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Application of Double Laplace Transform Decomposition Method on System of Partial Differential Equations

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Abstract
In this paper, the double Laplace transform decomposition method is used to find the exact solution of the system linear and nonlinear of partial differential equations subject to the initial conditions. Furthermore, two examples are illustrative to demonstrate the efficiency of the proposed method.
Keywords: initial Value Problem, Double Laplace Transform, Inverse Laplace Transform

1. Introduction

Many applications in sciences are modeled by linear and nonlinear partial differential equations. Systems of partial differential equations have attracted much attention in a variety of applied sciences. The general ideas and the essential features of these systems are of wide applicability. These systems were formally derived to describe wave propagation, to control the shallow water waves, and to examine the chemical reaction-diffusion model of Brusselator [1−6]. Integral transforms [7, 8] are extensively used in solving boundary value problems & integral equations. The problem related to partial differential equation commonly can be solved by using a special integral transform thus many authors solved the boundary value problems by using single Laplace Transform [9]. The Wave equation, Heat equation & Laplace's equations as three fundamental equations in mathematical Physics & occur in many branches of Physics, in applied mathematics as well as in Engineering. Eltayeb and Kilicman [10] has worked on the non-homogeneous wave equation with variable coefficients is solved by applying the double Laplace Transform. Recently in [11, 12] Aghili & Motahhari have applied the Double Laplace Transform to solve second order Linear Differential equation with constant coefficients and some properties were addressed. In [13] application of the double Laplace decomposition is extended to certain mathematical problems. The aim of this work is to solve system of partial differential equations by combining the modified domain decomposition techniques and the double Laplace transform method. Hence forth the different problems of initial value are solved without converting it into ordinary differential equation, & no need to find complete solution. So this proposed method is very reliable &convenient for solving system of partial differential equations. The double Laplace transform is defined as

$$F(p,s) = \mathcal{L}_x\mathcal{L}_t[f(x,t)] = \int_0^\infty e^{-px}\left(\int_0^\infty e^{-st} f(x,t)dt\right)dx \tag{1}$$

Wherex, t are real variables and p, s are complex values and further double Laplace transform of the first-order partial derivatives is given by

$$\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial f(x,t)}{\partial t}\right] = sF(p,s) - F(p,0) \tag{2}$$

Similarly, the double Laplace transforms for second partial derivatives with respect to x and t are defined by

$$\mathcal{L}_x\mathcal{L}_t\left[\frac{\partial^2 f(x,t)}{\partial^2 t}\right] = s^2F(p,s) - sF(p,0) - \frac{\partial F(p,0)}{\partial t} \tag{3}$$

WhereL_xL_t denotes the double Laplace transform with respect tox, t.

1. Analysis of Double Laplace Transform Decomposition Method

The main aim of this work is to discuss the use of modified double Laplace decomposition method to system of linear and nonlinear PDEs

Consider system of PDEs in operator form

$$\begin{aligned} L_1u(x,t)+R_1(u,v)+N_1(u,v) &= h_1(x,t) \\ L_2v(x,t)+R_2(u,v)+N_2(u,v) &= h_2(x,t) \end{aligned} \tag{4}$$

Subject to initial condition

$$\begin{aligned} u(x,0) &= f_1(x) \\ v(x,0) &= f_2(x) \end{aligned} \tag{5}$$

Where L_1 and L_2 are first order linear differential operators $L_1 = L_2 = \frac{\partial}{\partial t}$, $R_1$and R_2 are the remaining linear operator, N_1 and N_2 represents nonlinear terms. $h_1(x,t)$ And $h_2(x,t)$ are source terms.Applying double

Laplace transform on both sides of (4) and single Laplace transform is adopted and applied to equations (5)

$$\tilde{\mathcal{L}}\left(L_1u(x,t)+R_1(u,v)+N_1(u,v)\right)=\tilde{\mathcal{L}}\left(h_1(x,t)\right)$$
$$\tilde{\mathcal{L}}\left(L_2u(x,t)+R_2(u,v)+N_2(u,v)\right)=\tilde{\mathcal{L}}\left(h_2(x,t)\right)$$
$$\tilde{\mathcal{L}}\left(u(x,0)\right)=\tilde{\mathcal{L}}\left(f_1(x)\right)=F_1(p) \tag{6}$$
$$\tilde{\mathcal{L}}\left(v(x,0)\right)=\tilde{\mathcal{L}}\left(f_2(x)\right)=F_2(p)$$

where $\tilde{\mathcal{L}}=\mathcal{L}_x\mathcal{L}_t$is a double transform operator. The differential property (3)and initial conditions (6a) and (6b) are then used and gives

