Influence of Thermal Radiation on Stagnation Flow towards a Stretching Sheet with Induced Magnetic Field

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Abstract
In this study, we studied the effects of radiation and induced magnetic field effects on the flow past a stretching sheet. The governing equations of the flow are transformed into a set of nonlinear ordinary differential equations using self-suitable transformations, which are then solved numerically using Runge-Kutta based shooting technique. The effects of different non-dimensional parameters on velocity, induced magnetic field and temperature profiles along with the friction factor and local Nusselt number are discussed through graphs and tables. Results indicate that increase in either magnetic field parameter or stretching ratio parameter enhances the velocity of the flow.
Keywords: Stagnation point flow, Induced magnetic field, thermal radiation, stretching Sheet.

1. Introduction
The magneto hydrodynamics (MHD) deals with the motion of highly conducting fluids in the presence of magnetic field. The flow of the conducting fluid across the magnetic field causes the electric current, which has the capacity to change the influence of magnetic field and the flow behavior of the fluid. The magneto nanofluids have pivotal importance in the process of targeted drug release, asthma treatment, cancer therapy and in the construction of power generators. Flow due to stretching surface has many important applications in plastic and metal industries. Also, the glasses blowing, continuous casting of metals and spinning of fibers involve the concept of flow through stretching sheet.

Takhar et al. [1] discussed the effects of chemical reaction and magnetic field on viscous electrically conducting fluid on a stretching surface. The influence of thermal radiation on an incompressible stagnation point flow through a stretching sheet was analyzed by Pop et al. [2]. Singh and Singh [3] examined the impact of induced magnetic field on laminar natural convective flow in vertical concentric annuli. The influence of inclined magnetic field on free convection flow of dissipative fluid through a vertical channel was studied by Sandeep and Sugunamma [4]. Then after, Sandeep and Sugunamma [5] discussed the effects of thermal radiation and magnetic field on MHD flow through a moving vertical plate. From this paper, it is found that increase of thermal radiation reduces the fluid flow in the isothermal plate whereas it raises the fluid flow in ramped temperature. Heat transfer and Stagnation point flow past a stretching/shrinking surface in a micropolar fluid was characterised by Zaimi and Ishak [6]. Sugunamma et al. [7] discussed the effects of Magnetic field and Radiation on the flow of a nanofluid in a rotating frame. RamanaReddy et al. [8] studied the effects of thermal radiation, rotation and chemical reaction on MHD nanofluid flow along a stretching sheet. In this paper, it is found that thermal radiation and magnetic field parameters have proclivity to decrease the friction factor. Radiation and slip effects on magneto hydrodynamic stagnation point flow of nanofluid past a flat plate was examined by Haq et al. [9]. Raju et al. [10 analysed the heat and mass transfer in MHD casson nanofluid past a stretching sheet. From this paper, it is found that an inflation in exponential parameter and heat generation parameter intensify the heat transfer rate. The influence of Magnetic field on the boundary layer flow of nanofluids past a porous plate in a rotating frame was studied by Das et al. [11]. The effects of Homogeneous and Heterogeneous reactions on MHD flow between rotating plates with non linear thermal radiation were discussed by Ramana Reddy et al. [12]. The impact of cross-diffusion and aligned magnetic field of a nanofluid past an exponentially stretching sheet was discussed by Sulochana et al. [13]. Raju et al. [14] analyzed the optimal solutions for three-dimensional magneto hydrodynamic flow of nanofluid past a stretching sheet. Through this study, it is found that increase in the magnetic field parameter then decreases the heat transfer rate. By making use of all these articles we investigated the present study.

2. Mathematical Analysis
Consider an incompressible, electrically conducting stagnation-point flow towards a stretching sheet in the presence of induced magnetic field. The stretching sheet is considered along the $x-$ axis and $y-$axis is perpendicular to the sheet. It is assumed that the applied magnetic field is of uniform strength H_0. It is also assumed that induced magnetic field is applied in $y-$direction and the parallel component H_1 approaches the value $H_e = H_0$ in the free stream flow and normal component of the induced magnetic field H_2 vanishes near the wall. Under the above made assumptions the governing boundary layer equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0, \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+u_e(x)\frac{\partial u_e(x)}{\partial x}+\frac{\mu}{4\pi}\left(H_1\frac{\partial H_1}{\partial x}+H_2\frac{\partial H_1}{\partial y}-H_e\frac{\partial H_e}{\partial x}\right), \tag{3}$$

$$u\frac{\partial H_1}{\partial x}+v\frac{\partial H_1}{\partial y}=H_1\frac{\partial u}{\partial x}+H_2\frac{\partial u}{\partial y}+\alpha_1\frac{\partial^2 H_1}{\partial y^2}, \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

