

Numerical Computations of Radial Vibrations of Axially Polarized Piezoelectric Circular Cylinder

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Abstract:

Influence of the initial stresses on the frequency equation and the natural frequencies for radial vibrations of axially polarized piezoelectric circular cylinder have been taken into account.

The mechanical boundary conditions correspond to those of stress free lateral surfaces while the electrical boundary conditions correspond to those of open and short circuit are considered. The satisfaction of the boundary conditions lead to the frequency equation, in the form of determinant involving Bessel functions, have been taken into consideration. The roots of the frequency equations give the values of the characteristic circular frequency parameters of the first three modes for various geometries. These roots are numerically computed and programmed for numerical evaluation by "Bisection Method Iterations Technique (BMIT)" and presented graphically for various thickness of the circular cylinder and for different values of the initial stress. The effect of the initial stress on the natural frequencies are illustrated graphically for a transversely isotropic piezoelectric martial PZT−4 circular cylinder.

It is found that both the thickness of the circular cylinder and the initial stress have a substantial effect on the dispersion behavior.

The results obtained in this paper may be applied to the vibrations of annular accelerometers operating in the radial shear mode. Also, they have theoretical basis application and have meaningful design for piezoelectric probes and electro-acoustic devices in the nondestructive evaluation.

Keywords: Piezoelectricity, frequency equation, Transverse surface waves, Initial stress, Hexagonal crystals.

1. Introduction

Engineering materials called "smart materials" or "intelligent materials" recently have become a major focus of attention. In particular, piezoelastic (piezoelectric) materials have great promise for use in smart structural systems. The use of smart materials have become ever more important since the implementation of sophisticated functions in transducers is called for in today's technology. In a common definition, smart materials differ from ordinary materials in which they can perform two or several functions sometimes with a useful correlation or feedback mechanism between them. In the case of piezoelectric materials, they can be used for both sensor and actuator functions. When an external force acts on a piezoelastic materials, the stresses produce an electric potential within the material. Conversely, when an electric field is applied to a piezoelastic material, stresses are induced. The possibility exists, therefore, of determining the stresses in a piezoelastic material by measuring the electric potential, and then controlling the stresses by the action of an appropriate applied electric field [12], [20] and [26].

The piezoelectric materials, particularly piezoelectric ceramics, have been found to have wide applications in smart systems of aerospace, automotive, medical, and electronic fields due to the intrinsic coupling characteristics between their electric and mechanical fields [10, 20,24]. As the piezoelectric materials are being extensively used as actuators or transducers in technologies of smart and adaptive systems, the mechanical reliability and durability of these materials become increasing importance [14]. Since the investigation of mode vibrations in [3, 4, 14], the piezoelectric effect in hollow cylinders have been effectively employed in the fabrication of annular accelerometers, and other applications may arise wherever thickness shear elements such as delay line transducers of frequency control filters, are adapted to a cylindrical shape [23] and [25]. For the more historical development of piezoelectric problems and their applications in electromechanical devices (see Refs. [2], [14], [26]).

In the conventional surface wave devices, the acoustic wave propagates on a flat surface and the desired time delay is usually limited by the length of the crystal. It is conceivable that, if the surface wave is guided around a cylindrical body, one will be able to achieve a much longer time delay. In the design of signal filtering, the dispersive behavior of the surface waves becomes very important. The desirable dispersion can be obtained by changing the geometry of the substrate or adding metallic conducting films on top of the piezoelectric substrate [18].

Many electro-acoustic devices make use of surface waves propagating azimuthally in cylindric surface of a piezoelectric material. The study of such waves was initiated in [6, 24]. Effects of piezoelectricity on the properties of radial and transverse surface waves supported by a cylindrical surface of crystals are studied by many authors [2, 19, 21]. Subsequently, Wang et al [23] investigated the longitudinal wave propagation in piezoelectric coupled rods and Qian et al [18] have investigated scattering of elastic waves by cylinders in 1-3 piezocomposited. Dispersion phenomena in transversely isotropic piezo-electric plated with either short or open circuit boundary conditions has been treated by Guo et al [9]. Piezoelectric tubes and tubular composites for actuator and sensor applications are considered by Zhang et al [16]. Recently, Dong et al [5] studied wave propagation characteristics in piezoelectric cylindrical laminated shells under large deformation.

Free vibration resonant frequencies and mode shapes are fundamental information needed in device design. They have been obtained for a circular cylinder by many authors [15, 23]. Initial stresses are inseparable with surface acoustic wave (SAW) resonators for many reasons, including material processing stages and fabrication with thermal treatments inducing incompatible deformations and residual stresses and utilizing resonators as force and pressure sensors in many applications. Because of the importance of the initial stresses in applications, studies on the analysis of such effects have been done out with different objectives and approaches [1, 5, 6, 7, 11, 16, 17].

This paper deals with the radial vibrations of axially polarized piezoelectric circular cylinder and influence of the initial stresses on the frequency equation and its natural frequencies. The differential equations of piezoelectric radial motion were derived in terms of radial displacement and electric potential. The characteristic equation of radial vibration was obtained by applying mechanical and electric boundary conditions. The piezoelectric natural frequency of the fundamental mode was shown to increase as the radius of curvature decreased. It is found that both the thickness of the circular cylinder and the initial stress have a substantial effect on the dispersion behavior. Also, this study can offer theoretical basis and meaningful suggestion for the design of piezoelectric cylindrical transducers. A transducer polarized in the axial direction undergoes axial motion under the electric drive in the radial thickness direction, and is used as an aligner or a translator, as for example in a scanning tunnelling microscope [4]. A transducer polarized in the circumferential direction undergoes radial vibrations resulting from circumferential expansion and compression [13].

