Scaling of Transformer Ratio and Energy Gain in a Plasma Wakefield Acceleration Scheme Driven by Positron Beam

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Abstract
This paper focus on the response of the transformer ratio (R) and energy gain by the wakefield (ΔE) to change or variation in beam/plasma parameters in a plasma wakefield acceleration scheme driven by positron beam in a metre – scale plasma simulated in two dimension, using the plasma form whose particles are variably weighted to represent density ramp. The positron beam propagation regime followed the magnetically self-focus regime. $R < 2$ was achieved when the beam charge was altered, except for the case at $350pC$ that $R > 2$. The trend shows that R varies slightly with change in beam charge. ΔE increase with increase in beam charge, where at $350pC, \Delta E$ in excess of $60GeV$ was achieved. R increase with decrease in beam length (σ_z) to as high as 6.7 when $\sigma_z = 50\mu m$. But for $\sigma_z < 50\mu m$, R decrease adversely. Similarly, the energy gain increased with decrease in beam length, reaching a maximum value of $200.1GeV$ at $\sigma_z = 50\mu m$. For $\sigma_z < 50\mu m$ the energy gain decrease with further decrease in beam length. The transformer ratio increase non-uniformly with increase in plasma density (n_p). $R \geq 2$ and $\Delta E \geq 60GeV$ were achieved when $n_p > 3\times10^{21}m^{-3}$. Both R and ΔE decrease with increase in beam radius. Also ΔE and R decrease with increase in beam charge and increase with increase in plasma density for the two parameter optimization of the beam charge and plasma density. For this optimization $R > 2$ and $\Delta E > 60GeV$ were achieved. Also the two parameter optimization involving the beam charge and beam length shows that both ΔE and R increase with increase in beam length and decrease in beam charge. In this optimization, $R \geq 2$ and $\Delta E \geq 60GeV$ for each of the parameters varied.
Keywords: Transformer ratio, Energy Gain by the Wakefield, Beam/Plasma Parameters

1.0 Introduction

It is no longer news that ultrahigh accelerating field (longitudinal electric field gradient) and large focusing field gradient can be generated and sustained in plasma. This potential in plasma can advance the energy frontiers in accelerators, for possible application in many fields such as high energy and particle physics, condensed matter physics, industries, among others. The present day energy regime requirement has reached the Tera electron volt (TeV) level and in view of this, several researchers have worked extensively to unravel the promise and potentials imbedded in plasma based accelerator device with the aim of exploring its effectiveness and efficiency as a potential replacement of the conventional accelerators owing to the limitations inherent in the conventional accelerators. These works have proved several impressive accelerating field gradients with some orders of magnitude greater than that achieved in conventional accelerators, for instance, $1GVm^{-1}$ [1 – 4], $0.75GeVm^{-1}$ [5], $3.43GVm^{-1}$ [6], $2.9GVm^{-1}$ and $4.1GVm^{-1}$ [7], $2.0GVm^{-1}$ [8], $3.2\times10^{11}Vm^{-1}$ and $5.2\times10^{11}Vm^{-1}$ [9] among others have been reported in literatures, where in each case, different beam/plasma parameters were used. Though the plasma wakefield accelerator is still at the infant stage where several strategies are being explored through simulations and experiments so that its applicability as a possible replacement of the conventional accelerator is a reality.

But such possible application of the plasma wakefield accelerators (PWFAs) as a particle accelerator does not only depend on the extent of the electric field gradient achieved so far. It also depends on other configurations such as the transformer ratio (R) and energy gain (ΔE) by the wakefield behind the drive beam, the beam quality (such as energy spread and emittance of the beam) and the transverse stability of the beam as it traverse the plasma length. The PWFA can best be viewed as an energy transfer device where energy is transferred from the drive beam to the wakefield which is a critical component of PWFA worth investigating. An important parameter that specify the energy transfer in beam driven PWFA is the transformer ratio which is simply a ratio of the peak accelerating field (maximum electric field gradient) of the wakefield (charge oscillation or disturbance behind the drive beam) to the peak decelerating (retarding) field within the drive beam. It is a measure of the maximum energy gain by the wakefield behind the drive beam [10]. A witness beam or any charged particle will be accelerated to extremely large energy proportional to the energy gained by the wakefield, if injected in an appropriate region of the wakefield. The transformer ratio is only relevant when the drive beam propagated through the plasma is completely decelerated, because at this stage, it is believed that the drive beam has transferred all its energy to the wakefield [11]. Therefore, significant insight about the energy transferred from the drive beam to the wakefield can be assessed since the it determines the energy gain by the wakefield in plasma based accelerators. Due to the significance of the transformer ratio in beam driven PWFA scheme, several efforts and suggestions has been put in place on how to improve the transformer ratio above the theoretical limit of two, and such efforts yielded different values of transformer ratio. For instance, using a 1 – D square pulse where $n_b = \frac{n_0}{2}$, $R = 4$ was reported [12]. Also using a ramped bunch train and a phased bunch schemes, $R = 7.89$ and $R = 2.67$ respectively were reported [10]. $R = 1.2$ [13] and 1.88 [8] has been reported in literature. For instance, it has been suggested that the transformer ratio can be greater than or equals to two ($R \geq 2$) if the PWFA is operated in the nonlinear regime or by using asymmetric drive beam [11]. Shaping the drive beam so that $R \geq 2$ [14] and the use of more than one drive beam to cumulatively excite the plasma wakefield [7] has also been suggested.

In this paper, we present the response of R and ΔE (energy gain by the wakefield behind the drive beam) to change in beam/plasma parameter(s) in a plasma wakefield acceleration scheme driven by positron beam through a metre – scale plasma length in an underdense plasma whose density is variably weighted to represent density ramp with a view of assessing how our choice of beam – plasma parameters affects the magnitude of R and ΔE achievable, since the transformer ratio dependence on the shape of the drive beam and peak current has been critically examined in [11] and [15] respectively.

2.0 Basic Theoretical Background

2.1 Transformer Ratio Limit of a Symmetric Drive Beam and it Physical Interpretation

The upper limit to the transformer ratio for a symmetric drive beam in a plasma wakefield acceleration scheme can be proved to have a maximum value of 2 from the result of a linear wakefield excitation or impulse response in plasma acceleration. The longitudinal wakefield excited by a single symmetric drive beam in the linear regime [16] is given by

$$E_z(\xi) = \frac{e}{\varepsilon_0}\int_{-\infty}^{\xi} n_b(\xi')\cos\left[k_p(\xi - \xi')\right]d\xi' \tag{1}$$

The wakefield at a point ξ within the drive beam with a density distribution $n_b(\xi)$ is given by

$$E_z^-(\xi) = \frac{e}{\varepsilon_0}\int_{-\infty}^{\xi} n_b(\xi')\cos\left[k_p(\xi - \xi')\right]d\xi' \tag{2}$$

But we know that $cos(u + v) = cosucosv - sinusinv$, therefore

$$cos(k_p\xi - k_p\xi') = cosk_p\xi cosk_p\xi' - sink_p\xi sink_p\xi' \tag{3}$$

From equation (3), (2) becomes

$$E_z^-(\xi) = \frac{e}{\varepsilon_0}\int_{-\infty}^{\xi} n_b(\xi')\left[cosk_p\xi cosk_p\xi' - sink_p\xi sink_p\xi'\right]d\xi' \quad \text{or}$$

$$= \frac{e}{\varepsilon_0}\int_{-\infty}^{\xi} n_b(\xi') cosk_p\xi cosk_p\xi' d\xi' - \frac{e}{\varepsilon_0}\int_{-\infty}^{\xi} n_b(\xi') sink_p\xi sink_p\xi' d\xi' \quad \text{or}$$

$$E_z^-(\xi) = \frac{e}{\varepsilon_0} cosk_p\xi \int_{-\infty}^{\xi} n_b(\xi') cosk_p\xi' d\xi' - \frac{e}{\varepsilon_0} sink_p\xi \int_{-\infty}^{\xi} n_b(\xi') sin\,k_p\xi' d\xi' \tag{4}$$

As the drive beam traverses the plasma length, the core of the beam lose energy to the wakefield behind the drive beam. The beam is completely decelerated when it lose its energy completely to the wakefield, therefore assuming a symmetric drive beam where $\xi = 0$, at the centre so that equation (4) becomes

$$E_z^-(0) = \frac{e}{\varepsilon_0}\int_{-\infty}^{0} n_b(\xi') cosk_p\xi' d\xi' \tag{5}$$

