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Non-Newtonian Momentum Transfer Past an Isothermal Stretching Sheet with Applied Suction

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Abstract
The paper discusses the flow of an incompressible non-Newtonian fluid with heat and mass transfer due to stretching of a plane elastic surface in a saturated porous medium in the approximation of boundary layer theory. An exact analytical solution of non-linear MHD momentum equation governing the self-similar flow is given. The skin friction co-efficient decreases with increase in the visco-elastic parameter k_1 and increase in the values of both magnetic parameter and permeability parameter. The analysis of heat and mass transfer in this flow reveals that when the wall and ambient temperature held constant, temperature at a point increase with increase in k_1, k_2 and M_n .

I Introduction

The study of magneto convection flow of non-Newtonian fluids over a continuously moving porous wall has wide applications in technological and manufacturing processes in industries. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing these strips are sometimes stretched. Besides glass fibre an industries and paper production, manufacturing of polymeric sheets are some of the examples of practical applications of continuous moving flat surfaces.

Above cited numerous applications of non-Newtonian fluids have led to renewed interest among researchers to investigate visco-elastic boundary layer flow over a stretching plastic sheet (Rajgopal et. al., 1984, Dandapat et. al., (1989), Rollins and Vajravelu (1991), Anderson (1992) and Lawrence and Rao (1992), Char (1994), Rao (1996). Chaim (1982) considered the motion of power law fluid flow past a stretching sheet. Unlike the inelastic power law model, the Rivlin Erickson fluid studied by Siddappa and Khapate (1976) and Walters liquid B' considered by Siddappa and Abel (1986), Veena et. al., (2009), Shajahan et. al., (2008), Joshi et. al., (2007) in which both fluids exhibit normal stress differences in simple shear flows. A great deal of literature is available including those cited above on the two-dimensional visco-elastic boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from a fixed origin. However, Gupta and Gupta (1977) have pointed out that realistically stretching of the sheet may not necessarily by linear.

This situation was dealt by Kumaran and Ramanaiah (1996) in their work on boundary layer flow over a quadratic stretching sheet. But their work was confined to the viscous flow past a stretching sheet. McCormack and Crane (1973) have provided comprehensive discussion on boundary layer flow caused by stretching of an elastic flat sheet moving in its own plane with a velocity varying linearly with distance. Rajgopal, Na and Gupta (1984) analyzed the effects of visco-elasticity on the flow of a second order fluid with gradually fading memory and they arrived at the same governing boundary layer equation as that in [2,9,11,12 & 13]. The influence of uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching surface was studied by Pavlov (1974). MHD flow of visoc-elastic fluids was probably first considered by Sarpakaya (1961), while Anderson (1992) and Dandapat et. al. (1989) have obtained the similarity solutions of the boundary layer equation governing the flow in an elastic power-law fluid in the presence of external magnetic field.

Whereas Abel et. al., (1998), Veena et. al., (2007), Veena et. al., (2003), Rajgopal et. al., (2005) have obtained the non-similar solutions of viscous and visco-elastic boundary layer flow with suction blowing, steady and unsteady aspects with porosity and magnetic field past continuously moving stretching bodies.

Khan and Sanjayanand (2005) have studied the visco-elastic boundary layer flow and heat transfer over an exponential stretching sheet.

Sajid et. al., (2007), investigated the non-similar analytical solutions for MHD flow and heat transfer in a third order fluid past a stretching sheet.

Abel et. al., (2008), have analyzed the MHD flow of a visco-elastic fluid over a stretching sheet. Pantokratoras (2008) made a numerical investigation of MHD boundary layer flow with variable viscosity past a stretching surface

Motivated by all the above analyses in the present paper we are concerned with the study of combined effects of visco-elasticity, magnetic field and porous parameter over a continuously moving stretching surface in the presence of suction.

II. Flow analysis

An incompressible second-order fluid has a constitutive equation based on the assumptions of principle given by Coleman and NoLL [27] as

$$T=-pI+\mu A_1+\alpha_1 A_2+\alpha_2 A_1^2 \qquad ...(1)$$

Where T is the stress tensor, p the pressure, μ, α_1, α_2 are the material constants with $\alpha_1 < 0$ and A_1, A_2 are defined as

$$A_1 = (\text{grad } \nu) + (\text{grad } \nu)^T \qquad …(2)$$

$$A_2=\frac{d}{dt}A_1+A_1.\,\text{grad}\,\nu+(\text{grad}\,\nu)^T,\ A_1 \qquad …(3)$$

We considered the flow of a fluid obeying the constitution equation (1) displays normal stress differences in shear flow and is an approximation to a simple fluid in the sense of retardation. This model is applicable to some dilute polymer solutions and is valid at low rates of shear.