1. Basic Equations

The system of equations governing the vibrations of piezoelectric media with initial stresses (see, e.g., [16], [17]) is formulated in this section for transducers possessing cylindrical geometry.

The stress equations of motion, which includes the presence of initial stresses, are:

$$T_{ij,j} + (u_{i,k} T^{\circ}_{kj})_{,j} = \rho \ddot{u}_i \tag{1}$$

The Gauss's law of electrostatics without free charge is:

$$D_{i,i} = 0 \tag{2}$$

The strain-mechanical displacement relations are:

$$S_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j}) \tag{3}$$

The electric field - electric potential relations are:

$$E_i = -\varphi_{,i} \tag{4}$$

The linear piezoelectric constitutive relations are:

$$T_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k \,, D_i = e_{ikl} S_{kl} + \varepsilon_{lk} E_k \qquad (i, j, k, l = 1,2,3), \tag{5a,b}$$

Where, in the above, T_{ij}, T°_{kj}, u_i, D_i, S_{ij}, E_i are the components of stress, initial stresses, mechanical displacement, electric displacement, strain, and electric field, respectively; ρ and φ are the mass density and the electric potential; c_{ijkl} are the elastic moduli at constant electric field, ε_{ik} the dielectric coefficients at constant strain field and e_{kij} the piezoelectric coefficients. Common notational conventions are employed throughout, such as the comma for differentiation with respect to a space coordinate and a dot for differentiation with respect to time. Associating (r, θ, z) with $(1,2,3)$, respectively, the matrices of the elastic, piezoelectric, and dielectric constants for a ferroelectric ceramic poled in the z direction (crystal symmetry 6 mm) are:

$$\left[c_{pq}\right]=\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \qquad (6a)$$

$$c_{66}=\frac{1}{2}(c_{11}-c_{12}), \qquad (6b)$$

$$\left[e_{ip}\right]=\begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{33} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \qquad (6c)$$

$$\left[\varepsilon_{ij}\right]=\begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}. \qquad (6d)$$

Where we have a compact matrix notation [11]. This notation consists of replacing pairs of indices ij or kl by single indices p or q, where i, j, k and l take the values $1,2$ and 3, and p and q take the values $1, 2, 3, 4, 5$ and 6 according to:

ij or kl	11	22	33	23 or 32	31 or 13	12 or 21
p or q	1	2	3	4	5	6

3. Formulation of the Problem

We consider an infinite circular cylinder of inner radius a and outer radius b. The cylinder is made of ceramics with axial poling along the x_3 direction we choose (r,θ,z) to correspond to $(1,2,3)$ so that the poling direction corresponds to 3. The inner and outer surfaces are being electroded. There is no load applied, and we are interested in anti-plane axi-symmetric free vibrations [25]. The radial modes of an infinite cylinder are independent of θ and z, and have the circumferential mechanical displacement $u_\theta = 0$. Substituting from Eqs. (3), (4) and (6) into Eqs. (5) and the resulting expressions into Eqs. (1) and (2) leads to the governing equations:

$$(c_{44} + \tau^{\circ}_{rr}) + (\frac{\partial^2 u_z}{\partial z^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}) + e_{15}(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}) = \rho \frac{\partial^2 u_z}{\partial t^2}, \tag{7}$$

$$e_{15}(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}) - \varepsilon_{11}(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}) = 0, \tag{8}$$

with Eq. $(1)_2$ being satisfied identically. It is seen that the radial extensional mode u_r cannot be excited electrically, the free vibrations of which have been discussed by [25]. The radial shear mode u_z is, however, coupled to the electric potential φ and therefore examined below.

Eliminating φ from Eqs. (8) and (9), and assuming a steady-state solution in the forms:

$$u_z(r,t) = u_z(r)e^{i\omega t}, \qquad \varphi(r,t) = \varphi(r)e^{i\omega t}, \tag{9}$$

the following equation for u_z is obtained:

$$\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \lambda^2 u_z = 0, \tag{10}$$

Where $\lambda^2 = \rho\omega^2 / c^*_{44},$ (11a)

$$c^*_{44} = (c_{44} + \tau^{\circ}_{rr})(1 + k^2_{15}), \tag{11b}$$

and $k^2_{15} = e^2_{15} / (c_{44} + \tau^{\circ}_{rr})\varepsilon_{11}$ (11c)

where k^2_{15} is the square of the electromechanical coupling factor for shear (see, e.g., ref.[2]). The general solution of Eq. (10) is:

$$u_z = A_1 J_0(\lambda r) + A_2 Y_0(\lambda r), \tag{12}$$

Where J_0 and Y_0 are the zero-order Bessel functions of the first and second kind, respectively. Solving Eq. (8) for φ in view of Eq. (12),

$$\varphi = \frac{e_{15}}{\varepsilon^S_{11}}[A_1 J_0(\Omega r) + A_2 Y_0(\Omega r)] + A_3 \ln(r/b) + A_4. \tag{13}$$

4. Boundary Conditions

The following boundary and continuous conditions should be satisfied. It should be pointed out that two kinds of mechanical and electrical boundary conditions would be considered in this study. So the boundary condition may be written as:

a) Free tractions and unelectroded surfaces

If the cylindrical surfaces are free from tractions, i.e. the shearing stress must vanish.

