}_j \quad 1599$$

$$\frac{d\mathbb{T}_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18} (s_{(18)(j)})\mathbb{T}_{18}^*\mathbb{G}_j \quad 1600$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(3)}$  and  $(b_i'')^{(3)}$  belong to  $C^{(3)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable 1601

Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}(G_{23})^* = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 1602$$

$$\frac{d\mathbb{G}_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 1603$$

$$\frac{d\mathbb{G}_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 1604$$

$$\frac{d\mathbb{T}_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22} (s_{(20)(j)})\mathbb{T}_{20}^*\mathbb{G}_j \quad 1605$$

$$\frac{d\mathbb{T}_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22} (s_{(21)(j)})\mathbb{T}_{21}^*\mathbb{G}_j \quad 1606$$

$$\frac{d\mathbb{T}_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22} (s_{(22)(j)})\mathbb{T}_{22}^*\mathbb{G}_j \quad 1607$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(4)}$  and  $(b_i'')^{(4)}$  belong to  $C^{(4)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

1608

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}} (T_{25}^*) = (q_{25})^{(4)} \quad , \quad \frac{\partial (b_i'')^{(4)}}{\partial G_j} ((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^* \mathbb{T}_{25} \quad 1609$$

$$\frac{d\mathbb{G}_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^* \mathbb{T}_{25} \quad 1610$$

$$\frac{d\mathbb{G}_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^* \mathbb{T}_{25} \quad 1611$$

$$\frac{d\mathbb{T}_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^* \mathbb{G}_j \quad 1612$$

$$\frac{d\mathbb{T}_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^* \mathbb{G}_j \quad 1613$$

$$\frac{d\mathbb{T}_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^* \mathbb{G}_j \quad 1614$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(5)}$  and  $(b_i'')^{(5)}$  belong to  $C^{(5)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :- 1615

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}} (T_{29}^*) = (q_{29})^{(5)} \quad , \quad \frac{\partial (b_i'')^{(5)}}{\partial G_j} ((G_{31})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^* \mathbb{T}_{29} \quad 1616$$

$$\frac{d\mathbb{G}_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^* \mathbb{T}_{29} \quad 1617$$

$$\frac{d\mathbb{G}_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^* \mathbb{T}_{29} \quad 1618$$

$$\frac{d\mathbb{T}_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^* \mathbb{G}_j \quad 1619$$

$$\frac{d\mathbb{T}_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^* \mathbb{G}_j \quad 1620$$

$$\frac{d\mathbb{T}_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^* \mathbb{G}_j \quad 1621$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(6)}$  and  $(b_i'')^{(6)}$  belong to  $C^{(6)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable

Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :- 1622

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{33}''^{(6)})}{\partial T_{33}} (T_{33}^*) = (q_{33})^{(6)} \quad , \quad \frac{\partial (b_i''^{(6)})}{\partial G_j} ((G_{35})^*) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^* \mathbb{T}_{33} \quad 1623$$

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^* \mathbb{T}_{33} \quad 1624$$

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^* \mathbb{T}_{33} \quad 1625$$

$$\frac{d\mathbb{T}_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34} (s_{(32)(j)}) T_{32}^* \mathbb{G}_j \quad 1626$$

$$\frac{d\mathbb{T}_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34} (s_{(33)(j)}) T_{33}^* \mathbb{G}_j \quad 1627$$

$$\frac{d\mathbb{T}_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34} (s_{(34)(j)}) T_{34}^* \mathbb{G}_j \quad 1628$$

Obviously, these values represent an equilibrium solution of 79,20,36,22,23,

If the conditions of the previous theorem are satisfied and if the functions  $(a_i''^{(7)})$  and  $(b_i''^{(7)})$  belong to  $C^{(7)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :- 1629

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}''^{(7)})}{\partial T_{37}} (T_{37}^*) = (q_{37})^{(7)} \quad , \quad \frac{\partial (b_i''^{(7)})}{\partial G_j} ((G_{39})^{**}) = s_{ij}$$

Then taking into account equations(SOLUTIONAL) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^* \mathbb{T}_{37} \quad 1630$$

$$\frac{d\mathbb{G}_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^* \mathbb{T}_{37} \quad 1631$$

$$\frac{d\mathbb{G}_{38}}{dt} = -((a'_{38})^{(7)} + (p_{38})^{(7)})\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^* \mathbb{T}_{37} \quad 1632$$

$$\frac{d\mathbb{T}_{36}}{dt} = -((b'_{36})^{(7)} - (r_{36})^{(7)})\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38} (s_{(36)(j)}) T_{36}^* \mathbb{G}_j \quad 1633$$

$$\frac{d\mathbb{T}_{37}}{dt} = -((b'_{37})^{(7)} - (r_{37})^{(7)})\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38} (s_{(37)(j)}) T_{37}^* \mathbb{G}_j \quad 1634$$

$$\frac{d\mathbb{T}_{38}}{dt} = -((b'_{38})^{(7)} - (r_{38})^{(7)})\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38} (s_{(38)(j)}) T_{38}^* \mathbb{G}_j \quad 1635$$

If the conditions of the previous theorem are satisfied and if the functions  $(a_i''^{(8)})$  and  $(b_i''^{(8)})$  belong to  $C^{(8)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.



