Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol 13, 2013

The Dispersion Relation of Flexural Waves in a Magnetoelastic Anisotropic Circular Cylinder

Abo-el-nour N. Abd-alla [1,2] and Fatimah Alshaikh [2]
1. Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt.
2. Department of Mathematics, Faculty of Science, Jazan University, Jazan, Saudi Arabia.
* E-mail of the corresponding author: aboelnourabdalla@yahoo.com

Abstract

The objective of this paper is to investigate some aspects of dispersion relation of flexural waves propagation in a transversely isotropic hollow circular cylinder of infinite extent placed in a primary magnetic field. A frequency equation appropriate to the hollow circular cylinder is obtained by using the lame (Helmholtz) potentials for arbitrary values of the physical parameters involve as well as the primary magnetic field. Numerical calculations have been carried out when the cylinder is made of the material of Zinc Oxide. This study shows that waves in a solid body propagating under the influence of a superimposed magnetic field can differ significantly from those propagating in the absence of the magnetic field. Also, one may see that the effect of the primary magnetic field is to increase the values of the materials constants. Finally the results are given for different values of the primary magnetic field and presented graphically. The standard results of the previous investigations have also been deduced as particular cases.

Keywords: Natural frequencies, Magnetoelasticity, Flexural wave, transversely isotropic materials

1. Introduction

The analysis of flexural wave propagation in isotropic, homogeneous circular cylindrical shell according to the theory of elasticity has been done by many authors like: Greenspoon [1], Pao [2], Kumar [3], Hutchinson [4], Martin [5] and Honarvar et al. [6]. With the advancement of space research, it has become necessary to obtain a deep insight in the behavior of materials, especially of the anisotropic ones that are so frequently used in the missiles and other allied systems. Many authors such as: Mirsky [7], Prasad et al. [8], White et al. [9] and Tsai [10] have presented papers on the transversely isotropic cylindrical shells of infinite extent. Suhubi [11] Datta [12] and Abd-alla [13, 14] have been discussed a similar problems but in more general way where the magnetic field is taken in their considerations. For more details, one may go through recently published monographs of Nowacki [15], Moon [16], Parton et al [17], Maugin et al [18] and Auld [19]. Recently, the interaction of electromagnetic fields with the motion of a deformable solid is being received greater attention by many investigators. Therefore, many researchers have investigated the effect of the magnetic field on the wave propagation in anisotropic cylindrical materials. Barakati and Zhupanska [20] studied the effects of pulsed electromagnetic fields on the dynamic mechanical response of electrically conductive anisotropic plates. Dinzart and Sabar [21] presented numerical investigations into magneto-electro-elastic moduli responsible for the magneto-electric coupling as functions of the volume reaction and characteristics of the coated inclusions. Akbarovet al. [22] studied torsional wave dispersion in a three-layered (sandwich) hollow cylinder with finite initial strains. Chattopadhyay et al. [23] studied the propagation of horizontally polarized shear waves in an internal magnetoelastic monoclinic stratum with irregularity in lower interface. Tang and Xu [24] have applied the method of eigenfunction expansion to solve the problems of transient torsional vibration responses of finite, semi-infinite and infinite hollow cylinders. Acharya et al. [25] investigated the effect of the transverse isotropy and magnetic field on the interface waves in a conducting medium subject to the initial state of stress of the form of hydrostatic tension or compression. Petrov et al. [26] focused on the nature of ferromagnetic resonance (FMR) under the influence of acoustic oscillations with the same frequency as FMR. Mol'chenko et al. [27] constructed a two-dimensional nonlinear magnetoelastic model of a current-carrying orthotropic shell of revolution taking into account of finite orthotropic conductivity, permeability and permittivity. Abd-Alla and Abo-Dahab [28] studied the influence of the viscosity on reflection and refraction of plane shear elastic waves in two magnetized semi-infinite media. Selim [29] showed the effect of damping on the propagation of torsional waves in an initially stressed, dissipative, incompressible cylinder of infinite length. Dai and Wang [30] illustrated an analytical method to solve magneto-elastic wave propagation and perturbation of the magnetic field vector in an orthotropic laminated hollow cylinder with arbitrary thickness. Liu and Chang [31] investigated the interactive behaviors among transverse magnetic fields, axial loads and external forces of a magneto-elastic beam with general boundary conditions.

In this paper the materials were considered to be homogeneous and transversely isotropic. Plane waves propagation is discussed considering the solution of the equations of motion and the solution of the electro-magnetic equations of Maxwell in cylindrical coordinates. The treatments were carried out under the consideration of the displacement field, which does not depend on the vertical coordinate, may be written in terms of Lame potentials (sometimes is co-called Helmholtz potentials). A frequency equation concerning to the hollow circular cylinder is obtained by using the arbitrary values of the physical parameters involve as well as the primary magnetic field. Numerical calculations have been carried out for the cylinder is which is made up of zinc. This study shows that waves in a solid body propagating under the influence of a superimposed magnetic field can differ significantly from those propagating in the absence of the magnetic field. Also, one may see that the effect of the primary magnetic field is to increase the values of the material constants. Some of the results obtained in earlier works are obtained as particular cases of the more general results derived here. Finally, the results have been given for different values of the primary magnetic field and it has been presented graphically.

2. Basic equations

The problem being one of magneto-elasticity, the basic equations are those of electromagnetism and of elasticity. The Maxwell equations governing the electromagnetic field, are:

$$\vec{\nabla}\times\vec{H}=\frac{4\pi}{c}\vec{J}\ ,\quad \vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t},\quad \vec{\nabla}.\vec{B}=0,\quad \vec{B}=\mu_e\vec{H} \tag{1}$$

where the displacement current is neglected and Gaussian unites have been used. We have also Ohm's law in the following form:

$$\vec{J}=\sigma(\vec{E}+\frac{1}{c}\left(\frac{\partial\vec{u}}{\partial t}\times\vec{B}\right). \tag{2}$$

The needed strain-displacement relations are [8]:

$$e_{rr}=\frac{\partial u}{\partial r},\qquad e_{\theta\theta}=\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial r},\qquad e_{r\theta}=\frac{1}{2}\left[\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right]. \tag{3}$$

The constitutive stress-strain relations are [13]:

$$\tau_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz},\quad \tau_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta}+c_{13}e_{zz},\quad \tau_{r\theta}=c_{66}e_{r\theta}. \tag{4}$$

Taking into account the Lorentz body force, the stresses equations of motion become [9]:

$$\frac{\partial\tau_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{1}{r}(\tau_{rr}-\tau_{\theta\theta})+\frac{1}{c}(\vec{J}\wedge\vec{B})_r=\rho\frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$\frac{\partial\tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta}+\frac{2\tau_{r\theta}}{r}+\frac{1}{c}(\vec{J}\wedge\vec{B})_\theta=\rho\frac{\partial^2 v}{\partial t^2}. \tag{6}$$

