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Velocity and Thermal Slip Effects on Flow and Heat Transfer due to an Exponentially Stretching Sheet with Viscous Dissipation and Thermal Radiation

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Abstract
In this paper we have studied the flow and heat transfer characteristics of a viscous fluid in presence of viscous dissipation and thermal radiation due to an exponentially stretching sheet. The governing equations of this problem are basically highly non-linear partial differential equations which are converted into ordinary differential equations by using suitable similarity transformations techniques. These equations (pde's) are converted into system of ordinary differential equations of first order and solved numerically by ode45 solver in MATLAB software. The effects of governing parameters on flow and heat transfer analyzed and interpreted through graphs. Numerical values of skin friction coefficients and Nusselt number are calculated for different values of governing parameters and tabulated. Under some limiting conditions our current result agree very well with available results in the literature.

1. Introduction

As we know the flow and heat transfer characteristics due to stretching sheet has gained much more interest in past two decades. In view of many industrial applications such as Aero dynamic extrusion of plastic sheets, condensation process of metallic plates in a cooling bath and glass and polymer industries. Many authors analyzed flow and heat transfer characteristics due to linear and non linear stretching sheets. Some of the authors worked on exponential stretching sheet which are discussed below.

Parta et. al. [1] investigated the mixed convection flow and heat transfer of viscous fluid over an exponentially vertical stretching surface, they used shooting technique with numerical method to obtain solution of their governing boundary value problems and showed the velocity boundary layer thickness increased with increase of both mixed convection and viscous dissipation. Bidin and Nazar[2] obtained numerical solutions for boundary layer flow due to exponentially stretching sheet with thermal radiation. They used Keller box method for obtaining solutions of the problem and analyzed the effect of viscous dissipation and thermal radiation on temperature profile. Ishak[3] analyzed the effect of magnetic field on viscous flow and thermal radiation on temperature profile due to an exponentially stretching sheet. He investigated the local heat transfer rate at the surfaces decreases with increasing values of magnetic and radiation parameters and solved the bvp's by implicit finite difference method. Nadim et. al.[4]studied the effects of thermal radiation an boundary layer flow of Jeffrey fluid over an exponentially stretching surface and used two heating conditions namely prescribed surface temperature and prescribed wall heat flux. Swati and Gorla[5] investigate the effects of partial slip on the boundary layer flow due to exponentially stretching sheet in presence of thermal radiation, they obtained the solutions of governing equations numerically using shooting technique and showed velocity and temperature decreases with increasing values of slip parameter and temperature is decreasing with increasing values of thermal slip parameter. Kameshwaran et. al[6]studied the effect of radiation on hydromagnetic Newtonian liquid due to exponentially stretching sheet, they obtained the solution of boundary valued problem both numerical and analytical methods and showed increasing values of magnetic field results in thickening of concentration boundary layer , the combined and individual effects of magnetic parameter, radiation parameter and viscous dissipation parameter are increases the heat transfer rates. Nadeem and Lee[7] analyzed the flow and heat transfer charecteristics of nano fluid due to exponentially stretching sheet. They analyzed the effects of govering parameters on flow and heat transfer profiles and used the Homotopy analysis method. Swati et. al.[8] studied the effects of partial slip on chemical reaction in boundary layer flow due to an exponentially stretching sheet. They obtained the solution by numerical method using shooting technique and showed that fluid velocity and comcentration profiles decreases with increasing slip parameter. They also analyzed the effect of suction on flow, which results in decrease of velocity with increase of suction parameter. Jat and Chand[9] investigated the effects of magnetic field, viscous dissipation and radiation parameter on flow and heat transfer due to an exponentially stretching sheet and solved the bvp's numerically by standard technique. Hayat et. al[10] studied the magnetic effects on nano fluid flow in presence of porous medium with convective boundary conditions over an exponentially stretching sheet and solved the bvp's interms of series solution i,e Homotopy analysis method. Kumari and Nath[11]studied the mixed convection flow of Maxwell's fluid over an exponentially stretching sheet analyzed the effects of magnetic field and viscous dissipation used Chebjshev finite difference method for solving bvp's, they showed that the Nusselt number is slightly decreasing with increasing values of viscous dissipation where as skin friction coefficient slightly increases, Maxwell's fluid reduces skin friction coefficient and nusselt number where as buoyancy force enhance them. Shubhashini et. al.[12] investigated dual solution for mixed convection flow near the stagnation point region of nano fluid over an exponentially stretching /shrinking sheet, they solved the Bvp's by forth order Runge-Kutta method and interpreted that dual solution exists only for shrinking sheet condition. Beb et. al.[13] have given explicit numerical solution for unsteady hydromagnetic mixed convection of a nano fluid from an exponentially stretching sheet in porous medium. They solved the governing equations using Robust explicit finite difference method, they have given detail the study of stability and convergence analysis and showed that the flow is found to be accelerated with increasing thermal and species Grashaff's number. Malik et. al[14] analysed the flow of a Casson nano fluid over an vertical exponentially stretching cylinder, solved the governing system of equations by Runge-Kutta Felburg method and analyzed the governing parameter on the flow, temperature and concentration profiles. Nadeem and Hussain[15] investigated the heat transfer anlaysis of Williamson's fluid due to an exponentially stretching sheet and solved the problem with the help of optimal homotopy analysis method, the optimal convergence control parameter are also obtained for Prescribed surface temperature and prescribed wall heat flux. Choudhary et. al.[16] studied the thermal radiation effects on MHD boundary layer flow over an an exponentially stretching surface, numerical solutions obtained for the governing bvps. Mustaffa et. al.[17] studied the variable fluid properties on MHD fluid flow over an exponentially stretching sheet, solved the governing equations by 4th order Runge-Kutta method with shooting technique also carried out the mass transfer analysis. Sandeep and Sulochana[18] obtained dual solution for radiative MHD nanofluid over an exponentially stretching sheet with heat source / sink.

