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On the Extensional and Flexural of Generalized Thermoelastic Waves in an Anisotropic Plate

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Abstract

The propagation of extensional and flexural motions of generalized thermoelastic waves in a homogeneous, transversely isotropic plate of finite width is considered. The frequency equations for the plates in closed form and suitable mathematical conditions for symmetric and antisymmetric wave modes propagation are derived. Numerical calculations for three various theories of generalized thermoelasticity is carried out. In each case the real and imaginary parts of the frequency equation as a function of phase velocity for different values of thermal relaxation times are illustrated graphically. It is found that, the frequency equations of the extensional and flexural motions can be oscillate with respect to the medial of the plate. Moreover, it gets modified due to the thermal relaxation times and anisotropic effects. Finally, the results for the coupled thermoelasticity can be obtained as particular cases of the results by setting thermal relaxation times equal to zero

Keywords: Frequency equations; Extensional and flexural modes; Thermal relaxation times; Harmonic wave propagation

1. Introduction

Owing to the technological advances in recent years, plate elements are commonly selected as design components in many engineering structures, especially in the aerospace, marine and construction sectors, because of their ability to resist loads. With the evolution of light plate-structures, tremendous research interests in the vibration of the plates are generated. The negligence of considering vibration as a design factor can lead to excessive deflections and failures. The vibration design aspect is even more important in micro-machines such as electronic packaging, micro-robots, etc. because of their enhanced sensitivities to vibrations. Moreover, the frequency equation in anisotropic plates find use in many engineering structures and other areas of practical interest, such as slabs on columns, printed circuit boards or solar panels supported at a few points. With their potential applications of extensional and flexural modes of vibration of plates for considering the theories of generalized thermoelasticity has received considerable attention from researchers. The propagation of elastic waves in the layered media which are anisotropic in nature become very important and have long been of interest to researchers in many fields [1], [2], [3] and [5].
The coupled theory of thermoelasticity has been extended by including the thermal relaxation time in the constitutive equations with Lord and Shulman [6] and Green and Lindsay [4]. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity. This exists in the following differences between the two theories:
(i) The Lord-Şhulman (L-S) theory involves one relaxation time of thermoelastic process (τ_o) and that of Green and Lindsay (G-L) involves two relaxation times (τ_o, τ_1). (ii) The (L-S) energy equation involves first and second time derivatives of strain, whereas the corresponding equation in (G-L) theory needs only the first time derivative of strain. (iii) In the linearized case according to the approach of (G-L) theory the heat cannot propagate with finite speed unless the stresses depend on the temperature velocity, whereas according to (L-S) theory the heat can propagate with finite speed even though the stresses there are independent of the temperature velocity. (iv) The Lord-Şhulman (L-S) theory can not be obtained from Green and Lindsay (G-L) theory. Extensive theoretical efforts have been made so far to model thermoelastic waves in solids. The propagation of generalized thermoelastic waves in plates of an anisotropic media with different hypotheses has considered by [7], [10], [11], [12], [13], [15], [16] and [17].
In this paper, analysis for the propagation of thermoelastic waves in a homogenous transversely isotropic plate is carried out in the framework of the generalized theory of thermoelasticity. Commencing with a formal analysis of

waves in a heat-conducting layered plate of a transversely isotropic media, the frequency equation as function of the phase velocity of thermoelastic waves is obtained by invoking continuity at the interface and boundary of conditions on the surfaces of layered plate. Numerical solution of the frequency equations for a magnesium material is carried out for different values of relaxation times and illustrated graphically. Finally, when the two thermal relaxation times are neglected, one may get the results as in [8] and [9].

2. Formulation of the problem and its solution

We consider an infinite, homogeneous, transversely isotropic, thermally conducting elastic plate of thickness 2d initially at uniform temperature T_o. We take origin of the co-ordinate system (x_1, x_2, x_3) on the middle surface of the plate. The $x_1 - x_2$ plane is chosen to coincide with the middle surface and x_3-axis normal to it along the thickness. The fundamental system of field equations consists of the equations of motion for homogeneous anisotropic generalized thermoelasticity in the absence of body forces and heat sources are given by:

$$\sigma_{ij,j} = \rho \ddot{u}_i, \tag{1}$$

$$K_{ij} T_{,ij} - \rho C_e (\dot{T} + \tau_o \ddot{T}) = T_o \beta_{ij} (\dot{u}_{i,j} + \tau_o \delta_{ik} \ddot{u}_{i.j}), \tag{2}$$

$$\sigma_{ij} = c_{ijkl} e_{kl} - \beta_{ij} (T + \tau_1 \delta_{1k} \dot{T}), \qquad \beta_{ij} = c_{ijkl} \alpha_{kl}, \; e_{ij} = (u_{i,j} + u_{j.i})/2. \tag{3}$$

