

Time Dependant Momentum Operator and Time Evolution of Free Quantum Particles

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Abstract

In spite of outstanding achievement of the quantum theory, the theory is under never-ending judgment due to probabilistic character and certain unresolved paradoxes arising from the very fundamentals. Additionally, it is debated that a more sophisticated theory beyond the quantum theory emerges as a need for a better understanding of the atomic and sub atomic worlds. This paper, for that reason, focuses on the very fundamental structure of the quantum theory such as the physical and philosophical meanings of the wave functions, the probability and the Schrödinger Wave Equation (SWE). Specifically deals with the time evolution of the free quantum particles. The work just arises from the difference of the classical and quantum waves, specifically the comparison of the time and space dependence of classical waves and Schrödinger waves, for the free space, indicates that time dependence shows a clear dissimilarity. In order to tackle the problem, a time dependent momentum operator, for the first time, is defined and consequently an alternative TDSWE is straightforwardly derived and its philosophical implications have been discussed. Additionally, time and position dependence of the probability functions, relating free particles, have been physically and philosophically discussed and some alternative approaches and comments are forwarded. Finally, the approaches, comments and mathematical equations are in good agreement with the standard quantum theory and interestingly go further possibly tracing some signs of new pathways to resolve the mysterious quantum world.

Keywords: Schrödinger wave equation, fundamentals of quantum mechanics, momentum operator, quantum probabilities, time evolution of quantum systems.

1. Introduction

Quantum theory is certainly one of the most victorious theories within the scientific world and opens the doors to countless technological applications through describing, correlating and predicting the behaviour of vast range of physical systems made up of molecules, atoms, nuclei and other elementary particles.

No experimental evidence has been detected so far conflicting the predictions of the theory flashing the extraordinary success (Dirac 1930; Neumann 1955; Feynman *et al.* 1965; Eisberg & Resnick 1974; Penrose 1989; Griffiths 2004). In spite of its exceptional success, the theory surprisingly faces certain conceptual difficulties such as interpretation of wave functions, understanding of probabilistic nature, time evolution of quantum systems and some paradoxes such as Einstein, Podolsky, Rosen (EPR) Paradox/Quantum Entanglement (Einstein *et al.* 1935; Aspect *et al.* 1982; Horodecki *et al.* 2009). Quantum Decoherence/Quantum Measurement Problem (Zurek 1982; Zurek 2003; Paz & Zurek 2001) and Quantum Zeno Effect (Misra & Sudarshan 1977; Koshino & Shimizu 2005). In spite of nearly 100 years, famous discussions, chiefly between Einstein and Bohr, have unfortunately not been completely completed and some numerable quantum physicists still believe some dilemma about the probabilistic nature of the quantum world and of course about the conceptual composition of quantum theory (Pais 1982; Born 1973; Bohr 1949). A number of conceptual and philosophical interpretations, of the quantum theory, have been developed in the past and most of them experience some support but more noticeably intelligent criticism. Philosophically speaking, quantum theory barely probes its tools to explore the atomic order/world however believing it as the only possible theory/method would surely be dumpy mentality and alternative approaches/means could un questionably be developed by other civilizations.

The complexity of the theory mainly originates from the unsatisfactory definition of the most fundamental concepts, namely 'time' and 'position'. Consequently, the paradoxes and conceptual difficulties of the quantum theory, to our view, regularly initiate from the lack of a clear scientific definition between 'time' and 'position'. Quantum theory basically assumes that 'time' and 'space' are continuous and completely independent variables

and no scientific relation between time and position can be defined no matter if the particle is free or confined. The theory also assumes that quantum particles are solid spheres; however the particles are also somehow accompanied by physical waves. Hence, the existence of spatially and temporally distributed matter waves prevents to define the position and time dependence of dynamical variables such as velocity, momentum and energy, in unfriendliness to classical physics. Instead, well defined time and position dependant waves and depending on the wave functions particle's the existence probabilities are elegantly defined. In order to overcome certain difficulties of existing theory a novel approach and reassessment in the formulation of quantum mechanics are underwent and a new formulation had been developed by de Broglie, Madelung and Bohm, generally referred to as Bohmian mechanics (De Broglie 1928; Madelung 1927; Bohm 1952; Holland 1993; Durr & Teufel 2009). In this formulation, quantum mechanics is a deterministic theory in which particles follow trajectories just as in classical mechanics. The Schrodinger Equation is supplemented by a simple equation relating the particle trajectory to the phase of the wave function. The statistical aspect of quantum mechanics is incorporated by postulating that the particle trajectories in an ensemble of identical quantum states are distributed with a probability density given by the square of the absolute value of the wave function. Once such a distribution is established it is preserved for all later times by the Bohmian dynamical laws (Bohm 1952; Holland 1993; Durr & Teufel 2009).

