Magnetohydrodynamic Radiative Casson Fluid Flow over a Stretching Sheet with Heat Source/Sink

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Abstract
The influence of heat source/sink and thermal radiation on steady magnetohydrodynamic flow of a Casson fluid past a permeable stretching sheet was analysed numerically. We considered nanofluid volume fraction on the boundary is submissive controlled. The transformed equations governing the flow are solved numerically using bvp5c Matlab package. Effects of non dimensional governing parameters on velocity, temperature and concentration profiles are discussed and presented through graphs. Results indicate that the enhancement in Brownian motion and thermophoresis parameters depreciates the nano particle concentration and increases the mass transfer rate.
Keywords: MHD, Stretching, Suction/Injection, Radiation, Heat source/sink.

1. Introduction

The non-Newtonian fluid model is one of the Casson fluid models which were introduced by Casson in 1995. It is based on the model structure and its behavior of both liquid and solid of a two-phase suspension that exhibits yield stress. Casson fluid is well known for shear thinning liquid which is formed to an infinite viscosity at zero, if the shear stress is less than the yield stress is applied to the fluid; it's like a solid, if the shear stress larger than yield stress is applied, and its starts to move. Examples of Casson fluid are as follows: Jelly, tomato sauce, honey, soup, concentrated fruit juices. Human blood also treated as Casson fluid. Many researchers have been more interest in Casson fluid model and explained the boundary layer problem. Animasaun et al. (2016) studied the unequal diffusivities case of homogeneous- heterogeneous reactions within viscoelastic fluid flow in the presence of induced magnetic field and nonlinear thermal radiation

Abbas et al. (2016) discussed the numerical solution of heat and mass transfer on boundary layer flow a stagnation point over a stretching sheet in the presence of thermal radiation. The steady non-Newtonian fluid of a boundary flow represent the power-law model over a stretching sheet with suction was investigated by Ishak et al. (2012). The motion of thermal conductivity and temperature dependent viscosity of steady incompressible non-Newtonian fluid flow over a stretching sheet have been analyzed by Animasaun (2015). Bhattacharyya (2013) discussed the steady boundary layer on heat transfer and Casson fluid towards a stretching sheet and concluded that thermal boundary layer thickness and velocity enhances the Casson fluid. Raju et al. (2016a-2016d) illustrated the heat and mass transfer characteristics of various MHD flows. Das et al. (2104) explained the effects of thermal radiation on unsteady boundary flow of a Nano fluid over a stretching sheet and found that, rise in Lewis number lead to reduce the nanoparticle volume fraction. Ellahi (2009) discussed the non-Newtonian flows and the effects of the slip boundary condition in a channel. Sulochana et al. (2015a,b and 2016a,b), Sandeep et al. (2012, 2013) and Sugunamma and Sandeep (2011) studied the heat and mass transfer characteristics of various fluids by considering different channels.

The study of Soret and Dufour effects on the MHD flow of a Casson fluid over a stretching sheet was investigated by Hayat et al. (2012). The momentum and heat transfer on MHD boundary layer flow over stretching surface in porous medium in the presence of non-uniform heat source/sink was studied by Raju et al. (2016). The buoyancy effects on heat transfer and MHD stagnation point flow of a Nanofluid past stretching sheet have been explained by Makinde et al. (2013) and found that reduced Nusselt number depreciate with magnetic and enhances with convective parameters. The partial slip effects on boundary layer flow past a stretching sheet in the presence of thermal radiation was explained by Mukhopadhyay and Gorla (2012). The MHD boundary layer flow of Casson fluid over an exponentially shrinking sheet was discussed by Nadeem et al. (2012). Pal (2010) investigated effect of magnetic field on heat transfer in the boundary layers over a stretching surface and found that Skin-friction coefficient decreases with depreciate value of temperature distribution. The boundary layer flow of Casson fluid towards an exponentially stretching surface in the presence of blowing at the surface was analysed by Pramanik (2014). Sulochana and Sandeep (2015) analyzed the dual solution of radiative convection flow of a nanofluid towards a stretching sheet in porous medium.

The influence of heat source/sink and thermal radiation on steady magnetohydrodynamic flow of a Casson fluid past a permeable stretching sheet was analysed numerically. We considered nanofluid volume fraction on the boundary is submissive controlled. The transformed equations governing the flow are solved numerically using bvp5c Matlab package. Effects of non dimensional governing parameters on velocity, temperature and concentration profiles are discussed and presented through graphs.

2. Mathematical formulation

Consider a steady, incompressible, two dimensional MHD flow of Casson fluid past a stretching sheet coinciding with the plane $y=0$ and the flow is assumed to be confined to $y>0$. The flow is along the x -axis where x is the coordinate measured along the stretching sheet and y - axis is normal to the surface. A transverse magnetic field B_0 is applied in the y -direction and the stretching sheet velocity is assumed as $u_w(x)=cx,$ where $c>0$ is a constant. The uniform temperature near the sheet assumed as T_w and the temperature, concentration far away from sheet assumed as T_∞, C_∞ respectively.