The electro-magnetic field equations in vacuum are given by [14]:

$$(\frac{\partial^2}{\partial t^2}-c^2\nabla^2)(\vec{E}^*,\vec{h}^*)=\vec{0},\quad \vec{\nabla}\wedge(\vec{E}^*,\vec{h}^*)=\frac{1}{c}\frac{\partial}{\partial t}(-\vec{h}^*,\vec{E}^*), \tag{7}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$, and the quantities with asterisk corresponding values in vacuum. Substituting the values of the stresses from (4) in the equations of motion (5) and (6). Using (1) and (2) the equations of magneto-elasticity for a transversely isotropic perfectly conducting elastic material are:

$$c_{11}[\frac{\partial^2 u}{\partial r^2}-\frac{u}{r^2}+\frac{1}{r}\frac{\partial u}{\partial r}]+c_{66}[\frac{1}{r}\frac{\partial^2 v}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial v}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2}]+c_{12}(\frac{1}{r}\frac{\partial^2 v}{\partial r\partial\theta})+ \frac{\mu_e}{4\pi}\{[rot\, rot(\vec{u}\wedge\vec{H}_o)]\wedge\vec{H}_o\}_r=\rho\frac{\partial^2 u}{\partial t^2}, \quad (8)$$

and

$$c_{66}[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{1}{r}\frac{\partial^2 u}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial u}{\partial\theta}]+c_{11}[\frac{1}{r^2}\frac{\partial^2 v}{\partial\theta^2}+\frac{1}{r^2}\frac{\partial u}{\partial\theta}]+c_{12}(\frac{1}{r}\frac{\partial^2 u}{\partial r\partial\theta})+ \frac{\mu_e}{4\pi}\{[rot\, rot(\vec{u}\wedge\vec{H}_o)]\wedge\vec{H}_o\}_\theta=\rho\frac{\partial^2 v}{\partial t^2}, \quad (9)$$

where, we have considered the following relations:

$$\vec{E}=-\frac{1}{c}(\frac{\partial\vec{u}}{\partial t}\wedge\vec{B}),\quad \vec{h}=\vec{\nabla}\wedge(\vec{u}\times\vec{H}_o),\quad \vec{H}_o=\vec{H}_o+\vec{h},\quad \vec{H}_o=(0,0,H_o),\quad (\mu_o\approx 1). \quad (10)$$

3. Solution by using Lame (Helmholtz) potentials

A circular cylindrical solid of transversely isotropic elastic material of inner and outer radii a and b, respectively and subjected to an axial magnetic field is considered. The material of the elastic cylinder is regarded as a perfect conductor and the regions inside and outside, it is assumed to be vacuum. With reference to the cylindrical coordinates (r,θ,z), the displacement field for the case of plane motions is written as:

$$u=u(r,\theta,t)\quad v=v(r,\theta,t),\quad w=0 \quad (11)$$

From mathematical point of view every sufficiently smooth vector field $\vec{u}=(u,v,0)$ may be decomposed into a gradient of a scalar potential $\Phi(r,\theta,t)$ and a rotation of a vector potential $\Psi(r,\theta,t)$ according to the following relations:

$$u(r,\theta,t)=\frac{\partial\Phi}{\partial r}+\frac{1}{r}\frac{\partial\Psi}{\partial\theta},\quad v(r,\theta,t)=\frac{1}{r}\frac{\partial\Phi}{\partial\theta}-\frac{\partial\Psi}{\partial r} \quad (12)$$

From (12) in (8) and (9), one may get

$$\frac{\partial}{\partial r}[(c_{11}+\rho\alpha^2)(\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2})-\rho\frac{\partial^2\Phi}{\partial t^2}]+ \frac{1}{r}\frac{\partial}{\partial\theta}[c_{66}(\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2})-\rho\frac{\partial^2\Psi}{\partial t^2}]=0, \quad (13)$$

$$\frac{1}{r}\frac{\partial}{\partial\theta}[(c_{11}+\rho\alpha^2)(\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2})-\rho\frac{\partial^2\Phi}{\partial t^2}]+ \frac{\partial}{\partial r}[c_{66}(\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2})-\rho\frac{\partial^2\Psi}{\partial t^2}]=0, \quad (14)$$

where $\alpha^2=\frac{\mu_o H_o^2}{4\pi\rho}$.

From equation (13) and (14), we have the following:

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}=\frac{1}{c_1^2}\frac{\partial^2\Phi}{\partial t^2}, \quad (15)$$

$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}=\frac{1}{c_2^2}\frac{\partial^2\Psi}{\partial t^2}, \tag{16}$$

where $c_1=\sqrt{(c_{11/\rho})+\alpha^2}$, $c_2=\sqrt{c_{66/\rho}}$.

Now, consider harmonic solutions $\Phi=\Phi(r,\theta,t)$ and $\Psi=\Psi(r,\theta,t)$) in the form

$$\Phi(r,\theta,t)=\phi(r)\cos(n\theta)\exp(i\omega t),\qquad \Psi(r,\theta,t)=\psi(r)\sin(n\theta)\exp(i\omega t), \tag{17}$$

where ω is the frequency of the vibrations and $n(n=0,1,2,...)$ is an integer indicating the number of circumferential waves. Substituting from Eqs. (17) into Eqs. (15) and (16), we obtain the well-known Bessel equations for $\phi(r)$ and $\psi(r)$:

$$r^2\frac{d^2\phi}{dr^2}+r\frac{d\phi}{dr}+\left(r^2\gamma_1^2-n^2\right)\phi=0, \tag{18}$$

$$r^2\frac{d^2\psi}{dr^2}+r\frac{d\psi}{dr}+\left(r^2\gamma_2^2-n^2\right)\psi=0, \tag{19}$$

where $\gamma_1^2=\omega^2/c_1^2$, $\gamma_2^2=\omega^2/c_2^2$.