The use of symbol δ_{lk} makes the above equations possible for three of generalized thermoelasticity materials. For $k = 1$, one obtains L-S (Lord and Shulman) theory where $\tau_1 = 0, \quad \tau_o > 0$ and for $k = 2$, G-L (Green and Lindsay) theory is considered with the thermal relaxation times τ_o and τ_1 satisfy the inequality $\tau_1 \geq \tau_o > 0$. While for the C-D (Classical Dynamical Coupled) theory, the thermal relaxation times satisfy $\tau_1 = \tau_o = 0, \quad \delta_{lk} = 0.$

The elastic coefficients c_{ijkl} are a fourth rank tensor which has $3^4 = 18$ constants. But since σ_{ij} and e_{ij} are symmetric tensors. So, the tensor c_{ijkl} remains unchanged under a permutation of i and j or k and l, i.e., $c_{ijkl} = c_{jikl}$; $c_{ijkl} = c_{ijlk}$. According to these symmetries, we are left with 36 independent elastic constants instead of 81. We now introduce a compact matrix notation [9]. Therefore, we report complete determinations of the matrices:

$$c_{ijkl} = c_{pq}, e_{ikl} = e_{ip} \text{ and } \sigma_{ij} = \sigma_p \tag{4}$$

(where $p, q = 1-6, 1 = 11, 2 = 22, 3 = 33, 4 = 23, 5 = 13$ and $6 = 12$).

We assume an infinite, homogeneous, transversely isotropic, thermally conducting elastic plate of thickness $2d$ initially at uniform temperature T_o. We consider the faces of the plate to be the planes $x_3 = \pm d$ referred to as a rectangular set of Cartesian axes $Ox_1x_2x_3$. We suppose that the x_1-axis to be in the direction of the propagation of waves so that all particles on a line parallel to x_2 -axis are equally displaced. Therefore, all the field quantities will be independent of x_2-coordinate. The motion is assumed to take place in the dimensions (x_1, x_2). Here, u_1, u_2 are the displacements of a point in the $x_1 - x_2$ directions, respectively. In view of the governing equations (1) and (2) in non-dimensional form can be rewritten as:

$$u_{1,11} + c_2 u_{1,33} + c_3 u_{3,13} - (T + \delta_{2k} \tau_1 \dot{T})_{,1} = \ddot{u}_1, \tag{5}$$

$$c_3 u_{1,13} + c_2 u_{3,11} + c_1 u_{3,33} - \overline{\beta} (T + \delta_{2k} \tau_1 \dot{T})_{,3} = \ddot{u}_3, \tag{6}$$

$$T_{,11} + \overline{K} T_{,33} - (\dot{T} + \tau_o \ddot{T}) = \varepsilon_1 [\dot{u}_{1,1} + \delta_{1k} \tau_0 \ddot{u}_{1,1} + \overline{\beta} (\dot{u}_{3,3} + \delta_{1k} \tau_o \ddot{u}_{3,3})]. \tag{7}$$

After excluding the asterisk (*) for convenience, comma notation is used for spatial derivatives and we have introduced the following dimensionless quantities:

$$(x_1^*, x_3^*) = \frac{v_1}{k_1}(x_1, x_3), \quad (u_1^*, u_3^*) = \frac{v_1^3 \rho}{k_1 \beta_1 T_o}(u_1, u_3), \quad (t^*, \tau_o^*, \tau_1^* = \frac{v_1^2}{k_1}(t, \tau_o, \tau_1),$$

$$c_1 = \frac{c_{33}}{c_{11}}, \quad c_2 = \frac{c_{44}}{2c_{11}}, \quad c_3 = \frac{c_{13} + 0.5c_{44}}{c_{11}}, \quad \overline{K} = \frac{K_3}{K_1}, \quad T^* = \frac{T}{T_o},$$

$$\overline{\beta} = \frac{\beta_3}{\beta_1}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho^2 C_e v_1^2}, \quad \omega_1^* = \frac{v_1^2}{k_1}. \tag{8}$$

where $v_1 = (c_{11}/\rho)^{1/2}$ is the velocity of longitudinal waves, $k_1 = K_1/(\rho C_e)$ is the thermal diffusivity in the x-direction, ε_1 is the thermoelastic coupling constant, ω_1^* is the characteristic frequency of the medium and τ_o^*, τ_1^* are the dimensionless thermal relaxation constants.