Since the peak decelerating field E_{max}^- corresponds to the point where the core of the drive beam has completely lose its energy to the wakefield, therefore $E_{max}^- = E_{max}^-(0)$, but if the drive beam has not lose its energy completely to the wakefield behind the drive beam, then $|E_{max}^-| > |E_z^-(0)|$. However for the case of any instance $|E_{max}^-| \geq |E_z^-(0)|$, therefore,

$$E_{max}^- = \frac{e}{\varepsilon_0}\int_{-\infty}^{0} n_b(\xi') cosk_p\xi' d\xi' \tag{6}$$

Also the wakefield at a point ξ behind the drive beam is given by

$$E_z^+(\xi) = \frac{e}{\varepsilon_0}\int_{-\infty}^{\infty} n_b(\xi')\cos\left[k_p(\xi - \xi')\right] d\xi' \quad \text{or}$$

$$E_z^+(\xi) = \frac{e}{\varepsilon_0} cosk_p\xi \int_{-\infty}^{\infty} n_b(\xi') cosk_p\xi' d\xi' - \frac{e}{\varepsilon_0} sink_p\xi \int_{-\infty}^{\infty} n_b(\xi') sin\,k_p\xi' d\xi' \tag{7}$$

Assuming the drive beam distribution n_b is symmetric, then from equation (7),

$$\int_{-\infty}^{\infty} n_b(\xi') sin\,k_p\xi' d\xi' = 0$$

Therefore equation (7) reduced to the form

$$E_z^+(\xi) = \frac{e}{\varepsilon_0} cosk_p\xi \int_{-\infty}^{\infty} n_b(\xi') cosk_p\xi' d\xi' \tag{8}$$

If the maximum amplitude is reached, then $cosk_p\xi = -1$, therefore equation (8) becomes

$$E_z^+(\xi) = -\frac{e}{\varepsilon_0}\int_{-\infty}^{\infty} n_b(\xi') cosk_p\xi' d\xi' \quad \text{or}$$

$$E_z^+(\xi) = -2\frac{e}{\varepsilon_0}\int_{-\infty}^{0} n_b(\xi')\cos k_p\xi'\, d\xi' \quad (9)$$

Using equation (6), (8) can be written as

$E_z^+ = -2E_{max}^-$ or

$E_z^+ = 2|E_z^-(0)| \leq 2|E_{max}^-|$

So that

$|E_{max}^+| \leq 2|E_{max}^-|$ or

$$\left|\frac{E_{max}^+}{E_{max}^-}\right| \leq 2 \quad (10)$$

Therefore, the transformer ratio (R) by definition (defined as the ratio of the peak accelerating field to peak decelerating field) is given as

$$R = \frac{E_{max}^+}{E_{max}^-} \quad (11)$$

From equation (10), the transformer ratio will have a maximum value of 2, if the core of the drive beam completely lose all its energy to the wakefield. At this point the drive beam will be completely decelerated as it traverse the plasma length whose density distribution (n_p) is uniform. The case where n_p or n_b is variably weighted to represent density ramp, may influence the magnitude of the transformer ratio. In this work, the plasma density n_p is variably weighted to represent density ramp through which a single symmetric Gaussian beam is driven in order to excite plasma wake.

The physical interpretation of R can best be understood if we assume that the drive beam has lose all its energy E_{drive} to the wakefield after traversing a plasma length L given by

$$L = \frac{E_{drive}}{E_{max}^-} \quad (12)$$

Because after this distance, the drive beam must have deposited all its energy to the wakefield behind the drive beam. Therefore a test charge injected in correspondence of E_{max}^+ or in an appropriate region of the wakefield behind the drive beam will gain energy (ΔE) of the form

$$\Delta E = E_{max}^+ L \quad (13)$$

Substituting equation (12) into (13), we get

$$\Delta E = \frac{E_{max}^+}{E_{max}^-} E_{drive} \quad (14)$$

Using equation (11), (14) can be written as

$$\Delta E = RE_{driver} \quad (15)$$

According to equation (15), the energy gain by the wakefield largely depend on the transformer ratio and the energy of the drive beam. Therefore, transformer ratio is a key parameter that specify the energy for particle acceleration in beam driven plasma wakefield acceleration scheme.

2.2 Two – D Particle – in – Cell (PIC) Simulations Using Vsim

The simulation package – Vsim, comprise an electromagnetic solver that uses the Yee algorithm and initBeam macro to set up the initial beam properties. The positron beam is set to travel with the speed of light as it traverses the plasma length (1m) and Match Absorbing Layers (MALs) are used on transverse side of the simulation window to absorb outgoing waves. The plasma is represented by macro particles and the particles are variably weighted to represent density ramp. Both the beam and plasma particles are moved by the Boris push, but the plasma ions are considered stationary in the simulation time frame.

Positron beam initializes the field using a speed of light frame Poisson equation solve where the fields are evolved using the Finite Difference Time Domain (FDTD) PIC [8]. The electromagnetic fields are then updated via FDTD Yee algorithm scheme. The positron beam is launched at $x = 0$ in the positive x – direction with the Lorentz boosted Poisson field to ensure that the simulation is self – consistent from start. The beam is allowed to propagate through a ramped plasma density distribution and the simulation is run for specific number of time step with data dumped periodically to Hierarchical Data Format version 5 (HDF5) files.

The simulation geometry consist of a grid size of $LX = 12\times10^{-3}m$ and $LY = 44\times10^{-4}m$ with the number of cells $\Delta x = 128$ and $\Delta y = 176$. Beam particles per cell = 16 and plasma particles per cell = 9. The total numbers of beam and plasma particles used are 360448 and 202752 respectively. The positron beam density $n_b = 6.61\times10^{21}m^{-3}$, exceeds the initial plasma density $n_p = 3\times10^{21}m^{-3}$, and the positron beam propagation regime followed the magnetically self – focused regime. The initial longitudinal and transverse dimension of the beam are $\sigma_z = 75\mu m$ and $\sigma_r = 20\mu m$.

The beam has an initial charge of $500pC$ and energy of $30GeV$. The electron plasma frequency is $\omega_p = 3.26\times10^7 s^{-1}$. The beam length (σ_z), beam radius (σ_r), plasma density (n_p) and beam charge were each varied at different instant while other parameters are kept constant. We also performed two parameter optimization by simultaneously varying the plasma density and beam charge as well as the beam length and beam charge while in each case other parameters were kept constant. For each case, the peak accelerating and its corresponding decelerating fields are assessed after full propagation of the positron beam (FPPB) through the background plasma so as to enable the computation of R using equation (11) and ΔE using equation (15). The parameters varied are shown in Table 1 and Figure 1 is a flow chart describing how we assessed R and ΔE.

Table 1: Beam and Plasma Parameter(s) Varied.

S/No	Parameter(s) Varied	Range	Variation Factor
1	Beam Charge	$500pC - 100pC$	$50pC$
2	Beam Length (σ_z)	$75\mu m - 35\mu m$	$5\mu m$
3	Beam Radius (σ_r)	$20\mu m - 38\mu m$	$2\mu m$
4	Plasma Density (n_p)	$3\times10^{21}m^{-3} - 11\times10^{21}m^{-3}$	$1\times10^{21}m^{-3}$
5	Beam Charge and Plasma Density (n_p)	$480pC - 320pC$ and $4\times10^{21}\text{m}^{-3} - 12\times10^{21}\text{m}^{-3}$	$20pC$ and $1\times10^{21}m^{-3}$
6	Beam Charge and Beam Length (σ_z)	$480pC - 320pC$ and $85\mu m - 165\mu m$	20pC and 10μm