The general solutions of the equations (18) and (19) may take the following form:

$$\phi(r)=A_1Z_n(\gamma_1 r)+B_1W_n(\gamma_1 r), \tag{20}$$

$$\psi(r)=A_2Z_n(\gamma_2 r)+B_2W_n(\gamma_2 r), \tag{21}$$

where A_1, A_2, B_1 and B_2 are constants of integration and for brevity Z_n of order n denote the Bessel function J_n or I_n and W_n also of order n denote the Bessel function Y_n or K_n according to the signs of γ_1^2,γ_2^2. From (20), (21) and (12), one obtains

$$u=\left[A_1L_{11}+B_1L_{12}+A_2L_{13}+B_2L_{14}\right]\cos(n\theta)\exp(i\omega t), \tag{22}$$

$$v=\left[A_1L_{21}+B_1L_{22}+A_2L_{23}+B_2L_{24}\right]\sin(n\theta)\exp(i\omega t), \tag{23}$$

where

$$L_{11}=\frac{n}{r}Z_n(\gamma_1 r)-\delta\gamma_1 Z_{n+1}(\gamma_1 r),\quad L_{13}=\frac{n}{r}Z_n(\gamma_2 r),\quad L_{21}=-\frac{n}{r}Z_n(\gamma_1 r),$$

$$L_{12}=\frac{n}{r}W_n(\gamma_1 r)-\gamma_1 W_{n+1}(\gamma_1 r),\quad L_{14}=\frac{n}{r}W_n(\gamma_2 r),\quad L_{22}=-\frac{n}{r}W_n(\gamma_1 r),$$

$$L_{23}=-\frac{n}{r}Z_n(\gamma_2 r)+\delta\gamma_2 Z_{n+1}(\gamma_2 r),\quad L_{24}=-\frac{n}{r}W_n(\gamma_2 r)+\gamma_2 W_{n+1}(\gamma_2 r).$$

where $\delta=1$ for $Z=J$ and $\delta=-1$ for $Z=I$.

The equation governing magnetic field in vacuum is

$$\left[r^2\frac{d^2}{dr^2}+r\frac{d}{dr}+k^2r^2-n^2\right]h_z^*=0, \tag{24}$$

where

$$\vec{h}^*(r,\theta,t)=h_z^*(r)\cos(n\theta)\exp(i\omega t)\vec{e}_z,\quad k^2=\omega^2/c^2. \tag{25}$$

Hence the magnetic field in vacuum is given by:

$$\vec{h}^{*} = \vec{e}_z \begin{cases} A_3 J_n(kr)\cos(n\theta)\exp(i\omega t) & , \quad r \le a \\ B_3 Y_n(kr)\cos(n\theta)\exp(i\omega t) & , \quad r \ge b \end{cases} \tag{26}$$

where A_3 and B_3 are arbitrary constants.

4. The boundary conditions and the frequency equation

For free motion, the boundary conditions require that the total stress vanishes and the continuity of the magnetic field on the surfaces $r = a, b$, i.e.,

$$\left.\begin{aligned} &\tau_{rr} + M_{rr} - M^{*}_{rr} = 0 \\ &\tau_{r\theta} + M_{r\theta} - M^{*}_{r\theta} = 0 \\ &h = h^{*} \end{aligned}\right\} \quad on \quad r = a, b \tag{27}$$

where

$$M_{ij} = \frac{1}{4\pi}\left[H_i h_j + H_j h_i - \left(\vec{H}\cdot\vec{h}\right)\delta_{ij}\right], \quad M^{*}_{ij} = \frac{1}{4\pi}\left[H_i h^{*}_j + H_j h^{*}_i - \left(\vec{H}\cdot\vec{h}^{*}\right)\delta_{ij}\right]$$

Substituting the values of the stresses and the magnetic field in the above boundary conditions and eliminating A_1, B_1, A_2, B_2, A_3 and B_3, we get the frequency equation which may be written in the following form:

$$\Delta = \left|X_{ij}\right| = 0, \quad i, j = 1,2,....6, \tag{28}$$

where

$$X_{11} = \gamma_1^2\left(\frac{H_o^2}{4\pi} + c_{11}\right)Z_{n+2}(\gamma_1 a) - \left((2n+1)c_{11} + c_{12} + 2(n+1)\frac{H_o^2}{4\pi}\right)\frac{\delta\gamma_1}{a}Z_{n+1}(\gamma_1 a) + \frac{n(n-1)}{a^2}(c_{11} - c_{12})Z_n(\gamma_1 a),$$

$$X_{12} = \gamma_1^2\left(\frac{H_o^2}{4\pi} + c_{11}\right)W_{n+2}(\gamma_1 a) - \left((2n+1)c_{11} + c_{12} + 2(n+1)\frac{H_o^2}{4\pi}\right)\frac{\gamma_1}{a}W_{n+1}(\gamma_1 a) + \frac{n(n-1)}{a^2}(c_{11} - c_{12})W_n(\gamma_1 a),$$

$$X_{13} = \frac{n}{a}(c_{11} - c_{12})\left(-\delta\gamma_2 Z_{n+1}(\gamma_2 a) + \frac{n-1}{a}Z_n(\gamma_2 a)\right),$$

$$X_{14} = \frac{n}{a}(c_{11} - c_{12})\left(-\gamma_2 W_{n+1}(\gamma_2 a) + \frac{n-1}{a}W_n(\gamma_2 a)\right),$$

$$X_{15} = \frac{H_o}{4\pi}J_n(ka), \qquad X_{16} = 0,$$

$$X_{21} = \gamma_1^2\left(\frac{H_o^2}{4\pi} + c_{11}\right)Z_{n+2}(\gamma_1 b) - \left((2n+1)c_{11} + c_{12} + 2(n+1)\frac{H_o^2}{4\pi}\right)\frac{\delta\gamma_1}{b}Z_{n+1}(\gamma_1 b) + \frac{n(n-1)}{b^2}(c_{11} - c_{12})Z_n(\gamma_1 b),$$