The stresses and temperature gradient relevant to our problem in the plate are:

$$\sigma_{33} = (c_3 - c_1)u_{1,1} + c_1 u_{3,3} - \overline{\beta}(T + \tau_1 \dot{T}), \qquad \sigma_{31} = \overline{\beta} T_o c_2 (u_{1,3} + u_{3,1}), \tag{9}$$

As we considering plane harmonic wave traveling in the x-direction therefore we may take the solutions for u_1, u_3 and T of Eqs. (5), (6) and (7) is follows:

$$\{u_1, u_3, T\} = \{f(z), g(z), h(z)\}\exp[i\zeta(x - ct)] \tag{10}$$

where $(c = \omega/\zeta)$ is the phase velocity, ζ and ω are the wave number, circular frequency, respectively and $i = \sqrt{-1}$.

Now using solutions (10) into equations (5), (6) and (7) we get

$$(c_2 D^2 - \zeta^2 + \zeta^2 c^2)f + ic_3 \zeta Dg - \tau_a \zeta h = 0, \tag{11}$$

$$ic_3 \zeta Df + (c_1 D^2 - c_2 \zeta^2 + \zeta^2 c^2)g + i\overline{\beta}\tau_a Dh = 0, \tag{12}$$

$$i\varepsilon_1 \tau c \zeta^2 f + \varepsilon_1 \overline{\beta} \tau c \zeta Dg + (\overline{K} D^2 - \zeta^2 + \zeta^2 c^2 \tau)h = 0 \tag{13}$$

where

$$D = \frac{\partial}{\partial x_3}, \qquad \tau = i + \delta \tau_o \zeta c, \qquad \tau_a = i + \tau_1 \zeta c. \tag{14}$$

The solutions of equations (11)-(13) can be written in the form:

$$f(z) = \sum_{j=1}^{3} [P_j \exp(-\zeta M_j z) - Q_j \exp(\zeta M_j z)], \tag{15}$$

$$g(z) = \sum_{j=1}^{3} m_j [P_j \exp(-\zeta M_j z) - Q_j \exp(\zeta M_j z)], \tag{16}$$

$$h(z) = \zeta \sum_{j=1}^{3} l_j [P_j \exp(-\zeta M_j z) - Q_j \exp(\zeta M_j z)] \tag{17}$$

where

$$m_j = \frac{[\overline{\beta}(c_2 M_j^2 + c^2 - 1) + c_3] M_j}{i[M_j^2(\overline{\beta} c_3 - c_1) + c_2 - c^2]}, \quad l_j = [c_2 M_j^2 + c^2 - 1) - ic_3 M_j m_j]/\tau_a, \tag{18}$$

P_j, Q_j $(j = 1,2,3)$ are arbitrary constants, and M_1, M_2 and M_3 are the roots of the following equation

$$M^6 + L_1 M^4 + L_2 M^2 + L_3 = 0 \tag{19}$$

where

$$L_1 = -[\bar{k}c_2(c_2 - c^2) + c_1c_2(1-\tau c/\zeta) + \bar{K}c_1(1-c^2) - \bar{K}c_3^2 + ic_2\varepsilon_1\beta^{-2}\tau_a\tau c/\zeta]/\bar{K}c_1c_2,$$

$$L_2 = [\{\bar{K}(c_2 - c^2) + c_1(1-\tau c/\zeta) + i\varepsilon_1\beta^{-2}\tau_a\tau c/\zeta\}(1-c^2) - c_3^2(1-\tau c^2) + c_2(c_2 - c^2)$$
$$\times(1-\tau c/\zeta) - i\varepsilon_1(2\bar{\beta}c_3 - c_1)\tau_a\tau c/\zeta]/\bar{K}c_1c_2,$$

$$L_3 = -[(1-c^2)(c_2 - c^2)(1-\tau c/\zeta) + i\varepsilon_1(c_2 - c^2)\tau_a\tau c/\zeta]/\bar{K}c_1c_2.$$

The displacement components and temperature of the plate become:

$$u_1 = \sum_{j=1}^{3}[P_j\exp(-\zeta M_j z) + Q_j\exp(\zeta M_j z)]\exp[i\zeta(x-ct], \tag{20}$$

$$u_3 = \sum_{j=1}^{3}m_j[P_j\exp(-\zeta M_j z) - Q_j\exp(\zeta M_j z)]\exp[i\zeta(x-ct], \tag{21}$$

$$T = \zeta\sum_{j=1}^{3}l_j[P_j\exp(-\zeta M_j z) + Q_j\exp(\zeta M_j z)]\exp[i\zeta(x-ct]. \tag{22}$$

3. Boundary conditions

The non-dimensional boundary conditions at the surfaces $x_3 = \pm d$ of the plate are given by:

(i)Mechanical conditions (stress-free surfaces)

$$\sigma_{33} = 0, \qquad \sigma_{13} = 0. \tag{23}$$

(ii) Thermal condition (thermally insulated)

$$T_{,3} = 0. \tag{24}$$

The use of equations (20), (21) and (22) in equations (23) and (24), with the help of equations (9), leads to a system of the following coupled equations for the arbitrary unknown coefficients P_1, P_2, P_3, Q_1, Q_2 and Q_3:

$$\sum_{j=1}^{3}(iF - c_1m_jM_j + i\bar{\beta}\tau_a l_j)[P_j\exp(-\zeta M_j d) + Q_j\exp(\zeta M_j d)]\exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(im_j - M_j)[P_j\exp(-\zeta M_j d) - Q_j\exp(\zeta M_j d)]\exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(-l_jM_j)[P_j\exp(-\zeta M_j d) + Q_j\exp(\zeta M_j d)]\exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(iF - c_1m_jM_j + i\bar{\beta}\tau_a l_j)[P_j\exp(\zeta M_j z) + Q_j\exp(-\zeta M_j d)]\exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(im_j - M_j)[P_j\exp(\zeta M_j d) - Q_j\exp(-\zeta M_j d)]\exp[i\zeta(x-ct] = 0,$$

$$\sum_{j=1}^{3}(-l_jM_j)[P_j\exp(\zeta M_j d) + Q_j\exp(-\zeta M_j d)]\exp[i\zeta(x-ct] = 0. \tag{25}$$

where $F = c_3 - c_2$. We notice that the above six equations which are coming from applying the boundary conditions (24) must be satisfied simultaneously.

4. Frequency equation

The system of equation (25) has a nontrivial solution if and only if the determinant of the coefficients amplitudes P_i and Q_i, where ($i = 1,2,3$) vanishes. After applying algebraic reductions and manipulations this leads to the frequency equation (also called dispersion equation or secular equation) for thermally insulated plate oscillations. The frequency equation which corresponds to the extensional and flexural motions of the plate with respect to the medial plane $x_3 = 0$ may be written as:

$$G_1\Delta_1 - G_2\Delta_2 + G_3\Delta_3 = 0, \quad G_1\Delta_1 - G_2\Delta_2 - G_3\Delta_3 = 0 \tag{26}$$

where we have used:

$$G_j = iF - c_1 m_j M_j + i\bar{\beta}\tau_a l_j, \quad S_j = im_j - M_j, \quad E_j = -l_j M_j, \tag{27}$$

$$\Delta_1 = S_2E_3 - S_3E_2, \quad \Delta_2 = S_1E_3 - S_3E_1, \quad \Delta_3 = S_1E_2 - S_2E_1, \tag{28}$$

with $j = 1,2,3$ and m_j and l_j are given in Eqs. (18).

The frequency equations (26) correspond to the extensional and flexural motions of the plate with respect to the medial plane $x_3 = 0$.

5. Numerical Results and discussion

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results. The materials chosen for this purpose is single crystal of magnesium, the physical data for which is given by [14]:

$$\rho = 1.74\times10^3\ Kgm^{-3},\ c_{11} = 0.5974\times10^{11}\ Nm^{-2},\ c_{12} = 0.2624\times10^{11}\ Nm^{-2},$$
$$c_{13} = 0.217\times10^{11}\ Nm^{-2},\ c_{33} = 0.617\times10^{11}\ Nm^{-2}, c_{44} = 0.3278\times10^{11}\ Nm^{-2},$$
$$\beta_1 = 2.68\times10^6\ Nm^{-2}\ deg^{-1},\ \beta_3 = 2.68\times10^6\ Nm^{-2}\ deg^{-1},\ T_o = 298\ deg,$$
$$C_e = 1.04\times10^3\ Jkg^{-1}\ deg^{-1},\ \omega_1^* = 3.58\times10^5\ s^{-1},\ \varepsilon_1 = 2.02\times10^{-2},$$
$$K_1 = 1.7\times10^2\ Wm^{-1}\ deg^{-1},\ K_2 = 1.7\times10^2\ Wm^{-1}\ deg^{-1}.$$

We restrict our attention to make the dimensionless phase velocity η and the dimensionless wave number ξ to be $\eta = \sqrt{\rho c^2 / c_{11}}$ and $\xi = \zeta d / 2$ respectively.

The complex roots of characteristics equation (19) have been computed with the help of Cardan's procedure, which are then employed to solve frequency equations (FE) (26). Then, the real and imaginary parts of the (FE) (26) for the extensional and flexural motions are obtained for the phase velocity η for different values of thermal relaxation times by utilizing iteration method and illustrated graphically in Figs. (1a,b) – (7a,b).