The fluid obeying (1) past a flat porous sheet coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are applied along x-axis so that the wall is stretched keeping the origin fixed in a uniform magnetic field.

The steady incompressible two dimensional boundary layer equations of motion for visco-elastic (Walters liquid B' in the presence of magnetic field and porous medium obtained by Bread and Walters [28] in usual notation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad …(4)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-v\frac{\partial^2 u}{\partial y_2}+k\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y_2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right]=0 \qquad …(5)$$

Where $\nu=\frac{\mu}{\rho}$, $k\frac{-\alpha 1}{\rho}$

In deriving the above equations it is assumed that in addition to the normal stress is of the same order of magnitude as that due to shear stresses. Thus both ν, k are of $0(\delta^2)$, δ being the boundary layer thickness.

The appropriate boundary conditions are

$u = bx$, $\nu = \nu_w$ at $y = 0$ …(6)

$u \rightarrow 0$ as $y \rightarrow \infty$, $b > 0$ (b is stretching rate …(7)

The flow is caused solely by the stretching of the wall and the free stream velocity being zero. Equations (1) & (2) admit self-similar solution of the stype.

$$u=bxf_\eta(\eta):\quad \nu=-(b\nu)^{1/2} f(\eta);\quad \eta=\left(\frac{b}{\nu}\right)^{1/2} y \qquad …(8)$$

Equation of continuity is satisfied identically and substituting equation (8) in equation (2) it converts to

$$f_\eta^2(\eta)-f(\eta)f_{\eta\eta}(\eta)-f_{\eta\eta\eta}(\eta)+k_1\,[2f_\eta(\eta)f_{\eta\eta\eta}(\eta)-f_{\eta\eta}{}^2(\eta)-f(\eta)f_{\eta\eta\eta}(\eta)]$$
$$+(k_2+Mn)f_\eta(\eta)=0 \qquad …(9)$$

Where suffix η denotes differentiation w.r.t. η and

$$k_1=\frac{kb}{\nu},k_2=\frac{\nu}{k'b};M_n=\frac{\sigma B_0^2}{\rho b} \qquad …(10)$$

and velocity components given in (8)

$$f_\eta(0)=1,\quad f(0)=\frac{-\nu_o}{\sqrt{b\nu}};\ f_\eta(\infty)=0, \qquad \text{v}…(11)$$

Equation (9) subjected to the three boundary conditions (11) was derived by author [6']. But since equation (9) is of fourth order and highly non-linear, satisfying only three boundary conditions of equation (11). This difficulty was over come in [6] by expanding $f(\eta)$ in a power series interms of visco-elastic parameter k_1, assuming k_1 as very small.. This of course, is valid for dilute polymer solutions.

Hence we observed one more boundary condition included in (11) to get unique exact analytical solution of equation (9) that isf Khan and Sanjayanand (2005) have studied the visco-elastic boundary layer flow and heat transfer over an exponential stretching sheet.

Sajid et. al., (2007), investigated the non-similar analytical solutions for MHD flow and heat transfer in a third order fluid past a stretching sheet.

Abel et. al., (2008), have analyzed the MHD flow of a visco-elastic fluid over a stretching sheet. Pantokratoras (2008) made a numerical investigation of MHD boundary layer flow with variable viscosity past a stretching surface $f_{\eta\eta}(0) = 0$.

Thus it is interesting to note that equation (9) has a solution of the form

$$f_\eta(\eta) = \exp(-\alpha\eta),\ \alpha > 0 \qquad \text{...(12)}$$

Satisfying the condition $f_\eta(0) = 1$ and $f_\eta(\infty) = 0$ in (11). Integrating (12) and using $f(o) = R = \frac{-V_w}{\sqrt{b\nu}}$, the suction parameter,

We obtain

$$f(\eta) = \frac{1-\exp(-\alpha\eta)}{\alpha} - \frac{V_w}{\sqrt{b\nu}} \qquad \text{...(13)}$$

where α is the positive root of cubic equation

$$\alpha^3 - A_3\alpha^2 + A_4\alpha - A_5 = 0 \qquad \text{...(14)}$$

is found by Graffe's root square method.