$$X_{22}=\gamma_1^2\left(\frac{H_o^2}{4\pi}+c_{11}\right)W_{n+2}(\gamma_1 b)-\left((2n+1)c_{11}+c_{12}+2(n+1)\frac{H_o^2}{4\pi}\right)\frac{\gamma_1}{b}W_{n+1}(\gamma_1 b)$$
$$+\frac{n(n-1)}{b^2}(c_{11}-c_{12})W_n(\gamma_1 b),$$
$$X_{23}=\frac{n}{b}(c_{11}-c_{12})\left(-\delta\gamma_2 Z_{n+1}(\gamma_2 b)+\frac{n-1}{b}Z_n(\gamma_2 b)\right)$$
$$X_{24}=\frac{n}{b}(c_{11}-c_{12})\left(-\gamma_2 W_{n+1}(\gamma_2 b)+\frac{n-1}{b}W_n(\gamma_2 b)\right),$$
$$X_{25}=0,\qquad X_{26}=\frac{H_o}{4\pi}Y_n(kb), \tag{29}$$
$$X_{31}=\frac{2n}{a}\delta\gamma_1 Z_{n+1}(\gamma_1 a)-\frac{2n(n-1)}{a^2}Z_n(\gamma_1 a),$$
$$X_{32}=\frac{2n}{a}\gamma_1 W_{n+1}(\gamma_1 a)-\frac{2n(n-1)}{a^2}W_n(\gamma_1 a),$$
$$X_{33}=-\gamma_2^2 Z_{n+2}(\gamma_2 a)+\frac{2n}{a}\delta\gamma_2 Z_{n+1}(\gamma_2 a)-\frac{2n(n-1)}{a^2}Z_n(\gamma_2 a)$$
$$X_{34}=-\gamma_2^2 W_{n+2}(\gamma_2 a)+\frac{2n}{a}\gamma_2 W_{n+1}(\gamma_2 a)-\frac{2n(n-1)}{a^2}W_n(\gamma_2 a)$$
$$X_{35}=0,\qquad X_{36}=0,$$
$$X_{41}=\frac{2n}{b}\delta\gamma_1 Z_{n+1}(\gamma_1 b)-\frac{2n(n-1)}{b^2}Z_n(\gamma_1 b),$$
$$X_{42}=\frac{2n}{b}\gamma_1 W_{n+1}(\gamma_1 b)-\frac{2n(n-1)}{b^2}W_n(\gamma_1 b),$$
$$X_{43}=-\gamma_2^2 Z_{n+2}(\gamma_2 b)+\frac{2n}{b}\delta\gamma_2 Z_{n+1}(\gamma_2 b)-\frac{2n(n-1)}{b^2}Z_n(\gamma_2 b),$$
$$X_{44}=-\gamma_2^2 W_{n+2}(\gamma_2 b)+\frac{2n}{b}\gamma_2 W_{n+1}(\gamma_2 b)-\frac{2n(n-1)}{b^2}W_n(\gamma_2 b),$$
$$X_{45}=0,\qquad X_{46}=0$$
$$X_{51}=H_o\left(\gamma_1^2 Z_{n+2}(\gamma_1 a)-\frac{2(n+1)}{a}\delta\gamma_1 Z_{n+1}(\gamma_1 a)\right),$$
$$X_{52}=H_o\left(\gamma_1^2 W_{n+2}(\gamma_1 a)-\frac{2(n+1)}{a}\gamma_1 W_{n+1}(\gamma_1 a)\right),$$
$$X_{53}=0,\qquad X_{54}=0,\qquad X_{55}=J_n(ka),\qquad X_{56}=0,$$

$$X_{61} = H_o\left(\gamma_1^2 Z_{n+2}(\gamma_1 b) - \frac{2(n+1)}{b}\delta\gamma_1 Z_{n+1}(\gamma_1 b)\right),$$

$$X_{62} = H_o\left(\gamma_1^2 W_{n+2}(\gamma_1 b) - \frac{2(n+1)}{b}\gamma_1 W_{n+1}(\gamma_1 b)\right),$$

$$X_{63} = 0, \qquad X_{64} = 0, \qquad X_{65} = 0, \qquad X_{66} = Y_n(kb).$$

5. The frequency equations of radial and torsional waves

When $n = 0$ and using the properties determinants, the equation (28) becomes

$$\Delta_1 \cdot \Delta_2 = 0 \tag{30}$$

i.e. the determinant equation ($\Delta = 0$), breaks into the product of sub determinants ($\Delta_1 \cdot \Delta_2 = 0$), which is satisfied if either $\Delta_1 = 0$ or $\Delta_2 = 0$, where

$$\Delta_1 = \begin{vmatrix} \overline{X}_{11} & \overline{X}_{12} & \overline{X}_{15} & 0 \\ \overline{X}_{21} & \overline{X}_{22} & 0 & \overline{X}_{26} \\ \overline{X}_{51} & \overline{X}_{52} & \overline{X}_{55} & 0 \\ \overline{X}_{61} & \overline{X}_{62} & 0 & \overline{X}_{66} \end{vmatrix} = 0 \qquad \Delta_2 = \begin{vmatrix} \overline{X}_{33} & \overline{X}_{34} \\ \overline{X}_{43} & \overline{X}_{44} \end{vmatrix} = 0 \tag{31}$$

The frequency equation (31), corresponds to magneto-elastic radial waves [13]. While the frequency equation (32), corresponds to the magneto-elastic torsional waves [7] and [11], where $\overline{X}_{ij}$ is given by (29) when $n = 0$.

6. The numerical results

For numerical calculations, we consider the following transformation

$$\Omega = \frac{\omega}{\omega_i}, \qquad \omega_i = \frac{c_2}{b}, \qquad \Omega_1 = \beta_1\Omega, \quad \beta_1 = \frac{c_2}{c_1}, \qquad \beta_2 = \frac{c_2}{c}, \qquad h = \frac{a}{b}. \tag{32}$$

The calculations of the roots of the frequency equation (28), represent a major task and requires a rather extensive effort of numerical computations. The computations have been carried out on an electronic computer for the case of Zinc Oxide which has the following physical constants [19].

$$\rho = 5.676\ gm/cm^3, \quad c_{13} = 10.51(10^{11})\ dyne/cm^2, \quad c_{66} = 4.25(10^{11}) dyne/cm^2,$$

$$c_{11} = 20.97(10^{11})\ dyne/cm^2,\ c_{33} = 21.09(10^{11})\ dyne/cm^2,\ c = 3(10^{10})\ cm/\sec,$$

$$c_{12} = 12.11(10^{11})\ dyne/cm^2,\ c_{44} = 4.25(10^{11})\ dyne/cm^2,\ \mu_o = 1\ Gauss/Oersted.$$

7. Discussion and conclusion

The non-dimensional frequency Ω versus the thickness $h = a/b$ are plotted in all the Figures (1-9) which illustrate the effects of the primary magnetic field on the flexural vibrations of a transversely isotropic circular cylinder for the value of non-dimensional wave number n.

The frequency equations (28) and (31) are solved numerically, and for this purpose a matrix determinant computation routine was used for different Ω and h along with a root finding method to refine steps close to its roots. For each pair (Ω and h) the frequency equations are solved using "interval halving" iteration technique [34].

Figures 1, 2 and 3 represent the first, second and third modes, respectively, of dimensionless frequency Ω for flexural vibrations versus different values of $h = a/b$ for value of $H_o = 10^5, 10^6, 10^7$, when n=1. In Fig. 1, the first mode of dimensionless frequency Ω decreases slightly as the ratio thickness h increases for ($H_o = 10^5, 10^6$ Oersted). While, it increases monotonically as function of h for the value of primary magnetic field increases as well ($H_o = 10^7$ Oersted). Figures 2 and 3 show that the effect of the values of the primary magnetic field ($H_o = 10^5, 10^6$ Oersted) on the second and third modes of flexural vibrations is very small and barely curves in these two cases coincide to each other. While, it increases nonlinearly as function of h for the value of ($H_o = 10^7$ Oersted). It is obvious from Figs. 5, 6 and 7 that the first, second and third modes increase with the decrease of the thickness h. While all of these modes increase when increasing the imposed magnetic field H_o. The first three modes of dimensionless frequency spectrum Ω versus different values of h are given in Figure 4 for the value of ($H_o = 10^7$ Oersted), when the circumferential wave number $n = 1$ is calculated and given in the form of graphs. Also, for $n = 0$ and when the value of ($H_o = 10^7$ Oersted), Figures 8 and 9 presents the first three modes of Ω for radial and torsion vibrations, respectively, against different values of h. It was found that in this case, the frequency Ω of torsion vibrations is not affected with the values of the primary magnetic field H_o.