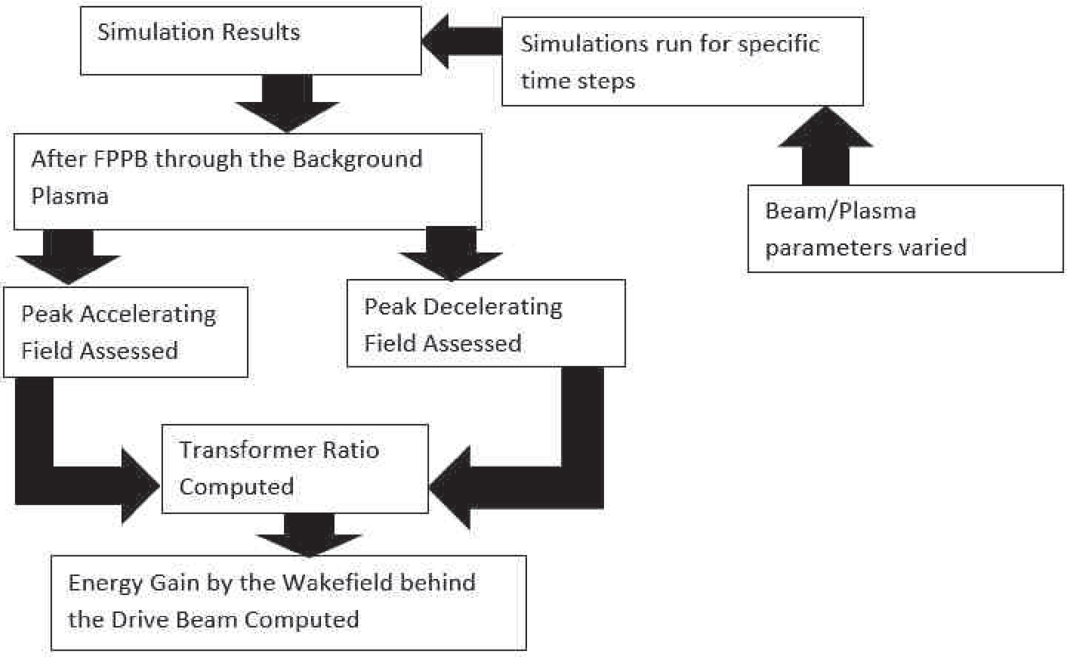


Figure 1: Flow Chart Illustrating the Evaluation of the Transformer Ratio and Energy Gain

3.0 Result and Discussion

Figure 2 shows how the transformer ratio varies with beam charge when reduced by a factor of $50pC$ from $500pC$ to $100pC$.

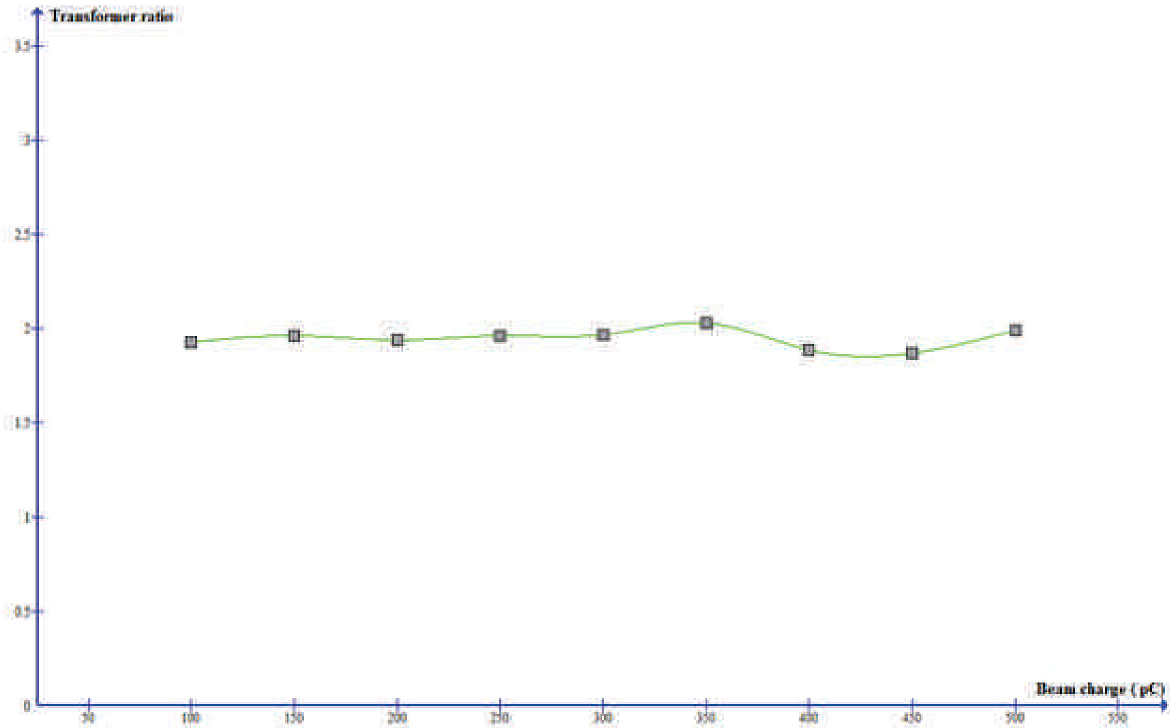


Figure 2: The Transformer Ratio against the Beam Charge when Reduced by a Factor of $50pC$ from $500pC$ to $100pC$.

The response of the transformer ratio to beam charge when reduced from $500pC$ to $100pC$ by a factor of $50pC$, shows that between $300pC$ and $100pC$, the transformer ratio is approximately steady and is slightly < 2. But the transformer ratio achieved is more than that reported in [8, 17]. But at $350pC$, the transformer ratio is > 2, and this contradict the linear theory prediction and the expected transformer ratio limit for a symmetric Gaussian beam $(R \leq 2)$. This is because a slight decrease in peak decelerating field was observed when the beam charge was altered to $350pC$, thus yielding high transformer ratio that is > 2, since the amplitude decelerating field is small. This is as expected of the transformer ratio in a plasma wakefield acceleration scheme operated in the under-dense (nonlinear) regime, as proposed by [11]. This decrease could be that the core of the beam lose large amount of its energy to the wakefield as it traverse the plasma length, the peak decelerating field observed. Meaning the energy loss by the core of the beam driving the plasma wake is high compared to other values of the beam charge that were altered while other parameters remain constant. Also the transformer ratio between $400pC$ and $500pC$ is less than 2.

Figure 3 shows how the energy gained by the wakefield behind the drive beam varies with beam charge when reduced by a factor of $50pC$ from $500pC$ to $100pC$.

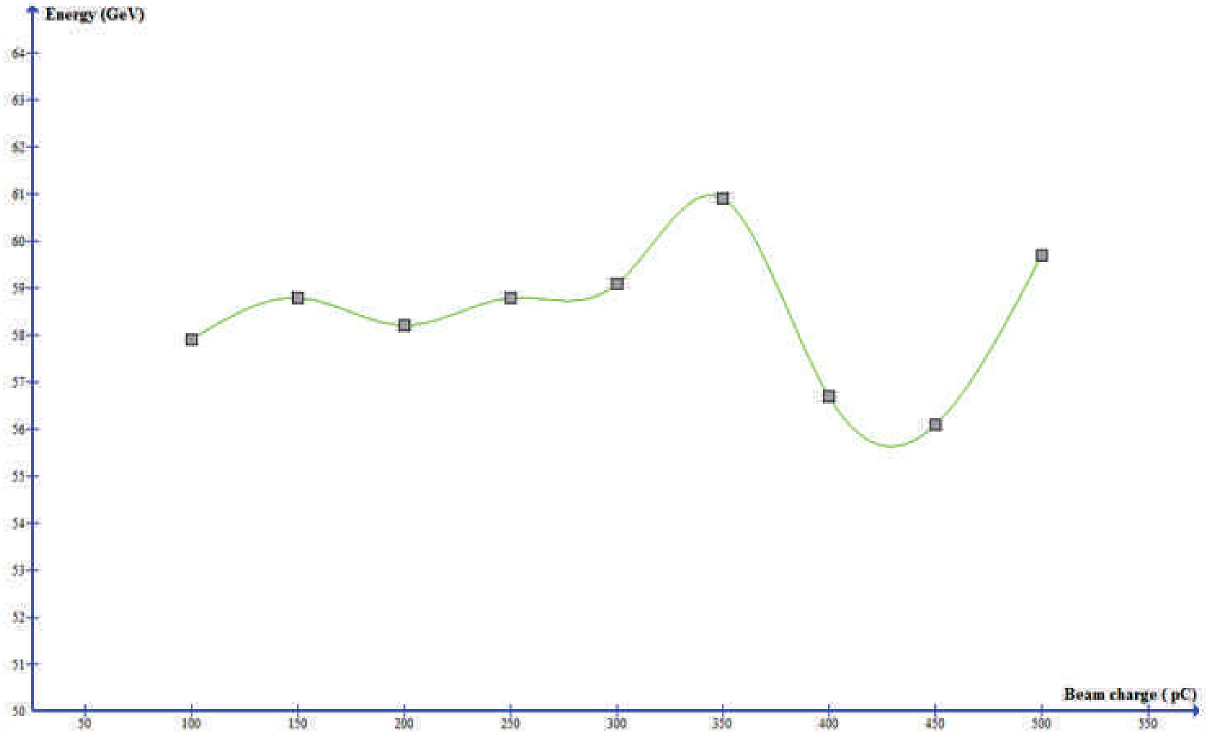


Figure 3: The Energy Gain against the Beam Charge when Reduced by a Factor of $50pC$ from $500pC$ to $100pC$.