On observing the above literature Authors did not discussed the combined effect of viscous dissipation and thermal radiation with velocity and temperature jump effects and also skin friction due to various values of slip parameter and wall temperature gradient profile for various governing parameters are discussed which are not done earlier.

2. Flow Analysis

A flow of viscous fluid is considered past an exponential stretching sheet. The continuity and momentum equations in the form of partial differential equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Here u and v are the velocity components along x-axis and y-axis respectively. $\upsilon=\frac{\mu}{\rho}$ is kinematic viscosity.

The boundary conditions of considered flow can be written as

$$u=U_0e^{x/L}+L\frac{\partial u}{\partial y}\ ,v=0\ \text{ at }\ y=0\quad\text{ and }\quad u\to\infty\ \text{ at } y\to\infty \tag{3}$$

Equations (1) and (2) can not be solved directly hence they are converted to ODE by using following similarity transformations which are given below.

$$u=U_0e^{x/L}f'(\eta),\ \ v=-\sqrt{\frac{\upsilon U_0}{2L}}\,e^{x/2L}\{f(\eta)+\eta f'(\eta)\}, \tag{4}$$

Equation (1) is satisfies identically by using (4) and equation(2) take the ordinary differential equation form as

$$f'''-2f''+ff''=0 \tag{5}$$

With corresponding to initial/boundary conditions

$$f(0)=0,f'(0)=1+\alpha f''(0)\quad \text{And } f'(\infty)\to 0 \tag{6}$$

Where $\alpha=N_1\sqrt{\frac{U_0\upsilon}{2L}}$ is the velocity slip parameter, Method of solving equation(5) with (6) is explained in the numerical method section.

3.Heat Analysis:

Here we have considered prescribed exponential surface temperature condition for heat transfer analysis. The governing energy equations are written as

$$\rho C_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=\kappa\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y} \tag{7}$$

Where T is the temperature of the flow field, κ is the thermal conductivity of the fluid, μ is the viscosity, ρ is the density, , C_p is the constant pressure, q_r is the radiative heat flux.

q_r is given (as Roseland approximations) as

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \qquad (8)$$

Where σ is Stefan-Boltzman constant and k^* is absorption coefficient and T^4 is given by

$$T^4 = 4T_\infty^3\, T - 3T_\infty^4 \qquad (9)$$

The PEST heating condition is given by

$$T = T_w + T_0 e^{x/2L} \quad \text{at} \quad y = 0 \quad \text{and} \quad T \to \infty \quad \text{at} \quad y \to \infty \qquad (10)$$

Using (4), (8), (9) and (10) in (7) we obtain,

$$\left(1 + \frac{4Nc}{3}\right)\theta'' + \Pr\left(f\theta' - f'\theta + Ef''^2\right) = 0 \qquad (11)$$

Where

$Pr = \frac{\mu C_p}{\kappa}$ is the Prandtl number , $E = \frac{U_0^2}{T_0 C_p}$ is the Eckert number, $Nc = \frac{4\sigma^* T_\infty^3}{k^* k}$ is the radiation parameter.