The real and imaginary parts of the frequency equations multiplied by 10^{-16} are plotted in Figs. (1a, 1b) and (2a, 2b) for the extensional and flexural motions respectively, versus the phase velocity η for (G-L) model (i.e., $\tau_1 = 5\tau_o$, $\tau_o = 0.1, 0.2, 0.3$). For the extensional and flexural motions in Figs. (1a, 1b), it is easy to see that Re(FE) and Im(FE) starts from zero as $\eta = 0$ and vary linearly until $\eta = 0.9$. After that, in the period $\eta = 0.9 - 2.0$, one may find from Fig. (1a) for extensional motions, that Re(FE) decreases nonlinearly as η increases when ($\tau_o = 0.2, 0.3$). But Re(FE) for flexural motion decreases slowly and attains a minimum value, then rises again in Fig. (1b) when ($\tau_o = 0.1$). While, for extensional and flexural motions in Figs. (2a, 2b), one may observe for extensional motion that Im(FE), in the period $\eta = 0.9 - 2.0$, increases nonlinearly as η increases, while the contrary happens for Im(FE) for flexural motion. In addition, Re(FE) and Im(FE) decrease due to increasing the thermal relaxation times for the all two pervious cases in the period ($\eta = 0.9 - 2.0$). Figs. (3a, 3b) and (4a, 4b) represent variations of the real and imaginary parts for extensional and flexural motions of (FE) multiplied by 10^{-25} with respect to the phase velocity η in case of (L-S) model for various values of the first thermal relaxation time τ_o (i.e. $\tau_1 = 0, \delta = 0, \tau_o = 0.1, 0.2, 0.3$). From Figs (3a, 3b) and (4a,4b), it is noted that the behavior and trend of the variations of Re(FE) and Im(FE) are almost similar as in case of Im(FE) for (G-L) model in Figs. (1a,1b) and (2a,2b). Figs. (5a, 5b) exhibit changes of Re(FE) and Im(FE) multiplied by 10^{-20} versus η in case of (C-D) model for extensional and flexural motions. The trend and behavior of these profiles are similar to that of previous Figures, while in this case, both of Re(FE) and Im(FE) are identical. This means that the value of (FE) are real only.

6. Conclusions

Analysis for the propagation of thermoelastic waves in arbitrary anisotropic plates, for extensional (symmetric) and flexural (antisymmetric) motions, is carried out in the framework of the generalized theory of thermoelasticity. The case of layered half space is considered. It is noted that the frequency equation of the waves gets modified due to the thermal and anisotropic effects and is also influenced by the thermal relaxation times. The increasing ratios of thermal relaxation times tend to increase the values of the frequency equation of different modes. When the phase velocity is small, it is seen that there is no change for Re(FE) and Im(FE) among the three various models of generalized thermoelasticity. The obtained solutions can be used for material systems of higher or less symmetries such as monoclinic, orthotropic, cubic, and isotropic as it is contained implicitly in the analysis.

