# Application Extended Tanh Method for Solving Nonlinear Generalized Ito System

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**Abstract**
In this paper, we generate new exact travelling wave solutions to the nonlinear generalized Ito system based on the extended tanh method with a computerized symbolic computation. It was obtained a various new solutions by applying this method and including solitons, kinks and plane periodic solutions. More importantly, for some equations, we also obtain other new and more general solutions at the same time. It is shown that the extended tanh method is straightforward and concise, and more powerful mathematical tool can be applied to other nonlinear partial differential equations arising in mathematical physics problems.
**Keywords:** Extended tanh method, nonlinear generalized Ito system.

## 1- Introduction

There is a wide class of partial differential equations of great significance to describe complex physical phenomena in various fields of science, such as fluid mechanics, solid state physics, plasma physics, plasma wave, chemical physics, condensed matter physics, optical fibers, biology, chemical kinematics, chemical physics and geochemistry. The phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear partial differential equations. Therefore, the researchers interested in studying and seeking to find exact solutions for them and made a great effort in it because of its importance in nonlinear science. A search for exactly solutions of nonlinear equations has been more interest in recent years because of the availability of symbolic computation Mathematica or Maple .These programs allow us to perform some complicated and differential calculations on a computer. Because of the complexity of the nonlinear wave equations there is no unified method to find all solutions of their equations. There are many methods to solve the various physical and engineering problems like as: A transformed rational function method [11, 23], Exp-Function method [3], extended mapping method [2], extended Jacobian Elliptic Functions expansion method [7], (G′/G)-expansion method [8, 24], modified simple equation method [9], F-Expansion method [1, 6], extended tanh method [5] and many other methods. The exact travelling wave solutions for these nonlinear partial differential equations including the soliton solutions, periodic solutions and rational solutions. Malfiet in [12, 13] subtracted the tanh method which is an efficient and powerful method and used to find the exact solutions for nonlinear partial differential methods. The tanh method applied by many works as in [15, 16,17,20] and by the references there in. Later, the extended tanh method, developed by Wazwaz [14, 18], is a direct and effective algebraic method for handling evolution equations so that one can apply it to models of various types of nonlinearity. The purpose of this paper is to apply the extended tanh method to solve nonlinear generalized Ito system and finding exact solution for this system.

## 2- Extended Tanh Method

Now, we illustrate the concept of the extended tanh method which used to solve partial differential equations. Wazwaz put the following steps to use this method as follows [19]:

Let we have a partial differential equation in terms of two variables $x, t$ in the form:

$$Q(u, u_t, u_x, u_{xx}, u_{xxx}, \dots) = 0 \tag{1}$$

Where $Q$ is a polynomial in $u(x,t)$ and its partial derivatives. By using the transformation $u(x,t) = U(\xi), \xi = \alpha x - \beta t,$ where $\alpha$ and $\beta$ are constants to be determined later, equation (1) transform to the ordinary differential equation in the form:

$$G(U, U', U'', U''', \dots) = 0 \tag{2}$$

Where $G$ is a polynomial in $U(\xi)$ and its derivatives.

Now, we use the new independent variable:

$$Y = tanh(\xi),\ \xi = \alpha x - \beta t \tag{3}$$

Which implies to changes of derivatives:

$$\frac{d}{d\xi} = (1 - Y^2)\frac{d}{dY},$$

$$\frac{d^2}{d\xi^2} = -2Y(1 - Y^2)\frac{d}{dY} + (1 - Y^2)^2\frac{d^2}{dY^2},$$

$$\frac{d^3}{d\xi^3} = 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} - 6Y(1 - Y^2)^2\frac{d^2}{dY^2} + (1 - Y^2)^3\frac{d^3}{dY^3} \tag{4}$$

The extended tanh method admits to use of the finite expansion:

$$u(x,t) = U(\xi) = \sum_{i=-m}^{m} a_i\, Y^i \tag{5}$$

Where $m$ is a positive integer which can be determined by balancing the highest order derivative term with the nonlinear terms in the resulting equation. If $m$ is not integer, then a transformation formula should be used to overcome this difficulty. Substituting equation (5) into equation (2) and collecting all the terms of the same power $Y^i, i = 0, \mp 1, \mp 2, \ldots$ and equating them to zero, we obtain a system of algebraic equations involving that $(a_i, i = -m, \ldots, 0, \ldots, m), \alpha, \beta$. Having determined these parameters from equation (5) we obtain an exact solution in a closed form.

