Soret and Dufour Effect on Unsteady Free Convective MHD Heat and Mass Transfer Flow with Variable Permeability, Heat Source and Thermal Diffusion

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Abstract
The present paper deals with the study of unsteady two dimensional free convection with heat and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a continuously moving infinite vertical plate under the influence of transverse magnetic field with variable permeability, heat source and thermal diffusion. The permeability of the porous medium fluctuates in time about a constant mean. The free stream velocity of the fluid vibrates about a mean constant value and the surface absorbs the fluid with constant velocity. Introducing the usual similarity transformations, the unsteady equations of momentum, energy and concentration are made similar. To obtain local similarity solutions of the problem, the equations are solved analytically after applying perturbation technique. The velocity field, temperature field, concentration field and skin friction co-efficient are shown graphically to observe the effects of various parameters entering in the problem. Finally a thorough discussion of different results are presented.

Introduction
Flows through porous media has become of principle interest in many scientific and engineering applications such as petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs and in water purification processes. Magneto hydrodynamic flows through porous media also have applications in meteorology, solar physics in an interaction of the geometric field with the fluids in the geothermal region, in cosmic fluid dynamics, astrophysics, geophysics and in the motion of earths core. In addition, from technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magneto spheres, aeronautics, chemical engineering and electronics, on account of their varied importance, these flows have been studied by many authors notable amongst them are Sherliff [1], Soundalgekar [2], Ferraro and Plumpton [3] and Cramer and Pai [4]. An extensive contribution on heat and mass transfer flow has been made by Gebhart and Pera [5] to highlight the insight of the phenomena. There after many authors have paid their attention towards the study of MHD free convection flow and mass transfer flows. Raptis et. al. [6] studied the mass transfer phenomena on the steady two dimensional flow through porous medium bounded by an infinite vertical plate with constant suction. Singh et. al., [7] investigated the effect of permeability variation on free convective flow in a porous medium bounded by a vertical porous wall when the permeability varies in a direction. Singh et. al., [8] discussed the effect of permeability variation on free convective flow through a porous medium past an infinite porous plate with constant suction. Helmy [9] studied the unsteady laminar free convective flow of an incompressible, viscous electrically conducting dusty fluid through a porous medium, bounded by an infinite vertical plane surface of constant temperature in the presence of uniform magnetic field, acting perpendicular to the surface. Recently again Singh et. al., [10] studied three dimensional fluctuating flow and heat transfer through a porous medium with variable permeability. Hassanian et. al., [11] in their work, discussed the effects of unsteady two dimensional free convection and mass transfer flow of a viscous, incompressible and electrically conducting fluid through a highly porous medium under the influence of a transverse magnetic field of uniform strength. More recently, Singh [12] studied about the effects of mass transfer on MHD free convection flow of a viscous fluid through a vertical channel using Laplace transform technique considering symmetrical heating and cooling of channel walls.

All the above mentioned authors have not studied the effect of heat source parameter and thermal diffusion, although the natural convection in enclosures has become increasingly important in engineering application. The study of Chu et al [13] represents a major contribution to the natural convection with concentrated heat sources. Besides, due to the fast growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy supplies to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen, helium) where thermal diffusion effect is found to be a magnitude that can not be neglected, which was investigated by Eckert and Drake [14].

In view of the application of heat source and thermal diffusion effect, Atul Kumar Singh [15] have studied two dimensional MHD free convection and mass transfer flow past an infinite vertical porous plate taking into account the combined effect of heat source and thermal diffusion in the presence of large suction. The similarity solutions are obtained [Hasen 16] by employing the perturbation technique.

Motivated by the above studies in the present paper it is proposed to study the effect of permeability variation on hydromagnetic flow through a porous medium in the presence of MHD free convection with heat and mass transfer flow. The porous medium is bounded by a vertical surface with constant temperature. This surface absorbs the fluid with a constant velocity and the free stream velocity of the fluid vibrates about a mean constant value. In unsteady mass diffusion equation the term corresponding to thermal diffusivity (DJ) is added following Sattar and Alam [17]. The permeability of the porous medium is assumed to be of the form

$$k'(t') = k'(1+\varepsilon e^{i\omega' t'}) \qquad \ldots(1)$$

where k′ is the mean permeability of the medium, ω' is the frequency of fluctuation, t′ is the time and t (<<1) is constant.