Figure 3 reveal that the energy gain by the wakefield increases with increase in beam charge. Though the energy gain by the wakefield is high, in excess of $57GeV$ for most values of the beam charge varied, but increases non – uniformly with increase in beam charge. When the beam charge was altered to $350pC$, the energy gain by the wakefield is more than twice the energy of the drive beam (in excess of $60GeV$) beyond which saturation occur where the energy gain by the wakefield decrease and increase in that succession.

Figure 4 shows how the transformer ratio varies with beam length when reduced by a factor of $5\mu m$ from $75\mu m$ to $35\mu m$.

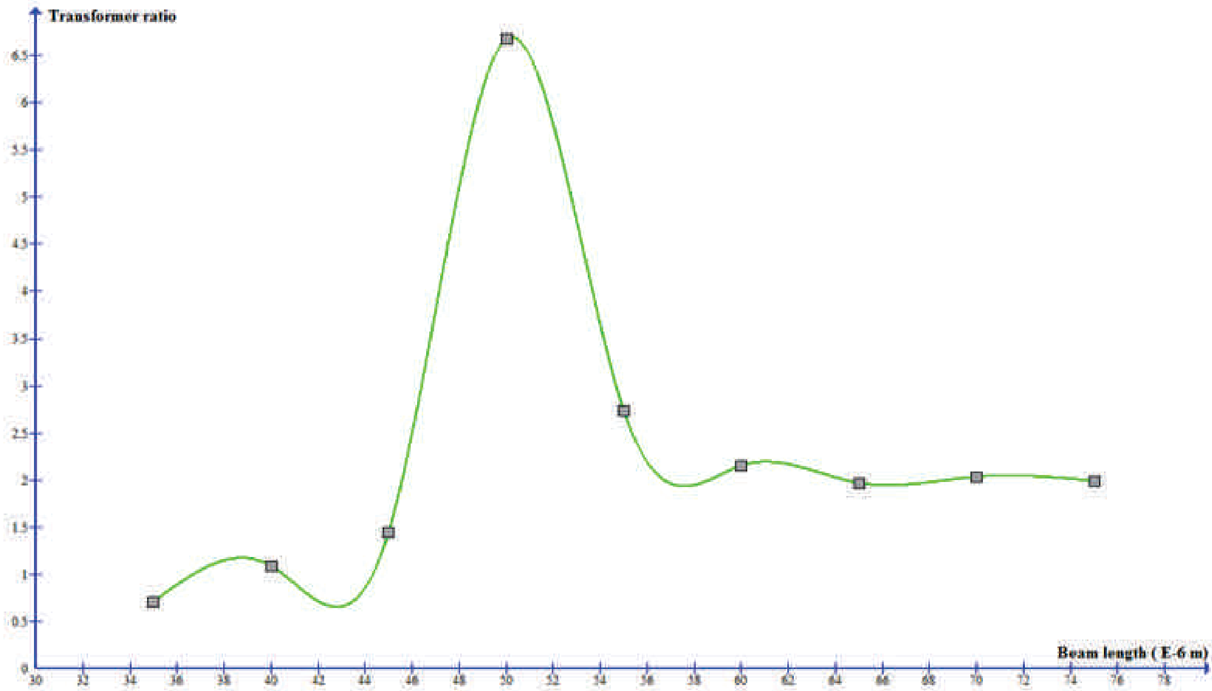


Figure 4: The Transformer Ratio as a Function of Beam Length when Reduced by a Factor of $5\mu m$ from $75\mu m$ to $35\mu m$.

ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.64, 2017

Figure 4 shows that the transformer ratio increases non – uniformly with decrease in beam length when altered between $75\mu m$ and $35\mu m$ by a factor of $5\mu m$. The trend shows a slow increase in transformer ratio between $75\mu m$ and $55\mu m$ and increase rapidly when the $\sigma_z = 50\mu m$, to as high as 6.7, more than that achieved in literature [12] using a 1-D square pulse where $n_b = \frac{n_0}{2}$, $L_b = 2.79\lambda_p$ and yields $R_t = 4$. In fact several suggestions on how to improve the transformer ratio above 2 has been proposed. Esarey *et al* [14] proposed shaping of the drive beam, and the use of non-symmetric drive beam and multiple drive beams to cumulatively increase the amplitude of the wakefield has been suggested [7, 18]. But in this case, a symmetric Gaussian beam propagated through plasma whose density is ramped yield a transformer ratio of 6.7. The basic parameters specifying the positron beam are: $30GeV$ energy, $\sigma_z = 50\mu m$, $\sigma_r = 20\mu m$ and $Q = 500pC$ propagated through a ramped plasma density of $3\times10^{21}m^{-3}$ yields high transformer ratio of this magnitude. This implies that a short positron beam propagated through plasma whose density is ramped will yield ultra-high transformer ratio. When the beam length was further decreased below $50\mu m$, the trend shows a rapid decrease in the transformer ratio. Meaning, as the beam approach a point-like form, the transformer ratio decrease rapidly, which suggest that beam lengths within such range will yield low transformer ratio.

Figure 5 shows how the energy gain by the wakefield behind the drive beam varies with beam length when reduced by a factor of $5\mu m$from $75\mu m$ to $35\mu m$.

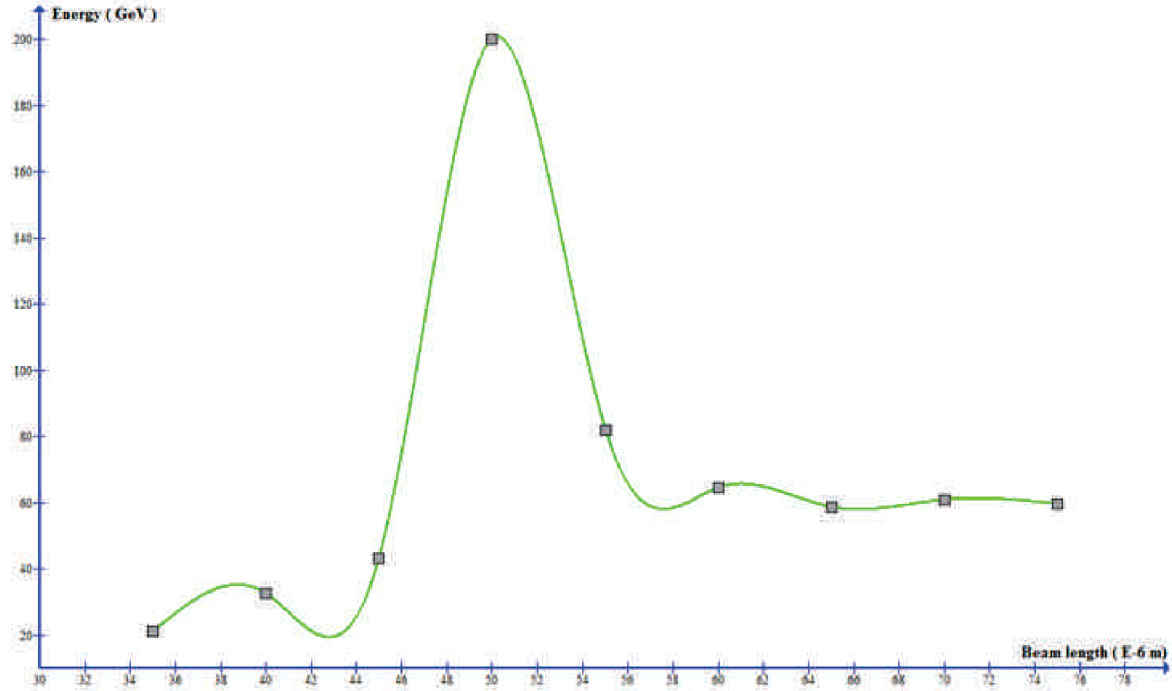


Figure 5: The Energy Gain as a Function of Beam Length when Reduced by a Factor of $5\mu m$ from $75\mu m$ to $35\mu m$.

From Figure 4, for the beam with longitudinal lengths $\sigma_z < 50\mu m$, the transformer ratio decrease rapidly, which suggest that beam lengths within such range will yield low transformer ratio as well as low energy gain as seen in figure 5 since the magnitude energy gain by the wakefield in beam driven PWFAs depend on transformer ratio. This explain why the trend of the transformer ratio for each beam/plasma parameter varied is similar to that of energy gain for that parameter. For $\sigma_z = 35\mu m$, the energy gain by the wakefield is $20GeV$, which is even less than the initial energy of the drive beam ($30GeV$) and is quite insignificant but for $\sigma_z = 50\mu m$, the energy gain by the wakefield is of the order of $200.1GeV$, quite impressive and is more than that reported in the literature [19], where energy doubling of $42GeV$ electrons in a metre-scale PWFA was demonstrated and energy gain of $85GeV$ was presented. The energy gain between $60\mu m$ and $55\mu m$ is also impressive because is more than twice the energy of the drive beam, while that between $65\mu m$ and $75\mu m$ is $59.7GeV$, almost double the energy of the drive beam.

