Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.57, 2016

Effect of Wiggler Magnetic Field on Second Harmonic Generation in Quantum Plasma

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This work was performed under the financial assistance from the UGC (India), under BSR fellowship award letter no. F.4-1/2006(BSR)/2007 (BSR) dated 22nd Oct., 2013.

Abstract
Second harmonic generation due to linearly polarized laser pulse propagating through quantum plasma immersed in a transverse wiggler magnetic field is studied using the quantum hydrodynamic (QHD) model. The effects associated with the Fermi pressure, the Bohm potential and the electron spin have been taken into account. Wiggler magnetic field plays both a dynamic role in producing the traverse harmonic current as well as kinematical role in ensuring phase-matching. The quantum dispersive effects also contribute to the intensity of second harmonics.
Keywords: Quantum plasma, Harmonic generation, Phase matching

1. Introduction

The study of electron-wave interaction in the presence of background plasma has attracted a lot of interest over the past few decades. The physical phenomenon of interaction of a high-intensity laser radiation with plasma leads to a number of relativistic and nonlinear effects such as self modulation, self-focusing, Raman scattering, and harmonic generation [1-2]. The generation of harmonic radiation is an important area of laser-plasma interaction for its value in many applications [3-10]. Harmonic generation offers an alternative source for short wavelength generation and important tool for diagnostics of nonlinear media. With the development of high intensity short pulse lasers, the electron motion becomes highly nonlinear, giving rise to nonlinearity rather than the anharmonicity of the bounded electron oscillation in atoms and molecules. Theory for coherent emission in the direction of propagation of laser beam, referred to as relativistic harmonic generation, has been established [1,11]. It pronounces that because of the mismatch between the phase velocities of the laser pulse and the generated harmonics under the collective response of the plasma, the conversion efficiency should be low unless a means for phase-matching [12] is applied. Matsumoto and Tanaka [13] have presented analysis of quasi phase-matched second-harmonic generation in the reflection by backward propagating interaction and showed that bistability appears in the generated second-harmonic power if the amount of phase mismatch is suitably chosen. Meyer and Zhu [14] claimed to have observed the second relativistic harmonic generated under the condition of beam filamentation. However, such second harmonic light has been later identified by several groups [15,16] to be associated with the transverse-density depression derived by laser self-channeling or filamentation, as is evident from its broad angular width caused by the plasma density gradient [14]. To increase the efficiency of the harmonic generation process which is significantly affected due to the phase mismatch between the fundamental and generated harmonics radiation several schemes have been proposed to make harmonic generation a resonant one. Singh et. al. [17] has shown that a density ripple in a plasma could be properly employed for resonant second harmonic generation. Nitikant and Sharma [18,19] found that wiggler magnetic field plays both a dynamic role in producing the transverse harmonic current and a kinematical one in ensuring phase- matching. Parashar and Pandey [20] proposed the employing of a density ripple to compensate for the momentum mismatch between the pump and second- harmonic wave in plasma and semiconductor respectively. Shkolnikov et al. [21] demonstrated the feasibility of optimal quasi phase matching for higher-order harmonic generation in gases and plasmas with modulated density. Rax and Fisch [22] studied phase modulated relativistic third harmonic generation employing resonant density modulation in plasma. Agrawal et al. [23] have studied resonant second harmonic generation of a millimeter wave in a plasma filled waveguide in the presence of a helical magnetic wiggler. Weissman et. al. [24] have studied second harmonic generation in Bragg-resonant quasi-phase-matched periodically segmented waveguides. Ding et al. [25] have developed a theory for quasi-phase-matched backward second and third harmonic generation in a periodically doped semiconductor.

All the above work has been done for classical plasma. Classical plasma physics has mainly focused on regimes of high temperatures and low densities, in which quantum mechanical effects play no role. Plasma where the density is quite high and the de–Broglie thermal wavelength associated with the charged particle i.e., $\lambda_B = \hbar/(2\pi m k_B T)$ approaches the electron Fermi wavelength λ_{Fe} and exceeds the electron Debye radius λ_{De}