Nomenclature

ρ	Constant mass density of the material	M_{ij}	Maxwell stress tensor
c_{ij}	Elastic material constants	μ_e	Magnetic permeability
$\vec{H}_o$	Initial constant magnetic field vector	ω	Angular frequency
$\vec{h}$	Perturbation of the magnetic field vector	t	Time
$\vec{E}$	Perturbation of the electric field vector	c	Velocity of light
τ_{ij}	Components of the stress tensor	σ	Electrical conductivity
e_{ij}	Components of the strain tensor	$\vec{J}$	Current density vector
$\vec{B}$	Magnetic induction	Φ, Ψ	Displacement potentials
u, v	Radial and tangential displacements		

References

[1] J.E. Greenspoon, "Flexural vibrations of a thick-walled hollow cylinder", Proceedings, Third U.S. National Congress: Applied Mechanics, pp.163-173 (1958).

[2] Y.H. Pao, "The dispersion of flexural waves in an elastic circular cylinder, Part II", J. Appl. Mech., 29, pp.61-64 (1962).

[3] R. Kumar, "The dispersion relation of flexural waves in an elastic circular cylinder", J. Appl. Mech., 39, pp.817-819 (1972).

[4] J.R. Hutchinson, "Axisymmetric flexural vibrations of a thick free circular plate", J. Appl. Mech. 46, pp.139-144 (1979).

[5] P.A. Martin, " On flexural waves in cylindrically anisotropic elastic rods", Int. J. solid and Structures 42, pp.2161-2179 (2005).

[6 F. Honarvar, E. Enjilela, A.N. Sinclair and S.A. Mirnezami, " Wave propagation in transversely isotropic cylinders", Int. J. solid and Structures 44, pp.5236-5246 (2007).

[7] I. Mirsky, "Wave propagation in transversely isotropic circular cylinders", part I: Theory, Part II: Numerical results, J. Acoust. Soc. Am., 37, pp.1016-1026, (1965).

[8] C. Prasad and R. K. Jain, "Vibrations of transversely isotropic cylindrical shells of finite length", J. Acoust. Soc. Am., 12 pp.1006-1009 (1965).

[9] J.E. White and C. Tongtaow, "Cylindrical waves in transversely isotropic media", J. Acoustic Soc. Am. 70(4), pp.1147-1155 (1981).

[10] Y.M. Tsai, "Longitudinal motion of a thick transversely isotropic hollow cylinder", Journal of Pressure Vessel Technology, 113, pp.585-589, (1991).

[11] E.S. Suhubi, "Small torsional oscillations of a circular cylinder with finite electrical conductivity in a constant axial magnetic field", Int. J. Eng. Sci., 2, pp.441-455 (1965).

[12] B.K. Datta, "On the stresses in the problem of magneto-elastic interaction on an infinite orthotropic medium with cylindrical hole", Ind. J. Theor. Phy., 33(4) pp.177-186 (1985).

[13] A.N. Abd-alla, "Magneto-elastic radial vibrations od a transversely isotropic hollow cylinder", Japan Journal of Industrial and Applied Mathematics, 14(3) pp.469-482 (1997).

[14] A.N. Abd-alla, "Natural frequencies of an anisotropic composite elastic cylinder placed in a magnetic field", Mec. Res. Com., 26(4), pp.397-405 (1999).

[15] W. Nowacki, "Magnetoelasticity," Chapter II, in Electromagnetic Interactions in Elastic Solids, edited by H. Parkus (Springer, Vienna, 1979), pp. 158-183.

[16] F.C. Moon, "Magneto-solid mechanics," John Wiley & Sons, New-York (1985).

[17] V.Z. Parton and B.A. Kudryavtsev, "Electromagnetoelasticity, Piezoelectrics and Electricity conducting solids", Gordon and Breach, New-York (1988).

[18] A.C. Eringen and G.A. Maugin, "Electrodynamics of Continua", Volumes I, II, Springer-Verlag, New York (1989).

[19] Auld B.A., (second ed.), (1990), Acoustic Fields and Waves in Solids vols. 1 and 2, Kreiger, Malabar, FL.

[20] Barakati A. and Zhupanska O.I., "Analysis of the effects of a pulsed electromagnetic field on the dynamic response of electrically conductive composites", Appl. Math. Modelling, doi: 10.1016/j.apm.2012.01.033 (2012).

[21] Dinzart F. and Sabar H., "Magneto-electro-elastic coated inclusion problem and its application to magnetic-piezoelectric composite materials", International Journal of Solids and Structures 48 pp.2393–2401 (2011).

[22] Akbarov, S. D., Kepceler T. and Mert Egilmez, M., " Torsional wave dispersion in a finitely pre-strained hollow sandwich circular cylinder", Journal of Sound and Vibration, 330(18-19), pp. 4519-4537 (2011).

[23] Chattopadhyay A., Gupta S. and Sahu S. A. , "Dispersion equation of magnetoelastic shear waves in irregular monoclinic layer", Applied mathematics and mechanics, 32(5), pp. 571-586 (2011).

[24] Tang, L. G. and Xu, X. M., " Transient torsional vibration responses of finite, semi-infinite and infinite hollow cylinders", Journal of Sound and Vibration, 329(8), pp.1089-1100 (2010).

[25] Acharya D.P., Roy I. and Sengupta S., "Effect of magnetic field and initial stress on the propagation of interface waves in transversely isotropic perfectly conducting media", Acta Mech

202, pp.35–45 (2009)
[26] Petrov V.M., Zibtsev V.V. and Srinivasan G., " Magnetoacoustic resonance in ferrite-ferroelectric nanopillars", Eur. Phys. J. B 71, pp.367–370 (2009).
[27] Mol'chenko L.V., Loos I. I., and Indiaminov R. Sh., " Determining the stress state of flexible orthotropic shells of revolution in magnetic field", International Applied Mechanics, 44(8), pp.882-891(2008).
[28] Abo-el-Nour N. Abd-Alla and S.M. Abo-Dahab, " The influence of the viscosity and the magnetic field on reflection and transmission of waves at interface between magneto-viscoelastic materials", Meccanica 43 pp.437–448 (2008).
[29] Selim, M. M, "Torsional waves propagation in an initially stressed dissipative cylinder", Applied Mathematical Sciences, 1(29), pp.1419 – 1427 (2007)
[30] Dai H.L., Wang X., " Magnetoelastodynamic stress and perturbation of magnetic field vector in an orthotropic laminated hollow cylinder", International Journal of Engineering Science 44 pp.365–378 (2006).
[31] Liu M.F. and Chang T.P., " Vibration analysis of a magneto-elastic beam with general boundary conditions subjected to axial load and external force", Journal of Sound and Vibration, 288(1-2), pp. 399-411 (2005).
[32] A.N. Abd-alla and Ibrahim Abbas, "Magnetoelastic longitudinal wave propagation in a transversely isotropic circular cylinder", J. Appl. Math. and Comput. 127(2-3) (2002), pp.347-360
[33] Rao S.S. "Applied numerical methods for engineers and scientists", Pearson Inc. U.K. (2001).