Subject to the boundary conditions

$$\left.\begin{aligned} &u=u_w(x)=cx, v=0, \frac{\partial H_1}{\partial y}=H_2=0, T=T_w \quad &&\text{at } y=0,\\ &u=u_e(x)=ax, H_1=H_e(x)=H_0x, T=T_\infty, &&\text{as } y\to\infty, \end{aligned}\right\} \tag{6}$$

where u,v are the velocity components of the fluid in x,y directions, H_1,H_2 are the magnetic components in x,y directions, ρ and μ are the fluid density and dynamic viscosity respectively, σ is the electrical conductivity, ρc_p is the specific heat capacitance, T is the fluid temperature, k is the effective thermal conductivity, σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively, α_1 is the magnetic diffusivity of the nanofluid, which is given by $\alpha_1=1/4\pi\sigma$.

To convert the governing equations into set of nonlinear ordinary differential equations, we now introduce the following similarity transformation.

$$\left.\begin{aligned} &u=cxf'(\eta), v=-\upsilon^{1/2}c^{1/2}f(\eta), \eta=\upsilon^{-1/2}c^{1/2}y, T=T_\infty(1+(\theta_w-1)\theta)\\ &H_1=H_0xg'(\eta), H_2=-H_0\upsilon^{1/2}c^{-1/2}g(\eta), \theta(\eta)=(T-T_\infty)/(T_w-T_\infty), \end{aligned}\right\} \tag{7}$$

Substituting equation (7) into (1)-(5), equations (1) and (2) satisfies automatically. Now, the equations (3)-(5) will be transformed into the following nonlinear coupled ordinary differential equations:

$$\left(1+\frac{1}{\beta}\right)f'''-\left(f'^2-ff''\right)+B\left(g'^2-gg''-1\right)+\left(\frac{a}{c}\right)^2=0, \tag{8}$$

$$\lambda g'''+fg''-f''g=0, \tag{9}$$

$$\left(1+\frac{4R}{3}\right)\theta''++\Pr f\theta'=0, \tag{10}$$

With the transformed boundary conditions

$$\left.\begin{aligned} &f=0, f'=1, g=0, g''=0, \theta=1, \quad &&\text{at } \eta=0,\\ &f'=a/c, g'=1, \theta=0, &&\text{as } \eta\to\infty, \end{aligned}\right\} \tag{11}$$

a,c are constants, B is the magnetic parameter, λ is the reciprocal magnetic Prandtl number, R is the radiation parameter, $\Pr$ the Prandtl number, which are represented below.

$$B=\frac{\mu H_0^2}{4\pi\rho c^2}, \lambda=\frac{1}{4\pi\sigma\upsilon}, R=\frac{4\sigma^* T_\infty^3}{kk^*}, \Pr=\frac{\mu c_p}{\rho}, \tag{12}$$

For physical quantities of engineering interest are the shear stress coefficient C_f (friction factor) and the local Nusselt number Nu_x are given by

$$\mathrm{Re}_x^{1/2} C_f=f''(0), \tag{13}$$

$$\mathrm{Re}_x^{-1/2}\, Nu_x = -\theta'(0), \tag{14}$$

Where Re_x is the local Reynolds number, given by

$$\mathrm{Re}_x = \frac{u_w x}{\upsilon},$$

3. Results and Discussion

The system of nonlinear ordinary differential equations (8) – (10) with the boundary conditions (11) has been solved numerically using Runge-Kutta based shooting technique. Further the effects of various physical parameters like radiation parameter R, magnetic field parameter B and stretching ratio a/c on velocity, induced magnetic field and temperature profiles. For numerical results we considered $\theta_w = 1.1,\ \mathrm{a/c} = 0.5,\ R = 1, \lambda = 0.1$. These values have been kept in common for the entire study except the varied values as shown in the respective figures and tables.

