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Double Diffusive Convection and Internal Heat Generation with Soret and Dufour Effects over an Accelerating Surface with Variable Viscosity and Permeability

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Abstract
The study deals with the coupled highly non-linear partial differential equations for the effects of internal heat generation (IHG), combined effects of Soret and Dufour with variable fluid properties like variable porosity, permeability and viscosity on thermal and diffusion mixed (free and forced) convection flow fluid through porous media over moving surface. The numerical computation has been carried out for other additional terms like internal heat generation (IHG), viscous dissipation and ohmic effect. We have drawn the numerical results and presented graphically for the flow characteristics like velocity, temperature and concentration for the effect of IHG, Soret and Dufour effects on mixed (free and forced) convective flow with variable fluid properties for various non-dimensional parameters namely prandtl number, Eckert number, porous parameter, viscosity parameter, Soret number and Dufour number of the problem using the shooting technique. In the absence of the additional terms and for constant viscosity our computational results are compared with the earlier works and found to be good agreement.
Keywords: Double diffusive, internal heat generation (IHG), Soret and Dufour effects, variable permeability, accelerating surface.

Nomenclature:

P_r :Prandtl number $\left(P_r = \dfrac{\rho \upsilon_\infty C_p}{k} \right)$

E :Eckert number $\left(E = \dfrac{b^2 x^2}{C_p (T_w - T_\infty)} \right)$

S_c :Schmidt number $\left(S_c = \dfrac{\upsilon_\infty}{D_m} \right)$

D_f :Dufour number $\left(D_f = \dfrac{D_m K_T (C_w - C_\infty)}{C_s C_p \upsilon_\infty (T_w - T_\infty)} \right)$

S_r :Soret number $\left(S_r = \dfrac{D_m K_T (T_w - T_\infty)}{\upsilon_\infty T_m (C_w - C_\infty)} \right)$

G_s :Temperature buoyancy parameter $\left(G_s = \dfrac{Gr_t}{\mathrm{Re}^2} \right)$

G_c :Mass buoyancy parameter $\left(G_c = \dfrac{Gr_m}{\mathrm{Re}^2} \right)$

Gr_t :Local temperature Grashof number $\left(Gr_t = \dfrac{g\beta (T_w - T_\infty) x^3}{\upsilon_\infty^{\ 2}} \right)$

Gr_m :Local mass Grashof number $\left(Gr_m = \dfrac{g\beta^* (C_w - C_\infty) x^3}{\upsilon_\infty^{\ 2}} \right)$

Re :Reynolds number $\left(\mathrm{Re} = \dfrac{u_0 x}{\upsilon} \right)$

σ :Local porous parameter $\left(\sigma = \dfrac{\upsilon}{b k_0} \right)$

α^* :Ratios of viscosities $\left(\alpha^* = \dfrac{\underline{\mu}}{\mu} \right)$

θ_r :Viscosity parameter in PSTC case $\left(\theta_r = \dfrac{T_r - T_\infty}{T_w - T_\infty} = -\dfrac{1}{\gamma(T_w - T_\infty)}\right)$

g_r :Viscosity parameter in PWHCF case $\left(g_r = \dfrac{T_r - T_\infty}{T_w - T_\infty} = -\dfrac{1}{\gamma(T_w - T_\infty)}\right)$

ε_0 :Porosity at the edge of the boundary layer

k_0 :Permeability at the edge of the boundary layer

η :Dimensionless similarity variable

d :Constant having value 3.0

d^* :Constant having value 1.5

$\theta(\eta)$:Dimensionless temperature parameter in PSTC case

$g(\eta)$:Dimensionless temperature parameter in PWHCF case

f :Non dimensionless stream function

r :Temperature parameter

s :Heat flux parameter

$H(\eta)$:Dimensionless concentration parameter in PSTC case

$h(\eta)$:Dimensionless concentration parameter in PWHCF case

g :Gravitational acceleration

1. Introduction

In the process of evaluating the characteristic behavior of the movement of the flow, temperature variations of the fluid and changes in the specious or concentration of the fluid flow depends on various forces(external or internal) and boundary conditions which influenced many researchers for the importance of understanding or investgating the natural phenomena occurs in nature and industrial or practical applications. In the past and recent, the study of fluid flow plays an important role for the development or analyze or understanding a physical system. Recently some of the interesting applications arises in science, engineering and industry like paper production, coating and polymer processing, drying of solids, drying process (food and wood products), petroleum industry, soil physics, food processing, geothermal reservoirs, filtration of solids from liquids, hydro-geology, chemical engineering, air conditioning, casting and welding, cloth industry, thermal insulation, enhanced oil recovery and many other.