**3- The nonlinear generalized Ito system:**

Now, we apply the extended tanh method to find the exact solution to the nonlinear generalized Ito system [1]:

$$u_t = v_x$$
$$v_t = -2v_{xxx} - 6(uv)_x - 12ww_x + 6p_x$$
$$w_t = w_{xxx} + 3uw_x$$
$$p_t = p_{xxx} + 3up_x \tag{6}$$

Many authors for different cases have obtained some exact and numerical solutions of the generalized Ito system [4, 10, 21, 22]. Now, we using new variable defined on the equation (5):

$$u(x,t) = U(\xi) = \sum_{i=-m}^{m} a_i\, Y^i,$$
$$v(x,t) = V(\xi) = \sum_{i=-n}^{n} b_i\, Y^i,$$
$$w(x,t) = W(\xi) = \sum_{i=-k}^{k} c_i\, Y^i,$$
$$p(x,t) = P(\xi) = \sum_{i=-d}^{d} r_i\, Y^i \tag{7}$$

Where $\xi = \alpha x - \beta t$, equation (6) transform from partial differential equation to ordinary differential equation as:

$$-\beta U' = \alpha V',$$
$$-\beta V' = -2\alpha^3 V''' - 6\alpha UV' - 6\alpha VU' - 12\alpha WW' + 6\alpha P',$$
$$-\beta W' = \alpha^3 W''' + 3\alpha UW',$$
$$-\beta P' = \alpha^3 P''' + 3\alpha UP' \tag{8}$$

Balancing $U'$ with $V'$ in the first equation, $V'''$ with the nonlinear term $UV'$ in the second equation, $W'''$ with $UW'$ in the third equation and $P'''$ with $UP'$ in the fourth equation in (8) leads to the following:

$$m = n, 4 + n - 1 = m + n + 1 = 2k + 1 = d + 1 \tag{9}$$

From this we get: $m = n = k = 2, d = 4$

Then,

$$U(\xi) = a_{-2}\, Y^{-2} + a_{-1}\, Y^{-1} + a_0 + a_1\, Y + a_2\, Y^2,$$
$$V(\xi) = b_{-2}\, Y^{-2} + b_{-1}\, Y^{-1} + b_0 + b_1\, Y + b_2\, Y^2,$$
$$W(\xi) = c_{-2}\, Y^{-2} + c_{-1}\, Y^{-1} + c_0 + c_1\, Y + c_2\, Y^2,$$
$$P(\xi) = r_{-4}\, Y^{-4} + r_{-3}\, Y^{-3} + r_{-2}\, Y^{-2} + r_{-1}\, Y^{-1} + r_0 + r_1\, Y + r_2\, Y^2 + r_3\, Y^3 + r_4\, Y^4 \tag{10}$$

Substituting equation (10) into equation (8) and collecting the coefficients of the powers $Y$ we get a system of algebraic equations as:

$$\beta a_1 + \alpha b_1 = 0,$$
$$2\alpha b_2 + 2\beta a_2 = 0,$$
$$\beta a_{-1} + \alpha b_{-1} + \beta a_1 + \alpha b_1 = 0,$$
$$\beta a_{-1} + \alpha b_{-1} = 0,$$
$$2\beta a_{-2} + 2\alpha b_{-2} = 0,$$
$$48\alpha^3 b_2 + 24\alpha a_2 b_2 + 24\alpha c_2^2 - 24\alpha r_4 = 0,$$
$$-18\alpha r_3 + 36\alpha c_2 c_1 + 18\alpha b_2 a_1 + 18\alpha b_1 a_2 + 12\alpha^3 b_1 = 0,$$
$$2\beta b_2 + 80\alpha^3 b_2 + 24\alpha a_2 b_2 + 24\alpha c_2^2 - 24\alpha c_0 c_2 - 12\alpha c_1^2 - 24\alpha r_4 + 12\alpha r_2 - 12\alpha a_0 b_2 - 12\alpha a_1 b_1 - 12\alpha b_0 a_2 = 0,$$
$$6\alpha a_{-1} b_2 + 6\alpha a_0 b_1 + 6\alpha a_2 b_{-1} - 18\alpha b_1 a_2 - 18\alpha b_2 a_1 + 6\alpha b_0 a_1 - \beta a_1 - 16\alpha^3 b_1 - 36\alpha c_1 c_2 + 12\alpha c_{-1} c_2 + 12\alpha c_0 c_1 + 18\alpha r_3 - 6\alpha r_1 = 0$$
$$-24\alpha c_0 c_2 + 12\alpha r_2 - 12\alpha c_1^2 - 12\alpha b_0 a_2 - 12\alpha b_1 a_1 + 2\beta b_2 + 32\alpha^3 b_2 - 12\alpha b_2 a_0 = 0$$

