Apodization Defocused Optical Imaging System with Different Apertures using Hanning Amplitude Filter

Ali H. Al-Hamadani[1] Ghada S. Karam[2] Abbas S. Al-Saedi[2*]
1.University of Technology, Laser & Optoelectronics Engineering Department, Baghdad, IRAQ
2.Al-Mustansiriya University, Physics Department, Baghdad, IRAQ

Abstract
The Optical system performance is characterized by the modulus and the phase of the amplitude impulse response (point spread function). An optical system with a nonuniform amplitude across its pupil called an apodized system. The motivation of apodizing a system is to increase the image quality of the system. In this paper, the influence of Hanning amplitude filters on the intensity of incoherent optical systems has been analyzed. A general expression for the PSF in the case of different square size pupil apertures (half diagonal= $\sqrt{\frac{\pi}{2}}$, 1) for a defocused optical system is considered. From the comparison between circular and square aperture it's found that the Hanning amplitude filters are effective in increasing the resolving power for an optical imaging system with half diagonal= $\sqrt{\frac{\pi}{2}}$ unlike the other square aperture with half diagonal= 1, and also found that the maximal intensity for square aperture with half diagonal= 1 remains better than the other square aperture.
Keywords: Hanning amplitude filters, PSF, Strehl ratio

I. INTRODUCTION

The optical features of an aberration-free defocused optical system used to image incoherently illuminated objects plays an important role in the optical system as a part of many image visualization system. The defocused images can be improved with the help of suitable amplitude filters (Thirupathi *et al*. 2014). There are several quality criteria to describe the performance of an optical system in the field of image science; the spatial resolution criterion, Strehl Ratio (SR) (Srisailam *et al*. 2014) (Karam 2005), and point spread function (PSF) is of the principal and earliest measures proposed for judging the quality of optical imaging systems (Sayanna *et al*. 2003). The intensity distribution of the image of a point object is generally known as the point spread function (PSF). The specific form of PSF for an ideal lens is known as the Airy pattern, the PSF which describes the performance of an optical imaging system can be shaped by apodization (Reddy & Sagar 2016). Apodization is a technique used for modifying the amplitude impulse response of an optical imaging system. Many works have been aimed at enhancement the optical system quality using apodization. Kumar *et al.* (2013) investigated the PSF of a defocused optical imaging system with circular aperture suffering from primary spherical aberration with Hanning amplitude filters. Venkanna & Sagar (2015) studied the influence of the filter which has been analyzed to estimate the optimum value of apodization for annular aperture. Venkanna *et al*. (2015; 2017) studied the effects of aberrations and aperture obscuration of optical systems with point objects. It is shown that in specific cases, the aberrated images can be improved with the help of appropriate amplitude filters, also in the case of circular aperture they studied the complex pupil function on the intensity point spread function of a coherent optical system. Wen *et al*. (2002) investigated analytically the PSF of an optical imaging system for solving image restoration problems with asymmetric apodization. Andra *et al*. (2016) studied primary energy based corollaries of point spread function with asymmetric apodization by using complex pupil function in the case of a three-zone aperture. In the present paper, we aimed to enhance the resolution of an optical system with a square pupil (Al-Hamadani & Hasan 2013) by using Hanning amplitude filters.

II. Mathematical Expression

The complex amplitude at the point (u, v) in the image plane of optical system can be expressed by using Fourier transform to pupil function *f(x,y)* (LaVeigne *et al*. 2008; NEIL *et al*. 2000)

$$A(u,v) = \int_y \int_x f(x,y) \cdot e^{i2\pi(ux+vy)} dxdy \qquad \text{....(1)}$$

Uu,v is the dimensionless coordinates

$$f(x,y) = \tau(x,y) \cdot e^{ikW(x,y)} \qquad \text{....(2)}$$

$\tau(x,y)$ represents the real amplitude distribution in exit pupil which called pupil transparency. In the present study, we considered the Hanning amplitude filter on the 1st. order and β is the apodizing parameter controlling the non-uniform transmission of the pupil.