$$sU(p,s)-U(p,0)+\tilde{\mathcal{L}}\left(R_1(u,v)\right)+\tilde{\mathcal{L}}\left(N_1(u,v)\right)=\tilde{\mathcal{L}}\left(h_1(x,t)\right)$$
$$sV(p,s)-V(p,0)+\tilde{\mathcal{L}}\left(R_2(u,v)\right)+\tilde{\mathcal{L}}\left(N_2(u,v)\right)=\tilde{\mathcal{L}}\left(h_2(x,t)\right)$$
$$U(p,s)=\frac{F_1(p)}{s}+\frac{1}{s}\tilde{\mathcal{L}}\left(R_1(u,v)+N_1(u,v)+h_1(x,t)\right) \tag{7}$$
$$V(p,s)=\frac{F_2(p)}{s}+\frac{1}{s}\tilde{\mathcal{L}}\left(R_2(u,v)+N_2(u,v)+h_2(x,t)\right)$$

Nowin Adomian Docomposition Method

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t)$$
$$v(x,t)=\sum_{n=0}^{\infty}v_n(x,t) \tag{8}$$

where the term u_n is to be recursively computed. The nonlinear operators can be defined as follows

$$N_1=\sum_{n=0}^{\infty}A_n$$
$$N_2=\sum_{n=0}^{\infty}B_n \tag{9}$$

where A_n and B_n are Adomian polynomials denoted by

$$A_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N_1\sum_{i=0}^{\infty}(\lambda^n u_n)\right)\right]_{\lambda=0}$$
$$B_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N_2\sum_{i=0}^{\infty}(\lambda^n u_n)\right)\right]_{\lambda=0} \tag{10}$$

Using double inverse Laplace transform for equation (7) and using equations (8), and (9) we have

$$\sum_{n=0}^{\infty}u_n(x,t)=f_1(x)+\mathcal{L}_p{}^{-1}\mathcal{L}_s{}^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_1\left[\left(\sum_{n=0}^{\infty}u_n(x,t),\sum_{n=0}^{\infty}v_n(x,t)\right)\right]+\sum_{n=0}^{\infty}A_n+h_1(x,t)\right)\right]$$
$$\sum_{n=0}^{\infty}v_n(x,t)=f_2(x)+\mathcal{L}_p{}^{-1}\mathcal{L}_s{}^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_2\left[\left(\sum_{n=0}^{\infty}u_n(x,t),\sum_{n=0}^{\infty}v_n(x,t)\right)\right]+\sum_{n=0}^{\infty}B_n+h_2(x,t)\right)\right]$$

Rearranging the terms

$$\sum_{n=0}^{\infty}u_n(x,t)=f_1(x)+\mathcal{L}_p{}^{-1}\mathcal{L}_s{}^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(h_1(x,t)\right)\right]$$
$$+\mathcal{L}_p{}^{-1}\mathcal{L}_s{}^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_1\left[\left(\sum_{n=0}^{\infty}u_n(x,t),\sum_{n=0}^{\infty}v_n(x,t)\right)\right]+\sum_{n=0}^{\infty}A_n\right)\right]$$

$$\sum_{n=0}^{\infty} v_n(x,t) = f_2(x) + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(h_2(x,t)\right)\right]$$
$$+\mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_2\left[\left(\sum_{n=0}^{\infty}u_n(x,t),\sum_{n=0}^{\infty}v_n(x,t)\right)\right]+\sum_{n=0}^{\infty}B_n\right)\right]$$

We define the following recursive formulas

$$u_0 = f_1(x) + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(h_1(x,t)\right)\right] = H(x,t)$$
$$u_{n+1} = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_1(u_n,v_n)+A_n\right)\right]$$

and

$$v_0 = f_2(x) + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(h_2(x,t)\right)\right] = W(x,t)$$
$$v_{n+1} = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_2(u_n,v_n)+B_n\right)\right]$$

Where the double inverse Laplace transform with respect to p, s is denoted by $\mathcal{L}_p^{-1}\mathcal{L}_s^{-1}$.
For modified Adomian decomposition method $H(x,t)$ and $W(x,t)$ are split into two parts. i.e.

$$H(x,t) = h_1(x,t) + h_2(x,t)$$
$$W(x,t) = w_1(x,t) + w_2(x,t)$$

Then the recursive formulas become

$$u_0 = h_1(x,t)$$
$$u_1 = h_2(x,t) + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_1(u_0,v_0)+A_0\right)\right] \tag{10}$$
$$u_{n+1} = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_1(u_n,v_n)+A_n\right)\right] n \geq 1$$

and

$$v_0 = w_1(x,t)$$
$$v_1 = w_2(x,t) + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_2(u_0,v_0)+B_0\right)\right] \tag{11}$$
$$v_{n+1} = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\tilde{\mathcal{L}}\left(R_2(u_n,v_n)+B_n\right)\right] n \geq 1$$

For the purpose of illustration of MDLADM, we consider system of linear and nonlinear partial differential equations.