 

$$-12\alpha c_{-1}c_2 - 12\alpha c_0c_1 - 12\alpha c_{-2}c_1 - 12\alpha c_0c_{-1} + 6\alpha r_1 + 6\alpha r_{-1} + \beta b_1 + 4\alpha^3 b_{-1} + 4\alpha^3 b_1 - 12\alpha b_2 a_{-1} - 6\alpha a_0 b_1 + 6\alpha a_2 b_{-1} - 6\alpha a_{-2} b_1 - 6\alpha a_0 b_{-1} + 6\alpha a_1 b_{-2} - 12\alpha b_{-1} a_2 - 6\alpha b_0 a_1 + 6\alpha b_2 a_{-1} - 6\alpha b_0 a_{-1} + \beta b_{-1} = 0,$$
$$2\beta b_{-2} + 32\alpha^3 b_{-2} - 12\alpha b_0 a_{-2} - 12\alpha c_{-1}^2 - 24\alpha c_0 c_{-2} + 12\alpha r_{-2} - 12\alpha a_{-1} b_{-1} - 12\alpha a_0 b_{-2} = 0,$$
$$12\alpha c_0 c_{-1} - 36\alpha c_{-2} c_{-1} - 6\alpha r_{-1} + 18\alpha r_{-3} - 18 a_{-2} b_{-1} - 18\alpha a_{-1} b_{-2} + 6\alpha b_{-2} a_1 + 6\alpha b_0 a_{-1} + 12\alpha c_{-2} c_1 - \beta b_{-1} - 16\alpha^3 b_{-1} + 6\alpha b_1 a_{-2} + 6\alpha a_0 b_{-1} = 0,$$
$$2\beta b_{-2} + 80\alpha^3 b_{-2} - 12\alpha a_{-1} b_{-1} - 12\alpha b_{-2} a_0 + 24\alpha a_{-2} b_{-2} - 12\alpha b_0 a_{-2} - 12\alpha c_{-1}^2 - 24\alpha c_0 c_{-2} + 24\alpha c_{-2}^2 + 12\alpha r_{-2} - 24\alpha r_{-4} = 0,$$
$$36\alpha c_{-2} c_{-1} - 18\alpha r_{-3} + 18\alpha b_{-2} a_{-1} + 18\alpha b_{-1} a_{-2} + 12\alpha^3 b_{-1} = 0,$$
$$48\alpha^3 b_{-2} + 24\alpha a_{-2} b_{-2} + 24\alpha c_{-2}^2 - 24\alpha r_{-4} = 0,$$
$$24\alpha^3 c_2 + 6\alpha a_2 c_2 = 0,$$
$$6\alpha^3 c_1 + 6\alpha a_1 c_2 + 3\alpha a_2 c_1 = 0,$$
$$40\alpha^3 c_2 + 6\alpha a_2 c_2 - 6\alpha a_0 c_2 - 3\alpha a_1 c_1 - 2\beta c_2 = 0,$$
$$8\alpha^3 c_1 + 6\alpha a_1 c_2 + 3\alpha a_2 c_1 - 6\alpha a_{-1} c_2 - 3\alpha a_0 c_1 + 3\alpha a_2 c_{-1} - \beta c_1 = 0,$$
$$-16\alpha^3 c_2 + 6\alpha a_0 c_2 - 3\alpha a_{-1} c_1 + 3\alpha a_1 c_{-1} + 6\alpha a_2 c_{-2} + 2\beta c_2 + 3\alpha a_1 c_1 - 6\alpha a_{-2} c_2 = 0,$$
$$-2\alpha^3 c_{-1} - 2\alpha^3 c_1 + 3\alpha a_0 c_1 - 3\alpha a_2 c_{-1} - 3\alpha a_{-2} c_1 + 3\alpha a_0 c_{-1} + 6\alpha a_1 c_{-2} + \beta c_1 + \beta c_{-1} + 6\alpha a_{-1} c_2 = 0,$$
$$-16\alpha^3 c_{-2} + 6\alpha a_{-2} c_2 + 3\alpha a_{-1} c_1 - 3\alpha a_1 c_{-1} - 6\alpha a_2 c_{-2} + 3\alpha a_{-1} c_{-1} + 6\alpha a_0 c_{-2} + 2\beta c_{-2} = 0,$$
$$8\alpha^3 c_{-1} + 3\alpha a_{-2} c_1 - 3\alpha a_0 c_{-1} - 6\alpha a_1 c_{-2} + 3\alpha a_{-2} c_{-1} + 6\alpha a_{-1} c_{-2} - \beta c_{-1} = 0,$$
$$40\alpha^3 c_{-2} - 3\alpha a_{-1} c_{-1} - 6\alpha a_0 c_{-2} + 6\alpha a_{-2} c_{-2} - 2\beta c_{-2} = 0,$$
$$-6\alpha^3 c_{-1} - 3\alpha c_{-1} a_{-2} - 6 a_{-1} c_{-2} = 0,$$
$$-24\alpha^3 c_{-2} - 6\alpha a_{-2} c_{-2} = 0,$$