This formulation gives quantum mechanics more of the features of classical statistical mechanics and it naturally leads to the question of why the particle trajectories should be distributed in any particular way. One point of view, arises from the similarities to statistical mechanics, is that there exists a natural or typical distribution of the trajectories (Durr & Teufel 2009; Durr *et al.* 1992; Shtanov 1997). Quantum theory, in general, describes two distinct physical concepts, namely dynamical variables/operators and wave functions/state vectors. Concerning the time evolution of a quantum system, two distinct approaches simply exist, namely Schrödinger and Heisenberg models/pictures (Rubin 2001; Merzbacher 1970). Schrödinger Model assumes that the wave functions/state vectors are time dependant but the dynamical variables/operators are constant and the model offers an equation, TDSWE, for the time evolution of the state vectors/wave functions. Heisenberg Model is, on the other hand, based on that the wave functions/state vectors are steady whereas the operators/dynamical variables are time dependant and the model offers the well known Heisenberg Equation. Independently from those pictures, ontological and epistemological approaches to the subject forwards that 'a quantum particle must exist at a specific point at a particular time' therefore the position of the particle could, somehow, be definable. Additionally, the very existence of any quantum particle in space and time must ontologically be conserved; hence both state vectors and operators should be, in the most general case, considered time dependant. In order to obtain a full picture of the atomic world means precise position at a given time, both time and position dependence of the operators ought to be defined. To our view, Einstein was/is right and it is the insufficiency of the theory and the theory ought to be progressed substantially to resolve the certain paradoxes and conceptual difficulties (Paris 1982). Philosophic and scientific discussion of this most fundamental theme is not the scope of the pre- sent paper, however will be handled in an upcoming paper.

In addition to those paradoxes and conceptual difficulties, there are some other fundamental insufficiencies and difficulties within the quantum theory. Quantum systems may have obviously been both 'closed' and 'open' in character and the problem must be treated accordingly (Breuer & Petruccione 2002; Muga *et al.* 2002). The physical properties of any quantum particle for open quantum systems are much more complicated due to having strongly time and position dependant energies; potential, kinetic or overall mechanical energy. The exchange energy, of the most quantum systems, seems to be negligible compared to the actual system energy maintaining the assumption of being a closed system is highly fulfilled. Open quantum systems, however, demonstrate very high exchange energies between the actual system and the 'environment' leading to very important applications in quantum optics, measurement in quantum mechanics, quantum statistical mechanics, quantum in- formation, quantum cosmology, some semi classical applications and recently the ultimate concept of human consciousness (Penrose 1994; Eisert & Wiseman 2007; Stapp 2009; Bohr 1928; Erol 2010; Erol 2010). Bohr emphasized that the border between classical and quantum worlds must be mobile so that even the ultimate apparatus the 'Human Consciousness' could in principle be measured and analyzed as a quantum object, provided that a suitable classical device could be found to carry out the task. The problem of open quantum systems is traditionally resolved by means of non-Markovian Stochastic Schrödinger Equation.

The approach basically considers that the present state of any quantum system is random but linked with the past and future states but can be predicted. Specific applications of the stochastic equation are known as von Neumann Equation or Quantum Liouville Equation and Lindblad Equation. Neumann Equation / Quantum Liouville Equation and Lindblad Equation describe the time evolution of a mixed quantum state by means of the

density matrix/density operator that is defined as the sum of all the possible probability densities (Lindblad 1976; Attal *et al.* 2006; Tarasov 2008). The Time Dependent Schrödinger Wave Equation (TDSWE) is widely used in quantum theory, especially in the study of physical phenomena for which the potential energy of the system explicitly depends on time (e.g., spectroscopic theory). The approaches above are all based on the position dependant Hamiltonian Operator which is defined in terms of position/space dependant momentum operator leading to a kinetic energy operator purely depends on position/space. However, open quantum systems surely consist of time dependant kinetic and potential energies. Therefore especially as in Hamilton Picture, time dependant momentum operator seems to be quite important and missing. Definition of 'time dependant momentum operator' is only possible if time dependant position is actually definable. Description of time position relation could undoubtedly influence the quantum theory and philosophy in the sense that as deterministic view strengthens, probabilistic view would weaken. The approach is scientifically very simple but philosophically complicated and would possibly be bridging the existing gap between the theory and certain paradoxes and conceptual difficulties of the quantum theory. The present work is based on this simple however vital estimation of realistic time dependant momentum operator. The work initially defines a realistic time dependant alternative momentum operator and Hamiltonian operator, leading to an TDSWE which can be employed from the simplest to the most complicated quantum systems.