$$T_{rz} = 0, \quad \text{for} \quad r = a, b, \tag{14}$$

If the electrodes are open for the electrical boundary conditions. This may express by

$$D_r = 0, \text{ for } \quad r = a, b \tag{15}$$

In which case, we have $A_3 = 0$, and

$$\Delta_1 = |a_{ij}| = 0 \tag{16}$$

b) Free tractions and electroded surfaces

The faces of the cylinder are traction- free. So, the shearing stress T_{rz} must vanish.

$$T_{rz} = 0, \quad \text{for} \quad r = a, b. \tag{17}$$

Applying an alternating voltage of potential $2\varphi_\circ e^{i\omega t}$ to the electroded cylindrical surfaces. Therefore, the electrical boundary conditions are:

$$\varphi = \varphi_\circ e^{i\omega t} \quad \text{at} \quad r = a, b. \tag{18}$$

Using the boundary conditions Eqs. (17) and (18), it can be shown that

$$\Delta_1 = k_{15}^2 \Delta_2, \tag{19}$$

Where

$$\Delta_2 = \frac{1}{\lambda \ln(h)} |b_{ij}| \tag{20}$$

c) Clamped and electroded surfaces

We consider the case when the two cylindrical surfaces are fixed and the two electrodes are shorted [25] . Then one may get:

$$u_z = 0, \quad \text{for} \quad r = a, b \tag{21}$$

$$\varphi = 0, \quad \text{for} \quad r = a, b \tag{22}$$

Which implies that $A_3 = 0, \quad A_4 = 0,$

These conditions lead to the following equation:

$$\Delta_3 = \left| d_{ij} \right| = 0 \tag{23}$$

The roots of Eqs. (16), (20) and (23) give the values of natural frequencies for the radial vibrations of axially polarized piezoelectric circular cylinder. It is easily noted that the frequency Eqs. (16), (20) and (23) involve the dimensions a, b of the cylinder and the elastic, electric and the initial stress constants. To simplify the calculation of the eigenvalues of Eqs. (16), (20) and (23), we confined our attention to make these quantities dimensionless, therefore we introduce the following transformations:

$$\begin{aligned} &\lambda = \frac{\omega}{c_1^*}, \qquad c_1^* = \sqrt{\frac{c_{44}^*}{\rho}}, \qquad \Omega_1 = \frac{\Omega}{\Omega_2}, \\ &h = \frac{a}{b}, \qquad \Omega_2 = \frac{\pi c_1^*}{b(1-h)}, \qquad \Omega = \frac{\omega \pi}{1-h}. \end{aligned} \tag{24}$$

where the elements of the determinants in Eqs. (16), ((20) and (23) are, respectively:

$$a_{11} = J_1(h\Omega_1), \qquad a_{12} = Y_1(h\Omega_1),$$

$$a_{21} = J_1(\Omega_1), \qquad a_{22} = Y_1(\Omega_1).$$

and

$$b_{11} = J_0(h\Omega_1) - J_0(\Omega_1), \qquad b_{12} = J_1(h\Omega_1) - hJ_1(\Omega_1),$$

$$b_{21} = Y_0(h\Omega_1) - Y_0(\Omega_1), \qquad b_{22} = Y_1(h\Omega_1) - hY_1(\Omega_1).$$

and

$$d_{11} = J_0(h\Omega_1), \qquad d_{12} = Y_0(h\Omega_1),$$

$$d_{21} = J_0(\Omega_1), \qquad d_{22} = Y_0(\Omega_1).$$

5. Numerical Results and Discussion

A computer program has been written for numerical evaluation of the frequency equations (16), (20) and (23). Numerical calculations have been carried out for a PZT-4 which possesses class 6mm symmetry. The computational piezoelectric material parameters are taken from Ref. [3] which are summarized in the following Table 1.

Table 1: the physical constants of ceramic PZT-4	
(Ceramic)PZT-4	Units

$\rho = 7500$	kg/m^3
$c_{44} = 2.6\times 10^{10}$	N/m^2
$e_{15} = 10.5$	C/m^2
$\in_{11} = 5.8X10^{-9}$	F/m

The calculation of the roots of the dispersion Eqs. (16), (20) and (23) which may be regarded as function of $n, h, \Omega, T_{rr}^{\circ}$,(we will replace T_{rr}° by σ for simplicity) represents a major task of this work. The dimensionless frequency Ω is calculated and plotted against the thickness ratio $h = a/b$ for three different values of initial stresses σ for the first three modes and when (n=0,1).

We have adopted the following iterative procedure for numerical computations. For a fixed value of l we evaluate the determinant, which is presented in the left hand side of Eqs. (16), (20) and (23) for various values of the unknown quantity Ω, commencing with the initial value zero and each time adding a fixed but small increment to that unknown quantity till the value of the determinant changes its sign. Then the "bisection method also, known as method of having the interval or Bolzano method" is applied to get the located root. With this root as the initial value, the procedure is repeated to find the next root [8].

The roots of the frequency equations give the values of the characteristic circular frequency parameters of the first three modes for various geometries. These roots are presented graphically for various thickness of the circular cylinder and for different values of the initial stress.

Figures (1), (2) and (3) represent the first, second and third modes, respectively, of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3)\times 10^{11}$ for the boundary conditions are free tractions and open circuit. It is seen that the circular frequency monotonically increases with diminution the thickness of the circular cylinder, while it becomes worth less with augmentation of the values initial stresses.

The general form of the dispersion curves illustrated in Figs.(1), (2), and (3) are very similar to those presented in Figs. (5), (6) and (7), for the boundary conditions are free tractions and close circuit, as well as which are also shown in Figs. (9), (10) and (11) for the boundary conditions are clamped surfaces and open circuit.