$$120\alpha^3 r_4 + 12\alpha a_2 r_4 = 0,$$
$$-60\alpha^3 r_3 - 12\alpha a_1 r_4 - 9\alpha a_2 r_3 = 0,$$
$$248\alpha^3 r_4 - 24\alpha^3 r_2 + 12\alpha a_2 r_4 - 6\alpha a_2 r_2 - 12\alpha a_0 r_4 - 9\alpha a_1 r_3 - 4\beta r_4 = 0,$$
$$114\alpha^3 r_3 - 6\alpha^3 r_1 + 12\alpha a_1 r_4 + 9\alpha a_2 r_3 - 12\alpha a_{-1} r_4 - 9\alpha a_0 r_3 - 3\alpha a_2 r_1 - 6\alpha a_1 r_2 - 3\beta r_3 = 0,$$
$$40\alpha^3 r_2 - 152\alpha^3 r_4 + 12\alpha a_0 r_4 + 9\alpha a_1 r_3 + 6\alpha a_2 r_2 - 12\alpha a_{-2} r_4 - 9\alpha a_{-1} r_3 - 6\alpha a_0 r_2 - 3\alpha a_1 r_1 - 2\beta r_2 + 4\beta r_3 = 0,$$
$$8\alpha^3 r_1 - 60\alpha^3 r_3 + 12\alpha a_{-1} r_4 + 9\alpha a_0 r_3 + 6\alpha a_1 r_2 + 3\alpha a_2 r_1 - 9\alpha a_{-2} r_3 - 6\alpha a_{-1} r_2 - 3\alpha a_0 r_1 + 3\alpha a_2 r_{-1} + 3\beta r_3 - \beta r_1 = 0,$$
$$-16\alpha^3 r_2 + 24\alpha^3 r_4 - 3\alpha a_{-1} r_1 + 12\alpha r_4 a_{-2} + 9\alpha a_{-1} r_3 + 6\alpha a_0 r_2 - 6\alpha a_{-2} r_2 + 3\alpha a_1 r_{-1} + 6\alpha a_2 r_{-2} + 2\beta r_2 + 3\alpha a_1 r_1 = 0,$$
$$-2\alpha^3 r_{-1} - 2\alpha^3 r_1 + 6\alpha^3 r_{-3} + 6\alpha^3 r_3 + 9\alpha a_{-2} r_3 + 6\alpha a_{-1} r_2 + 3\alpha a_0 r_1 - 3\alpha a_{-2} r_1 + 3\alpha a_0 r_{-1} + 6\alpha a_1 r_{-2} + 9\alpha a_2 r_{-3} + \beta r_1 + \beta r_{-1} - 3\alpha a_2 r_{-1} = 0,$$
$$-16\alpha^3 r_{-2} + 24\alpha^3 r_{-4} + 6\alpha a_{-2} r_2 + 3\alpha a_{-1} r_1 - 3\alpha a_1 r_{-1} - 6\alpha a_2 r_{-2} + 3\alpha a_{-1} r_{-1} + 6\alpha a_0 r_{-2} + 9\alpha a_1 r_{-3} + 12\alpha a_2 r_{-4} + 2\beta r_{-2} = 0,$$
$$60\alpha^3 r_{-3} + 8\alpha^3 r_{-1} + 3\alpha a_{-2} r_1 - 3\alpha a_0 r_{-1} - 6\alpha a_1 r_{-2} - 9\alpha a_2 r_{-3} + 3\alpha a_{-2} r_{-1} + 6\alpha a_{-1} r_{-2} + 9\alpha a_0 r_{-3} + 12\alpha a_1 r_{-4} - \beta r_{-1} + 3\beta r_{-3} = 0,$$
$$-152\alpha^3 r_{-4} + 40\alpha^3 r_{-2} - 3\alpha a_{-1} r_{-1} - 6\alpha a_0 r_{-2} - 9\alpha a_1 r_{-3} - 12\alpha a_2 r_{-4} + 6\alpha a_{-2} r_{-2} + 9\alpha a_{-1} r_{-3} + 12\alpha a_0 r_{-4} - 2\beta r_{-2} + 4\beta r_{-4} = 0,$$
$$114\alpha^3 r_{-3} - 6\alpha^3 r_{-1} - 3\alpha a_{-2} r_{-1} - 6\alpha a_{-1} r_{-2} - 9\alpha a_0 r_{-3} - 12\alpha r_{-4} a_1 + 9\alpha a_{-2} r_{-3} + 12\alpha a_{-1} r_{-4} - 3\beta r_{-3} = 0,$$
$$248\alpha^3 r_{-4} - 24\alpha^3 r_{-2} - 6\alpha a_{-2} r_{-2} - 9\alpha a_{-1} r_{-3} - 12\alpha a_0 r_{-4} + 12\alpha a_{-2} r_{-4} - 4\beta r_{-4} = 0,$$
$$-60\alpha^3 r_{-3} - 9\alpha a_{-2} r_{-3} - 12\alpha a_{-1} r_{-4} = 0,$$
$$-120\alpha^3 r_{-4} - 12\alpha a_{-2} r_{-4} = 0 \qquad (11)$$