Mathematical formulation

Consider an unsteady two dimensional MHD free convection and mass transfer flow of an incompressible viscous fluid through a highly porous medium bounded by non-conducting continuously infinite vertical plate with constant suction.

Introducing the Cartesian co-ordinate system, x-axis is chosen along the plate in the direction of flow and opposite to the direction of gravity and y-axis normal to it.

A uniform magnetic field is applied normal to the flow region. All the fluid properties are assumed constant except that the influence of density variation with temperature is considered only in the body force term and the permeability of the porous is a function of t. The applied magnetic field is of uniform strength and is traversally to the direction of the flow. The magnetic Reynolds number of the flow is taken so small that the induced magnetic field can be neglected.

The physical variables are functions of y′ and the time t′ only and therefore the equations expressing the conservation of mass momentum, energy and the equation of mass transfer within a concentration boundary layer are given as follows. In addition the analysis is based on the following assumptions:

1. The plate temperature is instantly raised from T_∞ to T_w where T_∞ is the temperature of the uniform flow, T′ is the temperature of the fluid in the free stream.
2. The concentration is instantly raised from C_∞ to C(x), where C_∞ is concentration of the uniform flow.
3. The induced magnetic field is assumed to be negligible, so that $\overline{B} = (0, B_o(x), 0)$.
4. The equation of conservation of electric charge is $\vec{\nabla}.\vec{J} = 0$, where $\vec{J} = \left(J_x, J_y, J_z\right)$.
5. The joule heating and viscous dissipation terms are assumed to be negligible.
6. The magnetic field applied is insufficient to cause joule heating so that the term due to electrical dissipation and internal heat generation is neglected in the energy equation.
7. The density is a linear function of temperature and species concentration.

Besides u′, v′ are the components of velocity in the x′ and y′ directing , g is the acceleration due to gravity, ρ the density of the fluid, p the pressure, μ the viscosity, B_o the magnetic induction, σ the electrical conductivity of the fluid. D and C_p are the co-efficient of chemical molecular diffusivity and the specific heat of the fluid at constant pressure respectively and k is the thermal conductivity of the fluid in the equation.

Thus equations expressing the conservation of mass, momentum and energy and the equation of mass transfer with thermal diffusivity, D_T and molecular diffusivity within the concentration boundary layer are given by

$$\frac{\partial v'}{\partial t'} = O \qquad \ldots(2)$$

$$\rho\left(\frac{\partial u'}{\partial t} + v'\frac{\partial u}{\partial y'}\right) = -\frac{\partial p}{\partial x} + \rho g + \mu\frac{\partial^2 u^1}{\partial y^2} - \frac{\mu}{k_1^{'}(t')}u' - \sigma B_o^2 u' - k_3 u^2 \qquad \ldots(3)$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \qquad \ldots(4)$$

$$\frac{\partial c'}{\partial t'} + v'\frac{\partial c'}{\partial y'} = D_M\frac{\partial^2 c'}{\partial y'^2} + D_T\frac{\partial^2 c'}{\partial y'^2} \qquad \ldots(5)$$

where g is the acceleration due to gravity, T′, C′, D_M, C_p and D_T are the temperature of the fluid in the free stream, the corresponding concentration, the co-efficient of chemical molecular diffusivity, the specific heat

and the thermal diffusivity of the fluid at constant pressure respectively, k is the thermal conductivity and k_3 is the inertia parameter.
Integrating equation (2) we get

$$v^1 = -v_0 \quad \ldots(6)$$

where $v_0 > 0$ is a constant and negative sign indicates the suction which is towards the plate.