Figure 6 show how the transformer ratio vary with plasma density when increased by a factor of $1\times10^{21}m^{-3}$ from $3\times10^{21}m^{-3}$to $11\times10^{21}m^{-3}$.

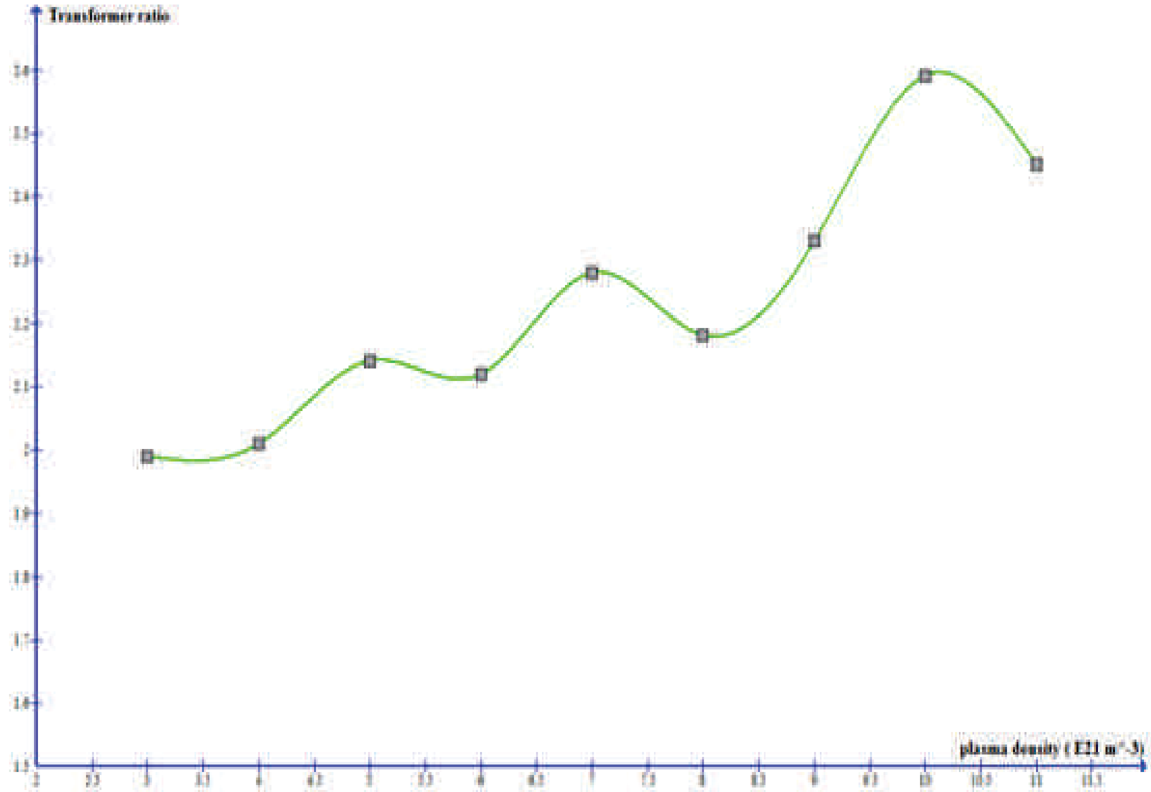


Figure 6: The Transformer Ratio against Plasma Density when Increased by a Factor of $\mathbf{1{\times}10^{21}m^{-3}}$ from $\mathbf{3{\times}10^{21}m^{-3}}$to $\mathbf{11{\times}10^{21}m^{-3}}$.

Figure 6 depicts the dependence of the transformer ratio on the plasma density (n_p) when altered from $3{\times}10^{21}m^{-3}$ to $11{\times}10^{21}m^{-3}$ by a factor of $1{\times}10^{21}m^{-3}$ while other parameters remain constant. The trend shows a non-uniform increase in transformer ratio with increase in plasma density. Beyond this density $3{\times}10^{21}m^{-3}$, the transformer ratio achieved is ≥ 2. This simply imply that upward ramped in plasma density yields transformer ratio that is > 2, which contradicts the idea of operating in the nonlinear regime $(n_b > n_p)$ to achieve transformer ratio that is > 2 [11], because between $7{\times}10^{21}m^{-3}$ and $11{\times}10^{21}m^{-3}$, the initial plasma density is more than the positron beam density $(n_p > n_b)$ and this is the linear regime. Therefore the magnitude of transformer ratio achievable in beam driven PWFAs does not only depend on whether it operated in nonlinear or linear regimes but also on the structural configuration of the plasma density.

Figure 7 shows how the energy gain by the wakefield behind the drive beam varies with plasma density when increased by a factor of $1{\times}10^{21}m^{-3}$ from $3{\times}10^{21}m^{-3}$to $11{\times}10^{21}m^{-3}$.

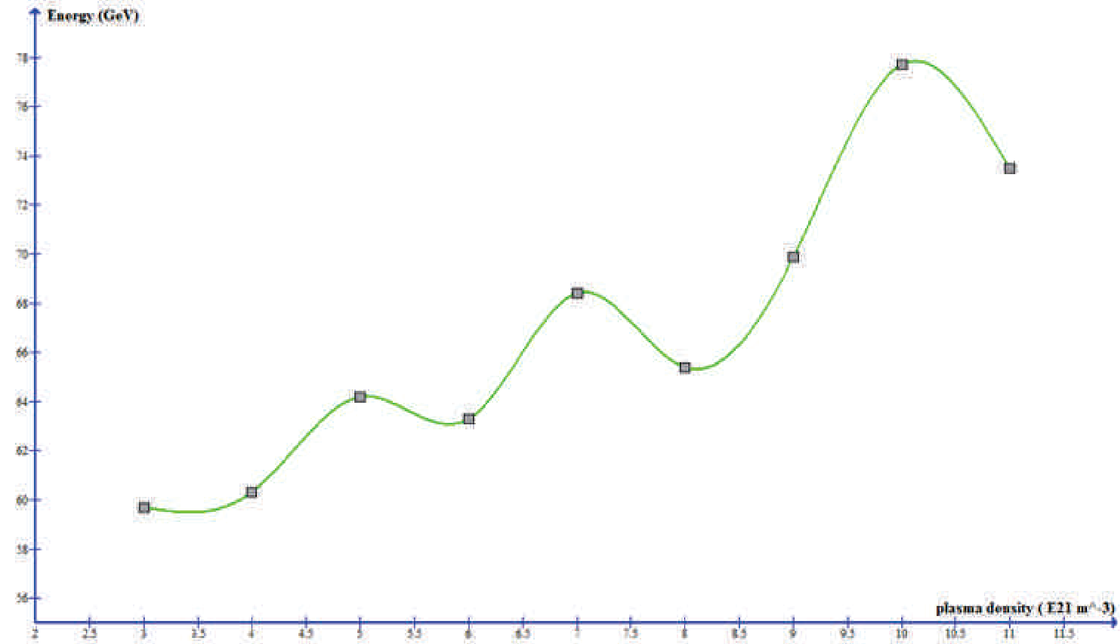


Figure 7: The Energy Gain as a Function of Plasma Density when Increased by a Factor of $1\times10^{21}m^{-3}$ from $3\times10^{21}m^{-3}$to $11\times10^{21}m^{-3}$.

The implication of the improved transformer ratio with increase in plasma density is seen in figure 6 where the energy gain by the wakefield behind the drive beam for $n_p \geq 4\times10^{21}m^{-3}$, is $\geq 60GeV$. This energy double and/or exceeds double the energy of the drive beam. The trend shows that the energy gain by the wakefield increase non-uniformly with increase plasma density, up to $10\times10^{21}m^{-3}$, beyond which it decrease.

Figure 8 shows how the transformer ratio varies with the beam radius when increased by a factor of $2\mu m$ from $22\mu m$ to $38\mu m$.