$(viz., \lambda_B \sim \lambda_{Fe} > \lambda_{De})$, the study of quantum effects becomes important. Furthermore, the quantum effects associated with the strong density correlation start playing a significant role when λ_B is of the same order or larger than the average inter-particle distance $(\sim n_o^{-1/3})$, i.e., $n_o \lambda_B^3 \geq 1$ hold in degenerate plasma. However, the other condition for degeneracy is that the Fermi temperture (T_F) which is related to the equilibrium density (n_o) of the charged particles must be greater than the thermal temperture (T) of the system. The high-density, low-temperature quantum Fermi plasma is significantly different from the low-density, high-temperature "classical plasma" obeying Maxwell-Boltzman distribution. During the last decade, there have been many papers devoted to influence of spin on dynamics of plasma [26-28]. Recently, the quantum kinetic studying of the waves in plasma made [29]. The growing interest in investigating new aspects of dense quantum plasmas motivated by its potential applications in modern technology e.g., microelectronics devices, quantum plasma echoes, metallic nanostructures, metal clusters, thin metal films, quantum well and quantum dots, nano-plasmonic devices, quantum x-ray free electron lasers, in super dense astrophysical environment (e.g. in the interior of Jupiter, white dwarfs, and neutron stars), in high intensity laser produced plasmas, in metallic nanostructures , in nonlinear quantum optics, in dusty plasmas and in next generation of laser based plasma compression experiment (LBPC) etc.

The present paper deals with the analysis of second harmonic generation in quantum plasma in the presence of a wiggler magnetic field. The effect of quantum Bohm potential, Fermi pressure and electron spin has been analyzed. The recently developed quantum hydrodynamic (QHD) model [30, 31] has been used to describe the interaction phenomena of second harmonic generation in quantum plasma. The QHD model consists of a set of equations describing the transport of charge density, momentum (including the Bohm potential) and energy in a charged particle system interacting through a self consistent electrostatic potential. QHD model is a macroscopic model and application is limited to those systems that are large compared to Fermi length of the species in the system. The advantages of the QHD model over kinetic descriptions are its numerical efficiency, the direct use of the macroscopic variables of interest such as momentum and energy and the easy way the boundary conditions are implemented.

In second harmonic generation, two photons of energy $\hbar\omega_o$ and momentum $\hbar\vec{k}_o$ combine to produce a photon of second harmonic radiation of energy $\hbar\omega_2$ under the phase matching condition $\hbar\vec{k}_2 = 2\hbar\vec{k}_o$, where $\vec{k}_o, \omega_o$ and $\vec{k}_2, \omega_2$ are the frequency and wave number of fundamental and second harmonic wave respectively. Since quantum plasma is a highly dispersive medium, the phase matching conditions are not satisfied, thereby making the process non-resonant. If the process is made a resonant- one the efficiency of the process can be enhanced significantly. Our main focus is to enhance the second harmonic generation in quantum plasma by satisfying the phase matching condition in the presence of wiggler magnetic field. The wiggler provides additional momentum to make process resonant which leads to enhance the efficiency of harmonic generation. The laser imparts an oscillatory velocity to plasma electrons and exerts a ponderomotive force on them at $(2\omega_o, 2\vec{k}_o)$. As the plasma electrons acquire oscillatory velocity at the second harmonic, the wiggler magnetic field beats with it to produce a second harmonic current at $(2\omega_o, 2\vec{k}_o + \vec{k}_\omega)$, which drives the second harmonic radiation.

2. Nonlinear current density (using QHD Model)

Consider a linearly polarized laser beam with electric field,

$$\vec{E}_y = \hat{y}E_o \exp(k_o z - \omega_o t) \tag{1}$$

propagating through high density quantum plasma, where propagation constant $k_o = \omega_o (1 - \omega_p^2/\omega_o^2)^{1/2} / c$. An external wiggler magnetic field $\vec{B}_w = \hat{y}B_{ow} \exp(k_w z)$ is applied in the transverse direction. The plasma frequency and the wiggler frequency are being defined as $\omega_p = \left[n_o e^2 /(m\varepsilon_o)\right]^{1/2}$ and $(\omega_{ow} = eB_{ow} / m.c)$ respectively. The interaction dynamics is governed by the following set of QHD equations.

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}(\vec{v}\times\vec{B})\right] - \frac{v_F^2}{n_o^2}\frac{\vec{\nabla} n^3}{n} + \frac{\hbar^2}{2m^2}\vec{\nabla}\left(\frac{1}{\sqrt{n}}\vec{\nabla}^2\sqrt{n}\right) - \frac{2\mu_B}{m\hbar}\vec{S}.\nabla\vec{B}, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \vec{v}.\vec{\nabla}\right)\vec{S} = \left(\frac{2\mu_B}{\hbar}\right)\left(\vec{B}\times\vec{S}\right), \tag{3}$$

$$\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{v}) = 0. \tag{4}$$

where, $\vec{v}$ is the velocity, $\hbar$ is the Planck's constant divided by 2π, v_F is the Fermi velocity and $\vec{S}$ is the spin angular momentum with $|S_o| = \hbar/2$ and $\mu = (-g/2)\mu_B$,with $g = 2.0023193$ and $\mu_B = e\hbar/2m$ being the Bohr magneton. The third term on the right–hand side of eq. (2) denotes the Fermi electron pressure. The fourth term is the quantum Bohm force and is due to the quantum corrections in the density fluctuation. The last term is the spin contribution to the momentum. The above equations are applicable even when different spin states are well represented by a macroscopic average. The wave equation for the current source is.