The respective boundary conditions of the problem are $y' = 0,\ u' = 0,\ T' = T'_w,\ c' = c_w'$ $y' \to \infty, \quad u' \to U(1+\varepsilon e^{i\omega t})$

$$T' \to T'_\infty,\ c' \to c'_\infty \quad \ldots(7)$$

where T'_w and T'_∞ are the surface temperature and the fluid temperature, c'_w and c'_∞ are the corresponding concentrations, U the constant velocity, ω' – the frequency of vibration of the fluid and ε a constant. In the free stream, equation (3) converts to

$$\rho\frac{dU'}{dt'} = -\frac{\partial p}{\partial x'} - \rho_\infty g - \frac{\mu}{k'_1(t')}U' - \sigma B_o^2 U' - k_3 v'^2 \quad \ldots(8)$$

Eliminating $\frac{\partial p}{\partial x'}$ term from equations (3) and (8) we obtain

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = \rho\frac{\partial u'}{\partial t'} + g(\rho_\infty - \rho) + \mu\frac{\partial^2 u'}{\partial y'^2} + \frac{\mu}{k'(t')}$$

$$(U'-u') + \rho B_0^2 (U'-u') + k_3 (U'-u')^2 \quad \ldots(9)$$

By making use of the following equation for density

$$(\rho_\infty - \rho) = \rho\beta(T' - T'_\infty) + \rho\beta^*(c' - c'_\infty) \quad \ldots(10)$$

which is linear function of temperature and species concentration with β as the volumetric co-efficient of thermal expansion, β* the volumetric co-efficient of diffusion expansion and ρ_∞ is the density of the fluid far away from the surface.
Making use of equation (6) in (4), (5) and (9) these equations take the following form.

$$\frac{\partial u'}{\partial t'} - v_o\frac{\partial u'}{\partial y'} = \frac{du'}{\partial t'} + g\beta(T' - T'_\infty) + g\beta^*(c' - c'_\infty) + \gamma\frac{\partial^2 u^2}{\partial y'^2}$$

$$+\frac{\nu}{k'_1(t')}(U'-u') + \sigma B_o^2 (U'-u') + k_3 (U'-u')^2 \quad \ldots(11)$$

$$\frac{\partial T'}{\partial t'} - v_o\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \quad \ldots(12)$$

$$\frac{\partial c'}{\partial t'} - v_o\frac{\partial c'}{\partial y'} = D_M\frac{\partial^2 c'}{\partial y'^2} + D_T\frac{\partial^2 T'}{\partial y'^2} \quad \ldots(13)$$

where ν = μ/ρ is the co-efficient of kinematic viscosity.
Introducing the following non-dimensional quantities:

$$y = \frac{y' v_o}{\nu},\ u = \frac{u'}{U},\ t = \frac{t' v_o^2}{\nu},\ w = \frac{\nu w'}{v_o^2},\ T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ C = \frac{c' - c'_\infty}{c'_w - c'_\infty},$$

$$Gr = \frac{\nu g\beta(T'_w = T'_\infty)}{UV_o^2}, G_m = \frac{\nu g\beta^*(c'_w - c'_\infty)}{UV_o^2}, k = \frac{v_o^2}{\nu^2}k', Sc = \frac{\nu}{D_M}$$

$$P_r = \frac{\rho\nu c_p}{k}, M = \frac{\sigma B_o^2 \nu}{\rho v_o^2} \quad \ldots(14)$$

Introducing all the above dimensionless quantities in equations (11), (12) and (13), they reduce to

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{dU^*}{dt} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{k(1+\varepsilon e^{i\omega t})}(U^* - u)$$

$$+M(U^*-u)+G_r T+G_m C+k_3(U^*-u)^2 \quad \ldots(15)$$

$$P_r\left(\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}\right)=\frac{\partial^2 T}{\partial y^2} \quad \ldots(16)$$

and

$$Sc\left(\frac{\partial c}{\partial t}-\frac{\partial c}{\partial y}\right)=\frac{\partial^2 c}{\partial y^2}+S_o\frac{\partial^2 T}{\partial y^2} \quad \ldots(17)$$

$$S_o=\frac{T_w-T_\infty}{C_w-C_\infty}$$

where $U' = \nu(1+\varepsilon e^{i\omega' t'})$ …(18)

The corresponding boundary conditions transfer to

$y = 0;\ u = 0,\ T = 1,\ C = 1$

$y \rightarrow \infty;\ u \rightarrow 1+\varepsilon e^{i\omega t},\ T = 0,\ C = 0$ …(19)