τ (x,y) = cos $\left(\pi\beta(x^2+y^2)^{1/2}\right)$, β = (0, 0.25, 0.50, 0.75, 1) ....*

(x, y): represent exit pupil coordinates
$e^{ikW(x,y)}$: represents wavefront of aberration function.
$w(x, y)$: represents aberration function (Braat & Haver 2008)

$$W(x,y) = w_{20}(x^2+y^2) + w_{40}(x^2+y^2)^2 + ... \quad ....(3)$$

Where W_{20} represents the defocus coefficient, W_{40} represents the coefficient of primary aberration
The intensity PSF, G(u, v) which is the real measurable quantity can be obtained by taking the squared modulus of $A(u,v)$. Thus,

$$\mathrm{G}(u,v) = \mathrm{N}\,|A(u,v)|^2 \quad ....(4)$$

Where N is the normalizing factor and will be computed using MATHCAD,
Let $z = 2\pi u$ and $m = 2\pi v$, which represent the dimensional coordinates. Then equation (4) can be written as

$$\mathrm{G}(z,m) = N\left|\int_y \int_x \tau(x,y)\, e^{ikw\,(x,y)}\, e^{i(zx+my)} dxdy\right|^2 \quad ....(5)$$

Because of the symmetric of intensity distribution on the two axes (z, m), so we can reduce it to one axis only, let ($m = 0$), then the equation (5) can be written as,

$$\mathrm{G}(z) = N\left|\int_y \int_x \tau(x,y)\, e^{ikw\,(x,y)}\, e^{i(zx)} dxdy\right|^2 \quad ...(6)$$

For an optical system with square and circular aperture, eq6 become for:

1- Circular aperture,

$$\mathrm{G}(z) = N\left|\int_{-1}^{1} \int_{-\sqrt{1-Y^2}}^{\sqrt{1-Y^2}} \tau\,(\mathrm{x,y})\, e^{ikw}\, e^{izx} dxdy\right|^2 \quad ...(7)$$

2- Square aperture with half diagonal =$\sqrt{\frac{\pi}{2}}$,

$$\mathrm{G}(z) = N\left|\int_{-\sqrt{\pi}/2}^{\sqrt{\pi}/2} \int_{-\sqrt{\pi}/2}^{\sqrt{\pi}/2} \tau\,(\mathrm{x,y}) e^{ikw}\, e^{izx} dxdy\right|^2 \quad ....(8)$$

3- Square aperture with half diagonal =1,

$$\mathrm{G}(z) = N\left|\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \tau\,(\mathrm{x,y}) e^{ikw}\, e^{izx} dxdy\right|^2 \quad ....(9)$$

III.Numerical Results

The results of the effect of Hanning amplitude filters apodization on intensity distribution in the image plane for an optical system with circular and squared aperture have been obtained from equations (7,8,9) as a function of dimensionless coordinates z by using Mathcad. The performance of the efficiency of Hanning function is described by the PSF of a circular and squared (half diagonal = $\sqrt{\frac{\pi}{2}}$, half diagonal= 1) apertures and they depicted in figs (1) to (3). In all these figures, the airy PSF is represented by the blue curve. It can be seen from these figures that the central PSF peak is boarded and lowered with increasing the value of β. However, on the other hand, the degree of apodization β is very effective on the width of central maxima (resolving power) in the presence of defocus w_{20}, its found that the best-resolving power can be obtained at β =0.25.