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}(\vec{r},t) = \frac{4\pi}{c^2}\frac{\partial \vec{J}}{\partial t}. \tag{5}$$

where, $\vec{J}$ is the current density.

Perturbative expansion of the set of QHD equations governing the electron plasma dynamics for the first order of the electromagnetic field gives

$$\frac{\partial \vec{v}^{(1)}}{\partial t} = -\frac{e}{m}\vec{E}^{(1)} - \frac{v_F^2}{n_o}\vec{\nabla}.n^{(1)} + \frac{\hbar^2}{4m^2 n_o}\vec{\nabla}(\vec{\nabla}^2 n^{(1)}) - \frac{2\mu_B}{m\hbar}\vec{S}_o.(\vec{\nabla}B^{(1)}), \tag{6}$$

$$\frac{\partial \vec{S}^{(1)}}{\partial t} = \frac{2\mu_B}{\hbar}(\vec{B}^{(1)} \times S_o), \tag{7}$$

$$\frac{\partial n^{(1)}}{\partial t} + \left(n_o\vec{\nabla}.\vec{v}^{(1)} + \vec{\nabla}n_o.\vec{v}^{(1)}\right) = 0. \tag{8}$$

From the equation of motion eq. (6), the components of the quiver velocity imparted to the plasma electron are written as,

$$\vec{v}_x^{(1)} = v_{1x}^{(1)}E_o \exp i(k_o z - \omega_o t). \tag{9}$$

$$\vec{v}_y^{(1)} = \left[v_{1y}^{(1)} + v_{2y}^{(1)}\exp i(k_w z)\right]E_o \exp i(k_o z - \omega_o t). \tag{10}$$

$$\vec{v}_z^{(1)} = \left[v_{1z}^{(1)}\exp i(k_w z)\right]E_o \exp i(k_w z - \omega_o t) \tag{11}$$

where,

$$\Omega_q = v_F^2 + \frac{\hbar^2 k_o^2}{4m^2}, \quad v_{1x}^{(1)} = \frac{\Omega_q k_o \eta_{1x}^{(1)}}{n_o\omega_o} + \frac{\mu_B S_o k_o}{m\hbar\omega_o}, \quad v_{1y}^{(1)} = \left[\frac{e}{mi\omega_o} + \frac{\Omega_q k_o^2 \eta_{1y}^{(1)}}{n_o\omega_o}\right],$$

$$v_{2y}^{(1)} = \left[\frac{2S_o k_\omega B_{ow}}{i\hbar^2\omega_o^2} + \frac{k_o\Omega_q\eta_{2y}^{(1)}}{n_o\omega_o}\right], \quad \text{and} \quad v_{1z}^{(1)} = \left[\frac{k_o\Omega_q}{n_o\omega_o^2}\left\{\omega_o\eta_{1z}^{(1)} - i\omega_{ow}\eta_{1x}^{(1)}\right\} - \frac{i\omega_{ow}S_o\mu_B k_o}{m\hbar\omega_o^2}\right].$$

From the above equations, it is evident that wiggler field effects the motion of electron both in longitudinal as well as in the transverse direction. Due to the above oscillations the first order perturbations in electron density are,

$$n_x^{(1)} = \eta_{1x}^{(1)}E_o \exp i(k_o z - \omega_o t). \tag{12}$$

$$n_y^{(1)} = \left[\eta_{1y}^{(1)} + \eta_{2y}^{(2)}\exp i(k_w z)\right]E_o \exp i(k_o z - \omega_o t). \tag{13}$$

$$n_z^{(1)} = \left[\eta_{1z}^{(1)}\exp i(k_w z)\right]E_o \exp i(k_o z - \omega_o t). \tag{14}$$

where the quantities $\eta_{1x}^{(1)}$, $\eta_{1y}^{(1)}$, $\eta_{2y}^{(2)}$ and $\eta_{1z}^{(1)}$ are obtained using continuity eq. (8), $\eta_{1x}^{(1)} = \frac{\mu_B S_o n_o k_o^2}{m\hbar\xi\omega_o^2}$,

$$\eta_{1y}^{(1)} = \frac{\mu_B S_o n_o k_o^2}{m\hbar\xi\omega_o^2}, \quad \eta_{2y}^{(1)} = \frac{2S_o n_o(k_o + k_w).k_w B_{ow}\mu_B^2}{i\xi\hbar^2\omega_o^3},$$

$$\eta_{1z}^{(1)} = \frac{i\mu_B S_o \omega_{ow}}{m\hbar\xi\omega_o^3}\left[-\frac{\Omega_q n_o k_o^4}{\xi\omega_o^2} - ik_o(k_o + k_w)\right], \text{ and } \xi = 1 - \frac{k_o^2\Omega_q}{\omega_o^{\;2}}.$$