To solve equations (15) to (17) we assume the perturbation solution for periodic and non-periodic terms of the following type

$$u(y,t)=u_o(y)+\in e^{i\omega t}u_1(y)+\ldots. \quad \ldots(20)$$

$$T(y,t)=T_o(y)+\in e^{i\omega t}T_1(y)+\ldots \quad \ldots(21)$$

$$C(y,t)=c_o(y)+\in e^{i\omega t}c_1(y)t\ldots. \quad \ldots(22)$$

substituting all these perturbations (20), (21) and (22) in (15), (16) and (17) we obtain the following set of differential equations

$$u_{o\eta\eta}+u_{o\eta}-\left(\frac{1}{k_2}+M+2k_3\right)=u_o(k_2 s')-G_r T_o-G_m C_o \quad \ldots(23)$$

$$u_{1\eta\eta}+u_{1\eta}-(s+i\omega)u_1=(s+i\omega)-\frac{u_o}{k_2}-G_r T_1-G_m C_1 \quad \ldots(24)$$

$$T_{o\eta\eta}+P_r T_{o\eta}=O \quad \ldots(25)$$

$$T_{1\eta\eta}+P_r T_{1\eta}-i\omega P_r T_1=O \quad \ldots(26)$$

$$C_{o\eta\eta}+ScC_{o\eta}+D_T\,T \quad \ldots(27)$$

$$C_{1\eta\eta}+Sc\,C_{1\eta}-Sc\,C_1\,i\omega+D_T\,T_1''=O \quad \ldots(28)$$

where suffix η denotes differentiation, w.r.t. it the corresponding boundary conditions (19) take the forms.

$$y=0,u_o=0,u_1=o,T_0=1,c_0=1,c_1=0,T_1=0 \quad \ldots(29)$$

$$y\rightarrow\infty;u_o\rightarrow 1,u_1\rightarrow 1,T_o\rightarrow o,T_1\rightarrow 0,c_o\rightarrow 0,c_1\rightarrow 0 \quad \ldots(30)$$

on solving the ordinary differential equations (23) to (28), the solutions of the equations (20) to (22) subjected to the boundary conditions (29) and (30) are now the expression for velocity in terms of the periodic and non-periodic parts is obtained as

$$u=u_o+\in\exp(i\omega t)[NR+iNI] \quad \ldots(31)$$

substituting Eulers formula for $e^{i\omega t} = \cos\omega t + i\sin\omega t$ we get

$$u=u_o+(NR\cos\omega t+NI\sin\omega t)\in \quad \ldots(32)$$

Again for $\omega t = \frac{\pi}{2}$, get the expression for transient velocity as

$$u=u_o\,NI \quad \ldots(33)$$

Then equation (31) takes the form

$$u=1+(M_1+M_2-1)\exp(R,y)-M_1\exp(-\Pr y)-M_2\exp(-Scy)$$
$$-\in[rA,\exp(rR_3y)+iA_2\exp(2R,y]$$

$$+iA_3 \exp(-P_r y) + iA_4 \exp(-S_c y) \qquad \ldots(34)$$

$$a_1 = \frac{1}{k_2} + Mn + 2k_3 + i\omega$$

$$a_2 = a_1 - i\omega$$

$$M_1 = \frac{G_r}{(P_r + R_1)(P_r + R_2)} \qquad M_2 = \frac{G_m}{(Sc + R_1)(Sc + R_2)}$$

$$R_1 = \frac{-1}{2} + \sqrt{\frac{8}{2}} \qquad S_1 = 1 + 4a_2$$

$$R_1 = \frac{-1\sqrt{S_1}}{2} \qquad R_2 = \frac{-1+\sqrt{S_1}}{2}$$

$$R_3 = \frac{-1+\sqrt{S_1 + 4i\omega}}{2}$$

$$A_1 = -(1 + A_2 + A_3 + A_4)$$

$$A_2 = \frac{1 + M_2 - M_1}{k_2\left[R_1^2 + R_1 - a_1\right]} \qquad A_3 = \frac{M_1}{k_2\left[P_o^2 + P_r - a_1\right]}$$

$$A_4 = \frac{M_2}{k_2\left[Sc^2 + Sc - a_1\right]}$$

where r-denotes real part and i-denotes the imaginary part.