Denote

**Definition of**  $\mathbb{G}_i, \mathbb{T}_i$  :-

1636

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}')^{(8)}}{\partial T_{41}} (T_{41}^*) = (q_{41})^{(8)} \quad , \quad \frac{\partial (b_{41}')^{(8)}}{\partial G_j} ( (G_{43})^* ) = s_{ij}$$

Then taking into account equations CONCATENATED EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -((a'_{40})^{(8)} + (p_{40})^{(8)})\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^* \mathbb{T}_{41} \quad 1637$$

$$\frac{d\mathbb{G}_{41}}{dt} = -((a'_{41})^{(8)} + (p_{41})^{(8)})\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^* \mathbb{T}_{41} \quad 1638$$

$$\frac{d\mathbb{G}_{42}}{dt} = -((a'_{42})^{(8)} + (p_{42})^{(8)})\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^* \mathbb{T}_{41} \quad 1639$$

$$\frac{d\mathbb{T}_{40}}{dt} = -((b'_{40})^{(8)} - (r_{40})^{(8)})\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42} (s_{(40)(j)})T_{40}^* \mathbb{G}_j \quad 1640$$

$$\frac{d\mathbb{T}_{41}}{dt} = -((b'_{41})^{(8)} - (r_{41})^{(8)})\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42} (s_{(41)(j)})T_{41}^* \mathbb{G}_j \quad 1641$$

$$\frac{d\mathbb{T}_{42}}{dt} = -((b'_{42})^{(8)} - (r_{42})^{(8)})\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42} (s_{(42)(j)})T_{42}^* \mathbb{G}_j \quad 1642$$

The characteristic equation of this system is

1643

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}) \{ (\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)} \\ & \left[ ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (q_{13})^{(1)} G_{13}^* \right] \\ & \left( (\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^* \\ & + \left( (\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)} \right) (q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)} (q_{14})^{(1)} G_{14}^* \\ & \left( (\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^* \\ & \left( (\lambda)^{(1)} \right)^2 + \left( (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda)^{(1)} \\ & \left( (\lambda)^{(1)} \right)^2 + \left( (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)} \right) (\lambda)^{(1)} \\ & + \left( (\lambda)^{(1)} \right)^2 + \left( (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda)^{(1)} (q_{15})^{(1)} G_{15} \\ & + \left( (\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)} \right) \left( (a_{15})^{(1)} (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (a_{15})^{(1)} (q_{13})^{(1)} G_{13}^* \right) \\ & \left( (\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^* \} = 0 \\ & + \\ & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}) \{ (\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)} \\ & \left[ ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (q_{16})^{(2)} G_{16}^* \right] \end{aligned}$$