Due to the presence of the magnetic field, electrons attain a spin angular moment. The dynamics of spin angular magnetic moment leads to dispersion. The components of first order perturbed spin angular momentum are obtained using eq. (3) as,

$$\vec{S}_x^{(1)} = \left[S_{1x}^{(1)} \exp i(k_w z)\right] E_o \exp i(k_o z - \omega_o t), \tag{15}$$

$$\vec{S}_y^{(1)} = S_{1y}^{(1)} E_o \exp i(k_o z - \omega_o t), \tag{16}$$

$$\vec{S}_z^{(1)} = S_{1z}^{(1)} E_o \exp i(k_o z - \omega_o t), \tag{17}$$

where,

$$S_{1x}^{(1)} = -\frac{2S_o B_{ow}\mu_B^2}{\omega_o^2\hbar^2}, \quad S_{1y}^{(1)} = \frac{\mu_B S_o}{i\hbar\omega_o}, \text{ and } \quad S_{1z}^{(1)} = \frac{i\mu_B S_o}{\hbar\omega_o}.$$

The perturbed density and spin motion of electrons due to oscillatory velocities generate oscillating current. The current density is the sum of conventional source current $(J_c = -nev)$ and the spin current due to spin magnetic moment $\left(J_s = -\frac{2\mu_B}{\hbar}\nabla(n\times\vec{S})\right)$, whose components are,

$$J_x^{(1)} = J_{cx}^{(1)} + J_{sx}^{(1)} = \left[J_{1x}^{(1)} + J_{2x}^{(1)} \exp i(k_w z)\right] E_o \exp i(k_o z - \omega_o t). \tag{18}$$

$$J_y^{(1)} = J_{cy}^{(1)} + J_{sy}^{(1)} = \left[J_{1y}^{(1)} + J_{2y}^{(1)} \exp i(k_w z)\right] E_o \exp i(k_o z - \omega_o t). \tag{19}$$

$$J_z^{(1)} = J_{cz}^{(1)} + J_{sz}^{(1)} = \left[J_{1z}^{(1)} + J_{2z}^{(1)} \exp i(k_w z)\right] E_o \exp i(k_o z - \omega_o t). \tag{20}$$

where,

$$J_{1x}^{(1)} = -en_o v_{1x}^{(1)} - \frac{2ik_o S_o \mu_B \eta_{1x}^{(1)}}{\hbar}, \quad J_{2x}^{(1)} = -\frac{2i(k_o + k_w) n_o \mu_B S_{1x}^{(1)}}{\hbar},$$

$$J_{1y}^{(1)} = -en_o v_{1y}^{(1)} - \frac{2ik_o \mu_B (n_o S_{1y}^{(1)} + S_o \eta_{1y}^{(1)})}{\hbar}, \quad J_{2y}^{(1)} = -en_o v_{2y}^{(1)} - \frac{2i(k_o + k_w) S_o \mu_B \eta_{2y}^{(1)}}{\hbar},$$

$$J_{1z}^{(1)} = -\frac{2ik_o \mu_B n_o S_{1z}^{(1)}}{\hbar}, \text{ and } J_{2z}^{(1)} = -en_o v_{1z}^{(1)} - \frac{2i(k_o + k_w) S_o \mu_B \eta_{1z}^{(1)}}{\hbar}.$$

Eqs. (18) - (20) contain the collective effects of the laser and magnetic fields on the plasma electrons. The first term in eqs. (18-20) arises due to the action of the radiation field on plasma electrons while the second term denotes the effect of wiggler field, under the influence of electron spin and other quantum effects. First order velocity beats with wiggler magnetic field at $(2\omega_o, 2\vec{k}_o + \vec{k}_w)$ to produce ponderomotive force, $F_p^{(2)}$. The plasma electrons acquire oscillatory velocity at $(2\omega_o, 2\vec{k}_o + \vec{k}_w)$ due to the force $F_p^{(2)}$, whose components are,

$$v_x^{(2)} = \left[v_{1x}^{(2)} \exp i(k_w z)\right] E_o^2 \exp 2i(k_o z - \omega_o t), \tag{21}$$

$$v_y^{(2)} = \left[v_{1y}^{(2)} \exp i(k_w z)\right] E_o^2 \exp 2i(k_o z - \omega_o t), \tag{22}$$

$$v_z^{(2)} = \left[v_{1z}^{(2)} + v_{2z}^{(2)} \exp i(k_w z)\right] E_o^2 \exp 2i(k_o z - \omega_o t), \tag{23}$$