Skin friction

The dimensionless skin friction co-efficient at the plate interms of its amplitude and phase is defined as

$$\tau_p = \left(\frac{\partial u}{\partial y}\right)_{y=o} = \left\{R_1(M_1 + M_2 - 1) + L_1 P_r + L_2\, Sc + \in |W_1| \cos(wt + \alpha)\right\} \qquad \ldots(35)$$

where $|N| = \sqrt{nR^2 + n_1 I^2}$ and $\frac{n_1 I}{n_2 R} = \tan\alpha$

and $n_1R + in_1I = A_1R_3 + A_2R_2 - A_3Pr - A_4$ SC graphical

Nomenclature

B_0	=	Magnetic Induction
$\overline{B}$	=	induced magnetic field
C_p	=	Chemical Molecular diffusivity
τ_p	=	Skin friction
D	=	Diffusivity
k	=	visco-elasticity
k′	=	mean permeability of the medium
P_r	=	Prandtl number
Re	=	Reynolds number
Sc	=	Schimidt number
c'_w and c'_∞	=	Fluid Temperature
P	=	Pressure
G	=	acceleration due to gravity
r	=	Real part
i	=	imaginary part
T	=	Temperature
T_w	=	Wall temperature
T_∞	=	Ambient temperature

T'_w and T'_∞	=	Surface temperature
u, ν	=	Velocity components
x,y	=	Co-ordinate system
k_1	=	Elastic parameter
k_3	=	inertia parameter
t′	=	time
T′, C′, D_M, D_T	=	the temperature of the fluid in the free stream

Greek symbols

ω'	=	frequency of fluctuation
α	=	Positive root of cubic equation
η	=	Dimensionless similarity variable
μ	=	Dynamic viscosity
ν	=	Kinematic viscosity
ρ	=	Density
τ	=	Dimensionless shear stress
u_∞	=	Free stream velocity
β*	=	the volumetric co-efficient of diffusion expansion
Σ	=	Electric Conductivity

Results and Discussion

To study the effects of inertia parameter k_3, magnetic field Mn, permeability parameter k_2, $\omega t = \pi/2$ with various values for w, Grashof number Gr and Gm and oscillations on flow field numerical values are computed from the obtained, analytical solution.

Figure 1 is the graph of variation of velocity distribution for different sets of values of all the above said parameters. We observe from the figure that velocity increase with increasing values of k_2 and decreases with increasing Mn, Gr, the concentration difference between the surface and the free stream and frequency ω.

In Fig. 2 & 3 we have plotted the fluctuating part of the velocity different sets of all said parameters. From the figures it is noticed that, the fluctuating part NR positive where the imaginary part of velocity NI remains negative. The real part of velocity NR increases with increasing the porosity of the medium while MI part decreases with increase in permeability.

The fluctuating part NR decreases with decrease in magnetic parameters Mn and the NI part increases in decreasing values of Mn.

The graph of phase of the skin friction is shown in figure 4 from the figure then is seen that tan α is positive for different sets of physical fluid parameter. Physical it reveals the fact that there is lag of phase that is the phase decrease with increasing values of k_2.

In Fig. 5 & 6 skin friction is presented and from the figure it is revealed that skin friction decreases with increase in values of inertia parameter k_3, magnetic parameter Mn and permeability parameter k_2.