The other motivation of the present work arises from the fact that no counterpart of the time dependant classical wave equation exists within the quantum theory. Waves accompanying the quantum particles are, naturally, matching with the classical counterparts. It is very well known that, space dependant classical wave equation is matched by the time independent and position dependant Schrödinger Wave Equation. The other part of the classical wave equation, namely time dependant part, is simply missing. Nevertheless, time dependant classical wave equation should naturally exist within the quantum world. The present work is also based on this most obvious mental picture.

2. What Do Schrödinger Waves Mean?

One of the most controversial issues of quantum mechanics is the 'physical meaning' of the matter/Schrödinger Waves. The issue undoubtedly has some philosophical concerns. Physics, in general, describe the natural events in terms of either 'particle' or 'wave' nature. In relation to this initiative, the quantum physics is fundamentally based on the wave particle duality of the atomic/micro world offered by the famous physicist de Broglie. De Broglie. The duality expresses that any particle with a mass of 'm' and the velocity of 'v', must be accompanied by a momentum of,

$$p = h/\lambda \tag{1}$$

where h is a well known universal constant known as the Planck's Constant and λ is the wave length of the accompanying waves. The idea of wave particle duality and how to describe a wave accompanying to a particle have been confusing subject matter and majority of the physicist starting from the foundations of quantum physics including the famous quantum physicist such as Schrödinger, Bohr and de Broglie felt uncomfortable on the issue. One of the main philosophical debates is based on the separation of the descriptions relating to the 'classical waves' and 'Schrödinger/matter waves'. The main question is; Should there be a difference between the classical wave character for example on a water surface and the waves accompanying an electron? In general, the difficulty originates from the wave nature of a quantum particle and the relation between the wave function and the probability distribution of a quantum particle. Specifically, the combination of a particle with a wave creates serious understanding problems. Most of the physicists, at this point, imagine that the particle is broken up into infinite number of smaller particles like the particles in classical media and then transfer to the wave nature accordingly. Nevertheless, it's well known from the Schrödinger Wave Equation that the particles has a mass of 'm' and never splits into smaller particles.

In order to understand and describe the waves accompanying a quantum particle, we consider the most general situation that is the free particle case. It is the textbook knowledge that the solution of one dimensional space Dependent Schrödinger Wave Equation for a free particle is given by,

$$\psi(x) = Ae^{ikx} + Be^{-ikx} = a\cos(kx) + ib\sin(kx) \tag{2}$$

where A is a complex constant describing the amplitude of the Schrödinger waves in the '+x' direction and similarly B is a complex constant describing the amplitude of the Schrödinger waves in the '-x' direction.

In this expression, the wave number is given by $k = 2\pi/\lambda$. Considering the classical waves and comparing the

quantum waves with the classical waves, the following arguments can be expressed on the physical meaning of the Schrödinger Waves:

- In the classical waves, every single point within the medium has a vibrating particle with a constant frequency and wave length, whereas the quantum mechanical counterpart comprises a single particle whose frequency must be changing with the wave number. In other words, the quantum cases must be dispersive.
- When the quantum particles transform to the wave nature, it must hold the particle as a hole with a mass of 'm' according to the Schrödinger's Wave Equation. In other words the particle cannot be broken up.
- The quantum particle must be vibrating with a constant frequency of $\omega = E/\hbar$ at every single point of x and time of t. Similarly the accompanying wave has a constant wave length of λ for any x and t point.
- The wave nature of a quantum particle, $\psi(x)$, can not be directly observed. This is due to the well known 'measurement problem' of quantum mechanics. More specifically the wave nature of the particle simply disappears when its position is measured. The only way to measure the wave nature can be achieved by considering a vast number of identical quantum systems and simultaneously measuring the position of the particles.
- The actual position of the quantum particle within the wave ought to be continuously changing to maintain the well known probability distribution functions. This is very important and indicates that the group velocity of the particle changes which leads to a certain dispersion relation.