With the corresponding boundary conditions are

$$\theta(0) = 1 + \beta\theta'(0) \quad \text{at} \quad \eta = 0 \qquad \text{and} \qquad \theta(\infty) \to 0 \quad \text{at} \quad \eta \to \infty \qquad (12)$$

And $\beta = D_1\sqrt{\frac{U_0}{2\nu L}}$ is the temperature jump parameter.

4. Numerical solution

The governing ordinary differential equations (5) and (11) and boundary conditions (6)and (12) are converted into first order system of differential equations with corresponding initial boundary conditions. The missing boundary condition $f''(0)$ and $\theta'(0)$ are obtained using the Newton-Rapson scheme, further the system of differential equations are solved by Runge-Kutta forth order integration scheme by MATLAB (ode45). Various numerical values of $f''(0)$ and $\theta'(0)$ for different set of parameters are calculated and tabulated. For brevity detailed explanation of method of solution is ignored.

5. Discussion of Results

In this paper we have investigated combined effect of thermal radiation and viscous dissipation in presence of velocity slip and thermal jump and calculated various numerical values of skin friction and wall temperature gradient are plotted and discussed.

Figure (1) and (2) are plotted for horizontal flow and velocity profiles for different values of velocity slip parameter and figure(3)-(7) are plotted for temperature profiles for different values of governing parameter. Further figure (8) is plotted for $f''(0)$ for different values of velocity slip parameter. Figure (9)-(16) are plotted for wall temperature gradient profile for different governing parameter and discussion of the result as follows.

Fig.(1) shows the effects of velocity slip parameter on flow profile with increasing values of α, the temperature is decreases initially and after certain distance from the sheet it increases.

Fig.(2) shows the effect of velocity slip parameter on velocity profile, on observing this the velocity profile decreases with increasing values of α . In presence of slip , the flow velocity close to sheet is no longer equal to the stretching velocity of the sheet. With increase of velocity slip fluid velocity decreases because of slip, the pulling of the stretching sheet can be partially transmitted to the fluid.

Fig(3) shows the effect of Eckert number on temperature profile, on observing this graph the thermal boundary layer thickness enhances with increase values of Ec. The effect of increasing the values of Ec in to increase temperature distribution in the flow region. This enhancement happen due to heat energy is stored in the fluid due to fractional heating.

Fig.(4) shows the effect of thermal radiation Nc on the temperature profile which shows that temperature profile increases with increase values of Nc. This is due to physical fact that thermal boundary layer thickness increases with increasing Nc

Figure.(5) shows the effect of Prandlt number Pr on temperature profile which shows that thermal boundary layer thickness decreases with increases value of Pr. The graph reveal that the increase of Pr results in decrease of temperature distribution this is because of decrease of thermal boundary layer thickness with increasing values of slow rate of thermal diffusion. The wall temperature is unity at the wall.

Figure (6) shows the effect of velocity slip parameter α on temperature profile, which shows that temperature profile increases with increase values of velocity slip parameter. Whereas Fig.(7) shows the effect of thermal slip parameter β on temperature profile, shows the thermal boundary layer thickness increases with increasing value of β. These slip effects on temperature profile, the less is transmitted to fluid form sheet, But as temperature gradient increases with increasing value of slip parameters α and β.

Fig.(8) shows the missing boundary condition profile $f''(0)$ for different values of slip parameter α it depicts that it increases with increasing values of α.

Fig.(10), Fig. (11) and Fig. (12) shows the missing boundary condition profile i.e. wall temperature gradient profile for different values of $\beta, Pr, Ec\ and\ Nc$ respectively, they shows that wall temperature gradient enhances with increasing values of $\beta, Pr, Ec\ and\ Nc$ respectivily. Similarly Figs.(13)(14)(15) and (16) shows the wall temperature gradient profile $\theta'(0)$ with respect to β, for different values of α, Ec, Pr and Nc respectively. Which shows wall temperature gradient profile increases with increasing values of respective parameter except Pr. Fig.(15) shows opposite results of other, i.e. wall temperature gradient profile decreases with increasing values of Pr.

The numerical values of $\theta'(0)$ for different values of various parameters are tabulated in Table-1. Wall temperature gradient enhances with increasing values of Ec, Nc, α, β and decreases with increasing values of Pr. Table-2, shows the comparison of results of $\theta'(0)$ for different values of Pr. On observing those values, we can conclude that our results are in good agreement with earlier results.

6. Conclusion

a) Radiation has capacity to enhance the momentum and boundary layer thickness.
b) Velocity profile decreases as increasing the velocity slip parameter.
c) Temperature profiles enhanced due to the increasing parametric values all governing parameter except Pr.
d) Exponential parameter improves the heat transfer rate and velocity profiles.