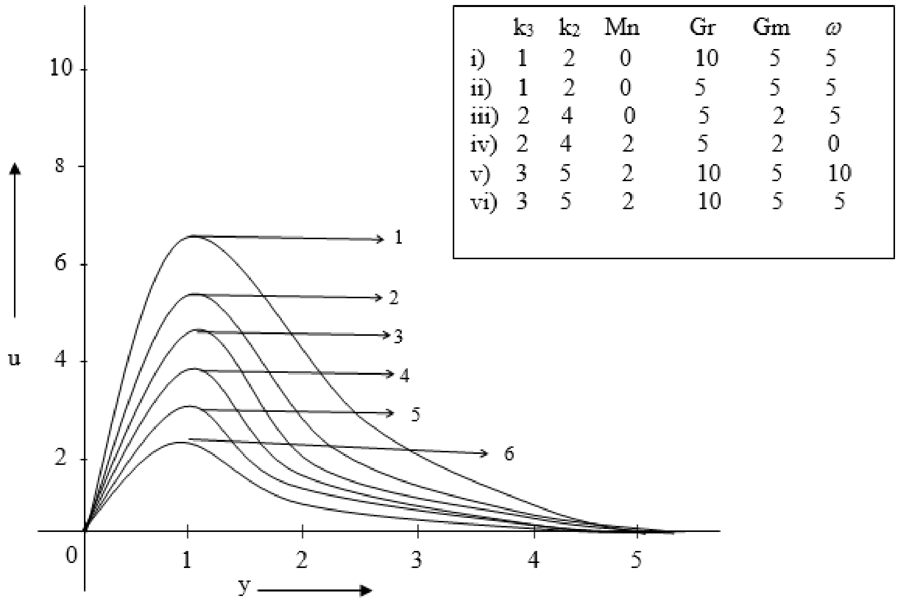

Fig1. Graph of velocity distribution Vs. y for various sets of values of k_2, Mn , Gr , Gm,

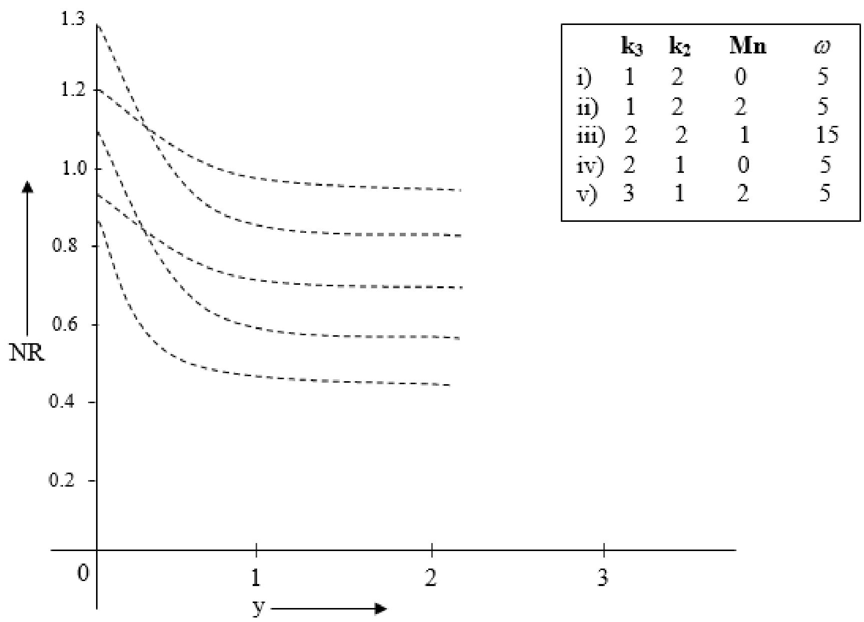

Fig2. Graph of velocity variation for the real fluctuating part for different sets of values of k_3, k_2, Mn, ω