3. Questioning Schrödinger Wave Equation

The critical view that gave birth to the Quantum Mechanics is the opinion of wave property of particles offered by de Broglie. The other very important view offered by Planck is the relation between the overall 'mechanical energy' and the 'frequency'. Those novel approaches opened new doors to the exciting world of quantum mechanics. The first equation allowed us to initiate the quantum mechanics was the equation of de Broglie, that is $p = \hbar k$ and the other equation offered by Planck is written as $E = \hbar \omega$, where \hbar is known as the reduced Planck's constant, p denotes the linear momentum, k denotes the wave number and ω is the angular frequency. Hence, any quantum particle with a mass of 'm' ought to be obeying the wave equation of Schrödinger, that is;

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x, t)\psi(x, t) \quad (3)$$

Questioning the time and position dependent Schrödinger Wave Equation (SWE) above, allows some fundamental properties to be underlined. First of all, SWE assumes that the 'space' and 'time' are 'continuous' free variables. This is actually philosophically important in the sense that the quantum world is based on 'discreteness'. The philosophical question at this point is; How far does the continuity goes concerning space and time? Assuming time and space continuous surely has some important effects on the entire theory. The other point ought to be underlined is that the mass of the particle in the equation represented by 'm' and the 'wholeness' of the particle is hold even though it transfers to the wave nature. The equation originally includes a potential energy term of $V(x, t)$ which is **both space and time dependent** in general case. However, the time dependence of the potential energy of the quantum particle is, in the most cases, excluded due to having 'time free' conserved physical systems. Though, especially 'open quantum systems' would normally have time dependent potential energy terms and to tackle the situation an equation directly including a time dependent potential energy seem to be a genuine necessitate.

In the SWE, time dependent overall mechanical energy of the quantum particle is expressed as the Time Dependent Hamiltonian Operator which is,

$$\hat{E} = i\hbar \frac{\partial}{\partial t} \quad (4)$$

Additionally, the kinetic energy operator for the particles is only expressed in terms of space which is, expressed as,

$$\hat{E}_k = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \quad (5)$$

It is very convenient, at this point, to articulate that directly 'time dependent kinetic energy operator' is

nonexistent. Should there be a time dependent kinetic energy operator, it would be very functional tool for especially ‘open quantum systems’.

One of the most interesting and counterintuitive issues of the quantum mechanics is the ‘wave nature’ of the particles. In one hand, there is the position and time dependant SWE describing the waves accompanying the quantum particles; on the other hand, we have the solidly known Classical Wave Equation (CWE). Philosophically speaking, the two wave equations somehow ought to be combined and united since they both describe the wave nature. More specifically, the wave function relating classical waves for instance water waves, sound waves or electromagnetic waves must be the same with the waves accompanying the quantum particles such as electrons, photons, atoms. It is the textbook knowledge that all classical waves must be obeying the Classical Wave Equation (CWE) given bellow,

$$\frac{\partial^2 \psi(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi(x, t)}{\partial t^2} \quad (6)$$

This equation, in fact, defines any classical wave in terms of time and position. If the classical wave equation is separated into the two parts, namely the space dependent and time dependent components, then the following equations can simply be found respectively;

$$\frac{\partial^2 \psi(x)}{\partial x^2} + k^2 \psi(x) = 0 \quad (7)$$

$$\frac{\partial^2 \psi(t)}{\partial t^2} + w^2 \psi(t) = 0 \quad (8)$$

The quantum mechanical equivalent of the classical equations above can simply be found by performing the space and time separation of the time and space dependant SWE. It is then straightforward to obtain the following textbook expressions,

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} [E - V(x)] \psi(x) = 0 \quad (9)$$

$$\frac{d}{dt} \psi(t) + iw \psi(t) = 0 \quad (10)$$

A brief examination and comparison of the classical and quantum mechanical equations immediately signify that the space dependent classical waves and Schrodinger waves are just identical and are defined with the same equations and solutions. However, the time dependent classical wave equation and Schrodinger wave equations are just dissimilar. In other words, time dependent classical waves are described with a second order differential equation, however, time dependent Schrodinger equation is just a first order differential equation. However, interestingly should one take the first derivative of the time dependent SWE, it’s straightforward to obtain the classical counterpart. That is to say, the first derivative is equal to;

$$\frac{\partial^2 \psi(t)}{\partial t^2} + iw \frac{\partial \psi(t)}{\partial t} = 0 \quad (11)$$

If one substitutes the first derivative from the time dependent SWE which is,

$$\frac{\partial \psi}{\partial t}(t) = -iw \psi(t) \quad (12)$$

It’s easy to acquire the classical equation, which is

$$\frac{\partial^2 \psi(t)}{\partial t^2} + w^2 \psi(t) = 0 \quad (13)$$

Table 1. The comparison of the Classical and the Quantum Cases for the space and time dependent waves.