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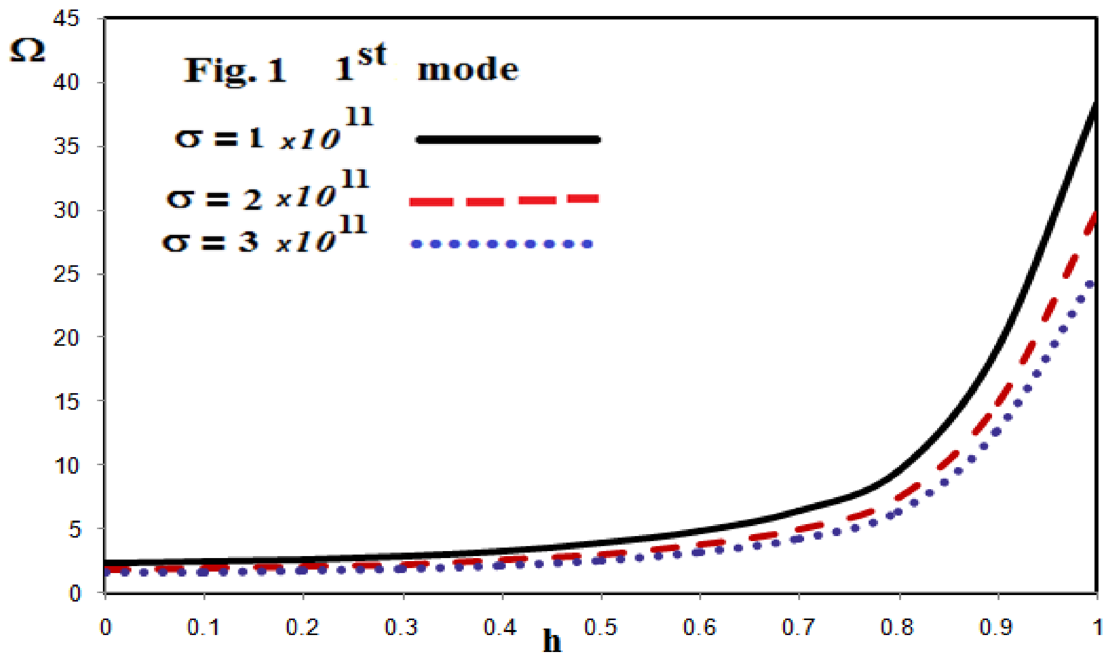

Figure (1), The first mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3) \times 10^{11}$ (Free tractions and unelectroded surfaces).

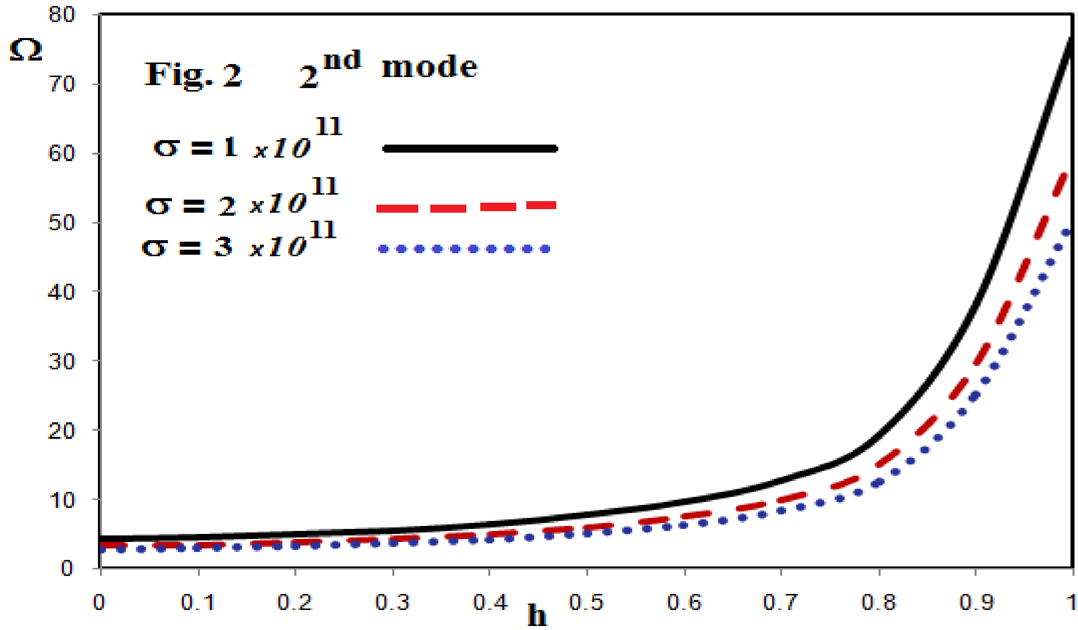

Figure (2), The second mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3) \times 10^{11}$ (Free tractions and unelectroded surfaces).