In the above mentioned applications several authors come across with the constant fluid properties i.e. without changin viscosity, permeability, porosity and diffusivity etc. i.e. which are not varying with respect to the physical quantities like position or time or temperature or concentration. Many authors have invesigated mathematically the above practical application problems by considering constant fluid properties over non-accelerating surface and which may fulfill the requirements of the experimental or industrial results. Later, the researchers developed these physical models by incorporating the accelerating surface with constant fluid properties. Various aspects of the above practical application problems for a gradual increase in understanding the problem of flow induced by a solid surface and continuous flat surface have been investigated by some of the authors name few [1-2]. The effect of the temperature heat sources or sinks in stagnant flow studied by Sparrow and Cess [3]. Several earlier research works has been carried out on non-moving surface with constant surface velocity and temperature but for many practical and industrial application problems the surface undergoes stretching and heating or cooling which is due to the fact that the changes of the surface velocity and temperature, which in investigated by Tsou[4]. Later the study has been carried out by the authors [5-7] on the stretching surface with constant surface temperature.

Several authors deals with the study of convective fluid flow in a or through flow permeable medium called porous medium taken a majore role for the scholors/researchers due to wide diverse range of engineering and industrial applications as mentioned below. We can observe the effect of porous media in the industrial applications like heat exchangers in high heat flux applications such as air conditioning, insulation of the heated body, electronic equipment, sensible heat storage beds, thermal energy storage, drying process (wood and food products) and filtration process etc. During the last few decades, several researchers [8-10] studied free convection, heat and mass transfer in a porous medium. Mass transfer effects on flow past an moving vertical plate has been already well studied [11-12]. Furthermore, the problem of heat and mass transfer of non-Newtonian fluids through porous media has become a very interesting subject for many research projects [13-15].

Eckert and Drake [16] explained the dufour and soret effect which helps in understanding the study between the mixtures of gases with modeling weight can be ignored in electronic technology field. Later Kafoussias and Williams [17] developed a similar effect on mixed (free forced) convective and mass transfer with variable fluid property like viscosity, which is a function of temperature. Further the same effect of dufour and soret over a vertical surface embedded in a porous media is investigated by Anghel et al [18]. Chen and Char [19] analyzed the effect of surface temperature power law and surface heat flux power law in the heat transfer characteristics of a continuous linear stretching surface. The non-Darcy free and forced convection along a vertical wall in a saturated porous medium has carried out by Lai and Kulacki [20]. Alabraba et al [21] explained the binary mixture of the laminar convection and hydromagnetic fluid flow along with radiative heat transfer.

Further Seddeek [22] invesigated his research work on hydro magnetic fluid flow and heat transfer of radiation and oscillatory boundary layer through porous media along with variable viscosity. The mass transfer and free convection flow of a viscous incompressible steady and electrically conducting fluid flow bounded by vertical infinite surface with constant suction velocity and constant heat flux through a highly porous medium was analyzed by Acharya et al [23]. Seddeek [24] have studied the effects of Soret and Dufour on mass transfer and mixed convective flow in the presence of suction and blowing over an accelerating surface along with a variable viscosity. Chandrasekhar and Namboodiri [25] have explained the behavior of velocity distribution and heat transfer of the fluid flow with variable permeability of the porous medium. Later Mohammadein and El-Shaer [26] have studied the importance of inertia effects of variable fluid properties through sparsely packed porous medium. Recently Dinesh et al [27-29] carried out their research work on mixed convection with fluid properties in terms of permeability, porosity and diffusivity over non-accelerating vertical plate for a more practical situation problem. Seddeek [30] carried out his research on free convection flow and the effects of thermal radiation and buoyancy in the presence of MHD with heat generation and variable viscosity over moving permeable surface. Recently Nalinakshi et al [31-34] have studied on mixed convection and effects of internal heat generation (IHG) with variable fluid properties like porosity and permeability on past a vertical heated plate. Sandeep et al [35-36] have studied the effect of cross diffusion with non-uniform heat sink/source and combined effect of non-uniform heat source/sink and viscous dissipation on MHD non-Newtonian fluid flow with Cattaneo-Christov heat flux. Very recently Sandeep et al [37] have also studied numerically about the unsteady Casson fluid flow past a stretching surface under chemical reaction with thermal radiation and cross diffusion.