Concept	Classical Case	Quantum Case
Space Dependent Waves	$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$	$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0, k^2 = \frac{2m}{\hbar^2}[E - V(x)]$
Space Dependent General Solution	$\psi(x) = Ae^{ikx} + Be^{-ikx}$	$\psi(x) = Ae^{ikx} + Be^{-ikx}$
Time Dependent Waves	$\frac{\partial^2\psi(t)}{\partial t^2} + w^2\psi(t) = 0$	$\frac{d}{dt}\psi(t) + iw\psi(t) = 0$
Time Dependent General Solution	$\psi(t) = Ae^{iwt} + Be^{-iwt}$	$\psi(t) = Be^{-iwt}$

4. Time Dependent Momentum and Kinetic Energy Operators

Quantum mechanics define the physical concepts by means of dynamical variables and dynamical variables are represented by pure mathematical operators. The mathematical operators, on the other hand, transform the wave functions to an altered one. The quantum operators are typically defined in terms of time or space derivatives and must be 'linear' and 'hermitical' to represent the physical world. Quantum theory purely relies on the two distinct concepts namely 'state vectors/wave functions' and 'physical concepts/dynamical variables' (Griffiths 2004; Merzbacher 1970). Therefore, spatial and temporal variation of the wave functions and dynamical variables are very crucial to be determined. Time evaluation of quantum mechanical concepts, namely dynamical variables and wave functions, is a fundamental issue and being handled differently in the sense that Heisenberg Picture considers that the operators are time dependant whereas the wave functions/state vectors are constant; alternatively Schrödinger Picture assumes that wave functions/state vectors being time dependant, but the operators are stationary (Griffiths 2004; Merzbacher 1970). Heisenberg Equation simply gives the time evaluation of a quantum mechanical operator and the approach is simply based on the instantaneous operator terms which means that time dependence of any quantum mechanical operator could be defined. Quantum Liouville Equation or Neumann Equation, on the other hand, describes time evaluation of the density matrix/operator which is the overall wave function of the quantum system. The equations are very similar in the sense that both derived from the time dependent SWE and both depend on the commutation relation between the density matrix and the Hamiltonian operator.

The existing quantum theory is based on the overall mechanical energy which is the sum of the kinetic and potential energies. Most of the physical systems comprised of time independent potential energies whereas for the systems which are based on time dependent potential energies than the time dependent SWE including directly time dependent potential energy operator seem to be a require.

The existing theory is based on the position dependent momentum operator, which is defined as;

$$\hat{P}(x) = -i\hbar \frac{\partial}{\partial x} \tag{14}$$

The important point at this stage is that the momentum operator is only defined as a function of the position. It is crystal obvious that for the system based on highly time dependent potential energy, the kinetic energy is also ought to be very strongly time dependent. Therefore definition of a time dependent momentum operator is something seems to be critical and would be extremely valuable if defined. In order to obtain directly time dependent momentum operator it's straightforward to multiply the both ends by means of $\partial x / \partial t$ and then it's found that;

$$\hat{P}(x) \frac{\partial x}{\partial t} = -i\hbar \frac{\partial}{\partial x} \frac{\partial x}{\partial t} \tag{15}$$

Using the classical velocity definition that is $v(t) = \partial x / \partial t$, the time dependent momentum operator transforms to the definition below;

$$\hat{p}(t) = \frac{-i\hbar}{v(t)} \frac{\partial}{\partial t} \quad (16)$$

This final equation defines the time dependent momentum operator in terms of the free time variable and time dependent velocity. At this point, if one uses the classical kinetic energy definition which is