By using Maple to solve this system, we get the following results:

$$1)\ a_{-2} = 0, \quad a_{-1} = 0, \quad a_0 = -\frac{-2\alpha^3 + \beta}{3\alpha}, \quad a_1 = 0, \quad a_2 = -2\alpha^2, \quad b_{-2} = 0,$$
$$b_{-1} = 0, \quad b_0 = -\frac{-c_1^2 + \beta^2 + 4\alpha^3\beta}{2\alpha^2}, \quad b_1 = 0, \quad b_2 = 2\alpha\beta, \quad c_{-2} = 0,$$
$$c_{-1} = 0, \quad c_0 = c_0, \quad c_1 = c_1, \quad c_2 = 0, \quad r_{-4} = 0, \quad r_{-3} = 0, \quad r_{-2} = 0,$$
$$r_{-1} = 0,\ r_0 = r_0,\ r_1 = 2c_0c_1, \quad r_2 = 0, \quad r_3 = 0,\ r_4 = 0,\ \alpha = \alpha, \quad \beta = \beta \qquad (12)$$

Substituting equation (12) into equation(10) , we get a solution of equation (6):

$$u(x,t) = -\frac{-2\alpha^3 + \beta}{3\alpha} - 2\alpha^2 \tanh^2(\alpha x - \beta t),$$
$$v(x,t) = -\frac{-c_1^2 + \beta^2 + 4\alpha^3\beta}{2\alpha^2} + 2\alpha\beta \tanh^2(\alpha x - \beta t),$$
$$w(x,t) = c_0 + c_1 \tanh(\alpha x - \beta t),$$
$$p(x,t) = r_0 + 2c_0c_1 \tanh(\alpha x - \beta t) \qquad (13)$$

2) $a_{-2}=0,\quad a_{-1}=0,\quad a_0=\frac{64\alpha^6-c_2^2}{24\alpha^4},\quad a_1=0,\quad a_2=-4\alpha^2,\quad b_{-2}=0,$

$b_{-1}=0,\quad b_0=b_0,\quad b_1=0,\quad b_2=\frac{c_2^2}{2\alpha^2},\quad c_{-2}=0,\quad c_{-1}=0,\quad c_0=c_0,$

$c_1=0,\quad c_2=c_2,\quad r_{-4}=0,\quad r_{-3}=0,\quad r_{-2}=0\ ,r_{-1}=0,\ r_0=r_0,\quad r_1=0\,,$

$$r_2=\frac{64\alpha^6c_0c_2-128\alpha^8b_0-c_2^4}{32\alpha^6},\quad r_3=0,\ r_4=0,\ \alpha=\alpha,\quad \beta=\frac{c_2^2}{8\alpha^3} \tag{14}$$