2. Numerical Examples

In this section, we apply the DLTDM to solve inhomogeneous linear and nonlinear system of partial differential equations.

Example 3.1 Consider the following system of linear partial differential equations [14]

$$\begin{aligned} u_t - v_x - (u-v) &= -2 \\ v_t + u_x - (u-v) &= -2 \end{aligned} \tag{12}$$

Subject to initial conditions

$$\begin{aligned} u(x,0) &= 1+e^x \\ v(x,0) &= -1+e^x \end{aligned} \tag{13}$$

Apply double Laplace transform on (12)

$$\tilde{\mathcal{L}}\left(u_t - v_x - (u-v)\right) = \tilde{\mathcal{L}}(-2)$$

$$\tilde{\mathcal{L}}\left(v_t + u_x - (u-v) = \right) = \tilde{\mathcal{L}}\left(-2\right)$$

Where $\tilde{\mathcal{L}}=\mathcal{L}_x\mathcal{L}_t$. Now using (3)

$$sU(p,s) - U(p,0) = \mathcal{L}_x\mathcal{L}_t\left(-2 + v_x + u - v\right)$$

$$sV(p,s) - V(p,0) = \mathcal{L}_x\mathcal{L}_t\left(-2 - u_x + u - v\right)$$

Using (13) and rearranging the terms

$$U(p,s) = \frac{1}{s(p-1)} + \frac{1}{ps} - \frac{2}{ps^2} + \frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(v_x + u - v\right)$$

$$V(p,s) = \frac{1}{s(p-1)} - \frac{1}{ps} - \frac{2}{ps^2} + \frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(-u_x + u - v\right) \tag{14}$$

Applying inverse double Laplace transform on (14)

$$u(x,t) = e^x + 1 - 2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(v_x + u - v\right)]$$

$$v(x,t) = e^x - 1 - 2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(-u_x + u - v\right)]$$

Recursive formula for modified Adomian decomposition method is

$$u_0(x,t) = 1 + e^x$$

$$u_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(v_{0x} + u_0 - v_0\right)]$$

$$u_{n+1}(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(v_{nx} + u_n - v_n\right)] \tag{15}$$

And

$$v_0(x,t) = -1 + e^x$$

$$v_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(-u_{0x} + u_0 - v_0\right)]$$

$$v_{n+1}(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(-u_{nx} + u_n - v_n\right)] \tag{16}$$

Components of solution are

$$\begin{cases} u_0(x,t) = 1 + e^x \\ v_0(x,t) = -1 + e^x \end{cases}$$

$$u_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(\left((1+e^x)\right)_x + 1 + e^x + 1 - e^x\right)]$$

$$u_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(2 + e^x\right)]$$

$$u_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(\frac{2}{ps} + \frac{1}{s(p-1)}\right)\right]$$

$$= -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left(\frac{2}{ps^2} + \frac{1}{s^2(p-1)}\right)$$

$$= te^x$$

$$v_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(\left((-1-e^x)\right)_x + 1 + e^x + 1 - e^x\right)]$$

$$v_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(2-e^x\right)]$$

$$v_1(x,t) = -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(\frac{2}{ps}-\frac{1}{s(p-1)}\right)\right]$$

$$= -2t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left(\frac{2}{ps^2}-\frac{1}{s^2(p-1)}\right)$$

$$= -te^x$$

$$u_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(v_{1x}+u_1-v_1\right)]$$

$$v_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\left(-u_{1x}+u_1-v_1\right)]$$

And

$$u_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t(-te^x+te^x+te^x)]$$

$$u_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(\frac{1}{s^2(p-1)}\right)\right]$$

$$= \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left(\frac{1}{s^3(p-1)}\right)$$

$$= \frac{t^2e^x}{2}$$

Similarly

$$v_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t(-te^x+te^x+te^x)]$$

$$v_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(\frac{1}{s^2(p-1)}\right)\right]$$

$$= \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left(\frac{1}{s^3(p-1)}\right)$$

$$= \frac{t^2e^x}{2}$$

Hence, the series solution is

$$u(x,t) = 1+e^x+te^x+\frac{t^2e^x}{2!}-........$$

$$= 1+e^{x+t}$$

$$v(x,t) = -1+e^x-te^x+\frac{t^2e^x}{2!}-........$$

$$= -1+e^{x-t}$$

Example 3.2 Consider the following system of nonlinear partial differential equations [14]

$$\begin{aligned} u_t + vu_x + u &= 1 \\ v_t - uv_x - v &= 1 \end{aligned} \qquad (17)$$