The equations that governs the present flow with above assumptions subject to the Boussinesq approximations can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho_f}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}-\frac{Q}{(\rho c_p)_{nf}}(T-T_\infty), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

The boundary conditions of equations (1) to (4) are

$$\begin{aligned}&u=U_w, v=0, T=T_w, D_B C'+(D_B/T_\infty)T'=0 \;\; at \;\; y=0,\\ &u\to 0,\;\; T\to T_\infty,\;\; C\to C_\infty \qquad as \;\; y\to\infty,\end{aligned} \tag{5}$$

In equation (5) prime denotes differentiation with respect to y ,where u and v are the velocity components in the x and y directions respectively, T is the temperature, C is the nanoparticles volume fraction, $\tau=(\rho c)_p/(\rho c)_f$, where $(\rho c)_p$ is the effective heat capacity of the nanoparticles, $(\rho c)_f$ is the heat capacity of the base fluid, $\alpha=k/(\rho c)_f$ is the thermal diffusivity of the fluid, ν is the kinematic viscosity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, ρ_f is fluid density, σ is electrical conductivity, β is the Casson parameter.

By using Roseland approximation, the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* is the Steffen Boltzmann constant and k^* is the mean absorption coefficient. Considering the temperature differences within the flow sufficiently small such that T^4 may be expressed as the linear function of temperature. Then expanding T^4 in Taylor series about T_∞ and neglecting higher-order terms takes the form

$$T^4 \cong 4T_\infty^3 T-3T_\infty^4, \tag{7}$$

In view of equations (6) & (7), equation (3) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{1}{(\rho c_p)_{nf}}\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}-\frac{Q}{(\rho c_p)_{nf}}(T-T_\infty), \tag{8}$$

By introducing the following similarity transforms

$$\psi=c^{1/2}\nu^{1/2}xf(\eta), \eta=c^{1/2}\nu^{-1/2}y,$$

$$\theta(\eta) = (T - T_\infty)/(T_w - T_\infty), \phi(\eta) = (C - C_\infty)/C_\infty, \quad (9)$$

Where the stream function ψ is defined as $u = \partial\psi / \partial y \; and \; v = -\partial\psi / \partial x$ which identically satisfies equation (1). Further $f(\eta), \theta(\eta) \; and \; \phi(\eta)$ are dimensionless stream, temperature and concentration functions respectively. Substituting equation (9) in to (2)-(4), we obtain

$$\left(1+\frac{1}{\beta}\right) f''' + ff'' - f'^2 - Mf' = 0, \quad (11)$$

$$\left(1+\frac{4}{3}R\right)\theta'' + Nb\theta'\phi' + Nt\theta'^2 + \Pr f\theta' - \Pr Q_H \theta = 0, \quad (12)$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + Lef\phi' = 0, \quad (13)$$

The boundary conditions (5) reduce to

$$\begin{aligned} & f(0) = 0, f'(0) = 1, \theta(0) = 1, Nb\phi'(0) + Nt\theta'(0) = 0, \\ & f'(\eta) \to 0, \theta(\eta) \to 0, \phi(\eta) \to 0 \;\; as \; n \to \infty, \end{aligned} \quad (14)$$

Here prime denotes differentiation with respect to η, where $\Pr = \nu/\alpha$ the Prandtl is number and $Le = \nu/D_B$ is the Lewis number, Nb is the Brownian motion parameter, Nt the Thermophoresis parameter and M is the magnetic field parameter, which are given as

$$S = v_w (c\nu)^{-1/2}, Nt = \frac{\tau D_T (T_w - T_\infty)}{\alpha T_\infty}, Nb = \frac{\tau D_B C_\infty}{\alpha}, R = \frac{4T_\infty^3 \sigma^*}{(\rho c_p)_{nf} \alpha k^*}, M = \frac{\sigma B_0^2}{c\rho_f}, \quad (15)$$

3. Results and Discussion

The system of nonlinear ordinary differential equations (11) to (13) with the boundary conditions (14) are solved numerically using bvp5c with Matlab package. The results obtained shows the influences of the non dimensional governing parameters, namely thermal radiation parameter R, magnetic field parameter M, Brownian motion parameter Nb, thermophoresis parameter Nt and on velocity, temperature and concentration profiles are investigated thoroughly.

Figs. 1-3 exhibits effect of magnetic field parameter on velocity temperature and concentration. It is evident that increase in magnetic field parameter decreases the velocity profiles. Generally, an increase in magnetic field generates the opposite force to the flow, called Lorentz force. This force causes to reduce the momentum boundary layer. It is also observed that an increase in magnetic field parameter increases the temperature and concentration. The reason behind this is increasing in magnetic field enhances the boundary layer thickness of thermal and concentration. The similar type of results has been observed for rising values of Casson parameter, which is displayed in Figs. 4-6.

Figs. 7 and 8 illustrate the effect of Radiation parameter on temperature and concentration profiles. It is clear that increasing in radiation causes increase in the fluid temperature. These show the domination of radiative heat transfer in Rosseland approximation. But radiation shows reverse action on concentration field. That is increase in radiation parameter causes to decrease in concentration profiles. This result can be explained by the fact that the decrease in the value of R means a decrease in the Rosseland radiation absorptivity k^*.