$\hat{E}_k(t) = \frac{\hat{p}^2(t)}{2m}$ and by substituting the definition above leads to the smart definition of time dependent kinetic energy operator, which is given by;

$$\hat{E}_k(t) = \frac{i\hbar}{2} \frac{\partial}{\partial t} \quad (17)$$

This expression is quite interesting in the sense that it is equal to the half of the textbook known time dependent Hamiltonian, which is

$$\hat{H}(t) = i\hbar \frac{\partial}{\partial t} \quad (18)$$

Using the time dependent kinetic energy operator and assuming that the overall Hamiltonian is given by the sum of the kinetic and potential energy operators, lead to the time dependent potential energy operator which can be given by;

$$\hat{V}(t) = \frac{i\hbar}{2} \frac{\partial}{\partial t} \quad (19)$$

This approach concludes that in Quantum Mechanics half of the time dependent Hamiltonian is equal to the kinetic energy operator and the other half equals to the time dependent potential energy operator. This is in general known as the Virial Theorem from classical physics. At this stage it's straightforward to obtain the time dependent SWE by simply substituting the definitions above within the well known Eigen function – Eigen value equation;

$$\hat{H}\psi(t) = E\psi(t) \quad (20)$$

The overall mechanical energy, E, is assumed to be fixed which means closed or conservative quantum systems are under consideration throughout. Substituting the overall time dependent Hamiltonian operator which is,

$$\hat{H} = \frac{i\hbar}{2} \frac{\partial}{\partial t} + V(t) \quad (21)$$

It's unproblematic to get

$$\frac{\partial\psi(t)}{\partial t} + i\frac{2}{\hbar}[E - V(t)]\psi(t) = 0 \quad (22)$$

Comparing this expression with the textbook time dependent SWE it's easy to extract the definition of angular frequency which can be found as,

$$w(t) = \frac{2}{\hbar}[E - V(t)] \quad (23)$$

it is very essential to express that, at this point, the frequency is naturally time dependent even though for the closed systems due to having $V(t)$ in the equation. This is something very important in terms of being an alternative approach to the time dependent Perturbation Theory. One then concludes that the evolution of any quantum system within time is in fact expressed by means of $w(t)$. Time dependent classical waves are described with a second order differential equation, nevertheless, time dependent Schrodinger equation is just first order differential equation.

However, interestingly enough, should one take the first derivative of the time dependent SWE, it's straightforward to obtain the classical counterpart. That is to say, the first derivative is equal to;

$$\frac{\partial^2 \psi(t)}{\partial t^2} + i\omega \frac{\partial \psi(t)}{\partial t} = 0 \quad (24)$$

If one substitutes the first derivative from the time dependent SWE which is,

$$\frac{\partial \psi(t)}{\partial t} = -i\omega \psi(t) \quad (25)$$

It's easy to get the classical equation, which is

$$\frac{\partial^2 \psi(t)}{\partial t^2} + \frac{4}{\hbar^2} [E - V(t)]^2 \psi(t) = 0 \quad (26)$$

The final equation meets two fundamental requirements expressed throughout. Alternatively, time dependant Schrödinger wave equation can directly be derived in the following way, The kinetic 'energy operator' given in the equation (16) can simply be substituted in the classical definition of the kinetic energy, $\hat{E}_k = \frac{p^2}{2m}$, then one can easily get the alternative formula of time dependant kinetic energy operator, that is,

$$\hat{E}_k = \frac{-\hbar^2}{2mv^2} \frac{\partial^2}{\partial t^2} \quad (27)$$

By simply using the standard equation of eigen function-eigen value that is (20) it is then straightforward to acquire the subsequent equation,

$$\frac{\partial^2}{\partial t^2} \psi(t) + \frac{2mv^2}{\hbar^2} [E - V(t)] \psi(t) = 0 \quad (28)$$

and of course the same equation of (26).

Firstly, the final ATDSWE simply contains a potential energy term which is directly time dependent. Hence any quantum system open or closed having a dynamic potential energy with explicitly time dependant potential energy can be analysed via this basic equation. This can be considered as an alternative approach to the well known time dependant perturbation theory. Secondly, the ATDSWE, is a second order differential equation which is in harmony with the classical counterpart. The angular frequency in this case is defined in general for the open quantum systems and can easily be reduced to the closed quantum systems. The main problem at this stage is of course to determine the time dependant kinetic energy of the quantum particle or alternatively time dependant potential and total mechanical energies of the quantum particle. Using the basic expression of kinetic energy, $E(t) - V(t) = T(t)$, leads to the result that, the quantum particles are accompanied by the Schrödinger/matter waves with frequencies totally depends only on the kinetic energy. This is evident that the wave frequency instantaneously varies as the kinetic energy changes as being the common property of the open quantum systems.