Thus from equation (13) we get remarkably simple exact analytical solution of equation (9) satisfying the boundary conditions (11). This gives the new velocity components u and v as

$$u = bx\exp(-\alpha\eta);\quad v = -\sqrt{\frac{b}{\rho}\upsilon}\left[\frac{1-\exp(-\alpha\eta)}{\alpha} - R\right] \qquad \text{...(15)}$$

Where 'α' is calculated from the equation (14) as

$$\alpha = \left[\frac{1+M_n+k_2}{1-k_1}\right]^{1/2} \qquad \text{...(16)}$$

Dimensionless shear stress τ at the wall is obtained as

$$\tau = f_{\eta\eta}(0) = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\left(\frac{1-k_1}{1+M_n+k_2}\right)^{1/2}$$

$$\tau = -\mu\left[\alpha^2 bx\right] \qquad \text{...(17)}$$

Which shows that τ vanishes when $k_1 = 1$. But this can never occur since the solution includes the combined effects of visco-elastic and magnetic forces.

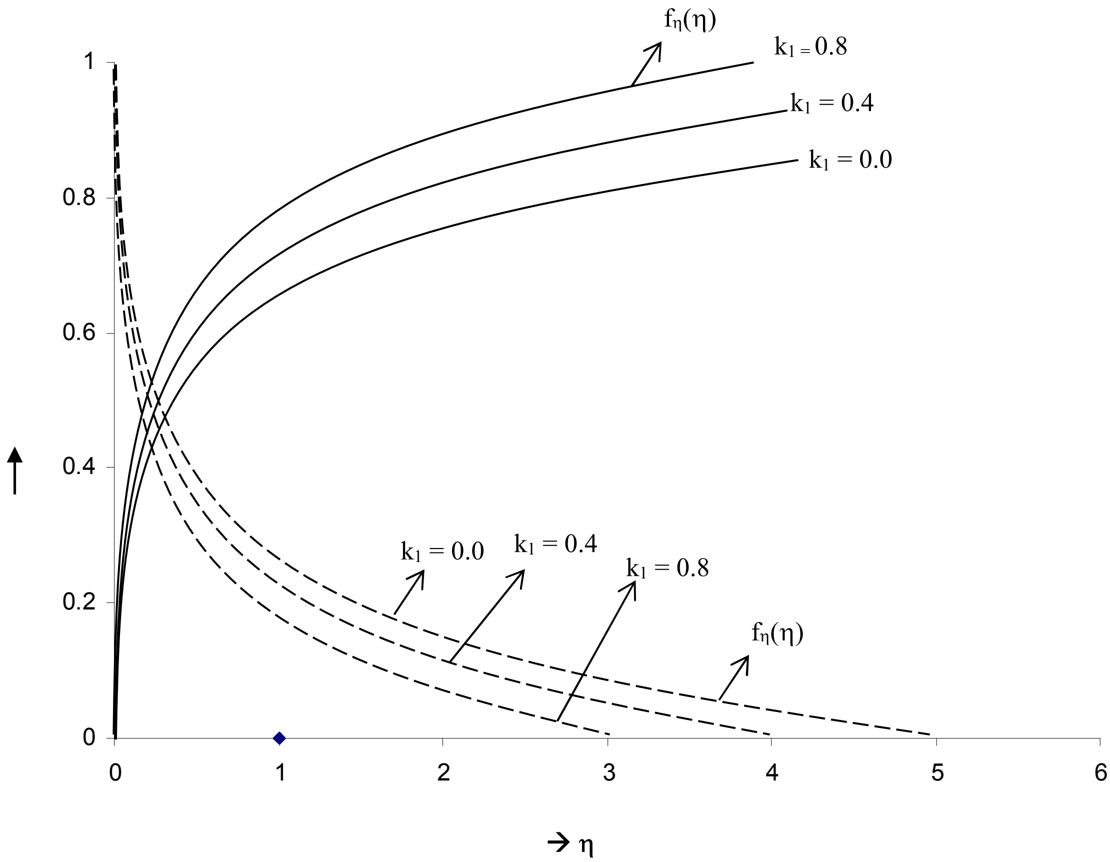

Fig. 1: Velocity variation in longitudinal and transverse directions for various values of visco-elastic parameter k_1 and fixed values of $k_2 = 2$ and Mn = 2, R = -0.424