where,

$$v_{1x}^{(2)} = \frac{\omega_{ow}}{2i\omega_o} v_{1z}^{(2)}, \quad v_{1y}^{(2)} = \frac{\omega_{ow} v_{1z}^{(1)}}{4i\omega_o} + \frac{\mu_B k_w B_{ow} S_{1y}^{(2)}}{\hbar\omega_o}, \quad v_{1z}^{(2)} = -\frac{ev_{1y}^{(1)}}{2i\omega_o mc},$$

and $v_{2z}^{(2)} = -\dfrac{ev_{2y}^{(1)}}{2i\omega_o mc}.$

The second order perturbed electron density and the spin magnetic moment are found to be,

$$n_x^{(2)} = \left[\eta_{1x}^{(2)} + \eta_{2x}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (24)$$

$$n_y^{(2)} = \left[\eta_{1y}^{(2)} + \eta_{2y}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (25)$$

$$n_z^{(2)} = \left[\eta_{1z}^{(2)} + \eta_{2z}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (26)$$

$$S_x^{(2)} = S_{1x}^{(2)} \exp i(k_w z) E_o^2 \exp i(k_o z - \omega_o t). \quad (27)$$

$$S_y^{(2)} = \left[S_{1y}^{(2)} + S_{2y}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (28)$$

$$S_z^{(2)} = \left[S_{1z}^{(2)} + S_{2z}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (29)$$

where,

$$\eta_{1x}^{(2)} = \frac{k_o \eta_{1x}^{(1)} v_{1x}^{(1)}}{\omega_o}, \ \eta_{2x}^{(2)} = \frac{n_o (2k_o + k_w) v_{1x}^{(2)}}{2\omega_o}, \ \eta_{1y}^{(2)} = \frac{k_o v_{1y}^{(1)} \eta_{1y}^{(1)}}{\omega_o},$$

$$\eta_{2y}^{(2)} = \frac{(2k_o + k_w)}{2\omega_o}\left[n_o v_{1y}^{(2)} + \eta_{1y}^{(1)} v_{2y}^{(1)} + \eta_{2y}^{(1)} v_{1y}^{(1)}\right], \ \eta_{1z}^{(2)} = \frac{n_o k_o v_{1z}^{(2)}}{\omega_o}, \ \eta_{2z}^{(2)} = \frac{n_o (2k_o + k_w) v_{2z}^{(2)}}{2\omega_o},$$

$$S_{1x}^{(2)} = \frac{(k_o + k_w) v_{1x}^{(1)} S_{1x}^{(1)}}{2\omega_o} - \frac{\mu_B B_{ow} S_{1z}^{(1)}}{i\hbar\omega_o}, \ S_{1y}^{(2)} = \frac{k_o v_{1y}^{(1)} S_{1y}^{(1)}}{2\omega_o} + \frac{\mu_B S_{1z}^{(1)}}{2i\hbar\omega_o}, \ S_{2y}^{(2)} = \frac{k_o v_{2y}^{(1)} S_{2y}^{(1)}}{2\omega_o},$$

$$S_{1z}^{(2)} = -\frac{\mu_B S_{1y}^{(1)}}{2i\hbar\omega_o}, \ \text{and} \ S_{2z}^{(2)} = -\frac{k_o v_{1z}^{(1)} S_{1z}^{(1)}}{2\omega_o}.$$

First order velocity betas with first order density perturbation and spin angular momentum perturbation to produce second harmonic currents at $(2\omega_o, 2\vec{k}_o + \vec{k}_\omega)$. The second order current densities are,

$$J_x^{(2)} = \left[J_{1x}^{(2)} + J_{2x}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (30)$$

$$J_y^{(2)} = \left[J_{1y}^{(2)} + J_{2y}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (31)$$

$$J_z^{(2)} = \left[J_{1z}^{(2)} + J_{2z}^{(2)} \exp i(k_w z)\right] E_o^2 \exp i(k_o z - \omega_o t). \quad (32)$$

where,

$$J_{1x}^{(2)} = -e\eta_{1x}^{(1)} v_{1x}^{(1)} - \frac{4i\mu_B S_o k_o \eta_{1x}^{(2)}}{\hbar}, \ J_{2x}^{(2)} = -\frac{2\mu_B}{\hbar}\left[i(2k_o + k_w)(S_o \eta_{2x}^{(2)} + n_o S_{1x}^{(2)} + \eta_{1x}^{(1)} S_{1x}^{(1)}\right],$$