In all the above practical applications we come across the variable fluid property (viscosity, permeability, thermal diffusion, solutal diffusion) which varies due to the physical parameters or quantities like position or time or temperature or concentration. As per our knowledge the researchers/scholars mentioned above are mainly concentrated to understand the fluid flow behavior like velocity, concentration and temperature over a non-accelerating surface or with varying one of the fluid properties like (viscosity, permeability, thermal diffusion, solutal diffusion). Also the variation of viscosity of the fluid changes with temperature or permeability of the porous medium varies with position or time which is observed in practical applications which are mentioned above and these changes affects the characteristics of the fluid flow. Hence the main novelty in our paper is to study mathematically the physical behavior of varying fluid properties (viscosity, porosity and permeability) and internal heat generation (IHG) along with accelerating boundary surface. To evaluate the variations will affect the flow behavior compared to the earlier works for constant viscosity or in the absence of variation of fluid properties and the present work satisfy exactly with the earlier works and found an exllent agreement of the numerical results.

2. Mathematical formulation

Consider a steady (independent of time), two-dimensional $(x \ \& \ y)$, combined effects of species and temperature for a viscous, incompressible and laminar fluid flow in the presence of uniform porous medium over an accelerating flat plate with internal heat generation, we have assumed that (a) the Bousinesque approximation is taken into account, (b) permeability and porosity are expressed in terms y-coordinate, (c) ohmic effect, combined effects of Soret and Dufour are considered, (d) all the molecular transport properties are constant except the variable viscosity, porosity and permeability, (e) The viscosity is inversely proportional to function of temperature and which is in the form of $\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma(T-T_\infty)\right]$ or $\frac{1}{\mu}=a(T-T_r)$ (see Lai and Kulacki [20]) where $a=\frac{\gamma}{\mu_\infty}$, $T_r=T_\infty-\frac{1}{\gamma}$, where $'a'$ *and* T_r are constants, In general $a>0$ for liquids and $a<0$ for gases.