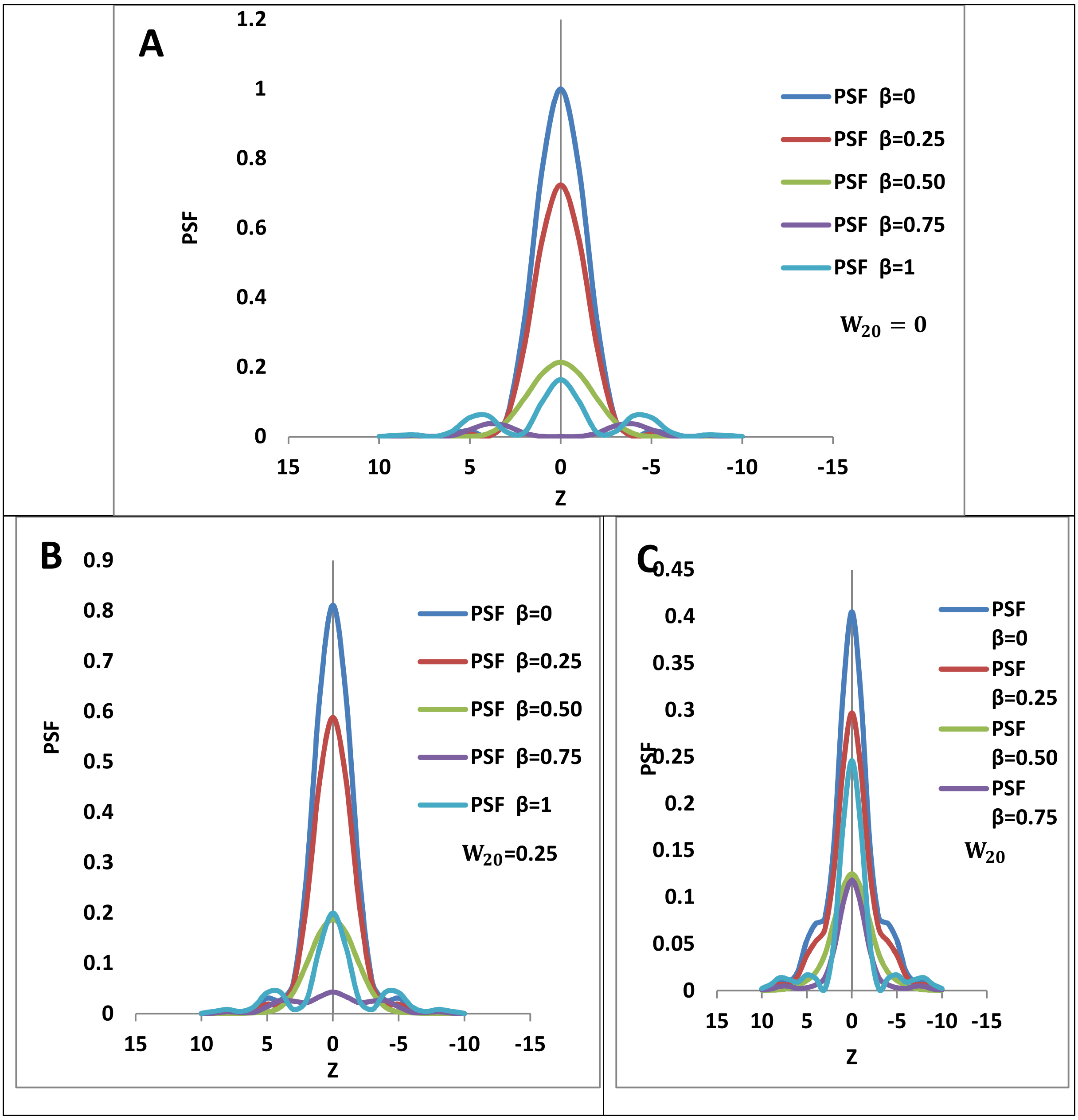

Figure.1 PSF profile of circular aperture

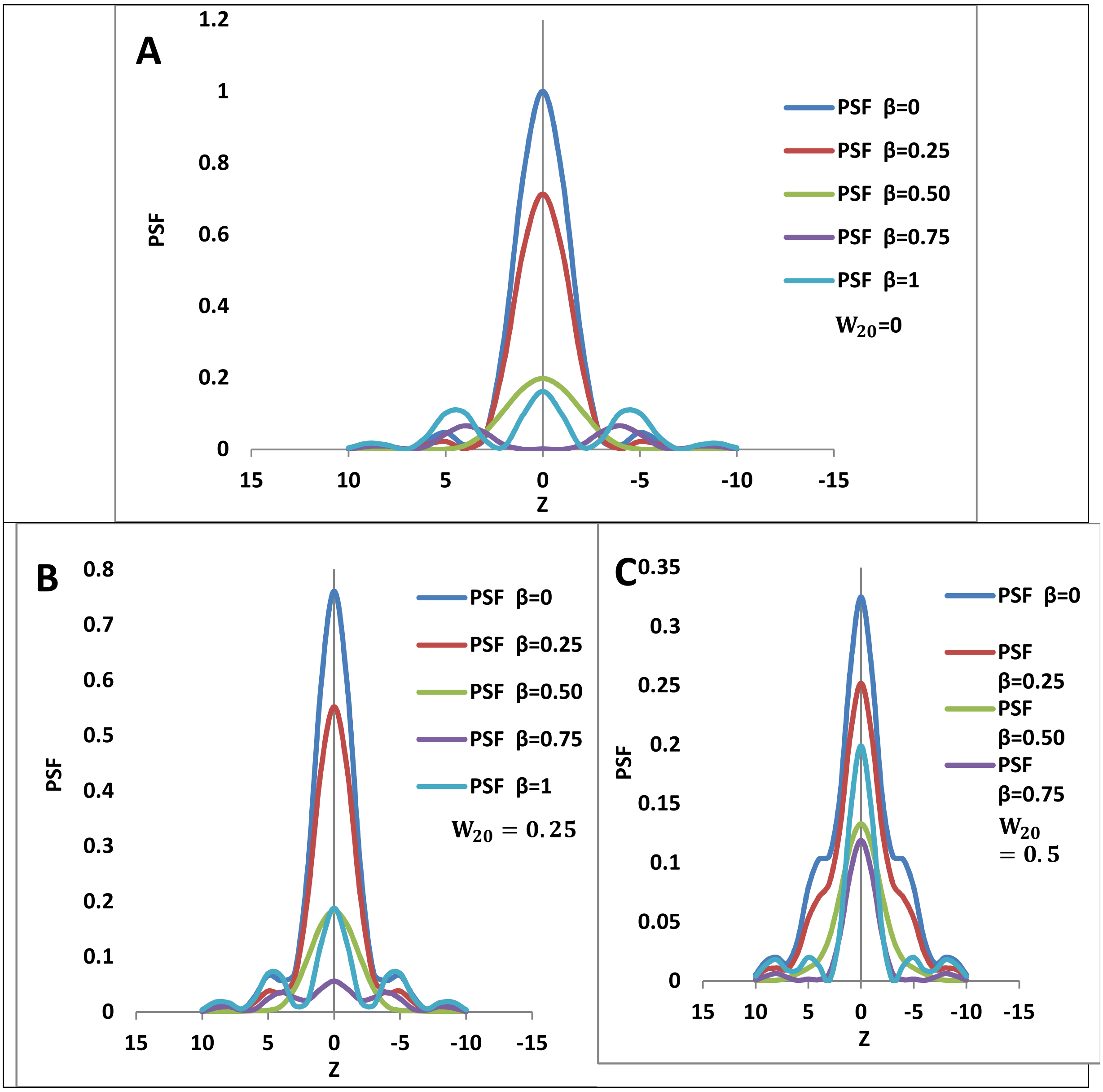

Figure2. PSF profile for square aperture of half diagonal = $\sqrt{\frac{\pi}{2}}$