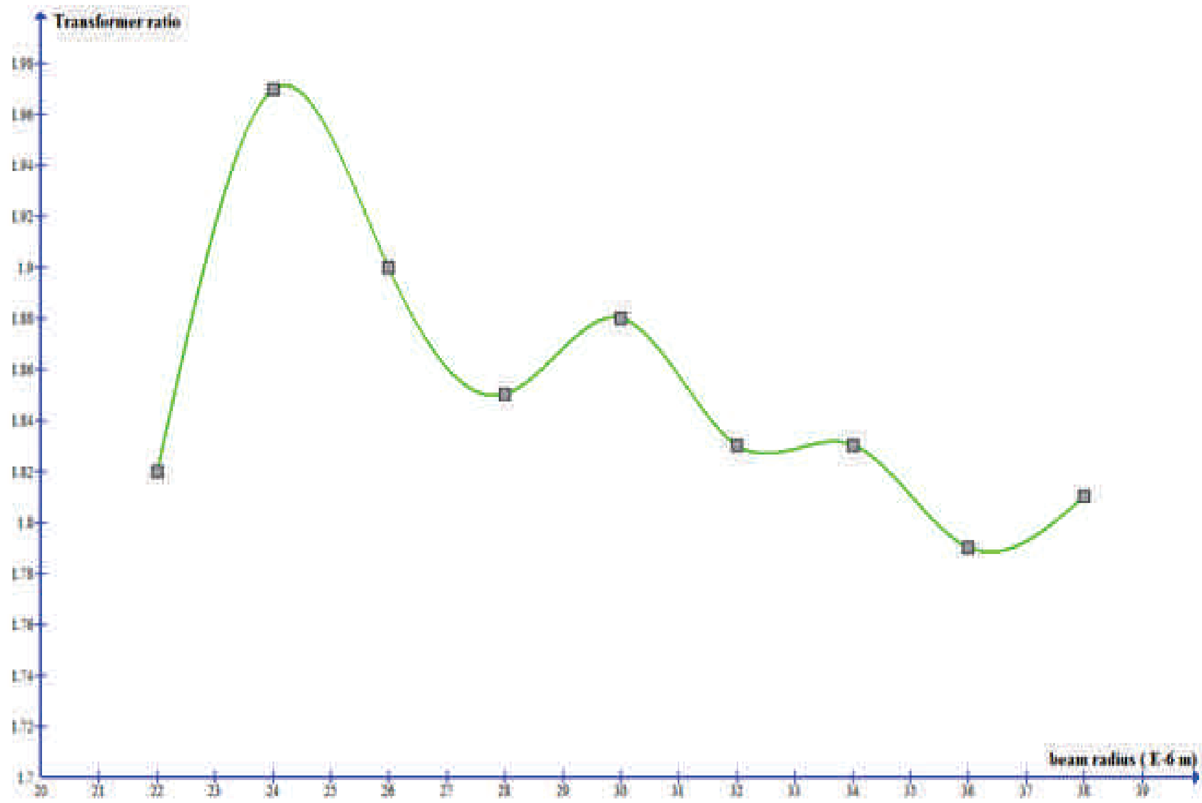


Figure 8: The Transformer Ratio against the Beam Radius when Increased by a Factor of $2\mu m$ from $22\mu m$ to $38\mu m$.

Figure 8 depicts the transformer ratio dependence on the beam radius when altered from $22\mu m$ to $38\mu m$ by a factor of $2\mu m$ while other parameters remain constant. The transformer ratio showed a nonlinear response when the beam radius is varied, but the trend shows that it reduces with increase in beam radius. The peak value of the transformer ratio achieved is < 2 as expected for a symmetric Gaussian beam propagated through plasma.

Figure 9 shows how the energy gain by the wakefield behind the drive beam varies with the beam radius when increased by a factor of $2\mu m$ from $22\mu m$ to $38\mu m$.

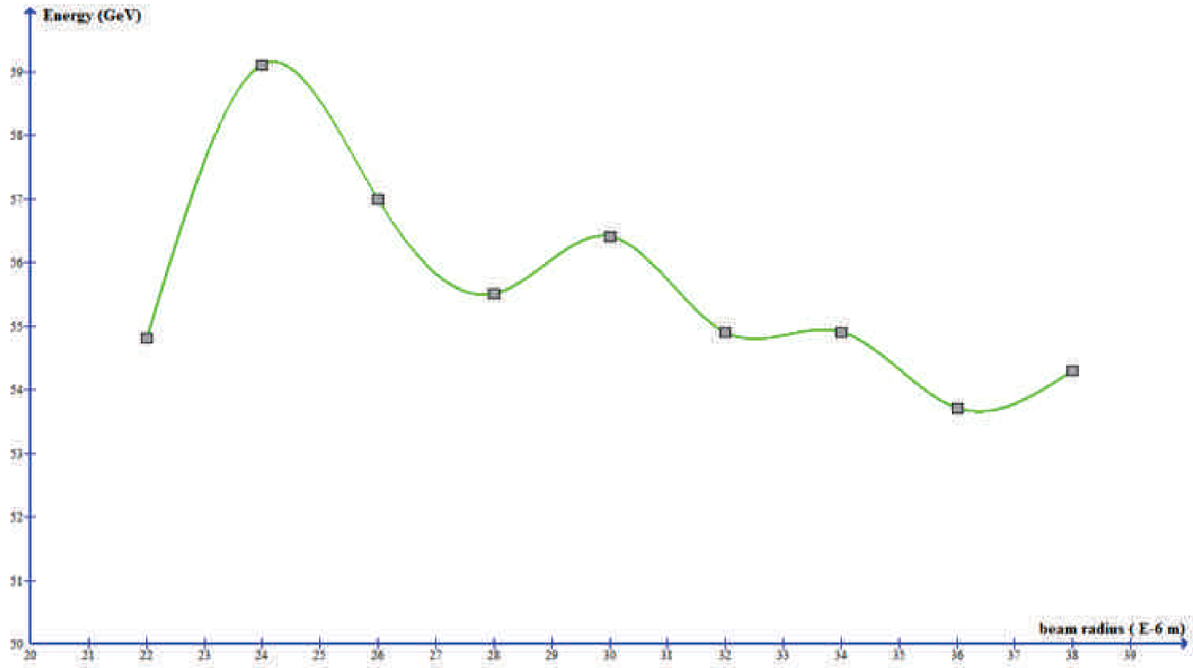
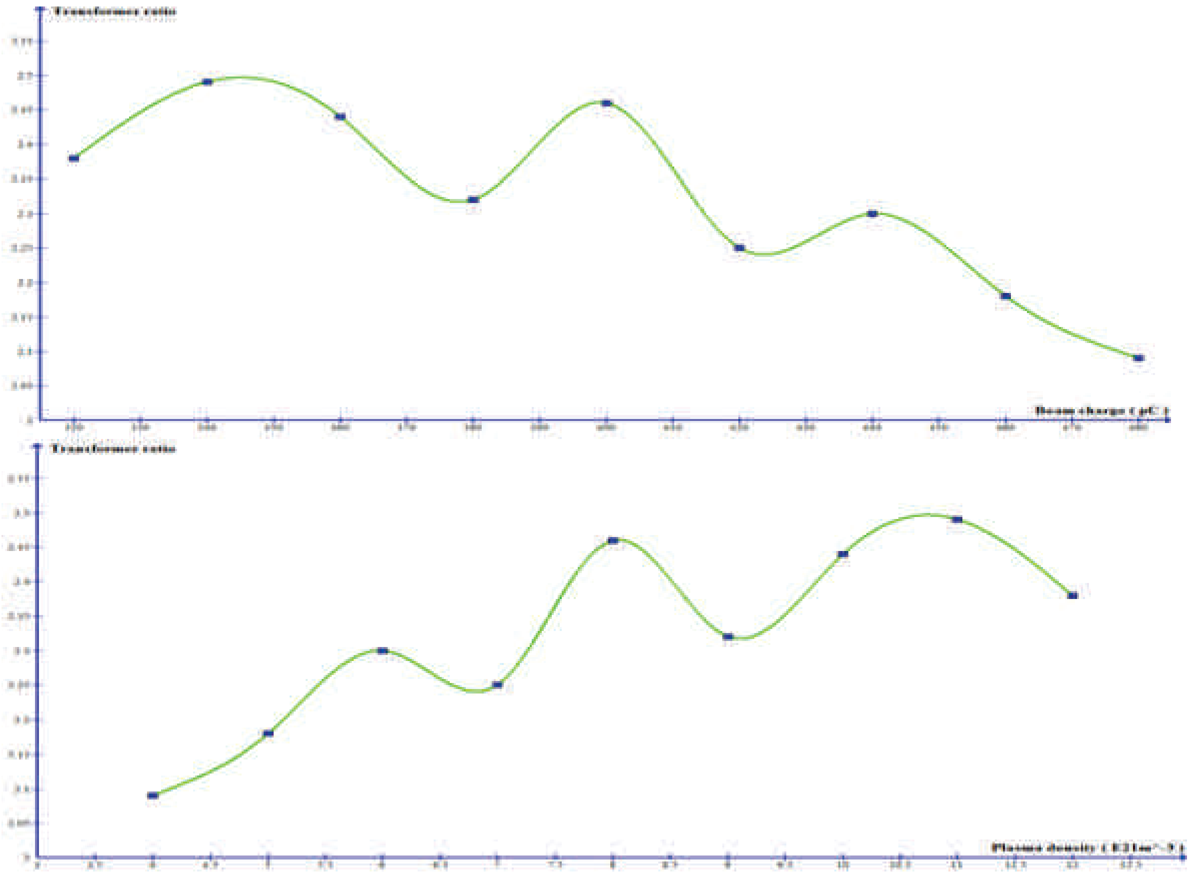


Figure 9: The Energy Gain as a Function of Beam Radius when Increased by a Factor of $2\mu m$ from $22\mu m$ to $38\mu m$.