Substituting equation (14) into equation(10) , we get another solution of equation (6):

$$u(x,t)=\frac{64\alpha^6-c_2^2}{24\alpha^4}-4\alpha^2\,tanh^2\left(\alpha x-\frac{c_2^2}{8\alpha^3}t\right),$$

$$v(x,t)=b_0+\frac{c_2^2}{2\alpha^2}tanh^2\left(\alpha x-\frac{c_2^2}{8\alpha^3}t\right),$$

$$w(x,t)=c_0+c_2\,tanh^2\left(\alpha x-\frac{c_2^2}{8\alpha^3}t\right),$$

$$p(x,t)=r_0+\frac{64\alpha^6c_0c_2-128\alpha^8b_0-c_2^4}{32\alpha^6}tanh^2\left(\alpha x-\frac{c_2^2}{8\alpha^3}t\right) \tag{15}$$

3) $a_{-2}=-2\alpha^2,\quad a_{-1}=0, a_0=a_0, a_1=0,\quad a_2=-2\alpha^2,\quad b_{-2}=-2\alpha^2(3a_0+4\alpha^2),$

$b_{-1}=0,\quad b_0=\frac{16\alpha^6+c_{-1}^2-9\alpha^2a_0^2}{2\alpha^2},\quad b_1=0,\quad b_2=-2\alpha^2(3a_0+4\alpha^2),$

$c_{-2}=0,\quad c_{-1}=c_{-1},\quad c_0=c_0,\quad c_1=c_{-1},\quad c_2=0,\quad r_{-4}=0,\quad r_{-3}=0,$

$r_{-2}=0\ ,r_{-1}=2c_0c_{-1},\ r_0=r_0,\quad r_1=2c_0c_{-1}\,,\quad r_2=0,$

$$r_3=0,\ r_4=0,\ \alpha=\alpha,\quad \beta=-3\alpha a_0-4\alpha^3 \tag{16}$$

Substituting equation (16) into equation(10) , we get another solution of equation (6):

$$u(x,t)=-2\alpha^2\,coth^2(\alpha x+(3\alpha a_0+4\alpha^3)t)+a_0-2\alpha^2\,tanh^2(\alpha x+(3\alpha a_0+4\alpha^3)t),$$

$$v(x,t)=-2\alpha^2(3a_0+4\alpha^2)\,coth^2(\alpha x+(3\alpha a_0+4\alpha^3)t)+\frac{16\alpha^6+c_{-1}^2-9\alpha^2a_0^2}{2\alpha^2}-2\alpha^2(3a_0+4\alpha^2)\,tanh^2(\alpha x+(3\alpha a_0+4\alpha^3)t),$$

$$w(x,t)=c_{-1}coth(\alpha x+(3\alpha a_0+4\alpha^3)t)+c_0+c_{-1}tanh(\alpha x+(3\alpha a_0+4\alpha^3)t),$$

$$p(x,t)=2c_0c_{-1}coth(\alpha x+(3\alpha a_0+4\alpha^3)t)+r_0+2c_0c_{-1}tanh(\alpha x+(3\alpha a_0+4\alpha^3)t) \tag{17}$$

4) $a_{-2}=-2\alpha^2,\quad a_{-1}=0, a_0=\frac{8\alpha^3-\beta}{3\alpha}, a_1=0,\quad a_2=-2\alpha^2,\quad b_{-2}=2\alpha\beta,$

$b_{-1}=0,\quad b_0=\frac{c_{-1}^2-\beta^2}{2\alpha^2},\quad b_1=0,\quad b_2=2\alpha\beta,\quad c_{-2}=0,\quad c_{-1}=c_{-1},$

$c_0=c_0,\quad c_1=-c_{-1},\quad c_2=0,\quad r_{-4}=0,\quad r_{-3}=0, r_{-2}=0\ ,r_{-1}=2c_0c_{-1},$

$$r_0=r_0,\quad r_1=-2c_0c_{-1}\,,\quad r_2=0, r_3=0,\ r_4=0,\ \alpha=\alpha,\quad \beta=\beta \tag{18}$$