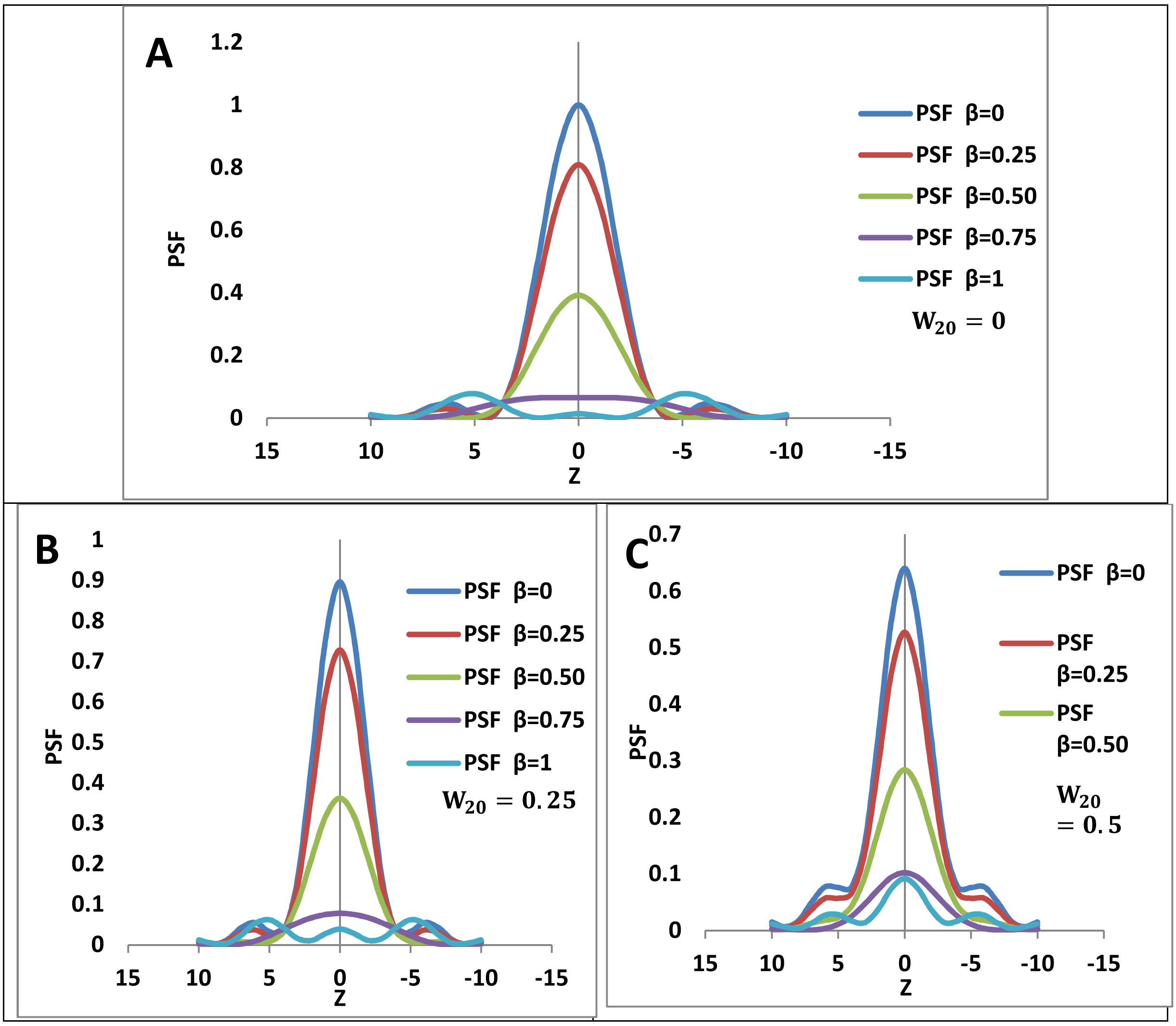

Figure.3 PSF profile for square aperture of half diagonal= 1

Figures (4,5,6) shows the intensity PSF for different apertures with defocus values. It's observed from the figures (4,5) the maxima PSF distribution is better for square aperture with half diagonal =1 than the other apertures for different values of defocus, while the resolving power for square aperture with half diagonal = $\sqrt{\frac{\pi}{2}}$ is the best.