$$J_{1y}^{(2)} = -e\eta_{1y}^{(1)} v_{1y}^{(1)} - \frac{2\mu_B}{\hbar}\left[2ik_o (S_o \eta_{1y}^{(2)} + n_o S_{1y}^{(2)} + \eta_{1y}^{(1)} S_{1y}^{(1)})\right],$$

$$J_{2y}^{(2)} = -(en_o v_{1y}^{(1)} + e\eta_{2y}^{(1)} v_{1y}^{(1)} + e\eta_{1y}^{(1)} v_{2y}^{(1)}) - \frac{2\mu_B}{\hbar}\left[2ik_o (S_o \eta_{2y}^{(2)} + n_o S_{2y}^{(2)} + \eta_{2y}^{(1)} S_{1y}^{(1)})\right],$$

$$J_{1z}^{(2)} = -en_o v_{1z}^{(2)} - \frac{2\mu_B}{\hbar}\left[2ik_o (S_o \eta_{1z}^{(2)} + n_o S_{1z}^{(2)} + \eta_{1y}^{(1)} S_{1y}^{(1)})\right],$$

$$J_{2z}^{(2)} = -en_o v_{2z}^{(2)} - \frac{2\mu_B}{\hbar}\left[2i(k_o + k_w)(S_o \eta_{2z}^{(2)} + n_o S_{2z}^{(2)} + \eta_{1z}^{(1)} S_{1z}^{(1)})\right]$$

Thus, the total second order nonlinear current (using eqs. (30-32)) is,

$$J_{2\omega_o, 2k_o + k_w}^{NL} = (J_{2x}^{(2)} + J_{2y}^{(2)} + J_{2z}^{(2)}) E_o^2 \exp i((2k_o + k_w)z - 2\omega_o t). \quad (33)$$

There also exists a self-consistent second harmonic field, $\vec{E}_{2\omega_o} = E_2 \exp i((2k_o + k_w)z - 2\omega_o t)$ due to which the linear current density is,

$$\vec{J}_{2\omega_o}^L = -\frac{n_o e^2 \vec{E}_{2\omega_o}}{2im\omega_o}. \quad (34)$$

3. Second harmonic field

The wave equation governing the generation of second harmonic is given by,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}_{2\omega o} = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\left(J^{NL}_{2\omega_o,\,2k_o+k_w} + \vec{J}^{L}_{2\omega_o}\right)E_o^2 \exp i((2k_o+k_w)z - 2\omega_o t).$$

On simplifying the above equation, we get the normalized amplitude

$$\frac{E_2}{E_o} = \frac{8\pi i\omega_o (J^{(2)}_{2x} + J^{(2)}_{2y} + J^{(2)}_{2z})}{c^2(2k_o+k_w)^2 - 4\omega_o^2 + \omega_p^2}.\left(\frac{a_o mc\omega_o}{e}\right). \tag{35}$$

where, $a_o = \left(\dfrac{eE_o}{mc\omega_o}\right)$.

The second harmonic power density can be written as

$$\vec{P}_2 = c/8\pi(\vec{E}_2^* \times \vec{H}_2) = \frac{(2k_o+k_w)c^2}{16\omega_o\pi}|E_2|^2 \tag{36}$$

and the fundamental power density can be written as,

$$\vec{P}_o = c/8\pi(\vec{E}_1^* \times \vec{H}_1) = \frac{k_o c^2}{8\omega_o\pi}|E_o|^2. \tag{37}$$

The ratio of second harmonic power density to that of the fundamental power density gives the efficiency of second harmonic generation as,

$$\left|\frac{P_2}{P_o}\right| = \left(\frac{2(2k_o+k_w)(mn_o\pi a_o^2c^2\omega_o^2}{k_o\omega_p^2}\right)\left[\frac{8\pi i\omega_o(J^{(2)}_{2x} + J^{(2)}_{2y} + J^{(2)}_{2z})}{c^2(2k_o+k_w)^2 - 4\omega_o^2 + \omega_p^2}\right]^2. \tag{38}$$

In figure 1, the power efficiency $(|P_2/P_o|)$ (in %) has been plotted as a function of normalized wiggler frequency (ω_{ow}/ω_o), for different values of plasma density. The figure shows that for a constant plasma density, the harmonic grows with an increase in the wiggler field. The maximum efficiency is attained at about $\omega_{ow}/\omega_o \approx 0.76$. Maximum efficiency appears at higher densities with increase in the wiggler field. The cut off value for the harmonic generation and the saturation value for magnetic field also increase with plasma density. The strong magnetic field and quantum effects both contribute to increase in the cut-off value of second harmonic generation.