Figs. 9 and 10 give the effect of the thermophoresis parameter on temperature and concentration profiles. It is noticed that the temperature profiles as well as the boundary layer thickness of the concentration field increases with increasing in *Nt*. Fig. 11 represents the effect of the Brownian motion parameter on concentration profiles. It is evident that the Brownian motion parameter have tendency to reduce the concentration field. The similar type of result has been observed for increasing values of heat source/sink parameter.

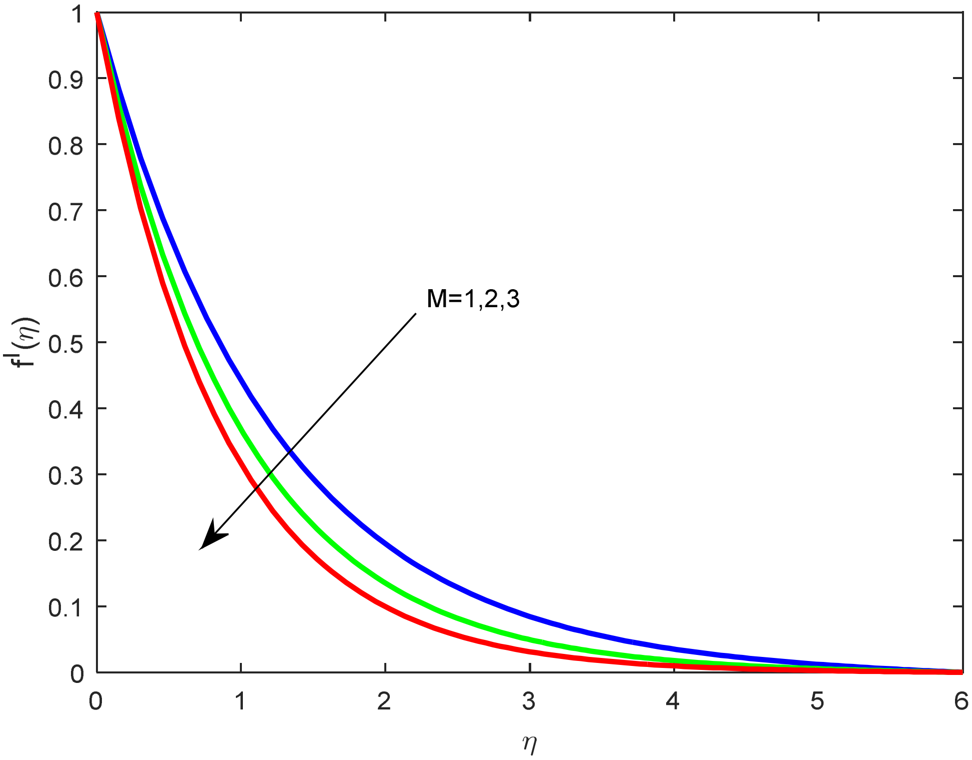

Fig.1 Velocity field for various values of M

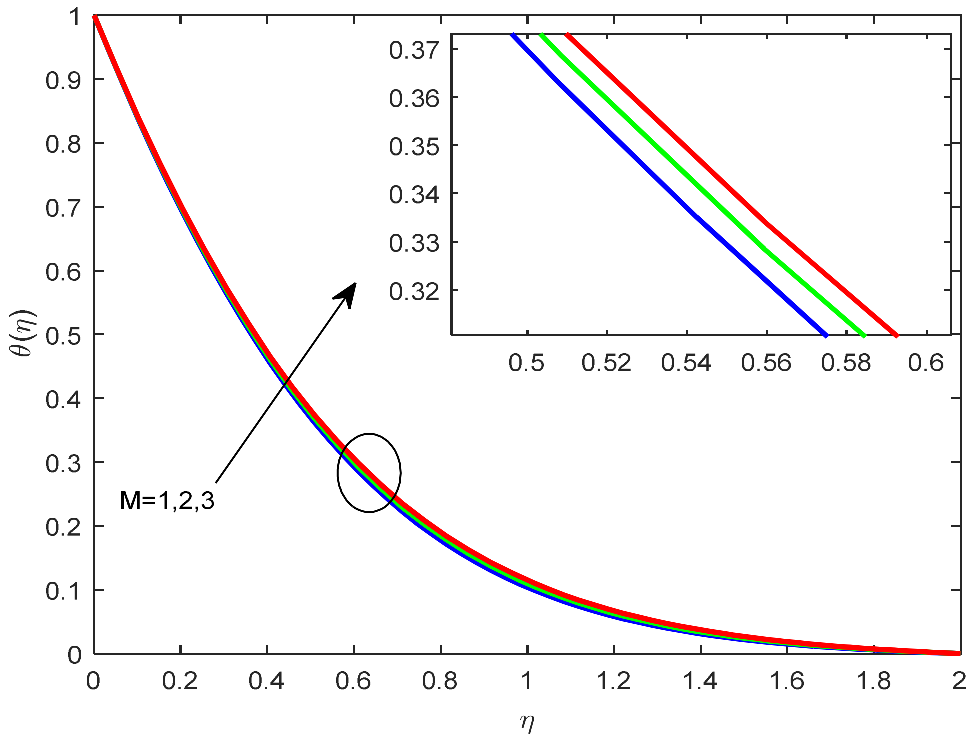

Fig.2 Temperature field for various values of M

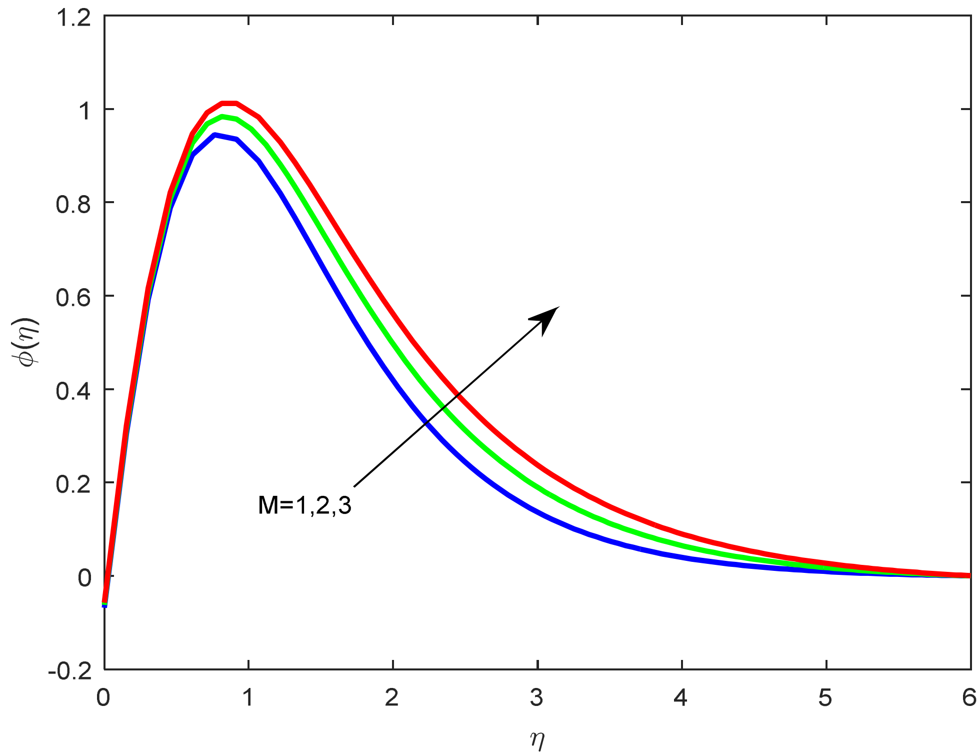

Fig.3 Concentration field for various values of M

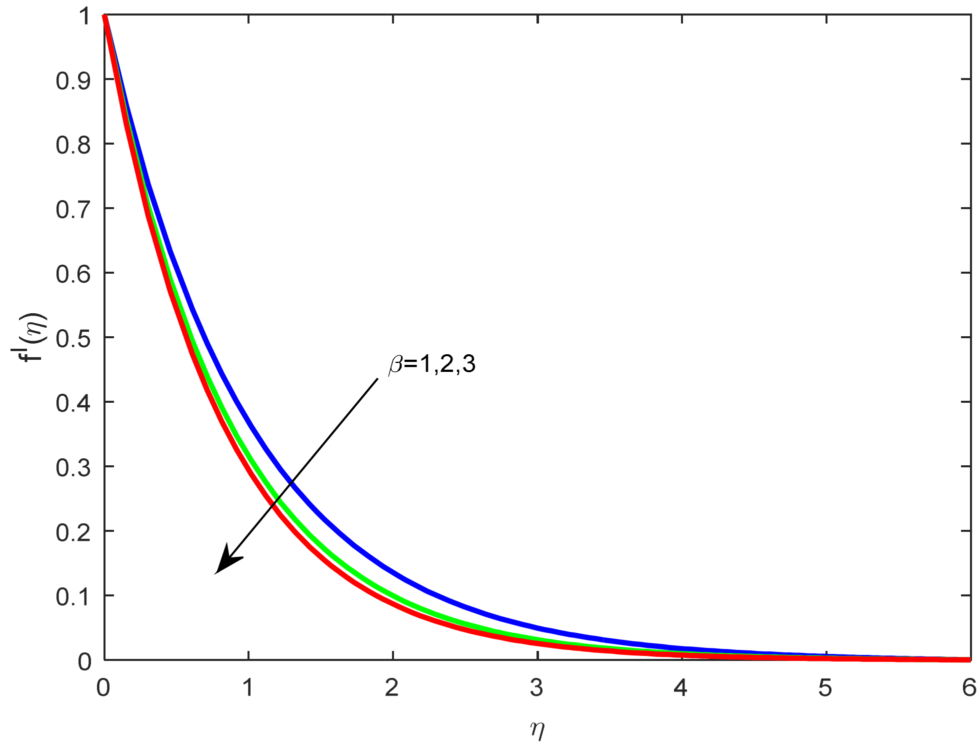

Fig.4 Velocity field for various values of β