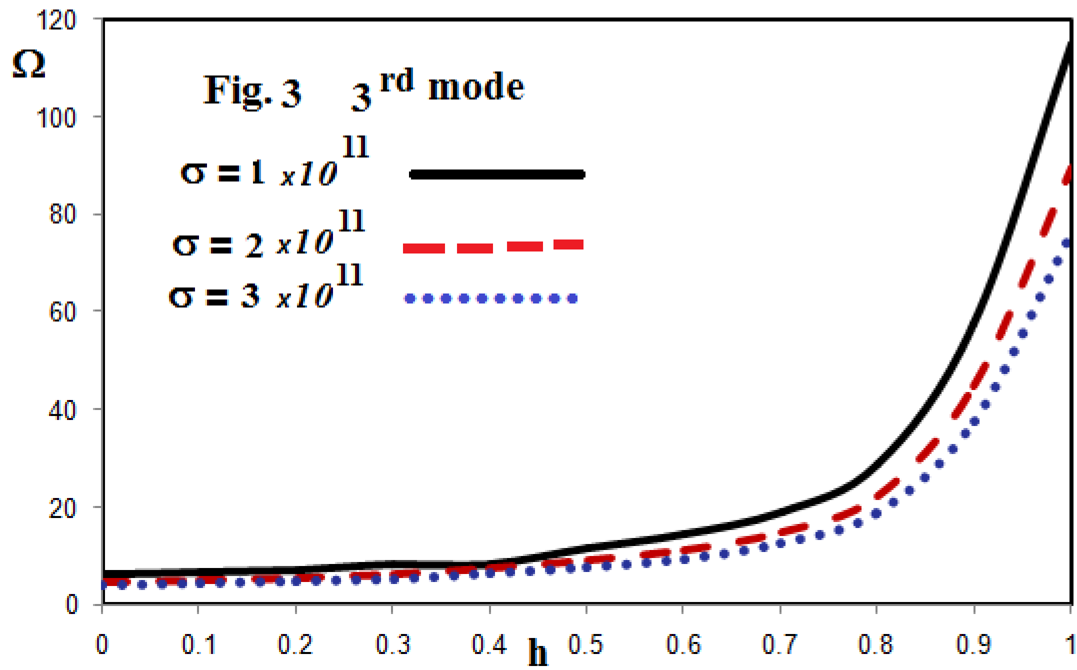

Figure (3), The third mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3) \times 10^{11}$ (Free tractions and unelectroded surfaces).

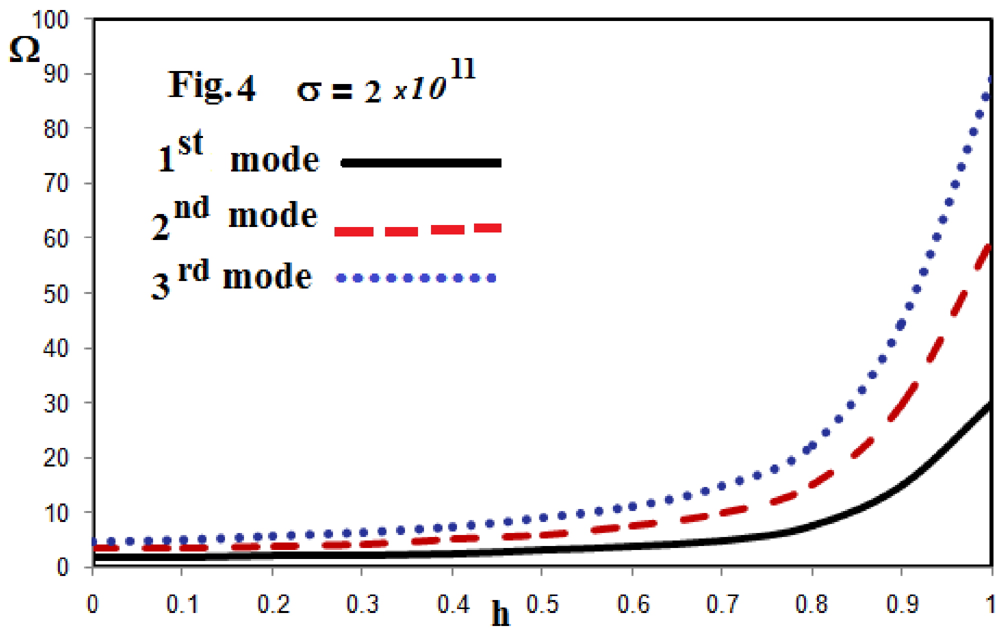

Figure (4), The first three modes of the natural frequency Ω versus h for the value of initial stresses $\sigma = 2\times 10^{11}$ (Free tractions and unelectroded surfaces).

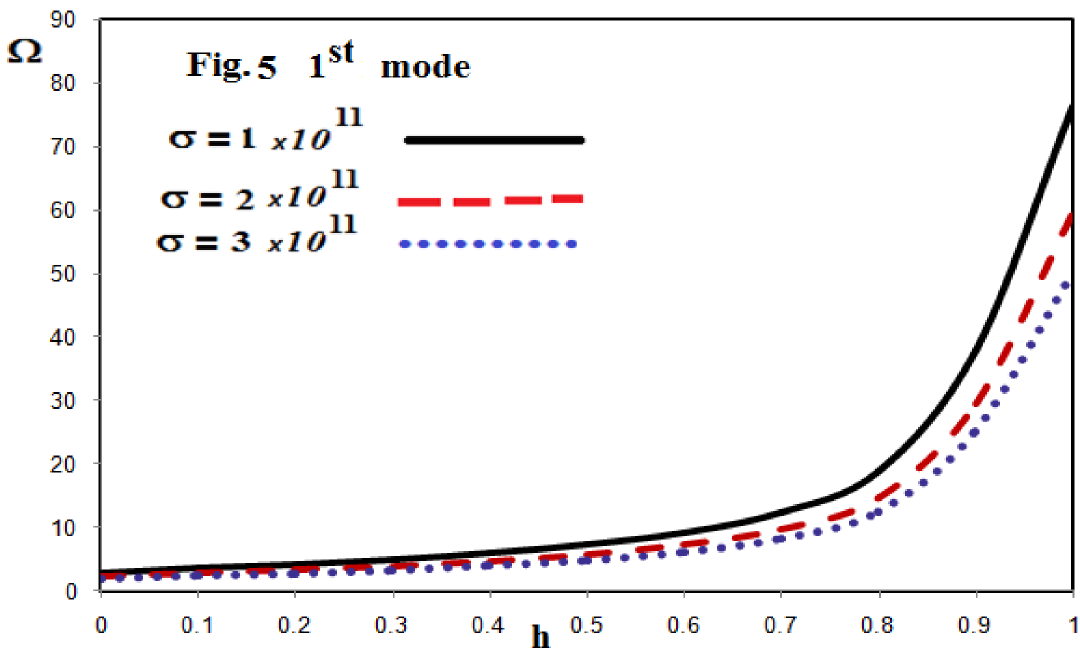

Figure (5), The first mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3)\times 10^{11}$ (Free tractions and electroded surfaces).