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Table-1
Comparative Values of wall temperature gradient for different governing parameters

	Bidin and Nazar with E=0(for two values of radiation parameter Nc)		Nadeem et. al. for PEST case with E=0,λ=0,B=0,Vw=0(for two values of radiation parameter Nc)		Swati and Gorla with λ=0,δ=0,S=0(for two values of radiation parameter Nc)		present Result with α=0,β=0,f(0)=0(for two values of radiation parameter Nc)	
Pr	0.5	1	0.5	1	0.5	1	0.5	1
1	0.6765	0.5315	0.68	0.534	0.6765	0.5315	0.67651	0.53124
2	1.0735	0.8627	1.073	0.863	1.0734	0.8626	1.07352	0.86277
3	1.3807	1.1214	1.381	1.121	1.3807	1.1213	1.38075	1.12143

Table-2
Values of wall temperature gradient for different governing parameters

E	Nc	Pr	α	β	-θ'(0)
0.5	0.5	1	0.5	0.5	-0.38983
1	0.5	1	0.5	0.5	-0.34905
2	0.5	1	0.5	0.5	-0.26748
0.5	0	1	0.5	0.5	-0.49975
0.5	1	1	0.5	0.5	-0.32304
0.5	1.5	1	0.5	0.5	-0.27741
0.5	0.5	1.5	0.5	0.5	-0.47647
0.5	0.5	2	0.5	0.5	-0.54053
0.5	0.5	3	0.5	0.5	-0.63237
0.5	0.5	1	0.2	0.5	-0.39982
0.5	0.5	1	0.4	0.5	-0.39387
0.5	0.5	1	0.6	0.5	-0.3856
0.5	0.5	1	0.5	0.2	-0.44766
0.5	0.5	1	0.5	0.4	-0.40737
0.5	0.5	1	0.5	0.6	-0.37375