The trend of the energy gain by the wakefield behind the drive beam seen in figure 9 shows a minimum energy gain of $54GeV$ and a maximum energy gain that is slightly in excess of $59GeV$, where the energy gain decrease with increase in beam radius.

Figure 10 shows how the transformer ratio varies with beam charge when reduced from $480pC$ to $320pC$ and the plasma density increased from $4\times10^{21}m^{-3}$ to $12\times10^{21}m^{-3}$simultaneously by a factor of $20pC$ and $1\times10^{21}m^{-3}$ respectively.

(a)



(b)

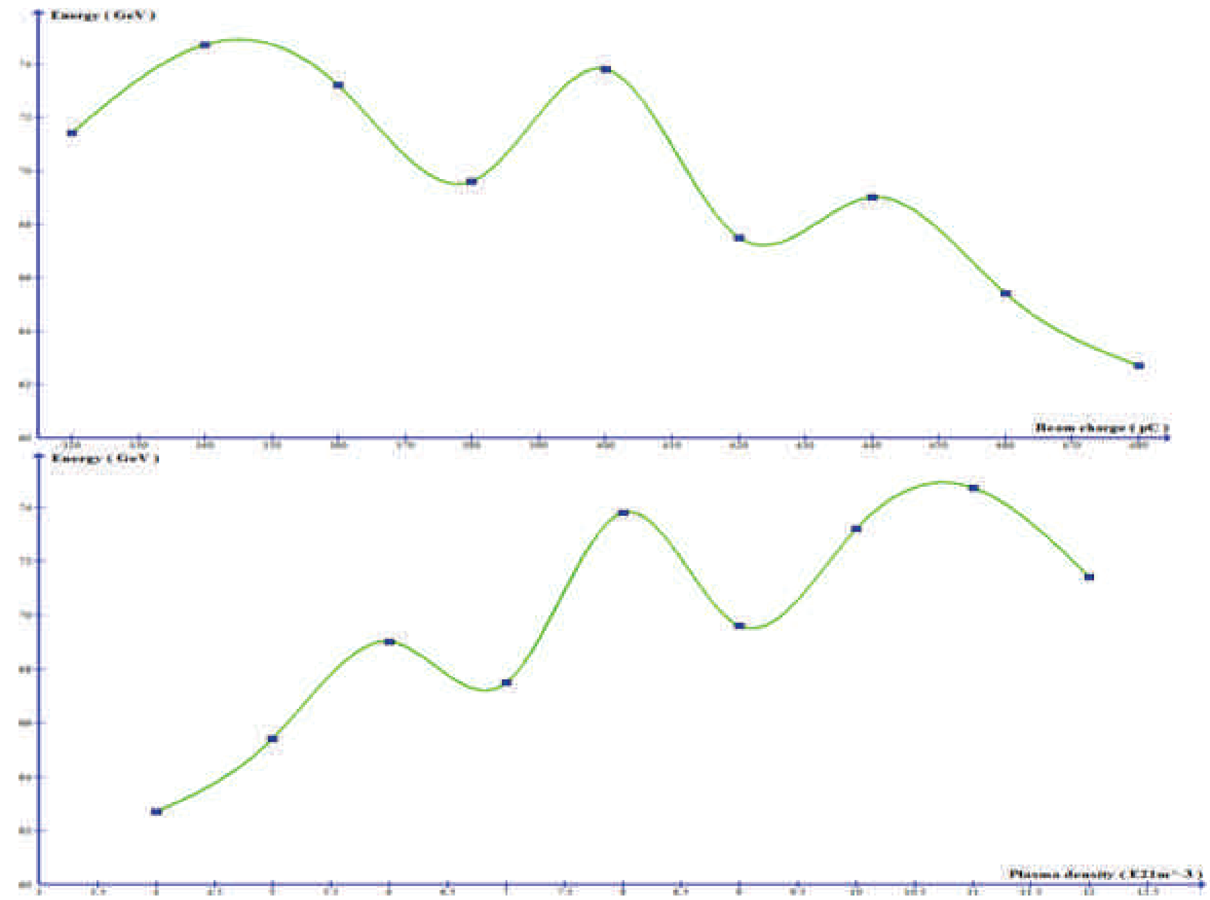
Figure 10: The Transformer Ratio as a Function of (a); Beam Charge when Reduced and (b) Plasma Density Increased Simultaneously by a Factor of $20pC$ and $1\times10^{21}m^{-3}$ Respectively.

Figure 10 depicts the dependence of the transformer ratio on the two parameter optimization of the beam charge when reduced from $480pC$ to $320pC$ and plasma density increased from $4\times10^{21}m^{-3}$ to $12\times10^{21}m^{-3}$ simultaneously by a factor of $20pC$ and $1\times10^{21}m^{-3}$ respectively. The trend shows that the transformer ratio decreases with decrease in beam charge and increase with increase in plasma density. One good feature of this two parameter optimization is that the transformer ratio is > 2 even when $n_p > n_b$, meaning increasing the plasma density whose particles are variably weighted to represent density ramp will yield high transformer ratio.

Figure 11 shows how the energy gain by the wakefield behind the drive beam varies with beam charge when reduced from $480pC$ to $320pC$and the plasma density when increased from $4\times10^{21}m^{-3}$ to $12\times10^{21}m^{-3}$ simultaneously by a factor of $20pC$ and $1\times10^{21}m^{-3}$ respectively.

ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.64, 2017

(a)



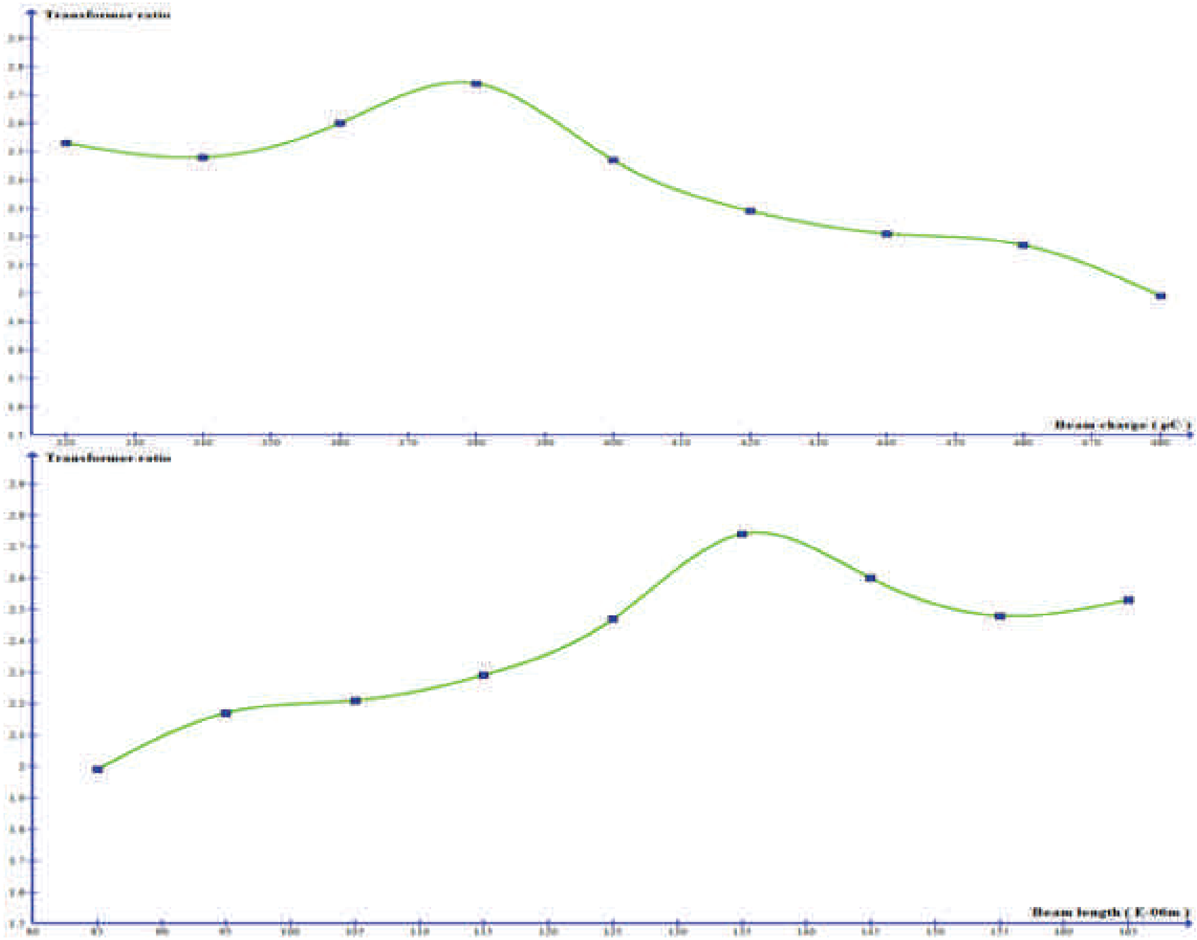
(b)