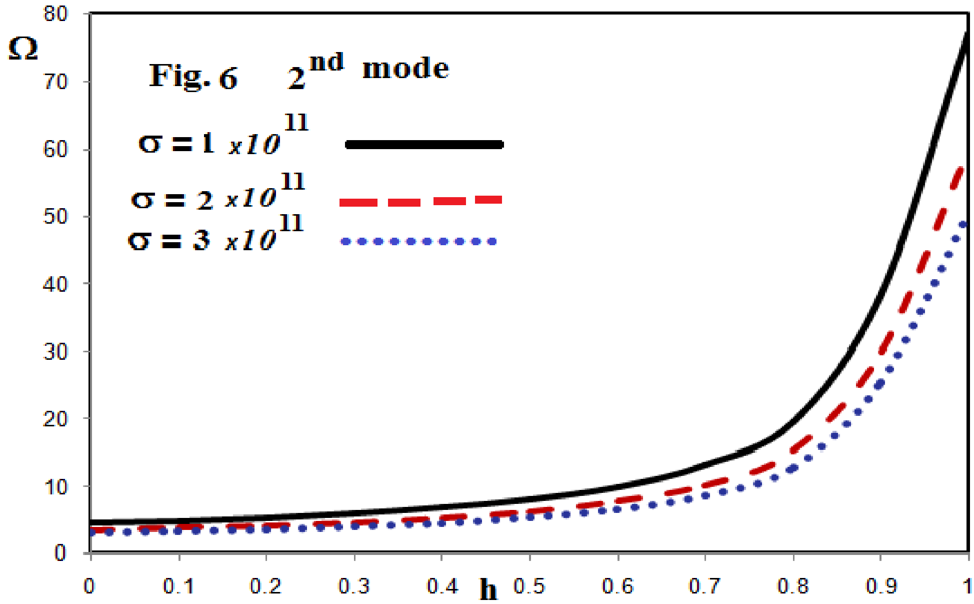

Figure (6), The second mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3)\times 10^{11}$ (Free tractions and electroded surfaces).

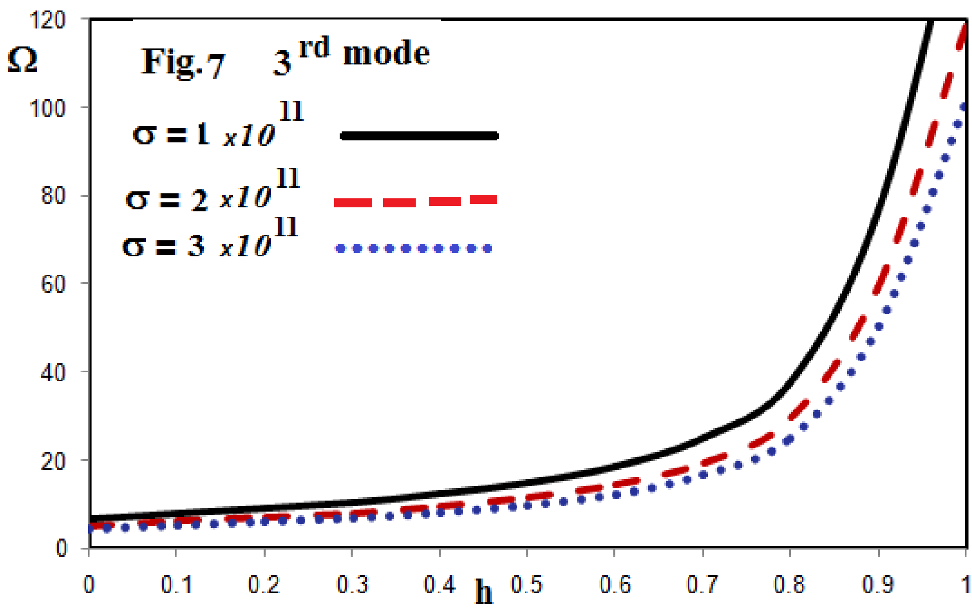

Figure (7), The third mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3)\times 10^{11}$ (Free tractions and electroded surfaces).