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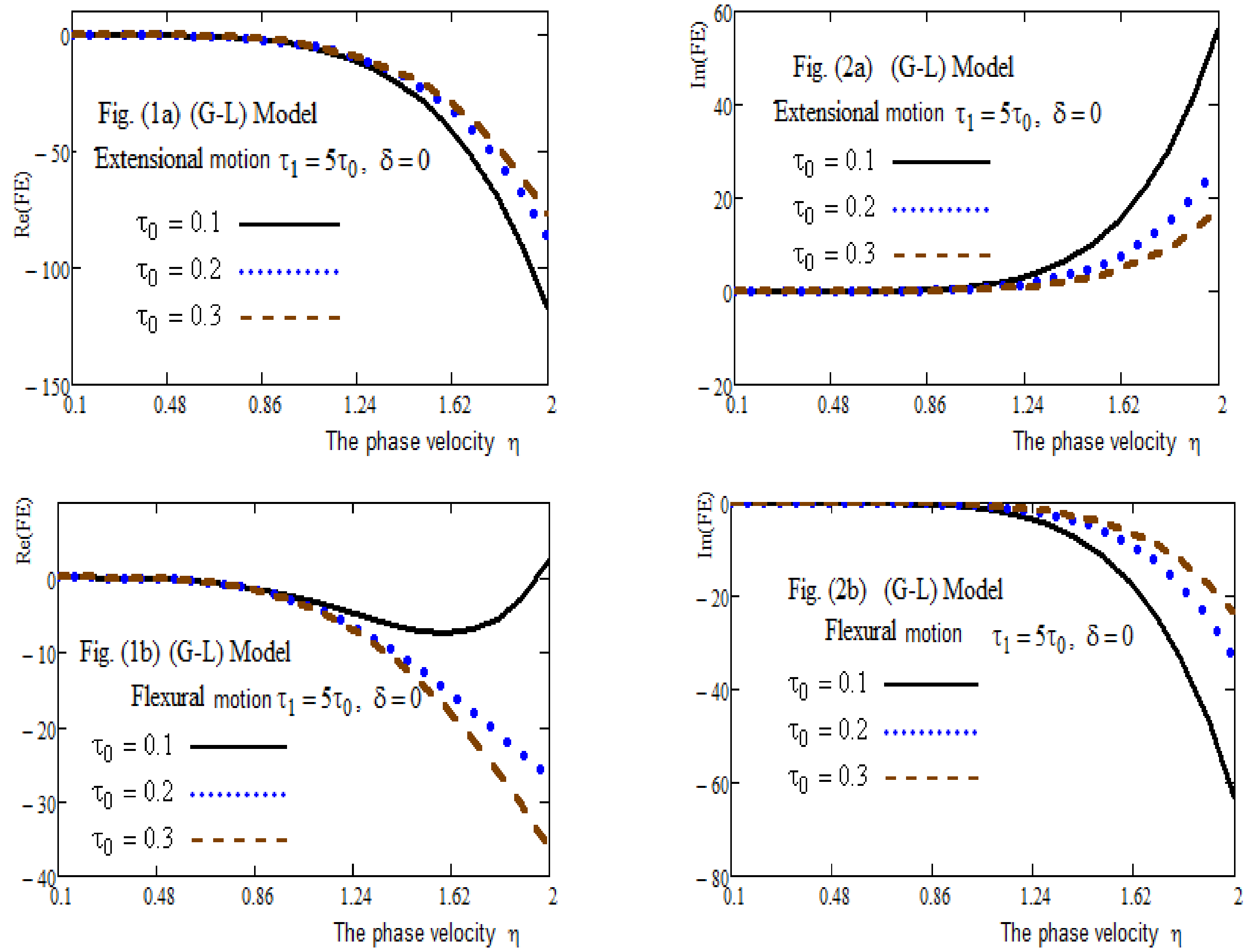
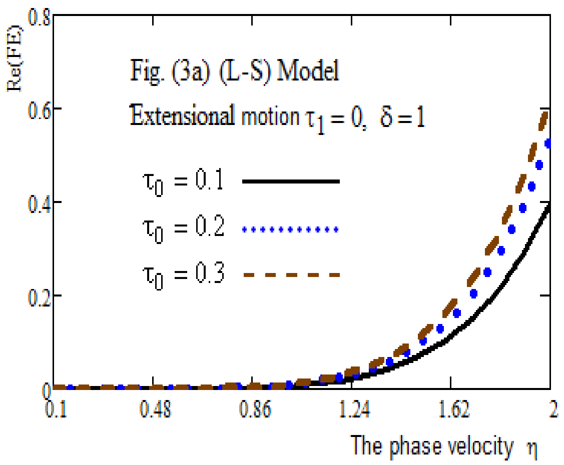
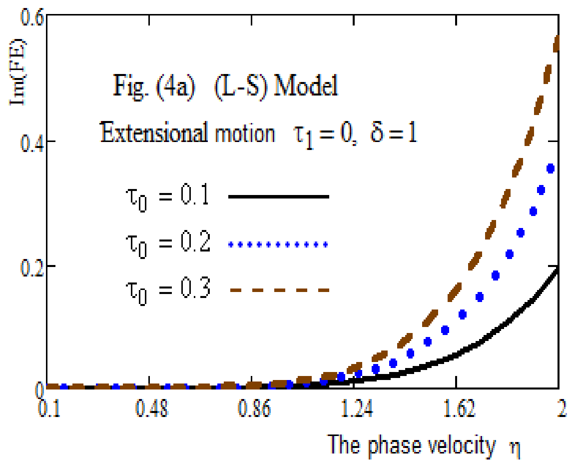
Nomenclature