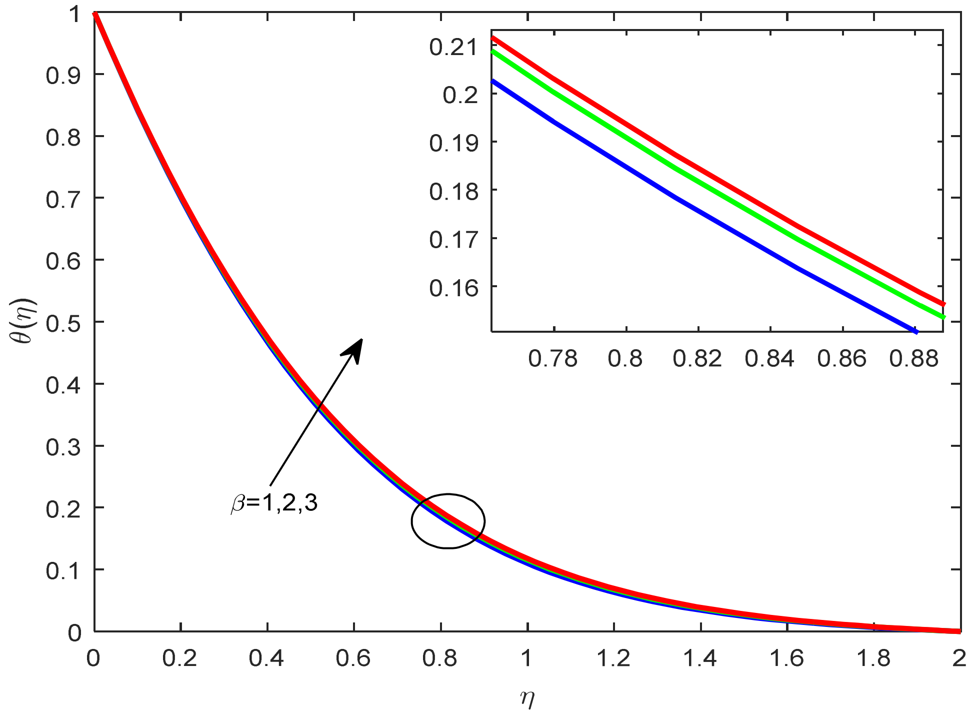

Fig.5 Temperature field for various values of β

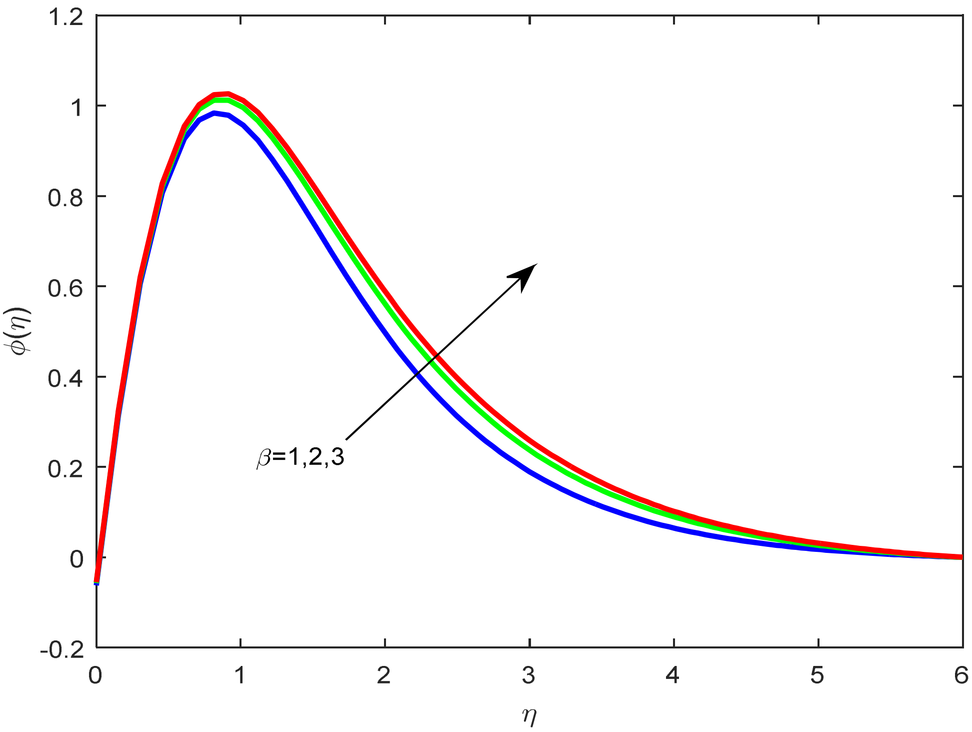

Fig.6 Concentration field for various values of β

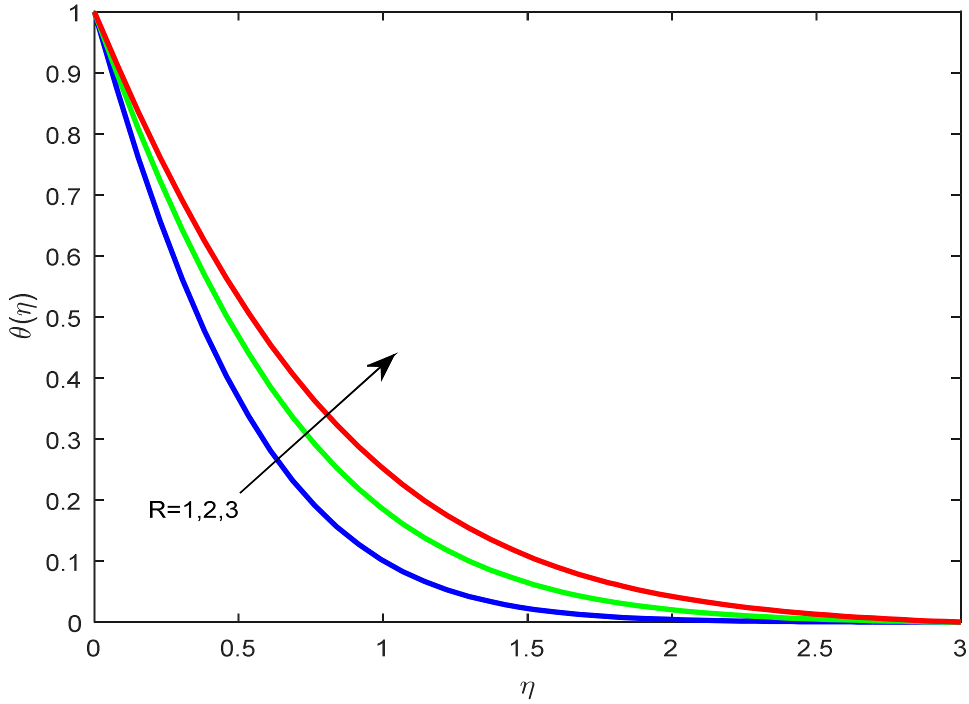

Fig.7 Temperature field for various values of R

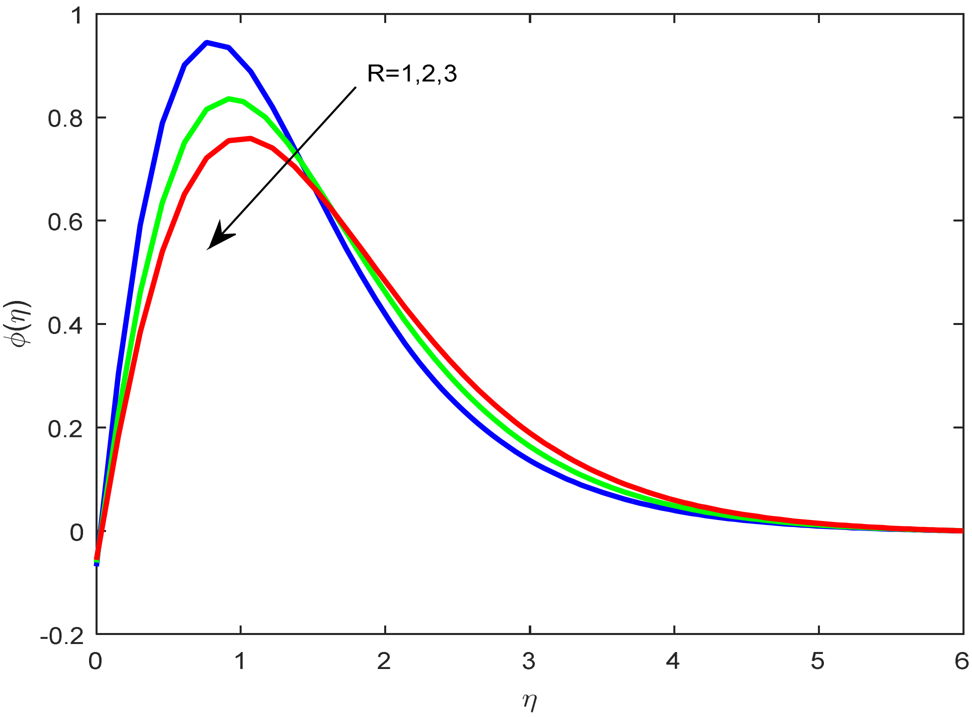

Fig.8 Concentration field for various values of R

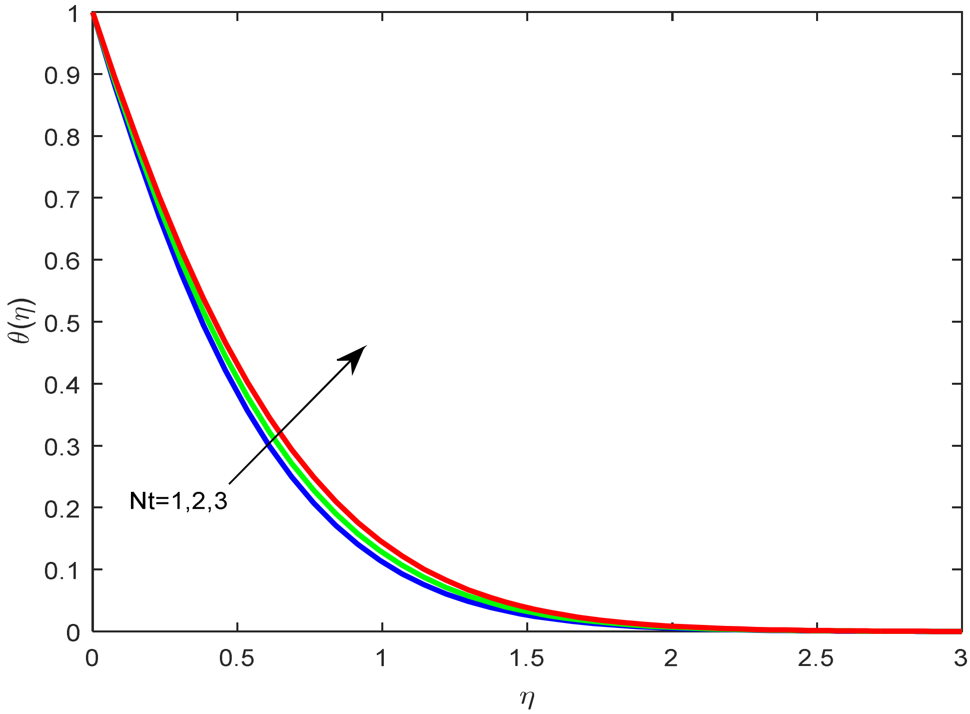