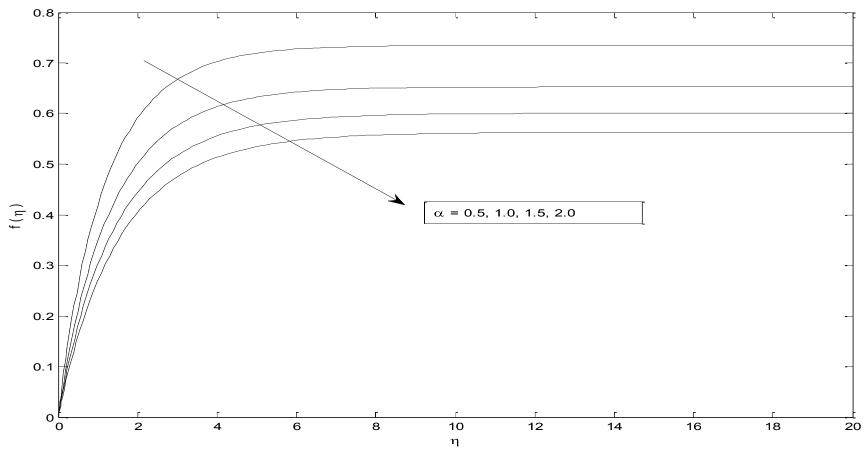


Fig.(1): flow profile f for different values of α

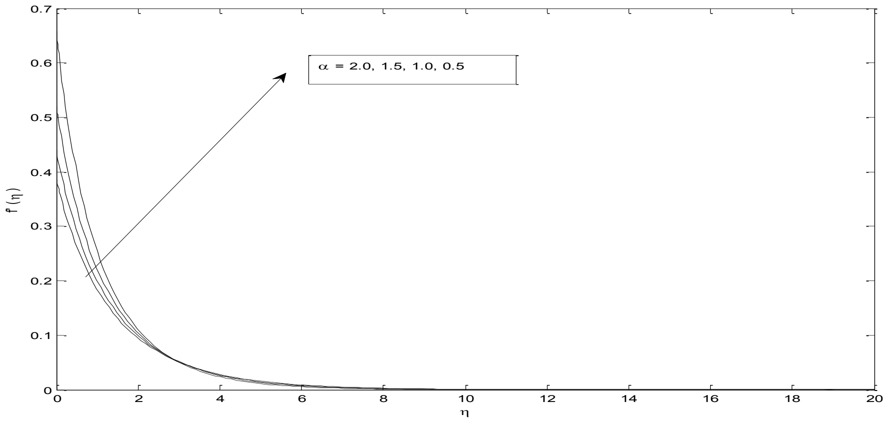


Fig.(2): Velocity profile f' for different values of α

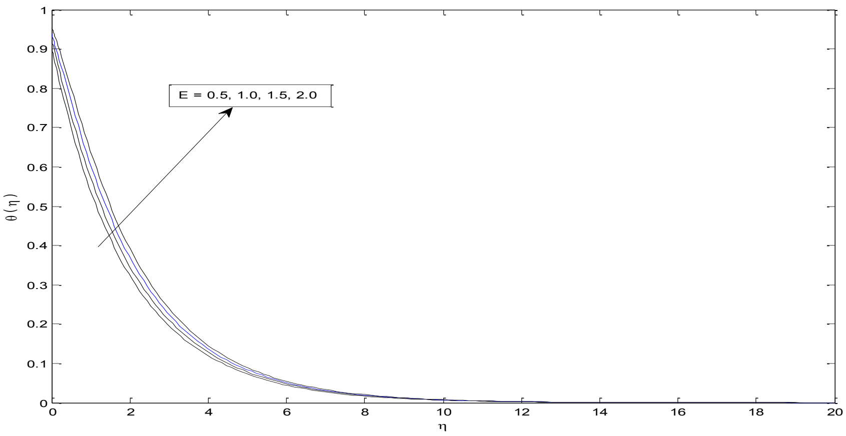


Fig.(3):Temperature profile for different values of E with $Nc = 0.5, \mathrm{Pr} = 1.0, \alpha = 0.\alpha 2, \beta = 0.2$

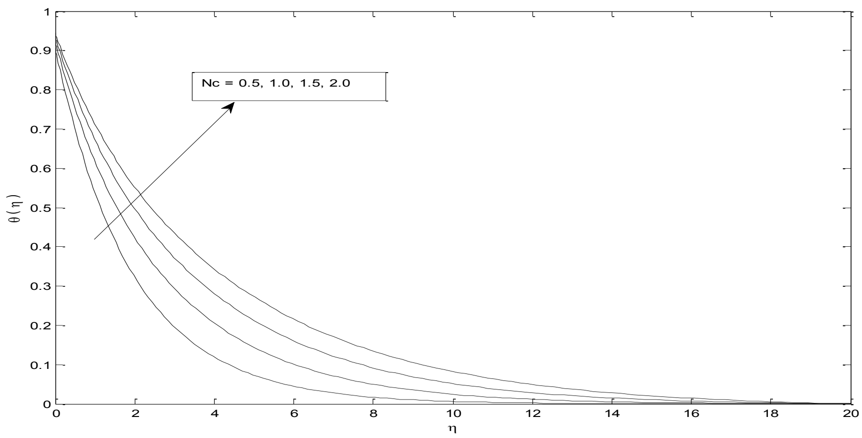


Fig.(4):Temperature profile for different values of Nc with $E = 0.5, \mathrm{Pr} = 1.0, \alpha = 0.2, \beta = 0.2$

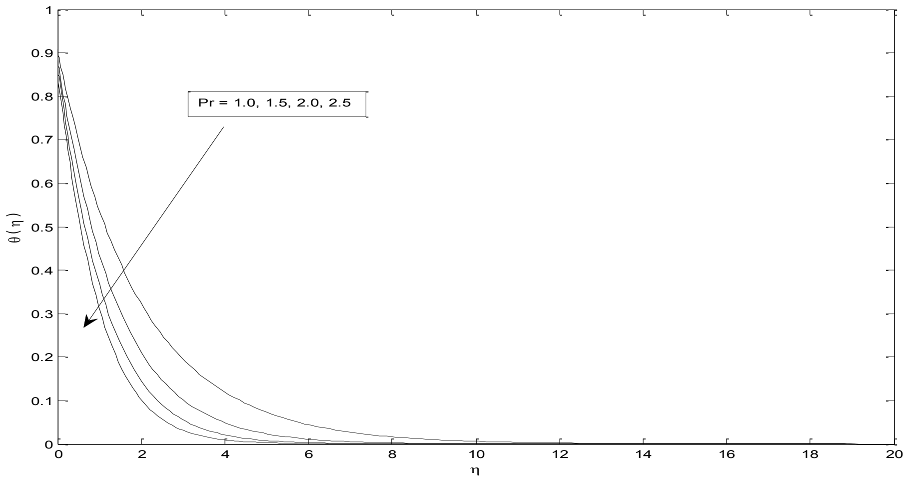


Fig.(5):Temperature profile for different values of Pr with $E = 0.5, \mathrm{Nc} = 0.5, \alpha = 0.2, \beta = 0.2$