Figs. 1 and 2 illustrate the effect of magnetic field parameter B on velocity and induced magnetic field profiles. It is found that an increase in the magnetic field parameter leads to velocity and induced magnetic field profiles. This is due to the hike in the retardation force acts along the flow.

Fig. 3 and 4 depict the influence of the stretching ratio parameter a/c on velocity and induced magnetic fields respectively. It is interesting to note that an increase in the value of a/c enhances the momentum boundary layer thickness and depreciates the induced magnetic field profiles. But we can observe an opposite results to the above with an increase in the reciprocal magnetic Prandtl number, which are displayed in Figs. 5 and 6.

The effect of radiation parameter R on temperature profiles of the flow is shown in Fig. 7. It is found that an increase in the radiation parameter leads to an increase in the temperature profiles of the flow. This may happen due to the fact that increase in radiation parameter generates heat to the fluid.

Finally we presented the effects of various physical parameters on friction factor (C_f) and local Nusselt number (Nu_x) in Table 1.

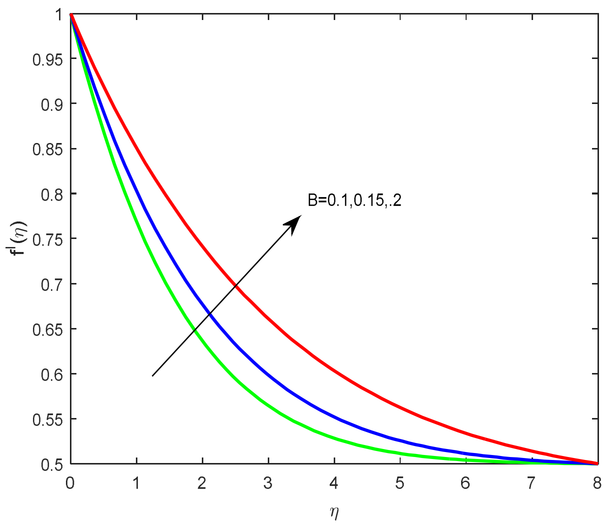

Fig.1 Velocity field for different values of B

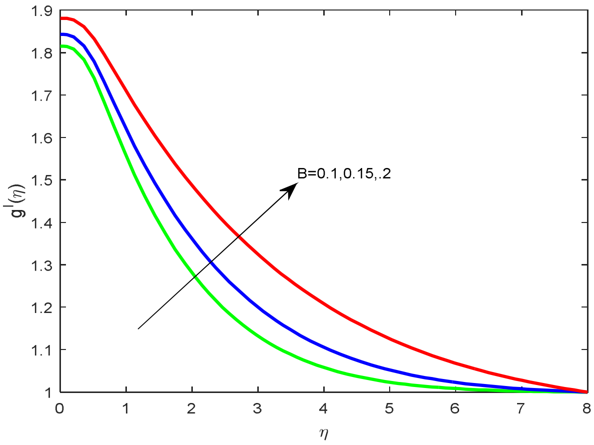