Then, the solution of equation (6):

$$u(x,t)=-2\alpha^2\,coth^2(\alpha x-\beta t)+\frac{8\alpha^3-\beta}{3\alpha}-2\alpha^2\,tanh^2(\alpha x-\beta t),$$

$$v(x,t)=2\alpha\beta\,coth^2(\alpha x-\beta t)+\frac{c_{-1}^2-\beta^2}{2\alpha^2}+2\alpha\beta\,tanh^2(\alpha x-\beta t),$$

$$w(x,t)=c_{-1}coth(\alpha x-\beta t)+c_0-c_{-1}tanh(\alpha x-\beta t),$$

$$p(x,t)=2c_0c_{-1}coth(\alpha x-\beta t)+r_0-2c_0c_{-1}tanh(\alpha x-\beta t) \tag{19}$$

5) $a_{-2}=-2\alpha^2,\quad a_{-1}=0, a_0=\frac{2\alpha^3-\beta}{3\alpha}, a_1=0,\quad a_2=0,\quad b_{-2}=2\alpha\beta,$

$b_{-1}=0,\quad b_0=\frac{c_{-1}^2-4\alpha^3\beta-\beta^2}{2\alpha^2},\quad b_1=0,\quad b_2=0,\quad c_{-2}=0,\quad c_{-1}=c_{-1},$

$c_0=c_0,\quad c_1=0,\quad c_2=0,\quad r_{-4}=0,\quad r_{-3}=0, r_{-2}=0\ ,r_{-1}=2c_0c_{-1},$

$$r_0=r_0,\quad r_1=0\,,\quad r_2=0, r_3=0,\ r_4=0,\ \alpha=\alpha,\quad \beta=\beta \tag{20}$$

Then, the solution of equation (6):

$$u(x,t)=-2\alpha^2\,coth^2(\alpha x-\beta t)+\frac{2\alpha^3-\beta}{3\alpha},$$

$$v(x,t)=2\alpha\beta\,coth^2(\alpha x-\beta t)+\frac{c_{-1}^2-4\alpha^3\beta-\beta^2}{2\alpha^2},$$

$$w(x,t)=c_{-1}coth(\alpha x-\beta t)+c_0,$$

$$p(x,t)=2c_0c_{-1}coth(\alpha x-\beta t)+r_0 \tag{21}$$

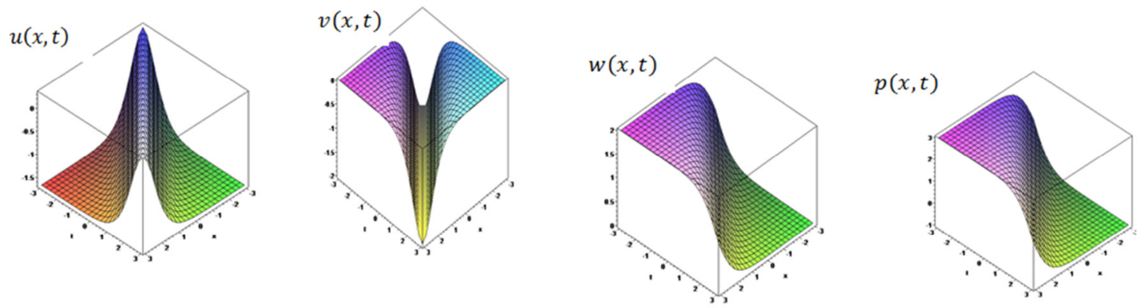


Fig.1: Exact solution of $u(x,t), v(x,t), w(x,t)$ and $p(x,t)$ for $-3 \leq x \leq 3, -3 \leq t \leq 3$ of Eq.(13).

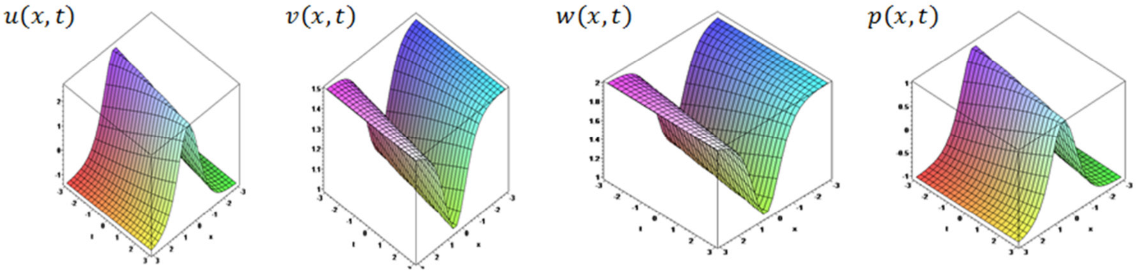


Fig.2: Exact solution of $u(x,t), v(x,t), w(x,t)$ and $p(x,t)$ for $-3 \leq x \leq 3, -3 \leq t \leq 3$ of Eq.(15).