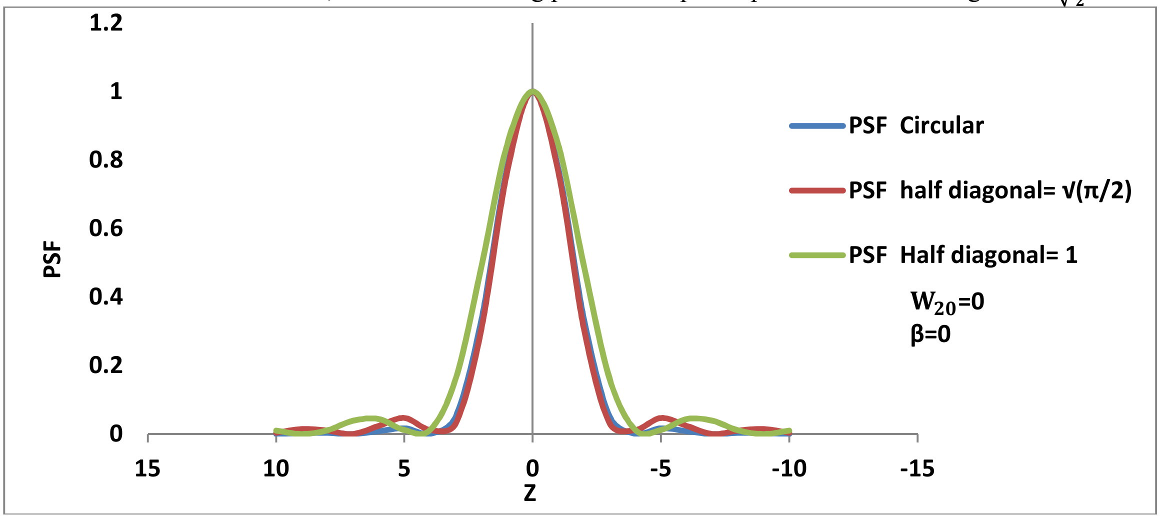

Fig 4. PSF profile of perfect system for different apertures

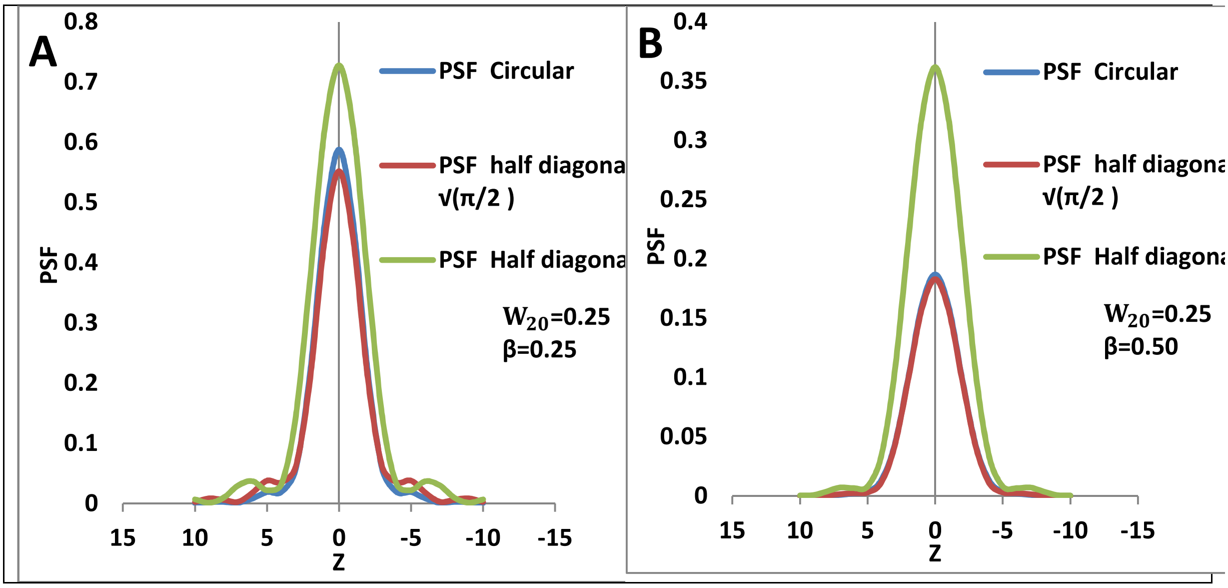

Fig 5. PSF profile of defocus =0.25 for different apertures

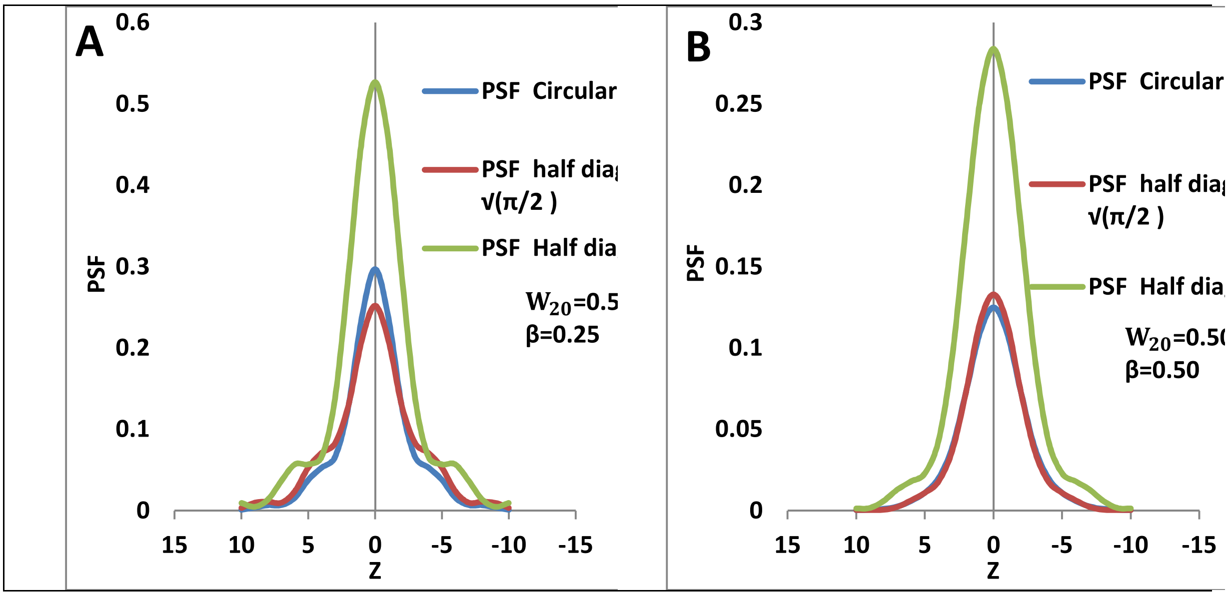

Fig 6. PSF profile of defocus =0.5 for different apertures

V. CONCLUSIONS

The present paper aimed at enhancing the resolution of an optical system with square aperture by using Hanning amplitude filters. It can be concluded that an additional improvement for resolution could be obtained by the use of complex pupil filters to perform apodization. The effect of Hanning apodization on the resolving power for square aperture with half diagonal = $\sqrt{\frac{\pi}{2}}$ is the more efficient, while the maxima PSF intensity peak is higher for square aperture with half diagonal =1.

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