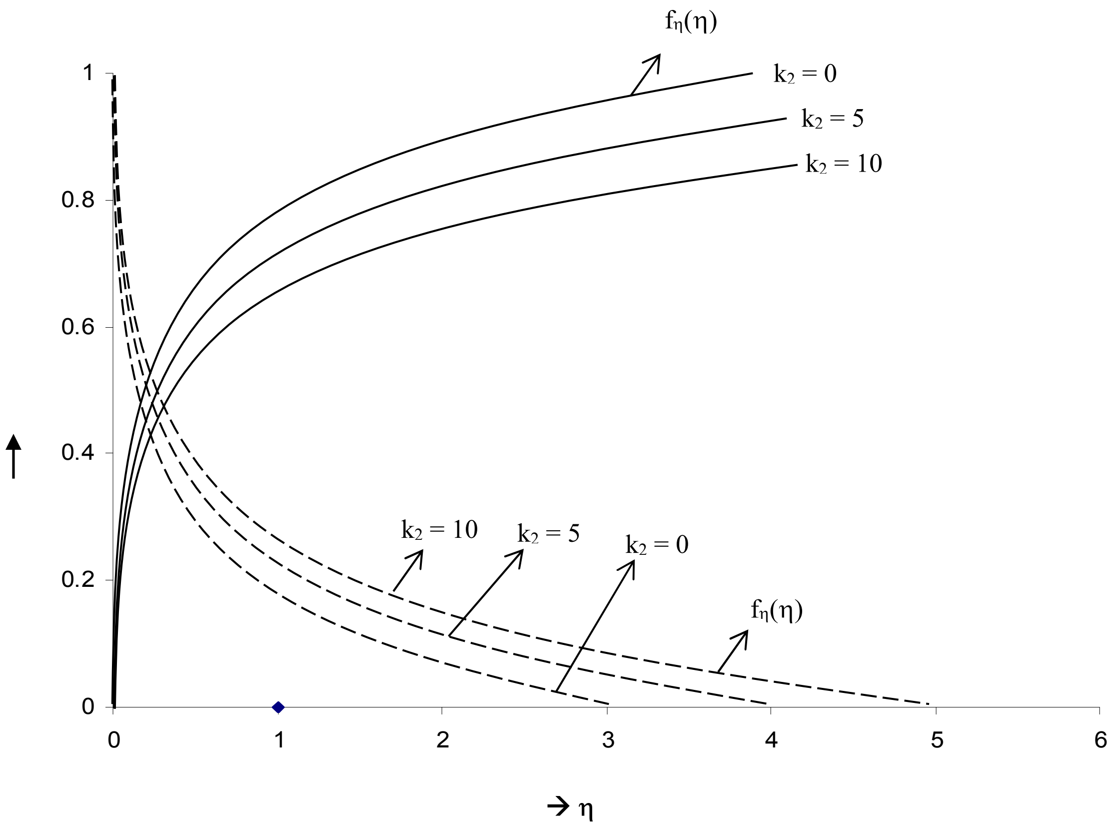

Fig. 2: Velocity profiles f(η) and $f_\eta(\eta)$ Vs. η for various values of permeability parameter k_2 = 0, 5, 10 and fixed values of k_1 = 0.4, Mn = 2, R = -0.424