5. Spatial Probability Density for Free Particles

The existing quantum theory offers that any quantum particle without any spatial restriction is accompanied by a standard wave function of,

$$\psi(x) = Ae^{ikx} + Be^{-ikx} = a\cos(kx) + ib\sin(kx) \quad (27)$$

where constant A represents the amplitude of waves going in the +x direction and B is the counterpart for -x direction. Similarly a and b denote the amplitudes for the real and imaginary parts of the travelling waves.

In the most general case a and b are assumed to be non equal. Employing some special conditions, such as $x=0$ and $x = \frac{\lambda}{4}$ leads to the determination of the constant a and b. Going through the process gives the following;

$$\text{If } x=0 \quad \text{then} \quad \psi(x) = \psi_0 = a \quad \text{and if } x = \frac{\lambda}{4} \quad \text{then} \quad \psi(x) = \psi_{\frac{\lambda}{4}} = ib.$$

In this case, the spatial probability density in one dimension, in general, is defined as;

$$P(x) = |\psi(x)|^2 \quad (28)$$

It is now straightforward to get the spatial probability density for the free particles which is bellow,

$$P(x) = |\psi_0|^2 \cos^2(kx) + \left| \psi_{\frac{\lambda}{4}} \right|^2 \sin^2(kx) \quad (29)$$

The finding probability per unit length around x is plotted as a function of the angle below.

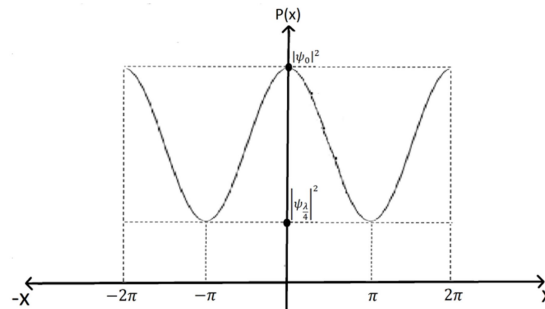


Figure 1. Periodic probability function of space.

According to the equation and of course the graph the spatial probability is a periodic function of space with a period of 2π . Due to be a periodic function, it is possible to say that all the information relating the probabilities is given within the $-\pi$ and π intersection. Hence the well known normalization condition can be maintained simply integrating the probability density from $-\pi$ to $+\pi$. Conducting the process gives that;

$$P = \int_{-\pi}^{\pi} \left[|\psi_0|^2 \cos^2(\theta) + \left| \psi_{\frac{\lambda}{4}} \right|^2 \sin^2(\theta) \right] d\theta = 1 \quad (30)$$

which leads to the equation of,

$$|\psi_0|^2 + \left| \psi_{\frac{\lambda}{4}} \right|^2 = \frac{1}{\pi} \quad (31)$$

Assuming ψ_0 and $\psi_{\frac{\lambda}{4}}$ are equal then it is straightforward to obtain $\psi_0 = \psi_{\frac{\lambda}{4}} = \frac{1}{\sqrt{2\pi}}$. This result is in accordance with the standard textbook wave function expression for the free particles. It is very interesting, at this point, to note that the probability density for a free particle changes between $|\psi_0|^2$ and $\left| \psi_{\frac{\lambda}{4}} \right|^2$. In the case of equality, the probability density does not change hence same at every point which means the particle exist with the same probability at any point. This is of course in harmony with the standard textbook knowledge.

6. Temporal Probability Density for Free Particles:

The quantum systems having time dependent Hamiltonian or Potential Energy is resolved by the well known 'time dependent perturbation theory'. This is the case, in general, for the open quantum systems. The quantum particles with very strongly and directly time dependent potential energies can alternatively be analysed by means of the ATDSWE accessible above. The equation offered above (19) is very useful in terms of including directly the time dependent potential energy term, $V(t)$.

The finding probability of a quantum particle on a fixed point of x , must be changing with time for the open quantum systems. Hence temporal probability density ought to be described clearly for both bound and free particles. The original definition and interpretation of Bohr for the probability density can be employed in the definition of 'temporal probability density' which is simply,

$$P(t) = |\psi(t)|^2 \quad (32)$$

Employing the time dependent SWE above, for the free particles, merely gives the time dependent wave function as follows,

$$\psi(t) = Ce^{iwt} + De^{-iwt} = c\cos(wt) + id\sin(wt) \quad (33)$$

where w denotes the angular frequency of vibrating quantum particle, C , D and c , d are constants determined by the physical boundary conditions in the backward and forward propagations, respectively. In the most general case, c and d are assumed to be non equal. Employing some special conditions, such as $t=0$ and $t = T/4$ leads to the determination of the constant c and d . Going through the process gives,

$$\text{if } t=0 \quad \text{then} \quad \psi(t) = \psi_0 = c \quad \text{and if} \quad t = \frac{T}{4} \quad \text{then} \quad \psi(t) = \psi_{\frac{T}{4}} = id.$$