Subject to initial conditions

$$u(x,0) = e^{x}$$
$$v(x,0) = e^{-x} \quad (18)$$

Apply double Laplace transform on (17)

$$\tilde{\mathcal{L}}\left(u_t + vu_x + u\right) = \tilde{\mathcal{L}}\left(1\right)$$
$$\tilde{\mathcal{L}}\left(v_t - uv_x - v\right) = \tilde{\mathcal{L}}\left(1\right)$$

Where $\tilde{\mathcal{L}}=\mathcal{L}_x\mathcal{L}_t$. Now using (3)

$$sU(p,s) - U(p,0) = \mathcal{L}_x\mathcal{L}_t\left(1 - vu_x - u\right)$$
$$sV(p,s) - V(p,0) = \mathcal{L}_x\mathcal{L}_t\left(1 + v + uv_x\right)$$

Using (18) and rearranging the terms

$$U(p,s) = \frac{1}{s(p-1)} + \frac{1}{ps^2} - \frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(vu_x + u)$$
$$V(p,s) = \frac{1}{s(p+1)} + \frac{1}{ps^2} + \frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(uv_x + v) \quad (19)$$

Applying inverse double Laplace transform on (19)

$$u(x,t) = e^{x} + t - \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(vu_x + u)\,]$$
$$v(x,t) = e^{-x} + t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(uv_x + v)\,]$$

Recursive formula for modified Adomian decomposition method is

$$u_0(x,t) = e^{x}$$
$$u_1(x,t) = t - \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(A_0 + u_0)\,]$$
$$u_{n+1}(x,t) = -\mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(A_n + u_n)\,] \quad (20)$$

and

$$v_0(x,t) = e^{-x}$$
$$v_1(x,t) = t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(B_0 + v_0)\,]$$
$$v_{n+1}(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(B_n + v_n)\,] \quad (21)$$

Components of solution are

$$\begin{cases} u_0(x,t) = e^{x} \\ v_0(x,t) = e^{-x} \end{cases}$$
$$u_1(x,t) = t - \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(A_0 + u_0)\,]$$
$$v_1(x,t) = t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(B_0 + v_0)\,]$$
$$A_0 = v_0(u_0)_x = 1,\;\; B_0 = u_0(v_0)_x = -1$$

Consequently, we have

$$u_1(x,t) = t - \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t\,(1 + e^{x})\,]$$

$$u_1(x,t) = t - \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(\frac{1}{ps} + \frac{1}{s(p-1)}\right)\right]$$
$$= t - \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left(\frac{1}{ps^2} + \frac{1}{s^2(p-1)}\right)$$
$$= -te^x$$
$$v_1(x,t) = t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t(-1+e^{-x})]$$
$$v_1(x,t) = t + \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(-\frac{1}{ps} + \frac{1}{s(p+1)}\right)\right]$$
$$= te^{-x}$$
$$u_2(x,t) = -\mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t(A_1 + u_1)]$$
$$v_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}[\frac{1}{s}\mathcal{L}_x\mathcal{L}_t(B_1 + v_1)]$$

where

$$A_1 = v_0(u_1)_x + v_1(u_0)_x = 0,\ B_1 = u_0(v_1)_x + u_1(v_0)_x = 0$$
$$u_2(x,t) = -\mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(-\frac{1}{s^2(p-1)}\right)\right]$$
$$= \frac{t^2e^x}{2}$$

and

$$v_2(x,t) = \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\left[\frac{1}{s}\left(\frac{1}{s^2(p+1)}\right)\right]$$
$$= \frac{t^2e^{-x}}{2}$$

Hence, the series solution is

$$u(x,t) = e^x - te^x + \frac{t^2e^x}{2!} - .... = e^{x-t}$$
$$v(x,t) = e^{-x} + te^{-x} + \frac{t^2e^{-x}}{2!} - .... = e^{t-x}$$

3. Conclusion

In this paper, we have successfully implemented the Modified double Laplace Adomian decomposition method on certain linear and nonlinear system of initial value problems. Numerical results of the considered three problems show that the method is capable of reducing the volume of computational work as compared to other methods.

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