The figure 2 is plotted for the power efficiency $(|P_2/P_o|)$ (in %) of second harmonic generation in magnetized quantum plasma as function of magnetic field strength ω_c/ω_o, where $\omega_c = eb/mc$ ($\vec{B}_y = \hat{y}b$ is static magnetic field), for the different values of normalized electron density. It is seen that the power efficiency variation is completely different in the phase-mismatched condition (in absence of wiggler). In the Phase mismatch condition, the second harmonic efficiency reduces with the increase in magnetic field and plasma density.

Figure 3 shows the variation of power efficiency $(|P_2/P_o|)$ (in %) of second harmonic generation as a function of normalized electric field parameter a_o at different densities. The resonant power efficiency increases sharply for lower values of intensity, however at higher values it saturates.

In addition, we can conclude that the efficiency for phase matched is distinguished from efficiency of phase mismatched. The power efficiency of second harmonic generation in phase mismatched condition is always below the phase matched one. It is worth mentioning that in low density plasma we need a super strong magnetic field to get maximum power efficiency of harmonic generation whereas in quantum plasma which is highly dense, the excitation of efficient harmonics becomes easy by applying lesser magnetic field strength. The quantum diffraction also enhance the harmonic generation. A balance between the plasma density and applied field is required to obtain optimum efficiency.

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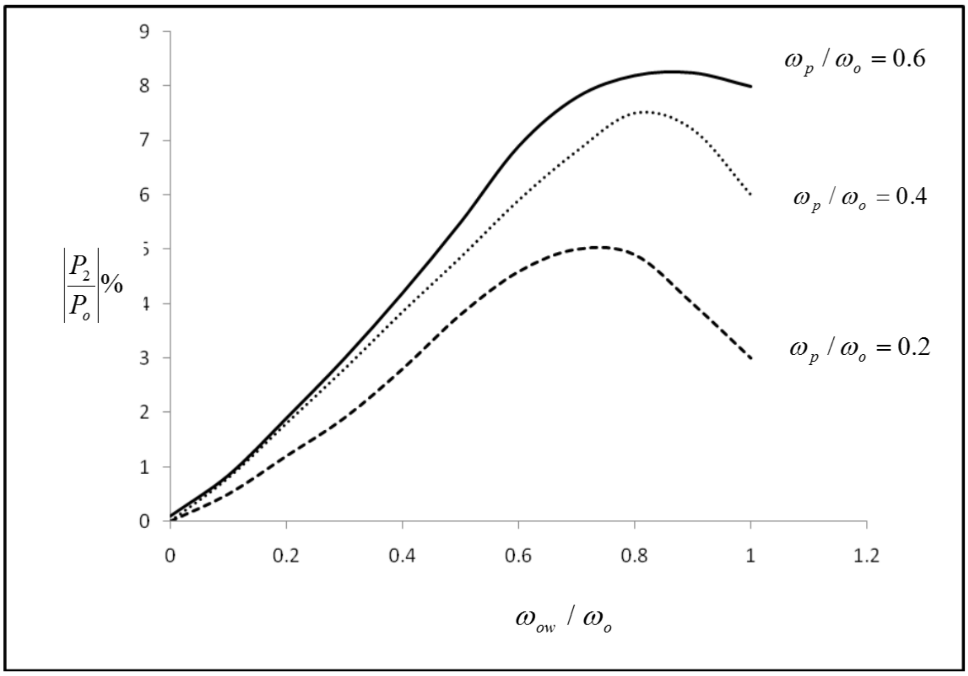

Figure 1: Variation of power efficiency $(|P_2 / P_o|)$ (in %) for phase-matched second harmonic with normalized wiggler frequency (ω_{ow} / ω_o) for $n_o = 10^{28} cm^3$, $a_o = 0.271$ and different values of normalized electron density.