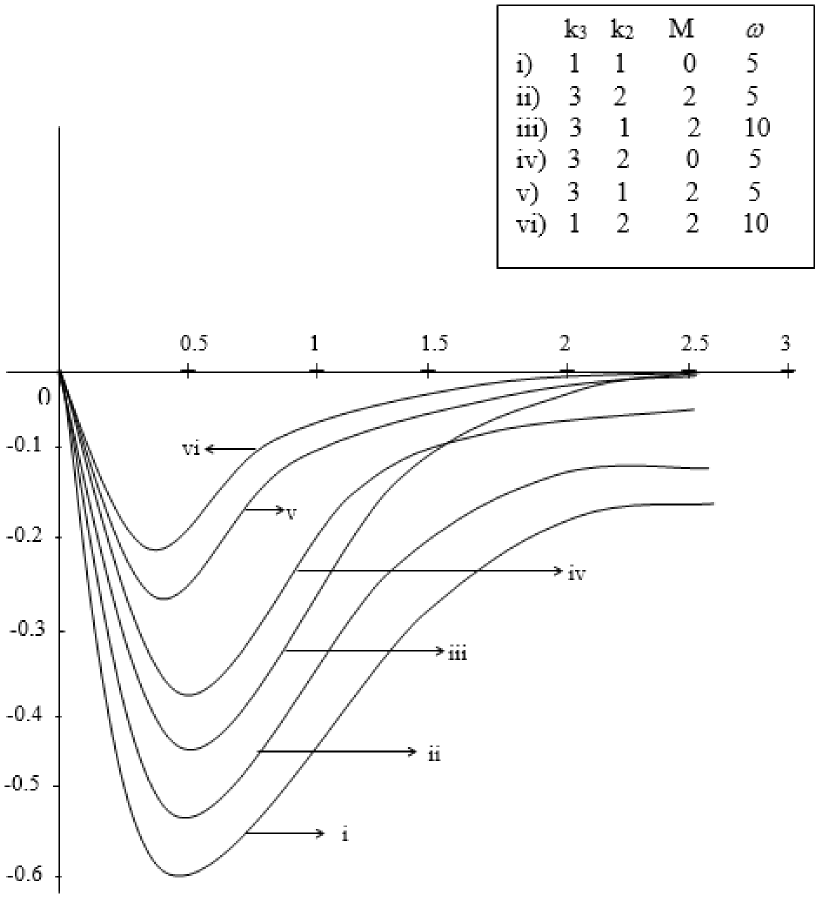

Fig3. Graph of velocity profiles for the imaginary fluctuating part.