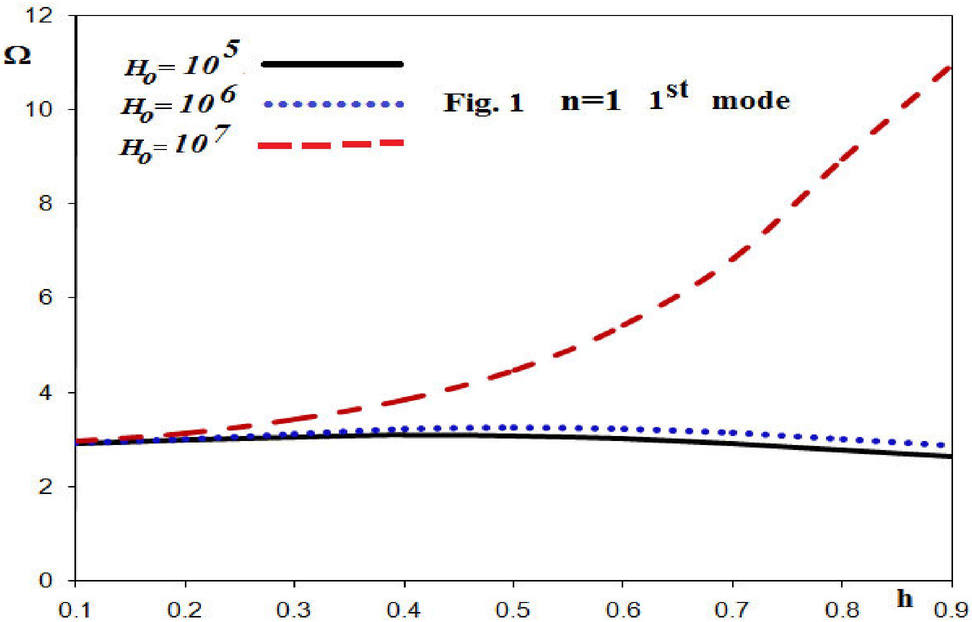

Figure 1 .The first mode of dimensionless frequency Ω for flexural vibrations versus different values of h=a/b for value of $H_o = 10^5, 10^6, 10^7$, when n=1.

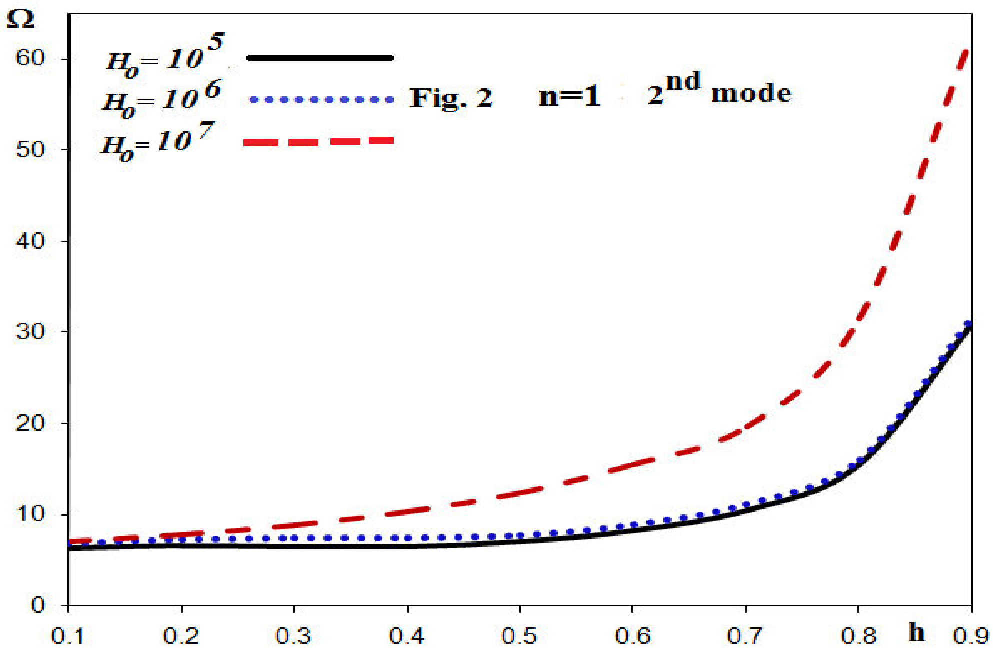

Figure 2 .The second mode of dimensionless frequency Ω for flexural vibrations versus different values of h=a/b for value of $H_o = 10^5, 10^6, 10^7$, when n=1.

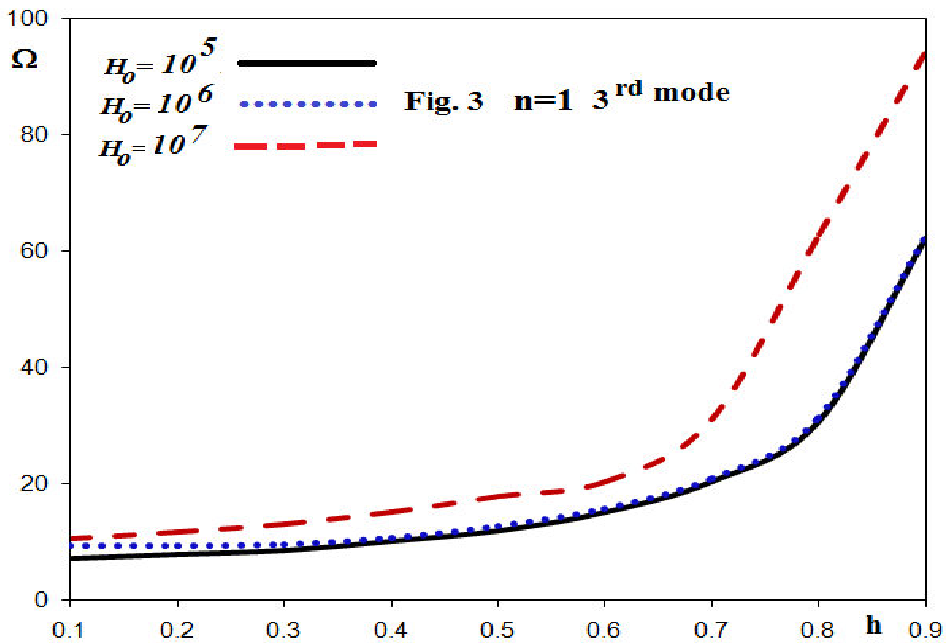

Figure 3 .The third mode of dimensionless frequency Ω for flexural vibrations versus different values of h=a/b for value of $H_o = 10^5, 10^6, 10^7$, when n=1.