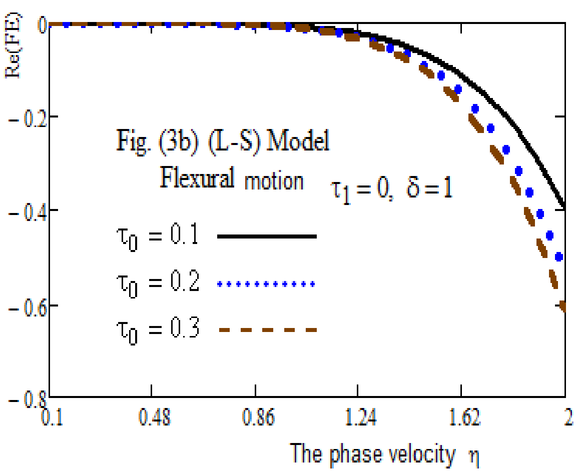
ρ	density,
t	time,
u_i	displacement in the x_i direction,
K_{ij}	thermal conductivities,
C_e	specific heat at constant strain,
τ_o , τ_1	thermal relaxation times,
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor,
β_{ij}	thermal moduli,
α_{kl}	coefficients of linear thermal expansion tensor,
T , T_o	Temperature and reference temperature,
c_{ijkl}	fourth-order tensor of the elasticity
δ_{ij}	Kronecker's delta,
v_1	velocity of longitudinal waves,
k_1	thermal diffusivity in the x-direction,
ε_1	thermoelastic coupling constant
ω_1^*	characteristic frequency of the medium
τ_o^* , τ_1^*	dimensionless thermal relaxation constants

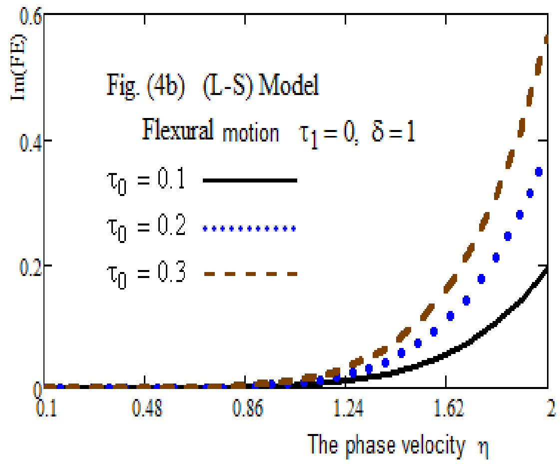


Figs. (1a) and (1b): The real parts of the frequency equation for the symmetric and antisymmetric motions multiplied by 10^{-16} versus the phase velocity for (G-L) model for different values τ_1.	Figs. (2a) and (2b): The imaginary parts of the frequency equation for the symmetric and antisymmetric motions multiplied by 10^{-16} versus the phase velocity for (G-L) model for different values τ_1.