Fig.2 Induced magnetic field for different values of B

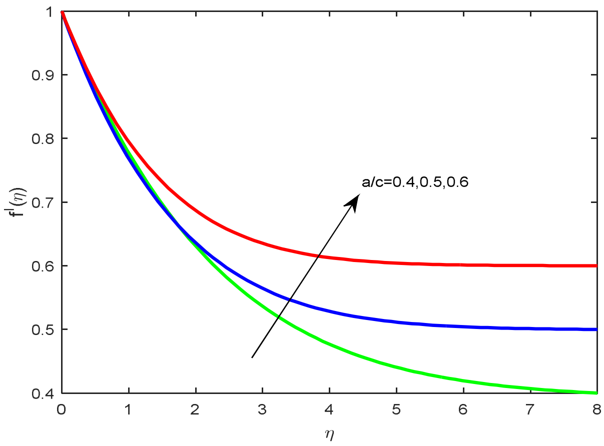

Fig.3 Velocity field for different values of a/c

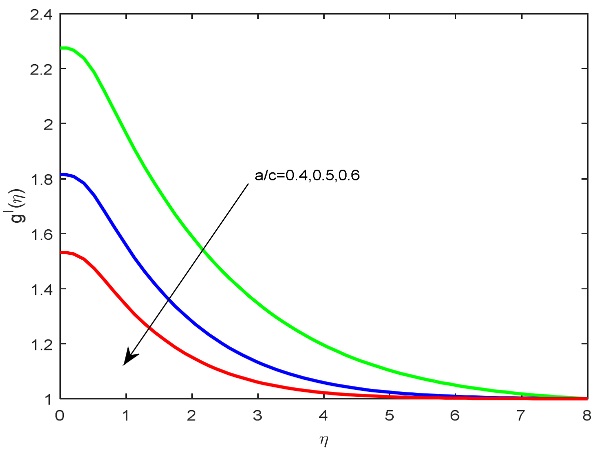

Fig.4 Induced magnetic field for various values of a/c

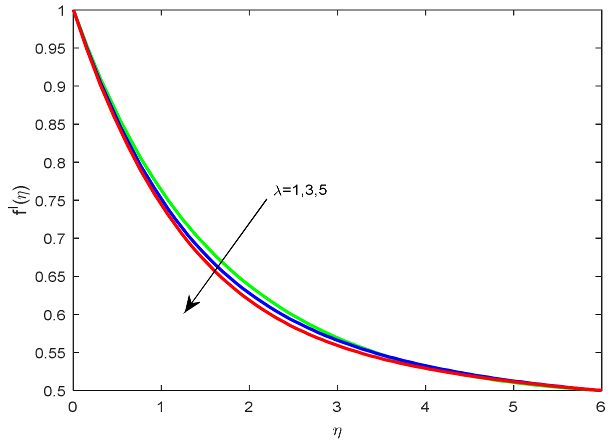

Fig.5 Velocity field for different values of λ

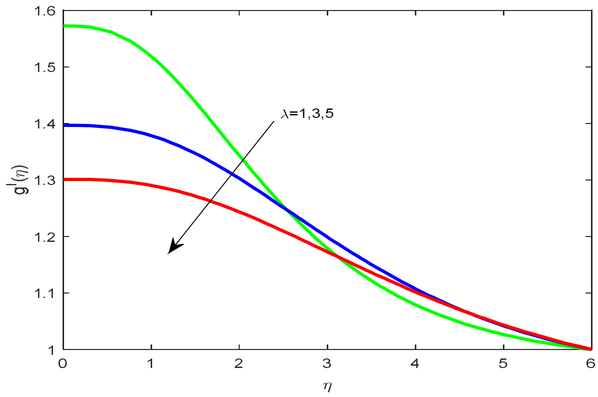

Fig.6 Induced magnetic field for different values of λ

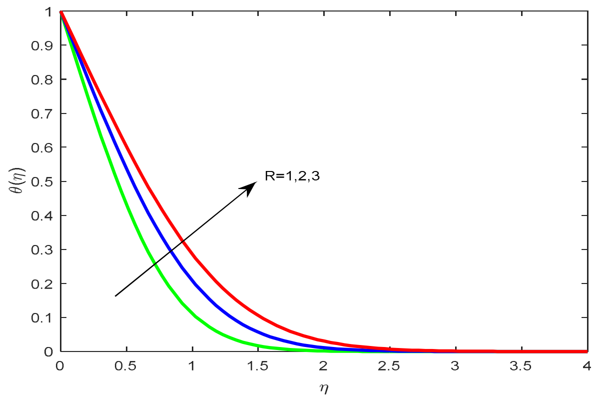

Fig.7 Temperature field for different values of R

Table 1 Variation in friction factor and Nusselt number

R	B	a/c	λ	C_f	Nu_x
1				-0.3091	1.2083
2				-0.3091	0.9465
3				-0.3091	0.8007
	0.5			-0.3027	1.2097
	1.0			-0.2495	1.2193
	1.5			-0.1803	1.2320
		0.4		-0.2784	1.2033
		0.5		-0.3027	1.2097
		0.6		-0.2754	1.2159
			0.5	-0.3211	1.2072
			1.0	-0.3404	1.2038
			1.5	-0.3518	1.2016

4. Conclusions

From the present study we are concluding the following results.

- Increase in magnetic field parameter enhances both velocity as well as induced magnetic field profiles.
- Increase in radiation parameter raises the temperature profiles.
- Reciprocal magnetic Prandtl number slowdowns the motion of the fluid.
- Stretching ratio parameter enhances the rate of heat transfer.

5. References

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