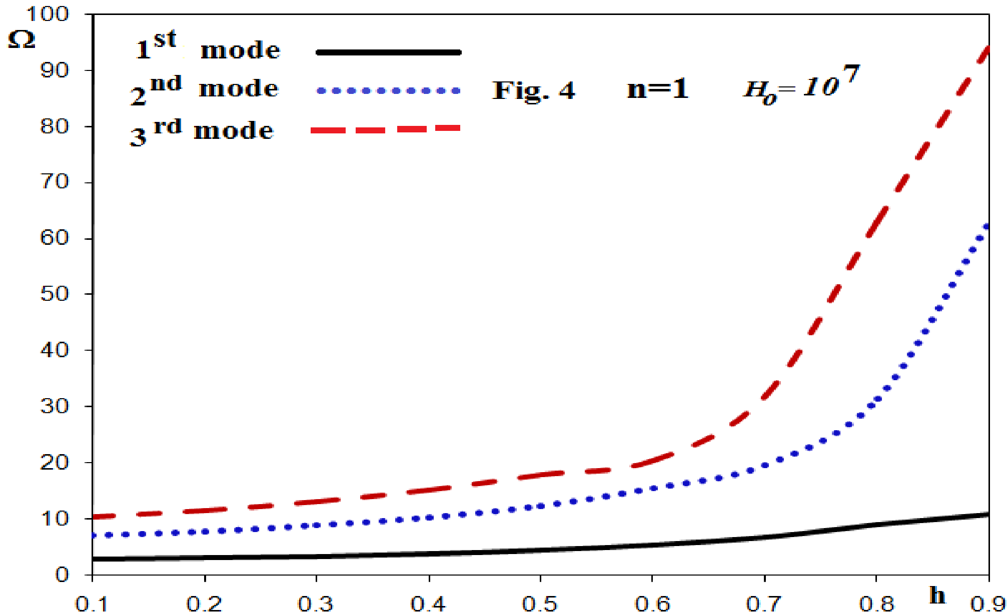

Figure 4 .The first three modes of dimensionless frequency Ω for flexural vibrations versus different values of h=a/b for value of $H_o = 10^7$, when n=1.

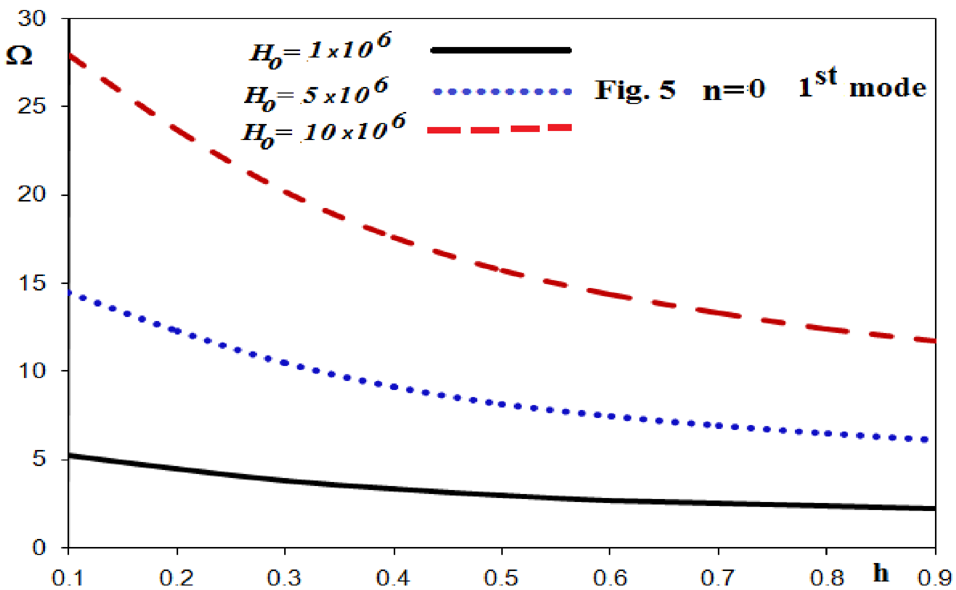

Figure 5. The first mode of dimensionless frequency Ω for radial vibrations versus different values of h=a/b for value of $H_o = (1, 5, 10)10^6$, when n=0.

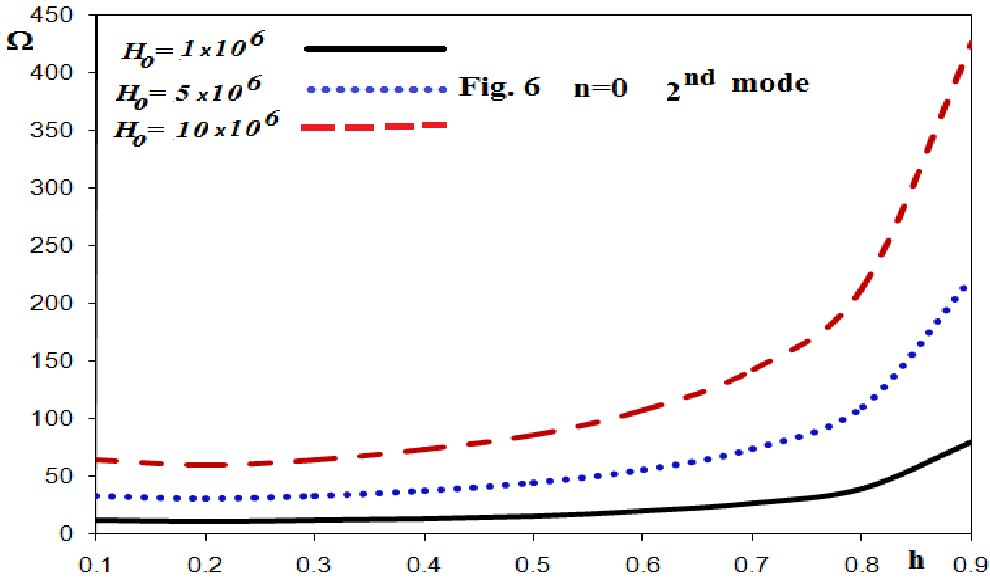

Figure 6. The second mode of dimensionless frequency Ω for radial vibrations versus different values of h=a/b for value of $H_o = (1, 5, 10)10^6$, when n=0.