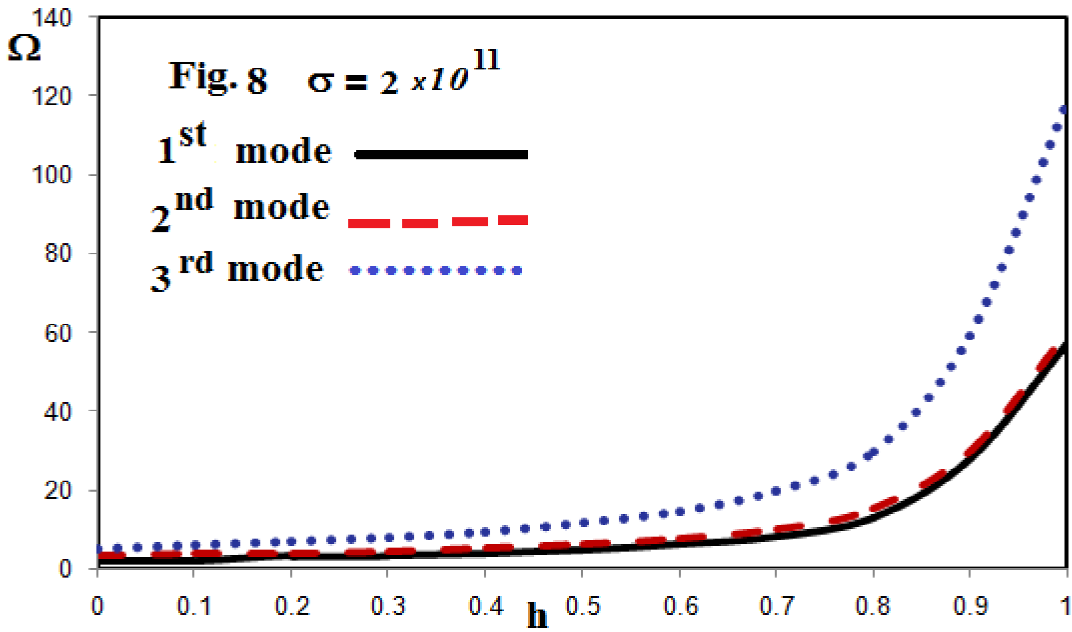

Figure (8), The first three modes of the natural frequency Ω versus h for the value of initial stresses

$\sigma = 2 \times 10^{11}$ (Free tractions and electroded surfaces).

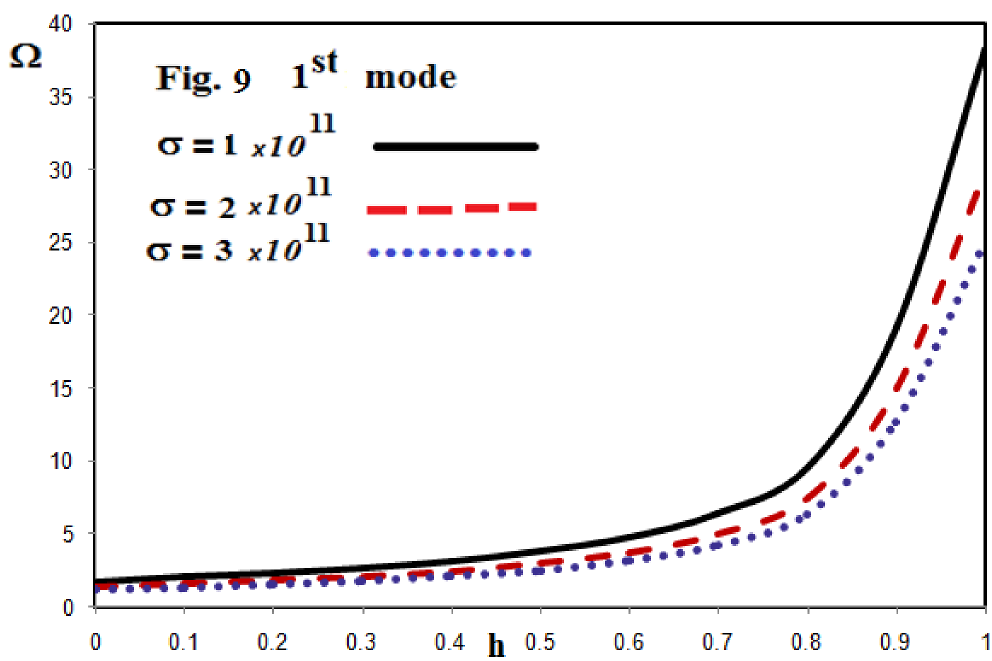

Figure (9), The first mode of the natural frequency Ω versus h for different values of initial stresses

$\sigma = (1,2,3) \times 10^{11}$ (Clamped and electroded surfaces).

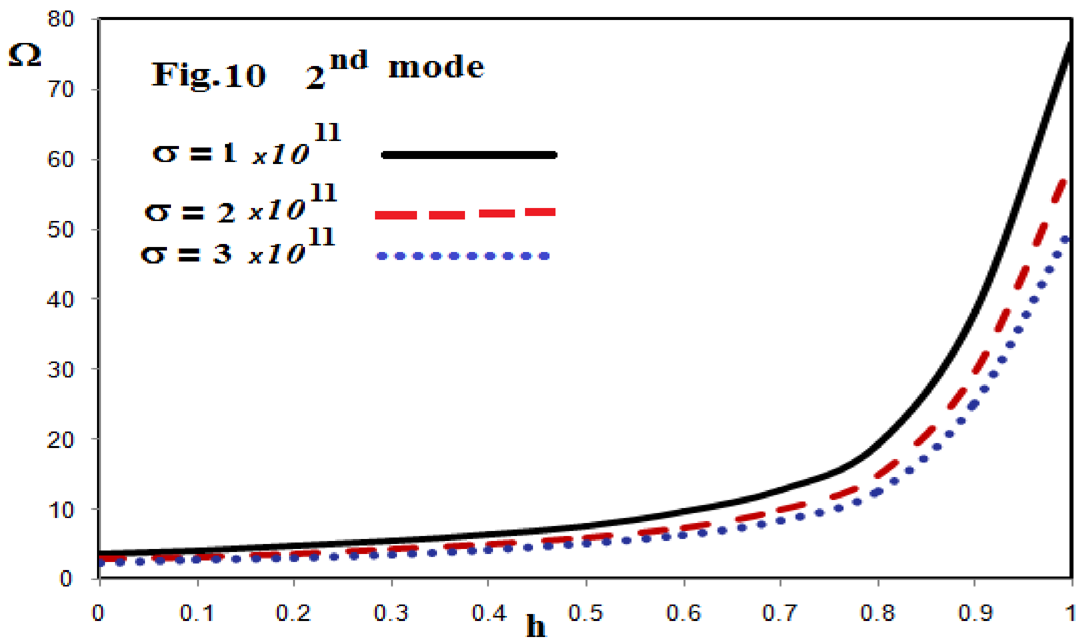

Figure (10), The second mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3) \times 10^{11}$ (Clamped and electroded surfaces).