It is very interesting to note that the general term has both forward and backward time propagation terms, exclusively different from the free particle solution of the ordinary TDSWE. Quantum systems/ particles are, in most cases, naturally confined in space but confinement in terms of time does not normally exist. Therefore boundary conditions for the time evolution of the quantum systems/particles is quite different compared to the spatial conditions. Spatial confinement of any quantum particle is just fulfilled naturally however temporal confinement is not possible. Philosophically speaking creation of any quantum system or particle is not possible to set the initial conditions. Instead, the quantum systems and particles just exist independently from time and no temporal initial conditions can normally be set.

In the most general case, the constants of C and D can be constant no equal then temporal probability density can be define as;

$$P(t) = |\psi_0|^2 \cos^2(wt) + \left| \psi_{\frac{T}{4}} \right|^2 \sin^2(wt) \quad (34)$$

Similar discussions to the space dependent probability density can be carried on for the time dependent case. The position dependent graph is then identical for the time dependence by simply substituting x with t and $P(x)$ with $P(t)$.

Considering the equation and of course the graph the temporal probability is a periodic function of time with period of 2π . Due to be a periodic function it is possible to say that all the information is given within the $-\pi$ and π intersection. Hence the well known normalization condition can be maintained simply integrating the probability density from $-\pi$ to $+\pi$. Conducting to process gives that;

$$P = \int_{-\pi}^{\pi} \left[|\psi_0|^2 \cos^2(\theta) + \left| \psi_{\frac{T}{4}} \right|^2 \sin^2(\theta) \right] d\theta = 1 \quad (35)$$

which leads to the equation of

$$|\psi_0|^2 + \left| \psi_{\frac{T}{4}} \right|^2 = \frac{1}{\pi} \quad (36)$$

Assuming ψ_0 and $\psi_{\frac{T}{4}}$ are equal then it is straightforward to extract $\psi_0 = \psi_{\frac{T}{4}} = \frac{1}{\sqrt{2\pi}}$. This result is in accordance with the standard textbook wave function expression for the free particles. It is very interesting at this point to note that the probability density for a free particle changes between $|\psi_0|^2$ and $\left| \psi_{\frac{T}{4}} \right|^2$ and in the case of equality the probability density does not change hence same at every point which means the particle exist with the same probability at any point. This is of course in harmony with the standard textbook knowledge.

7. Conclusions

The first conclusion is the definition of a ‘momentum operator’ in terms of directly time derivative; hence the operator basically describes the time evolution of momentum for a quantum particle. The pictures of Schrödinger and Heisenberg, without any doubt, analyze the counterintuitive structure of the quantum theory, however more realistic representation ought to consider simultaneous variation of both dynamical variables/operators and state vectors/wave functions with respect to time and space. Restricting the variation of them, as in the HM and SM, is not philosophically realistic. The momentum operator is simply based on the ‘time dependant position operator’ for the quantum particles and in fact would initiate deep philosophic discussions through. Time dependant

momentum operator specifically expresses that instantaneous position of any quantum particle could, in fact, be predictable. This conclusion, on its own, is revolutionary and supports Einstein in the sense that a novel approaches could be developed to move the quantum theory from purely probabilistic to more deterministic.

The second important conclusion is the suggestion of so called directly time dependent SWE. Fundamental time dependent SWE, which is the counterpart of the classical time dependant wave equation of classical mechanics, is simply derived by means of novel defined time dependant momentum operator'. The equation only requires time dependant kinetic energy or alternatively time dependant both potential and overall mechanical energies to estimate the time evolution of the Schrödinger wave functions. The equation can further be expanded to obtain the time evolution of the density matrix, as in the cases of NE/QLE or LE. The a time dependent SWE also expresses the basic relation between kinetic energy, or alternatively potential and total mechanical energies and angular frequency of the Schrödinger waves accompanying the quantum particles. The time evolution of the frequency is directly proportional to the actual time dependant kinetic energy of the quantum particles. It is also important to note that time dependant kinetic energy for the quantum particles is simply equal to the “zero point energy”, would surely ignite many philosophical and scientific discussions in the future.

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