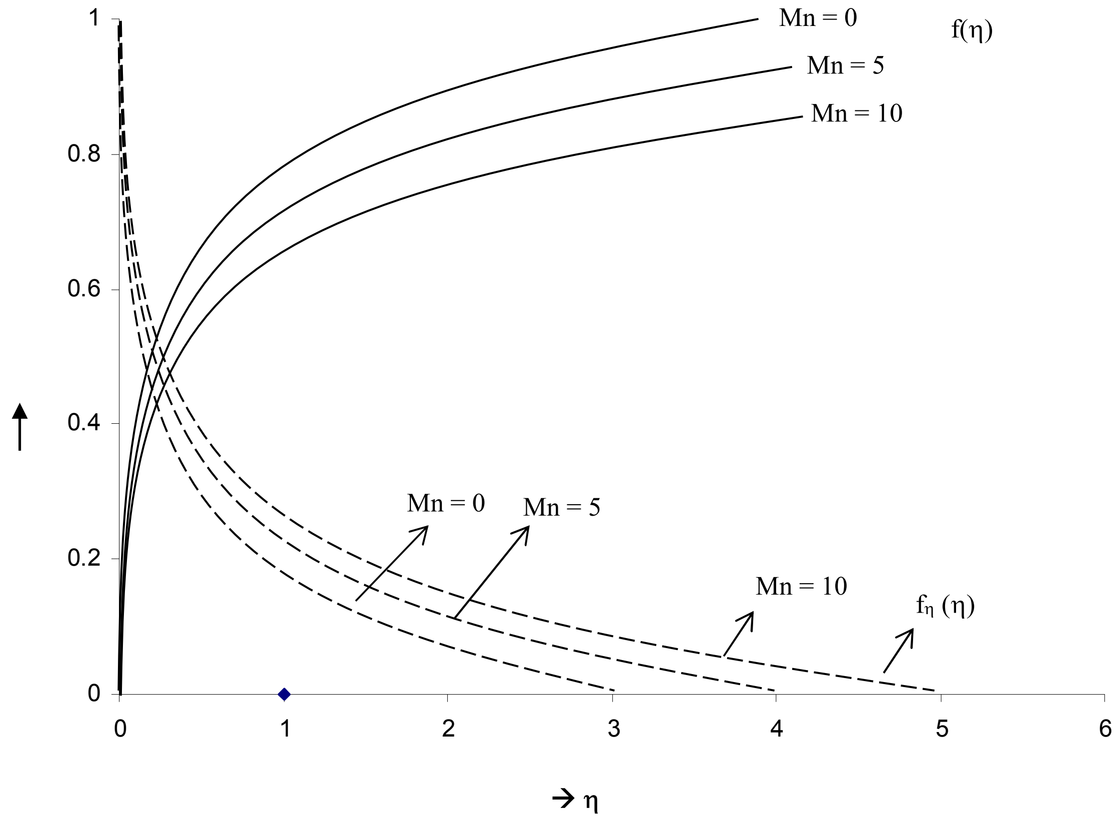

Fig. 3: Velocity distribution of f(η) and f_η(η) Vs. η for different values of Mn = 0,5,10 and fixed values of k_1 = 0.4, k_2 = 5, R = -0.424