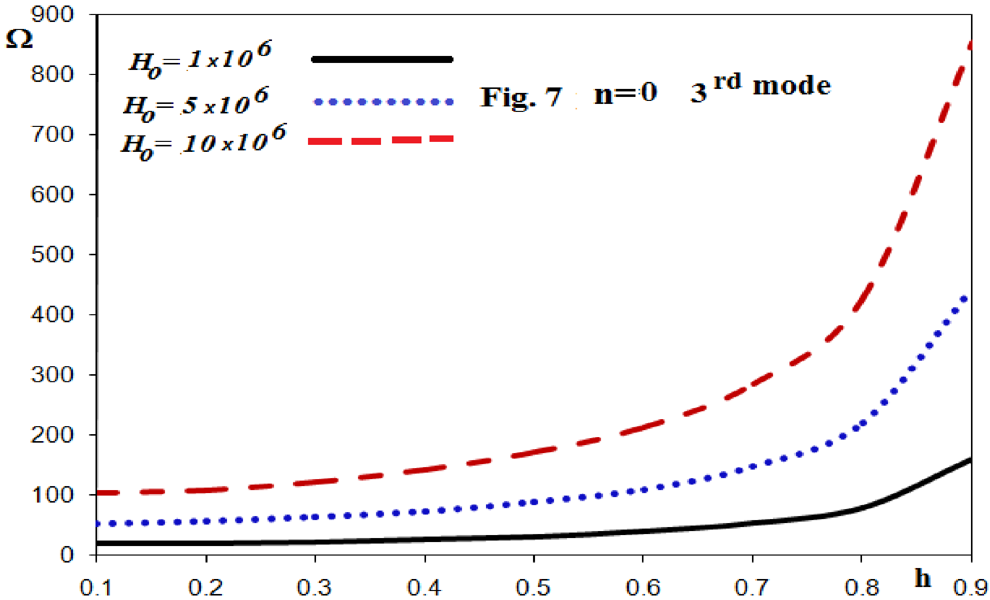

Figure 7. The third mode of dimensionless frequency Ω for radial vibrations versus different values of h=a/b for value of $H_o = (1, 5, 10)10^6$, when n=0.

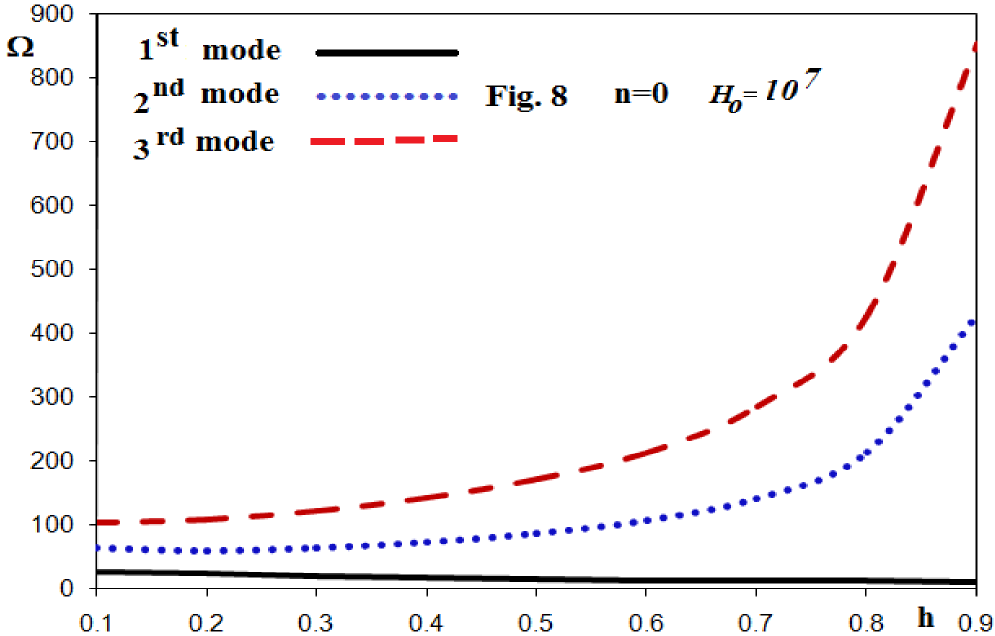

Figure 8 .The first three modes of dimensionless frequency Ω for radial vibrations versus different values of h=a/b for value of $H_o = 10^7$, when n=0.

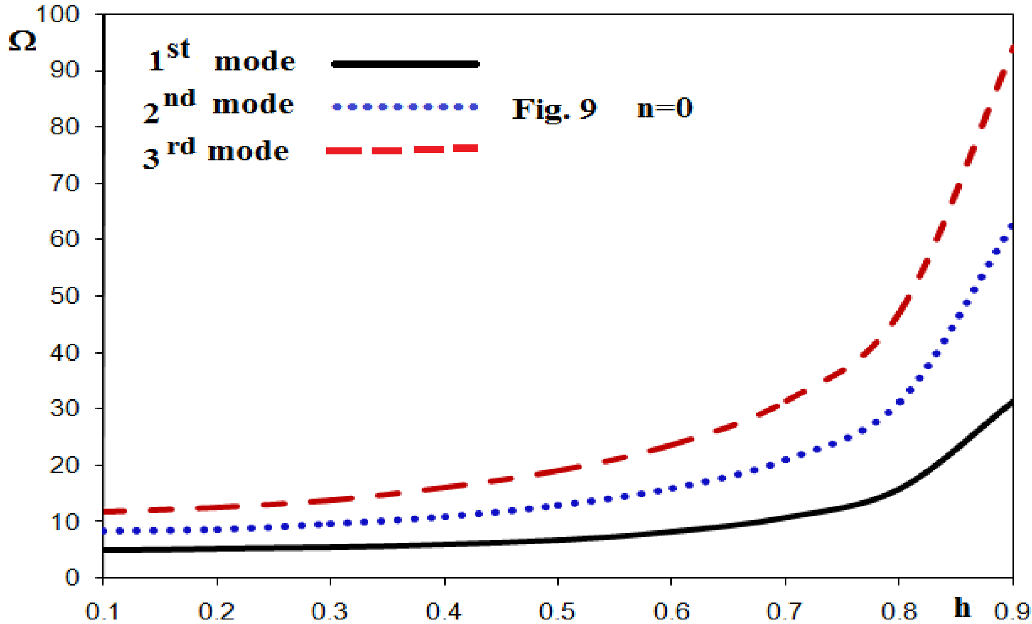

Figure 9 .The first three modes of dimensionless frequency Ω for torsion vibrations versus different values of h=a/b, when n=0.