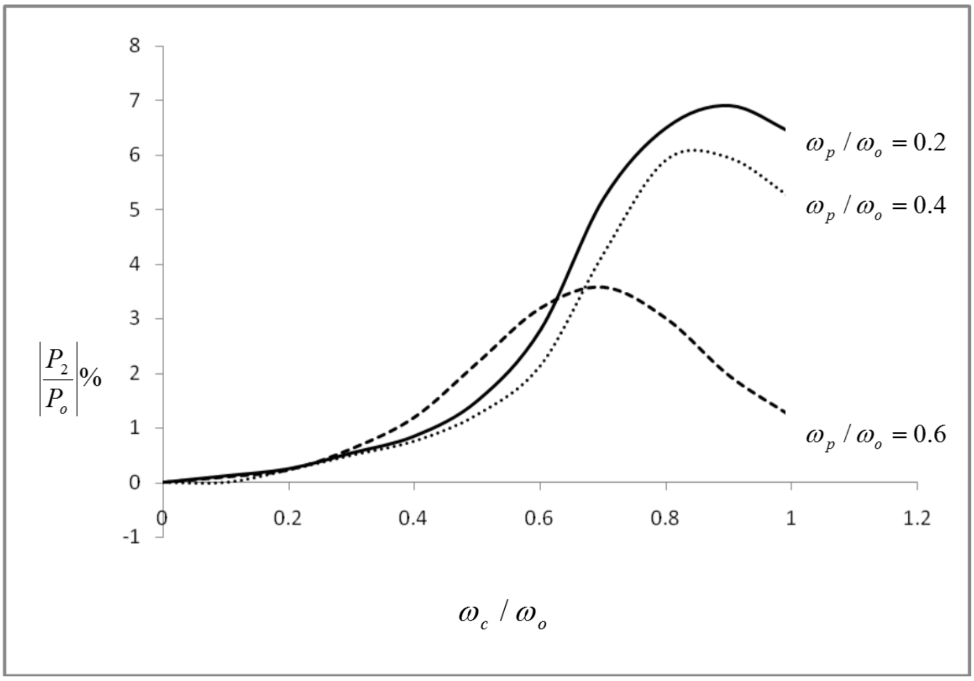

Figure 2: Variation of power efficiency $(|P_2 / P_o|)$ (in %) for phase-mismatched second harmonic with respect to the normalized wiggler frequency (ω_{ow} / ω_o) for for $n_o = 10^{28} cm^3$, $a_o = 0.271$ and different values of normalized electron density.

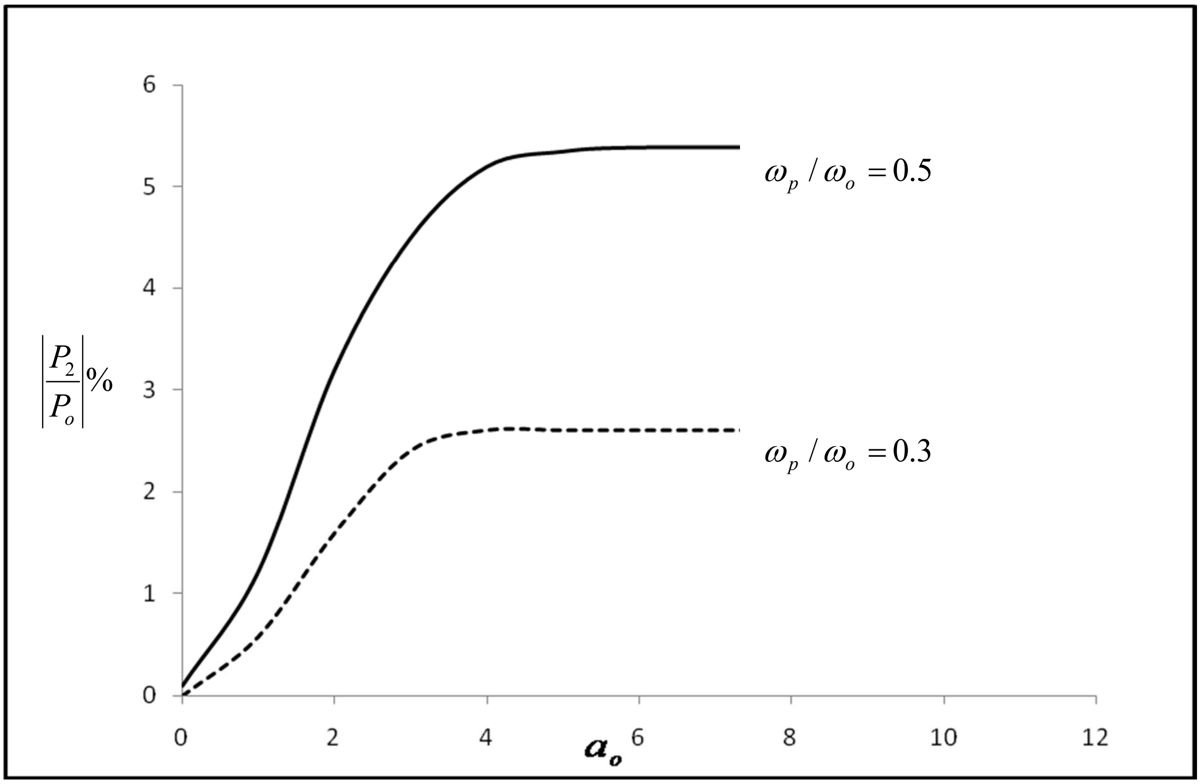

Figure 3: Variation of power efficiency $(|P_2 / P_o|)$ (in %) for phase-matched second harmonic with respect to the laser intensity (a_o) for $\omega_{ow} = 2.82 \times 10^{18}\, Hz$, $n_o = 10^{28}\, cm^3$ and different values of normalized electron density.