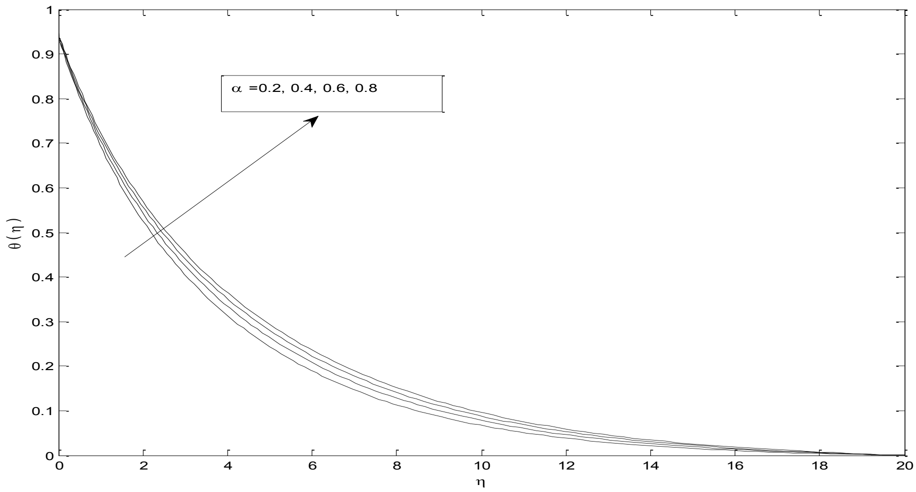


Fig.(6):Temperature profile for different values of α with $E = 0.5, \mathrm{Nc} = 0.5, Pr = 1.0, \beta = 0.2$

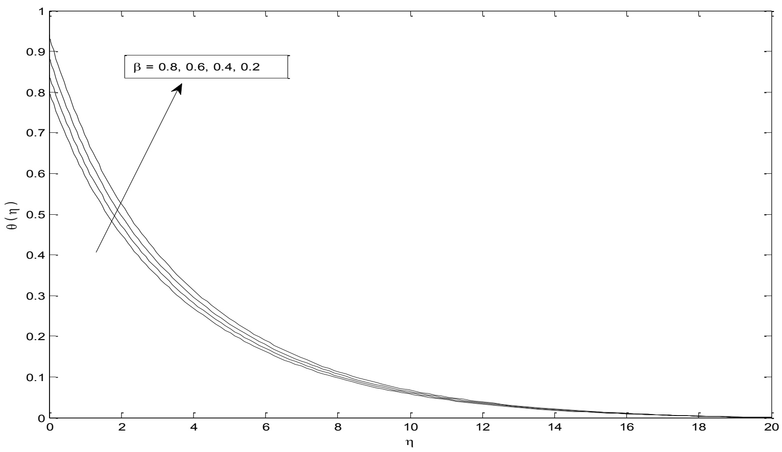


Fig.(7):Temperature profile for different values of β with $E = 0.5, \mathrm{Nc} = 0.5, \alpha = 0.2, Pr = 1.0$