Figure 11: The Energy Gain as a Function of (a); Beam Charge when Reduced and (b); Plasma Density when Increased Simultaneously by a Factor of $20pC$ and $1\times10^{21}m^{-3}$ Respectively.

The corresponding energy gain by the wakefield behind the drive beam is shown in figure 11. The trend shows that the energy gain by the wakefield increases with increase in plasma density and decrease in beam charge. The energy gain is more than twice the energy of the drive beam.

Figure 12 shows how the transformer ratio varies with beam charge when reduced from $480pC$ to $320pC$ and the beam length increased from $85\mu m$to $165\mu m$ simultaneously by a factor of $20pC$ and $10\mu m$ respectively.

(a)



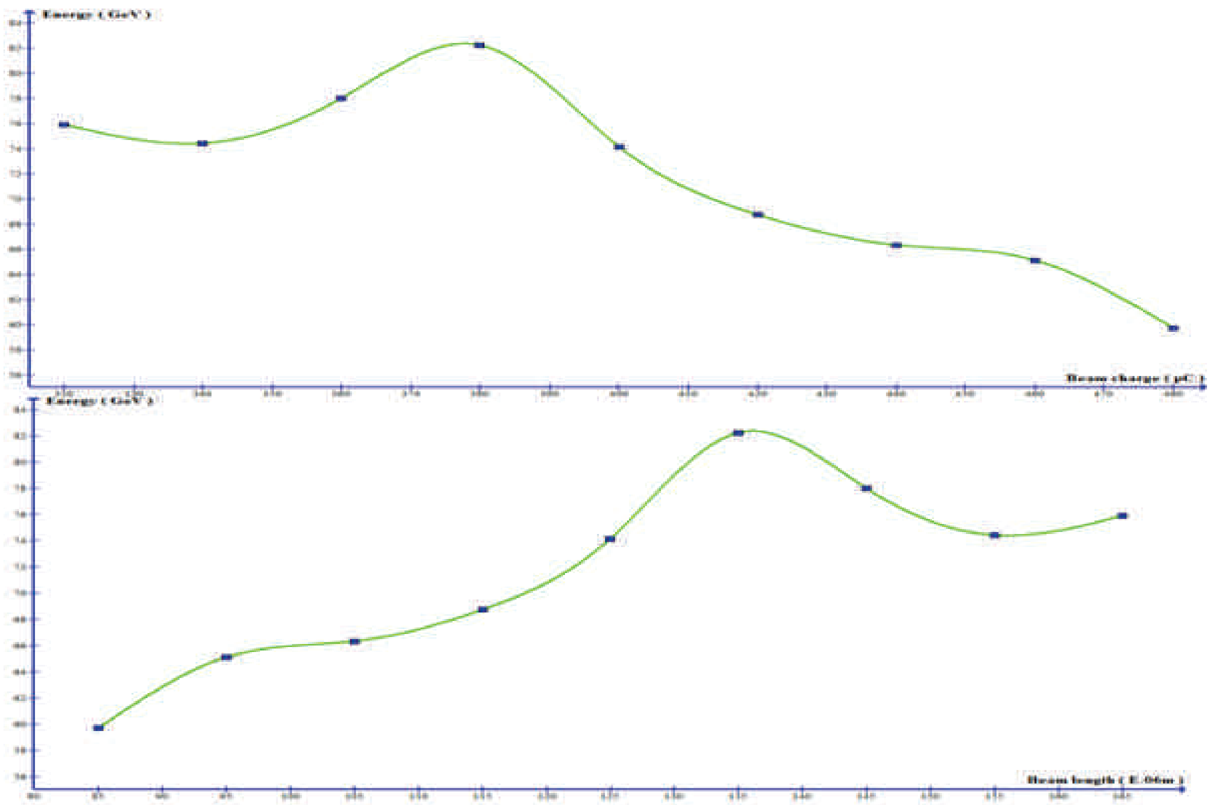
(b)

Figure 12: The Transformer Ratio as a Function of (a) Beam Charge when Reduced and (b) Beam Length Increased Simultaneously by a Factor of $20pC$ and $10\mu m$ Respectively.

Figure 12 depicts the transformer ratio as a function of two parameter optimization of the beam charge when reduced from $480pC$ to $320pC$ and beam length increased from $85\mu m$ to $165\mu m$ simultaneously by a factor of $20pC$ and $10\mu m$ respectively. The trend shows that the transformer ratio increase with increase in beam length and decrease in beam charge. In this optimization, the transformer ratio is ≥ 2 for each of the parameter varied. A two parameter optimization involving the beam length and plasma density [17] shows that as the beam length is shorten, the transformer ratio increase from 1 to a value of 1.6, which implies that as the bunch length is decreased, one benefits from both an increased accelerating field as well as higher transformer ratio. But the two parameter optimization of the beam charge and beam length shows that the transformer ratio increase with increase in beam length and decrease in beam charge.

Figure 13 shows how the energy gain by the wakefield behind the drive beam varies with beam charge when reduced from $480pC$ to $320pC$ and the beam length increased from $85\mu m$ to $165\mu m$ simultaneously by a factor of $20pC$ and $10\mu m$ respectively.

(a)



(b)

Figure 13: The Energy Gain as a Function of (a); Beam Charge when Reduced and (b); Beam Length Increased Simultaneously by a Factor of $20pC$ and $10\mu m$ Respectively

The corresponding energy gain by the wakefield presented in figure 13 shows that the minimum energy gain by the wakefield almost double the energy of the drive beam when $\sigma_z = 85\mu m$ and $Q = 480pC$, but the result in general is quite impressive since the energy gain by the wakefield is twice or even more than twice the energy of the drive beam for other values of the parameters optimized. The trend showed that the energy gain by the wakefield behind the drive beam increase with increase in beam length and decrease in beam charge.

4.0 Conclusion

Achieving a transformer ratio above the theoretical limit of two and energy gain by the wakefield that is twice or more or more than twice the energy of the drive beam is an important component in beam-plasma accelerator. Therefore, in this work, we considered the transformer ratio and energy gain response or dependence on beam/plasma parameters in a PWFA scheme driven by a single positron beam through a metre scale plasma whose density is variably weighted to represent density ramp.

To this end, the transformer ratio and energy gain response to beam/plasma parameters reveal that a short positron beam of longitudinal length, $50\mu m$ with other parameters used for this simulations yield extremely high transformer ratio of 6.67 and energy gain of the order of $200.1GeV$. When the plasma density was altered to $\geq 4\times10^{21}m^{-3}$ and the beam charge to $350pC$, the transformer ratios achieved for these parameters were > 2. Also the same transformer ratio value (greater than two) was achieved for the two parameter optimization of the plasma density and beam charge while that between beam length and beam charge yield, $R \geq 2$. The energy gain by the wakefield when the transformer ratio is above the value of two is more than twice the energy of the drive beam and is sustained through the plasma depletion length. These results are quite impressive because they constitute the major essence why the plasma wakefield accelerator is being investigated. But varying the beam radius did not yield any impressive result since the magnitude of the transformer ratio for the varied beam radius is < 2 and the energy gain by the wakefield is less than twice the energy of the drive beam.

Acknowledgement
We wish to thank Dr Raymond S. Anchaver for his contributions toward the successful completion of this work.

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