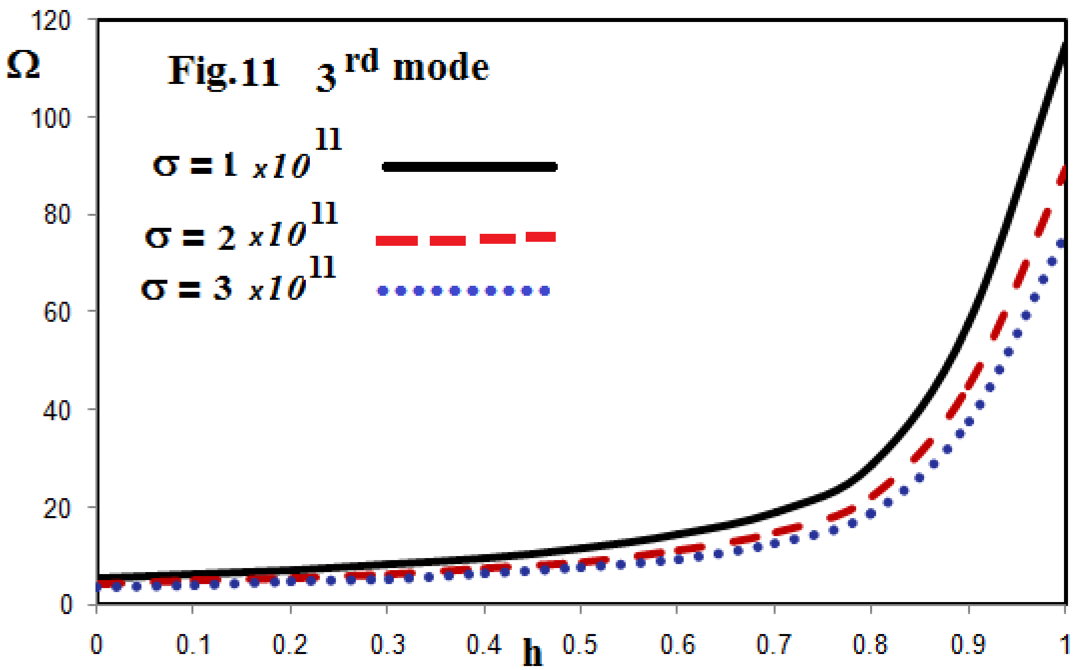

Figure (11), The third mode of the natural frequency Ω versus h for different values of initial stresses $\sigma = (1,2,3) \times 10^{11}$ (Clamped and electroded surfaces).

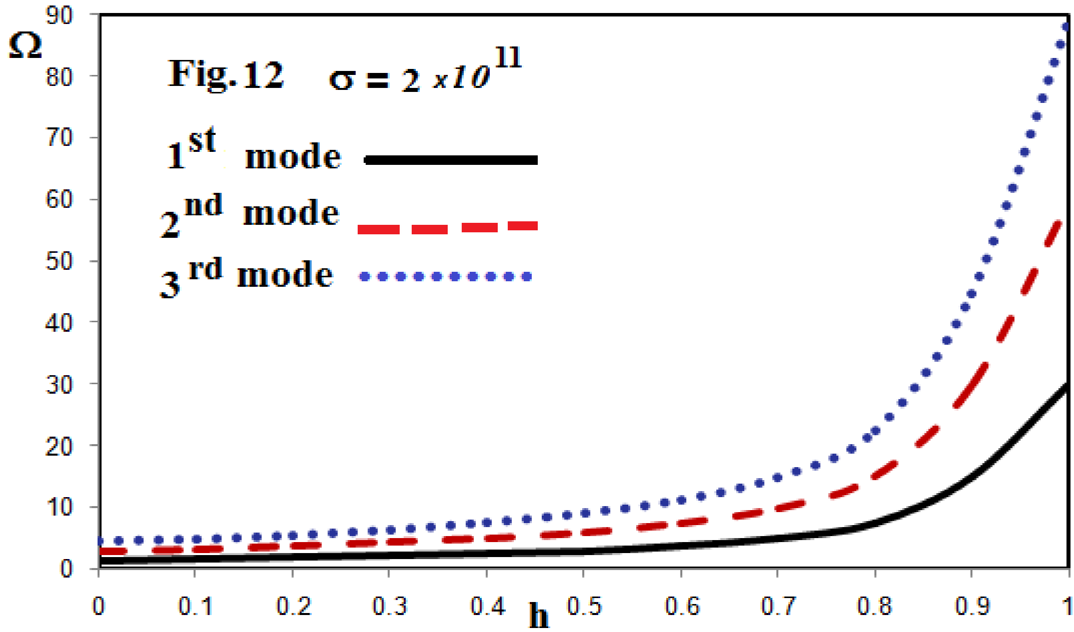

Figure (12), The first three modes of the natural frequency Ω versus h for the value of initial stresses $\sigma = 2\times10^{11}$ (Clamped and electroded surfaces).