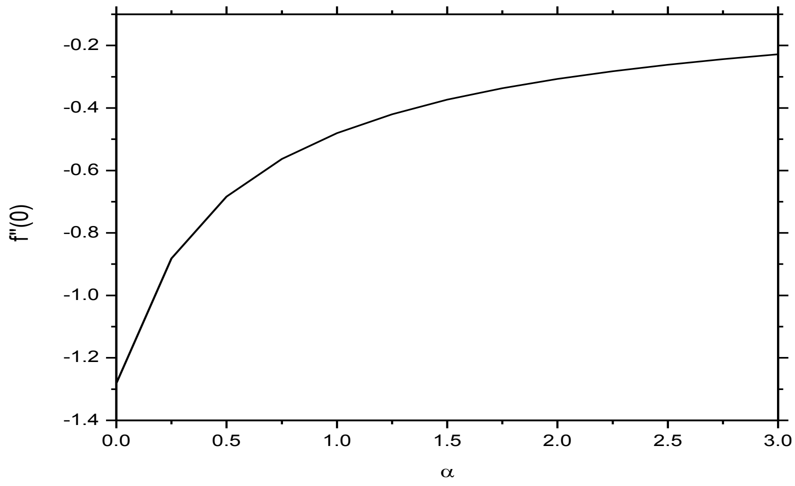


Fig. (8): profile of $f''(0)$ for different values of velocity slip parameter α

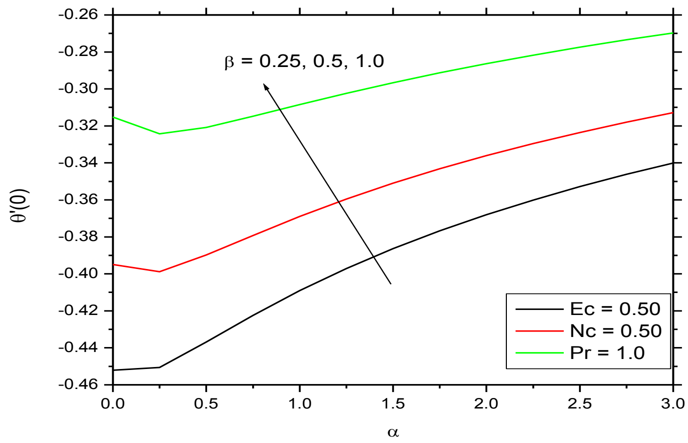


Fig.(9): profiles of $\theta'(0)$ for different values of velocity slip parameter α with $\beta = 0.25, 0.5, 1.0$

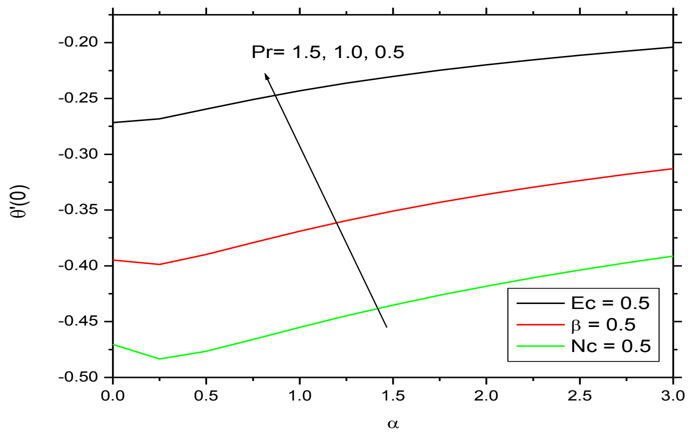


Fig.(10): profiles of $\theta'(0)$ for different values of velocity slip parameter α with $Pr = 0.5, 1.0, 1.5$

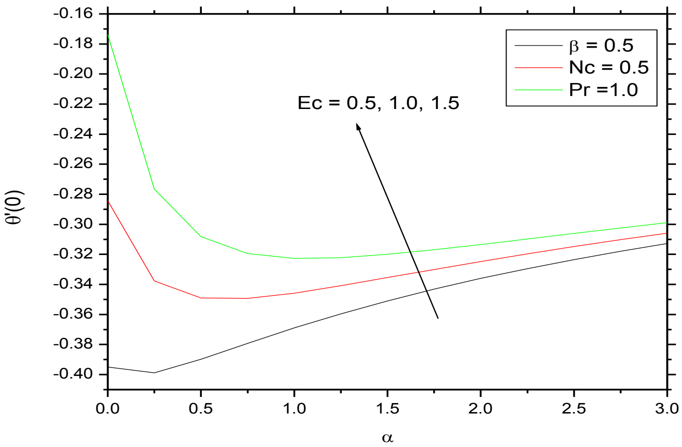


Fig.(11): profiles of $\theta'(0)$ for different values of velocity slip parameter α with $Ec = 0.5, 1.0, 1.5$

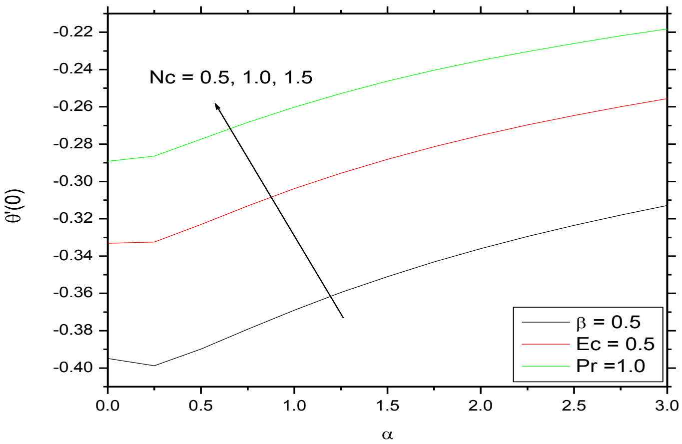
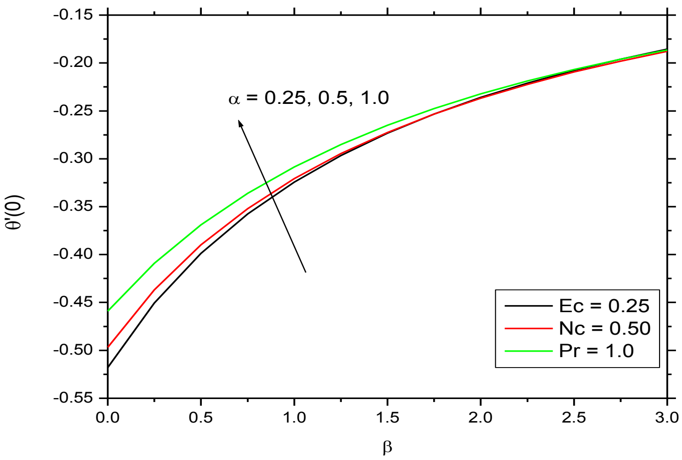