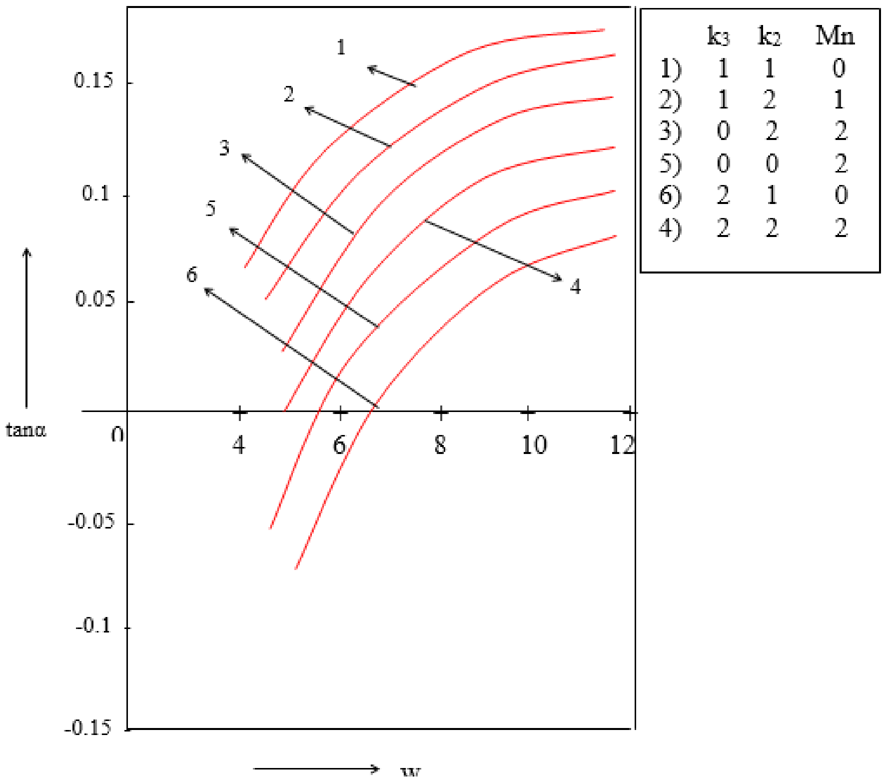

Fig4. Graphs of phase skin friction Vs. angular velocity

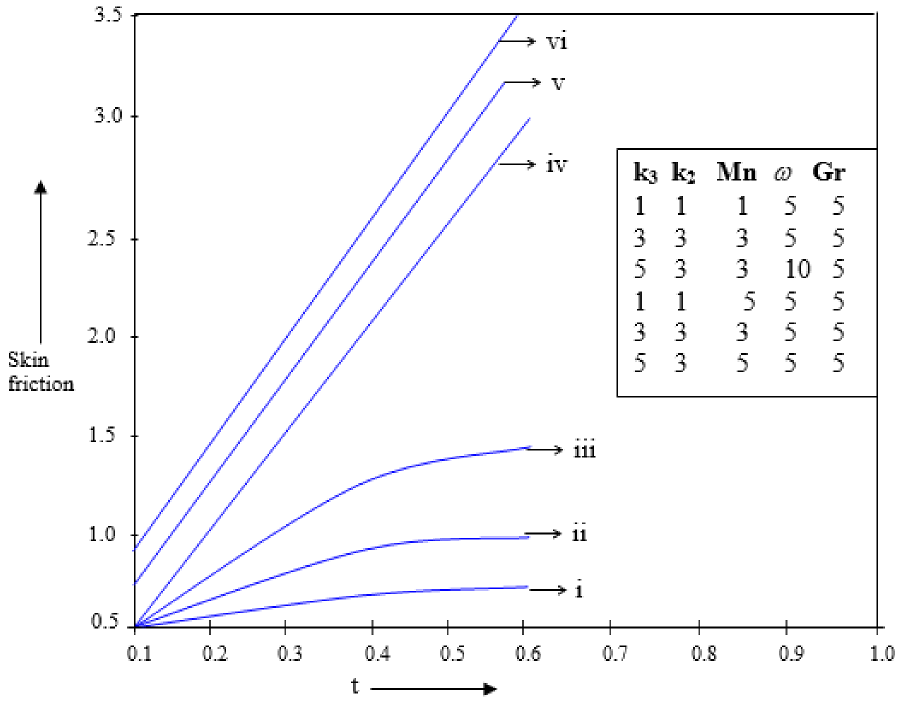

Fig5. Graph of Skin friction for different sets of values of k_3, k_2, Mn, ω

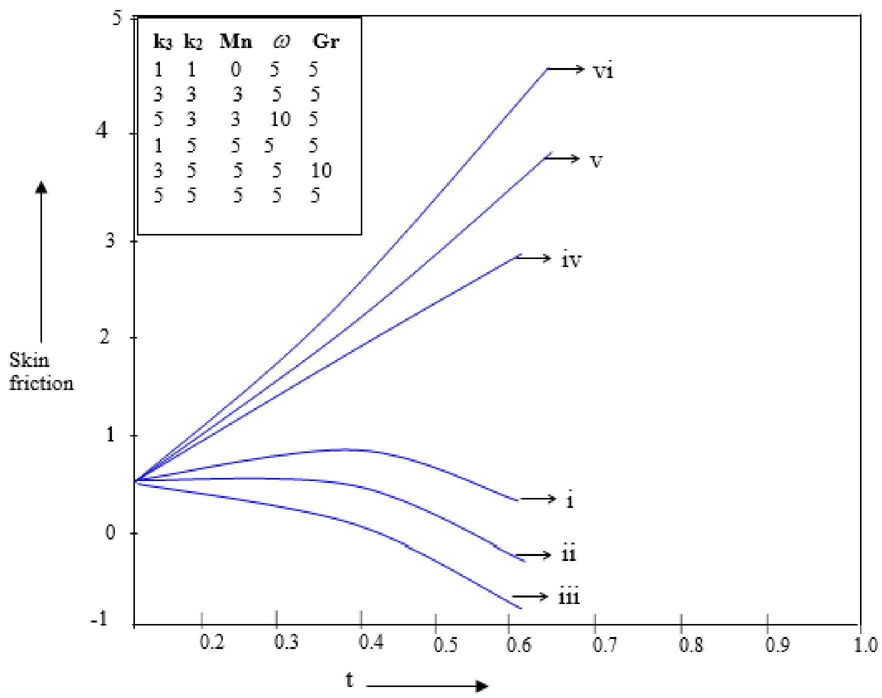

Fig6 . Skin friction variation versus time for different values of k_3, k_2, Mn, ω

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