Fig.9 Temperature field for various values of *Nt*

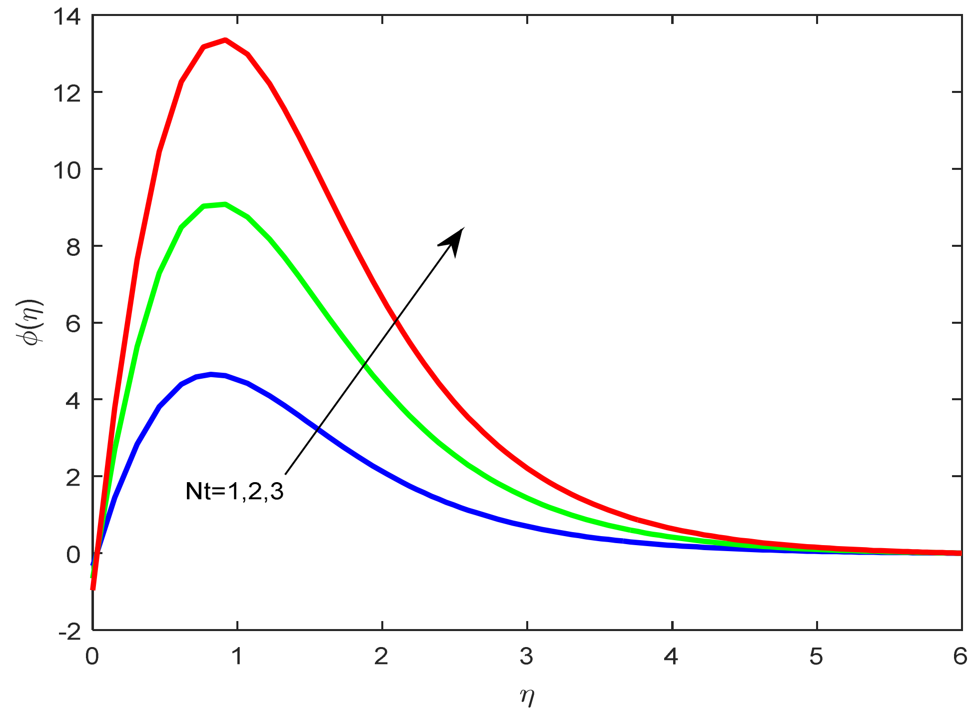

Fig.10 Concentration field for various values of *Nt*

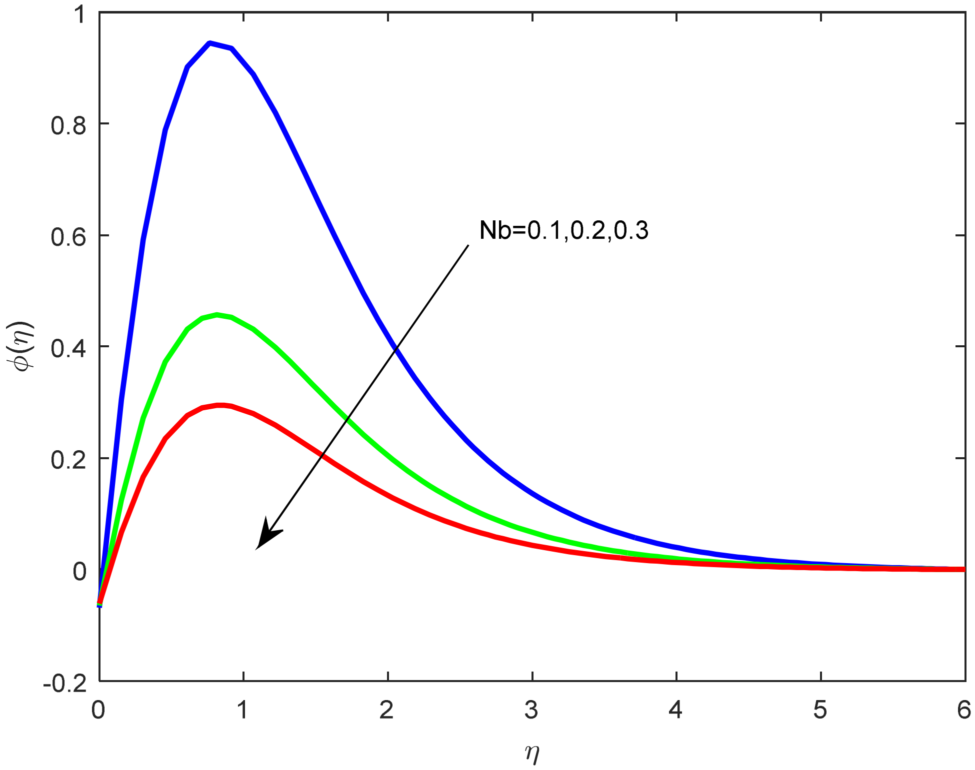

Fig.11 Concentration field for various values of *Nb*

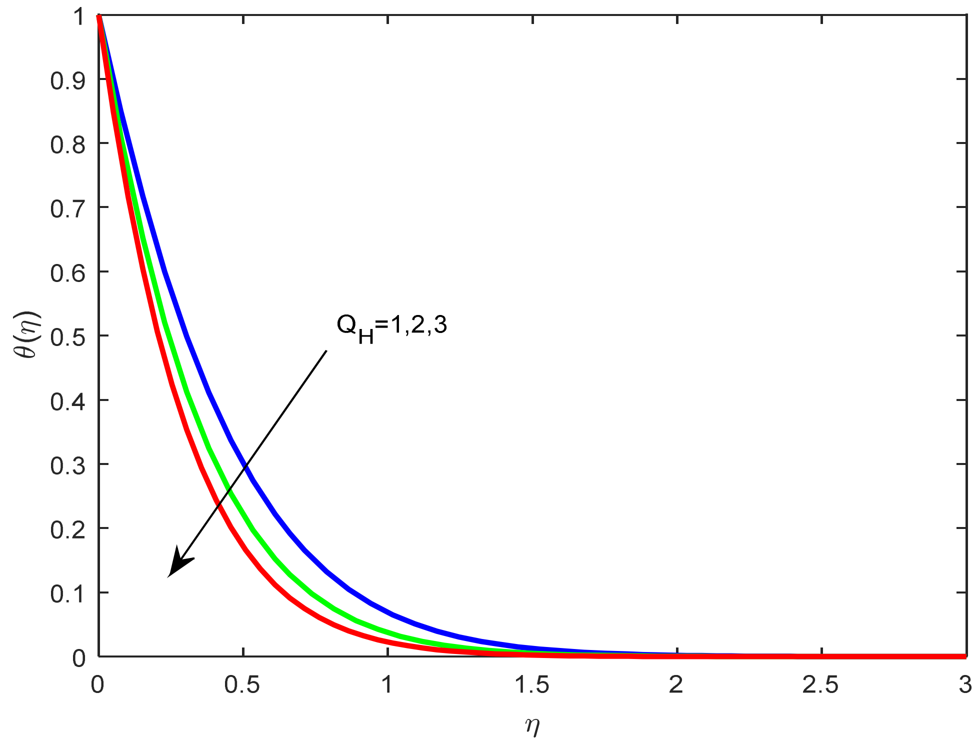

Fig.12 Temperature field for various values of heat source/sink

4. Conclusion

The influence of heat source/sink and thermal radiation on steady magnetohydrodynamic flow of a Casson fluid past a permeable stretching sheet was analysed numerically. We considered nanofluid volume fraction on the boundary is submissive controlled. The transformed equations governing the flow are solved numerically using bvp5c Matlab package. Effects of non dimensional governing parameters on velocity, temperature and concentration profiles are discussed and presented through graphs. Conclusions are as follows:

- Rising values of heat source/sink parameter reduces the temperature filed.
- Magnetic field parameter have tendency to increase the temperature and concentration fields and reduce the velocity field.
- Thermophoresis and Brownian motion parameters have shown reverse response in temperature and concentration filed.
- Radiation parameter has tendency to increase the temperature filed.

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