Fig.(12): profiles of $\theta'(0)$ for different values of velocity slip parameter α with $k = 0.5, 1.0, 1.5$



Fig(13): profiles of $\theta'(0)$ for different values of thermal slip parameter β with $\alpha = 0.25, 0.5, 1.0$

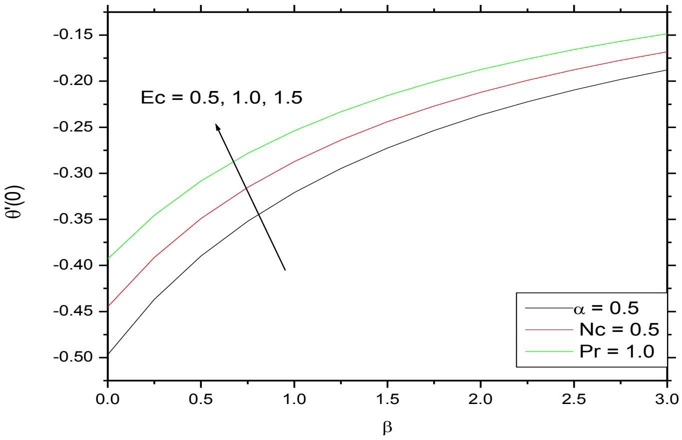
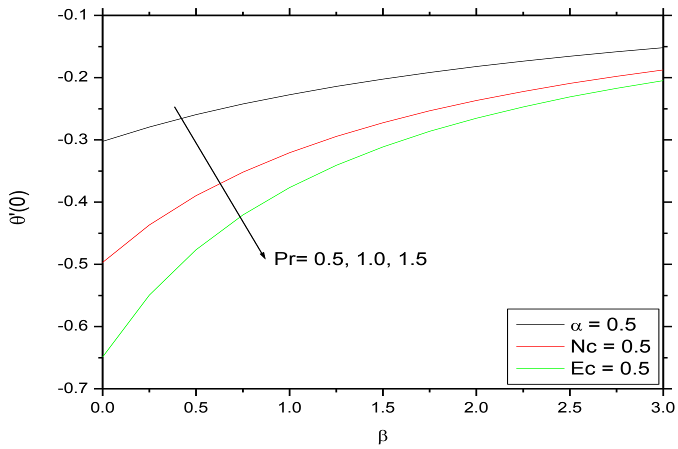
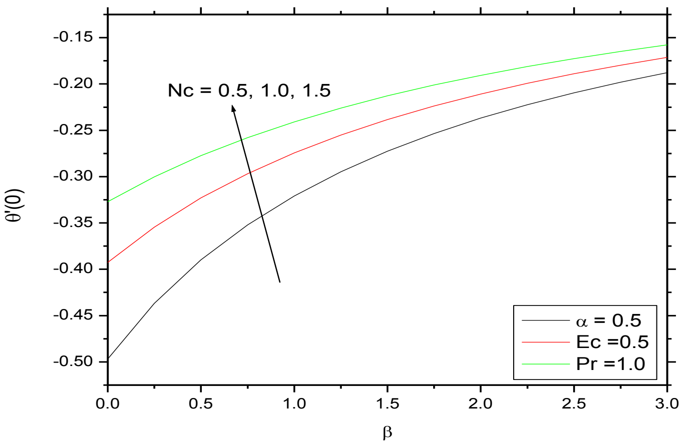


Fig.(14): profiles of $\theta'(0)$ for different values of thermal slip parameter β with $Ec = 0.5, 1.0, 1.5$



Fig(15): profiles of $\theta'(0)$ for different values of thermal slip parameter β with $Pr = 0.5, 1.0, 1.5$



Fig(16): profiles of $\theta'(0)$ for different values of thermal slip parameter β with $k = 0.5, 1.0, 1.5$