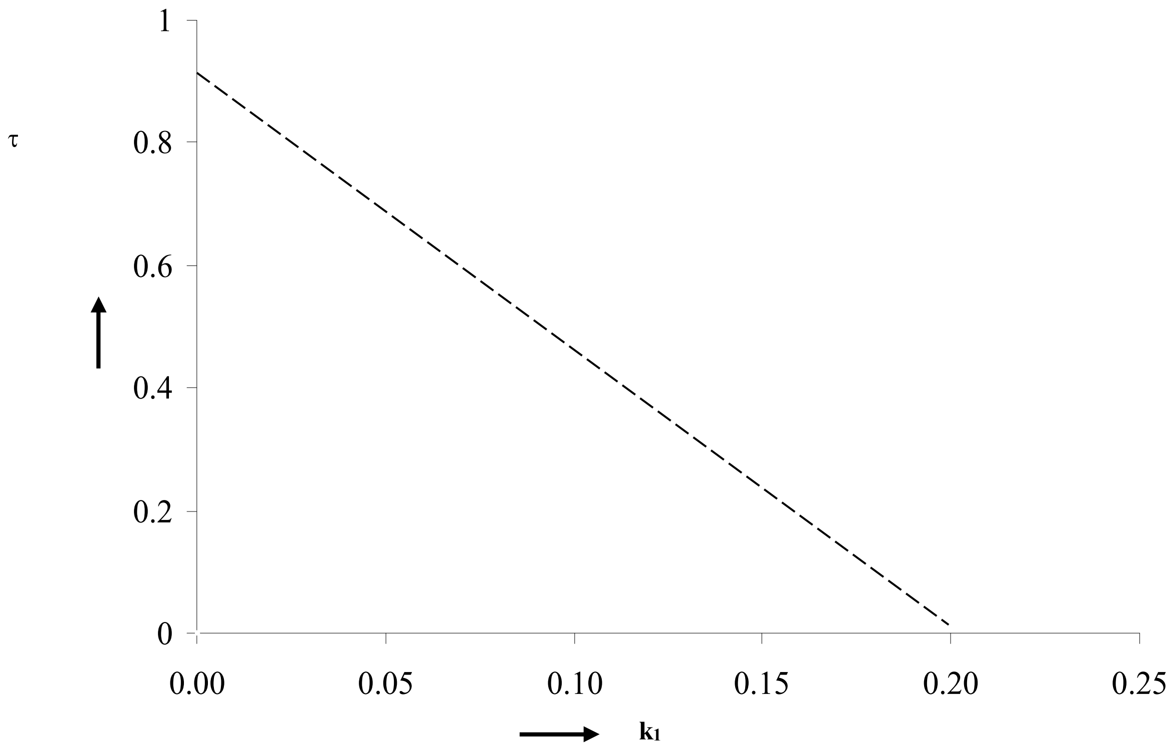

Fig. 4: Variation of skin friction co-efficient τ Vs. visco elastic parameter k_1 for values of k_2 = 5, Mn = 5, and R = 0.424

III Results and discussion

To explain the effects various physical concepts such as boundary layer thickness δ, stream wise velocity, wall friction co-efficient τ, visco-elasticity, permeability and magnetic parameter many graphs are drawn and discussions are made as follows.

Momentum boundary layer flow in a visco-elastic fluid past an exponentially stretching permeable sheet have been investigated in the present study. The governing basic boundary layer equation of momentum is highly non-linear and converted into ordinary differential equation by applying suitable similarity transformations.

Fig. 1 is the graph of velocity profiles $f(\eta)$ and $f_\eta(\eta)$ for various values of visco-elastic parameter for fixed values Mn and k_2, R. It is observed from the figure that the transverse flow velocity $f(\eta)$ is a decreasing function and longitudinal flow velocity $f_\eta(\eta)$ is an increasing function visco-elastic parameter k_1.

Fig. 2 and Fig. 3 depict the graphs of f (η) and $f_\eta(\eta)$ versus η for different values of permeability parameter k_2 and magnetic parameter Mn respectively. In both the cases velocity distribution f (η) along transverse direction is a decreasing function of k_2 and Mn where velocity profile $f_\eta(\eta)$ along longitudinal direction is an increasing function of k_2 and Mn respectively for the value of suction parameter R = -0.424.

Fig. 4 demonstrates the graph of skin friction co-efficient versus visco-elastic parameter k_1 for fixed values of k_2, R and Mn. From the figure it is noticed that the increase in k_1 leads to the decrease of skin friction. This is due to the fact that elastic property in visco-elastic fluid reduces the frictional force. This result has great significance in polymer processing industry, as the choice of higher order visco-elastic (Walters liquid B') fluid would reduce the power consumption for stretching the boundary sheet.

IV Conclusions

The distinguish feature of the above exact analytical results is that the effect of visco-elasticity, magnetic field and porosity are combined into the single parameter $\alpha \geqq 1$ with added suction V_w which is defined in equation (16). Since magnetic parameter Mn and permeability parameter k_2 are positive $Mn \geqq k_2 \geqq 0$ and $0 \leqq k_1 \leqq 1$, it can be concluded that an increase in values of Mn and k_2 have the same influence on the flow field as increased velocity. According to equations (15) and (17) the main effects of visco-elasticity, magnetic field and permeability are to reduce the velocity within the boundary layer and the external velocity normal to the sheet and also to reduce the boundary layer thickness while increasing in skin friction.

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NOMENCLATURE

A_3, A_4, A_5	-	Positive constants.
B_0	-	Induced magnetic field
k_1	-	Visco-elastic parameter
k_2	-	Permeability parameter
Mn	-	Magnetic parameter
x,y	-	Coordinate system
u,v	-	Components of velocity along x and y directions
R	-	Suction parameter.
p	-	Pressure
T	-	Stresstensor
b	-	Stretching rate.

Greek symbols

τ	-	Non-dimensional skin friction co-efficient
α	-	Positive root of cubic equation
μ	-	Limiting viscosity
η	-	Similarity variable
σ	-	Electrical conductivity
ψ	-	Stream function
ρ	-	Density
ν	-	Kinemetic viscosity
μ, α_1, α_2	-	Material constants