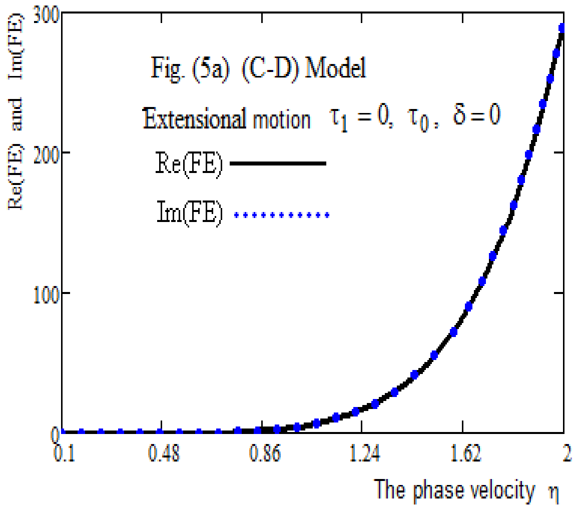


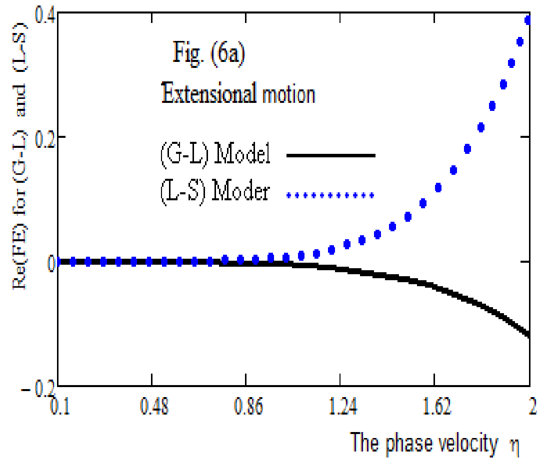


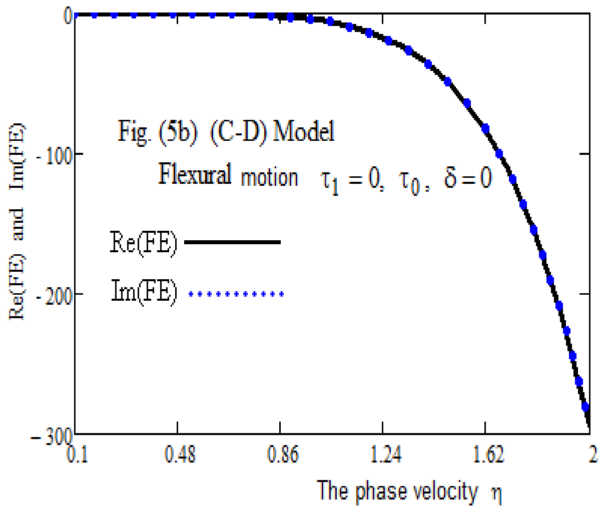


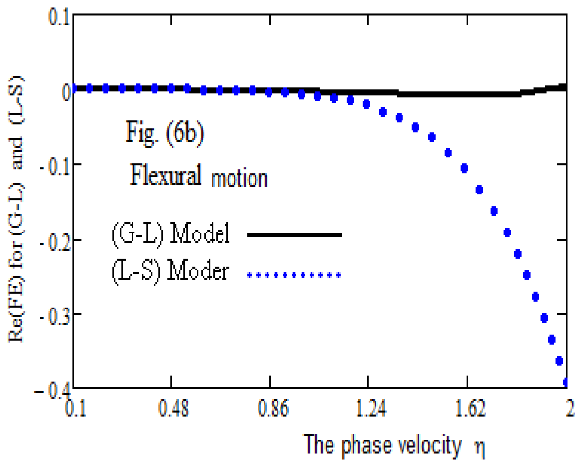


Figs. (3a) and (3b): The real parts of the frequency equation for the symmetric and antisymmetric motions multiplied by 10^{-25} versus the phase velocity for (L-S) model for different values τ_0.	Figs. (4a) and (4b): The imaginary parts of the frequency equation for the symmetric and antisymmetric motions multiplied by 10^{-25} versus the phase velocity for (L-S) model for different values τ_0.

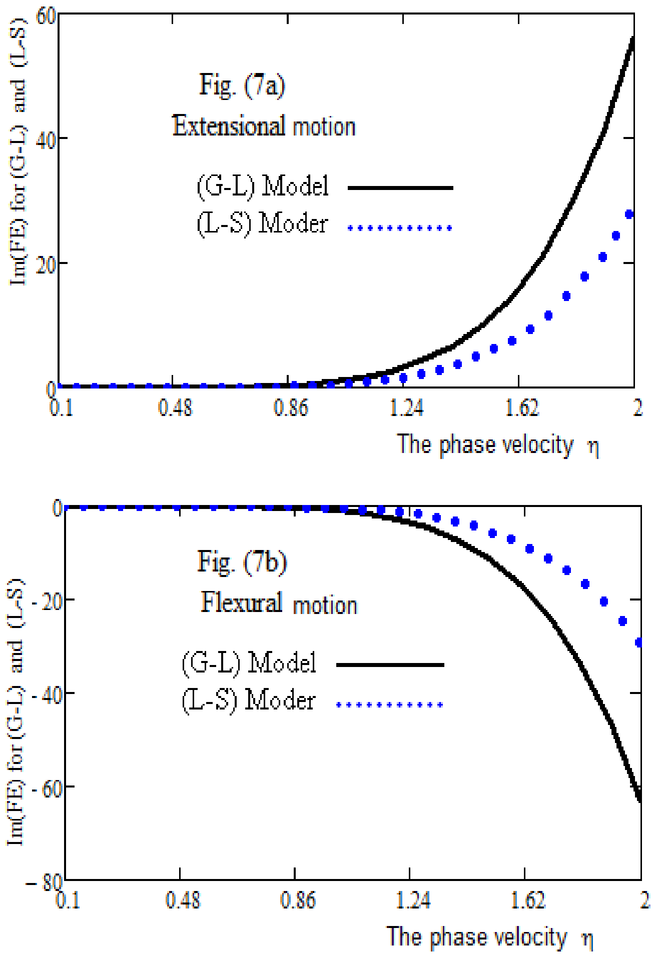








Figs. (5a) and (5b): The real and Imaginary parts of the frequency equation for the symmetric and antisymmetric motions multiplied by 10^{-20} versus the phase velocity for (C-D) model.	Figs. (6a) and (6b): The real parts of the frequency equation for (G-L) and (L-S) models multiplied by 10^{-19} multiplied by 10^{-25} respectively, for the symmetric and antisymmetric motions versus the phase velocity.



Figs. (7a) and (7b): The imaginary parts of the frequency equation for (G-L) and (L-S) models multiplied by 10^{-16} and multiplied by 10^{-21} respectively, for the symmetric and antisymmetric motions versus the phase velocity.