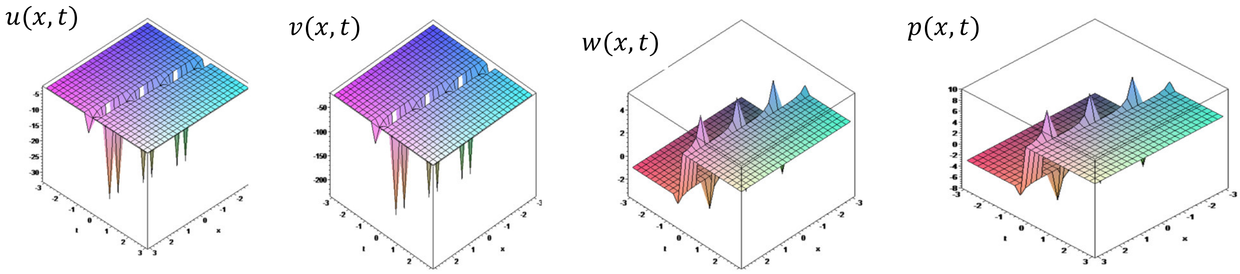


Fig.3: Exact solution of $u(x,t), v(x,t), w(x,t)$ and $p(x,t)$ for $-3 \leq x \leq 3, -3 \leq t \leq 3$ of Eq.(17).

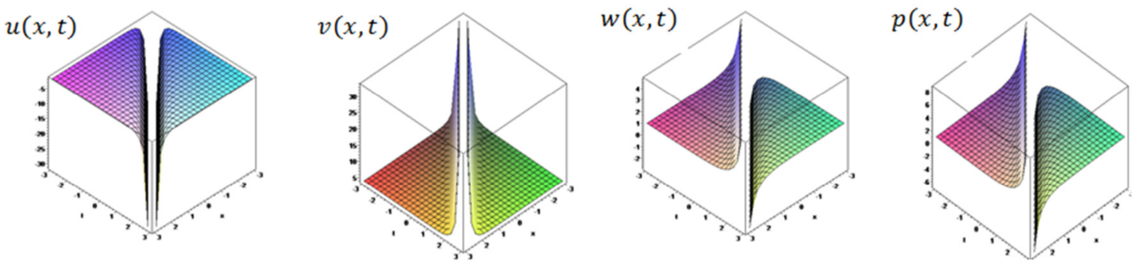


Fig.4: Exact solution of $u(x,t), v(x,t), w(x,t)$ and $p(x,t)$ for $-3 \leq x \leq 3, -3 \leq t \leq 3$ of Eq.(19).

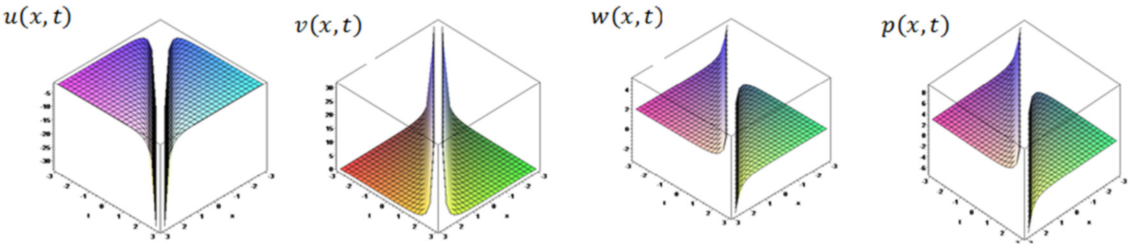


Fig.5: Exact solution of $u(x,t), v(x,t), w(x,t)$ and $p(x,t)$ for $-3 \leq x \leq 3, -3 \leq t \leq 3$ of Eq.(21).

 

## 4- Conclusion

We obtained various new exact solutions to the nonlinear generalized Ito system by using extended tanh method. The key idea behind our method is to express the solution as a polynomial in hyperbolic functions, then the solving of the variable coefficient partial differential equations implies solving a system containing ordinary differential equations and algebraic equations. The performance of this method is powerful and can be used to discuss nonlinear evolution equations and related models in scientific fields. The tedious computations associated with the algebraic calculations are facilitated using symbolic computation software such as Maple and Mathematica.

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