$$\begin{aligned}
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\
 & + \left( ((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \\
 & \left( ((\lambda)^{(2)})^2 + ( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} ) (\lambda)^{(2)} \right) \\
 & \left( ((\lambda)^{(2)})^2 + ( (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} ) (\lambda)^{(2)} \right) \\
 & + \left( ((\lambda)^{(2)})^2 + ( (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} ) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\
 & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) \left( (a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^* \right) \\
 & \left( ((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}T_{17}^* + (b_{17})^{(2)}s_{(16),(18)}T_{16}^* \right) \} = 0 \\
 & + \\
 & ((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)}) \{ ((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}) \\
 & \left[ \left( ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^* \right) \right] \\
 & \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(21)}T_{21}^* + (b_{21})^{(3)}s_{(20),(21)}T_{21}^* \right) \\
 & + \left( ((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G_{20}^* + (a_{20})^{(3)}(q_{21})^{(1)}G_{21}^* \right) \\
 & \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(20)}T_{21}^* + (b_{21})^{(3)}s_{(20),(20)}T_{20}^* \right) \\
 & \left( ((\lambda)^{(3)})^2 + ( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} ) (\lambda)^{(3)} \right) \\
 & \left( ((\lambda)^{(3)})^2 + ( (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} ) (\lambda)^{(3)} \right) \\
 & + \left( ((\lambda)^{(3)})^2 + ( (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} ) (\lambda)^{(3)} \right) (q_{22})^{(3)}G_{22} \\
 & + ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) \left( (a_{22})^{(3)}(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^* \right) \\
 & \left( ((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(22)}T_{21}^* + (b_{21})^{(3)}s_{(20),(22)}T_{20}^* \right) \} = 0 \\
 & + \\
 & ((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)}) \{ ((\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)}) \\
 & \left[ \left( ((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)})(q_{25})^{(4)}G_{25}^* + (a_{25})^{(4)}(q_{24})^{(4)}G_{24}^* \right) \right] \\
 & \left( ((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(25)}T_{25}^* + (b_{25})^{(4)}s_{(24),(25)}T_{25}^* \right) \\
 & + \left( ((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)})(q_{24})^{(4)}G_{24}^* + (a_{24})^{(4)}(q_{25})^{(4)}G_{25}^* \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( ((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})S_{(25),(24)}T_{25}^* + (b_{25})^{(4)}S_{(24),(24)}T_{24}^* \right) \\
 & \left( ((\lambda)^{(4)})^2 + ( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} ) (\lambda)^{(4)} \right) \\
 & \left( ((\lambda)^{(4)})^2 + ( (b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} ) (\lambda)^{(4)} \right) \\
 & + \left( ((\lambda)^{(4)})^2 + ( (a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} ) (\lambda)^{(4)} \right) (q_{26})^{(4)} G_{26} \\
 & + ((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}) \left( (a_{26})^{(4)}(q_{25})^{(4)}G_{25}^* + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^* \right) \\
 & \left( ((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})S_{(25),(26)}T_{25}^* + (b_{25})^{(4)}S_{(24),(26)}T_{24}^* \right) \} = 0 \\
 & + \\
 & \left( (\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)} \right) \{ (\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)} \} \\
 & \left[ \left( ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \right] \\
 & \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})S_{(29),(29)}T_{29}^* + (b_{29})^{(5)}S_{(28),(29)}T_{29}^* \right) \\
 & + \left( ((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)}) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \right) \\
 & \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})S_{(29),(28)}T_{29}^* + (b_{29})^{(5)}S_{(28),(28)}T_{28}^* \right) \\
 & \left( ((\lambda)^{(5)})^2 + ( (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} ) (\lambda)^{(5)} \right) \\
 & \left( ((\lambda)^{(5)})^2 + ( (b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)} ) (\lambda)^{(5)} \right) \\
 & + \left( ((\lambda)^{(5)})^2 + ( (a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} ) (\lambda)^{(5)} \right) (q_{30})^{(5)} G_{30} \\
 & + ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) \left( (a_{30})^{(5)}(q_{29})^{(5)}G_{29}^* + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^* \right) \\
 & \left( ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})S_{(29),(30)}T_{29}^* + (b_{29})^{(5)}S_{(28),(30)}T_{28}^* \right) \} = 0 \\
 & + \\
 & \left( (\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)} \right) \{ (\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)} \} \\
 & \left[ \left( ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \right] \\
 & \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})S_{(33),(33)}T_{33}^* + (b_{33})^{(6)}S_{(32),(33)}T_{33}^* \right) \\
 & + \left( ((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)}) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \right) \\
 & \left( ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})S_{(33),(32)}T_{33}^* + (b_{33})^{(6)}S_{(32),(32)}T_{32}^* \right) \\
 & \left( ((\lambda)^{(6)})^2 + ( (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} ) (\lambda)^{(6)} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( (\lambda)^{(6)} \right)^2 + \left( (b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & + \left( (\lambda)^{(6)} \right)^2 + \left( (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} (q_{34})^{(6)} G_{34} \\
 & + \left( (\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) \left( (a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\
 & \left( (\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(34)} T_{33}^* + (b_{33})^{(6)} s_{(32),(34)} T_{32}^* \} = 0 \\
 & + \\
 & \left( (\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)} \right) \{ (\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)} \} \\
 & \left[ \left( (\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)} \right) (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (q_{36})^{(7)} G_{36}^* \right] \\
 & \left( (\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(37)} T_{37}^* + (b_{37})^{(7)} s_{(36),(37)} T_{37}^* \\
 & + \left( (\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)} \right) (q_{36})^{(7)} G_{36}^* + (a_{36})^{(7)} (q_{37})^{(7)} G_{37}^* \\
 & \left( (\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(36)} T_{37}^* + (b_{37})^{(7)} s_{(36),(36)} T_{36}^* \\
 & \left( (\lambda)^{(7)} \right)^2 + \left( (a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} \\
 & \left( (\lambda)^{(7)} \right)^2 + \left( (b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)} \right) (\lambda)^{(7)} \\
 & + \left( (\lambda)^{(7)} \right)^2 + \left( (a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} (q_{38})^{(7)} G_{38} \\
 & + \left( (\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)} \right) \left( (a_{38})^{(7)} (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (a_{38})^{(7)} (q_{36})^{(7)} G_{36}^* \right) \\
 & \left( (\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)} \right) s_{(37),(38)} T_{37}^* + (b_{37})^{(7)} s_{(36),(38)} T_{36}^* \} = 0
 \end{aligned}$$

#### Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's Letters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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