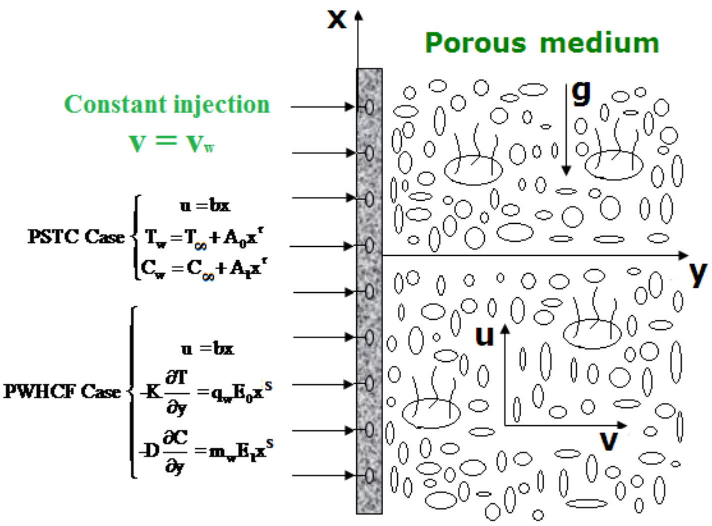

Fig. 1: Physical Configuration

The physical configuration of the fluid flow is depicted in the Fig. 1, where the velocity component u is considered along with x-direction and v is in the direction of y, the accelerating surface moves along the x-axis and both x-axis and y-axis are mutually perpendicular to each other. Under all the above assumptions the governing equations of motion are expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+g\beta\left(T-T_\infty\right)-g\beta^*\left(C-C_\infty\right)-\frac{\varepsilon(y)\bar{\mu}u}{\rho k(y)}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\bar{\mu}\varepsilon^2(y)u^2}{\rho C_p k(y)}+\frac{q'''}{\rho C_p}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where θ_r is a viscosity parameter, which is defined by (Seddeek [22]), following Chandrasekhara and Namboodiri [25], the variable permeability $k(\eta)$ and the variable porosity $\varepsilon(\eta)$ are given by

$$\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}=-\frac{1}{\gamma(T_w-T_\infty)} \tag{5}$$

$$k(\eta)=k_0(1+de^{-\eta}), \tag{6}$$

$$\varepsilon(\eta)=\varepsilon_0(1+d^*e^{-\eta}), \tag{7}$$

where ε_0 is the porosity and k_0 is the permeability of the porous medium at the edge of the boundary layer, $d=3.0$ and $d^*=1.5$ are considered as constants for variable permeability and porosity respectively and all other physical quantities have their standard meanings. The governing equations of the physical model are studeid under two different types of boundary conditions, which are the boundary at the accelerating surface and at the far away from the oscillatory surface in the porous medium. These two different types of boundary conditions have been discussed in the following cases: (i) Prescribed Surface Temperature and Concentration (PSTC) and (ii) Prescribed Wall Heat and Concentration Flux (PWHCF).

(i) Prescribed Surface Temperature and Concentration (PSTC)

In this case a polynomial of degree r has been maintained at the boundary on temperature of the vertical plate i.e. the accelerating temperature of the vertical plate T_w is considered in the form of $T_w=T_\infty+A_0x^r$, similarly the concentration of the accelerating wall C_w is assumed in the form of $C_w=C_\infty+A_1x^r$. The following are the expressions for boundary conditions on velocity, temperature and concentration fields:

$$u = bx,\ v = v_w,\ T_w = T_\infty + A_0 x^r,\ C_w = C_\infty + A_1 x^r \quad at\ y = 0 \tag{8}$$

$$u = 0,\ T = T_\infty,\ C = C_\infty \quad as\ y \to \infty \tag{9}$$

where r is the temperature parameter and if $r = 0$ then the thermal boundary conditions become isothermal, A_0, A_1 are arbitrary constants and b is the constant stretching rate, Since the Eqs. (1)-(4) are coupled highly non-linear partial differential equations, so it is difficult to get the solution of these equations using analytical methods or techniques. So we are adopting the numerical technique to solve them. Acharya et al [23] introduced the following similarity variable η and dimensionless parameters f, θ, H to solve these equations numerically.

$$\psi = (\upsilon b)^{\frac{1}{2}}\, x\, f(\eta), \quad \eta = \left(\frac{b}{\upsilon}\right)^{\frac{1}{2}} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad H(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{10}$$

the velocity equation (1) will satisfy automatically when the stream function $\psi(x, y)$ is defined in such way that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ and the corresponding velocity components are given by

$$u = bxf'(\eta), \quad v = -(b\upsilon)^{\frac{1}{2}} f(\eta). \tag{11}$$

Substituting Eqs. (10) & (11) in Eqs. (2)-(4) using Eqs. (6) & (7), we get the following transformed coupled non-linear ordinary differential equations.

$$f''' - \left(\frac{1}{\theta - \theta_r}\right)\theta' f'' - \left(\frac{\theta}{\theta_r} - 1\right)\left(ff'' - f'^2\right) + \alpha^* \varepsilon_0 \sigma \left(\frac{1 + d^* e^{-\eta}}{1 + de^{-\eta}}\right)\left(\frac{\theta}{\theta_r} - 1\right) f' = \left(\frac{\theta}{\theta_r} - 1\right)\left(G_s \theta - G_c H\right), \tag{12}$$

$$\theta'' - P_r f\, \theta' - r\, P_r f' \theta = -P_r D_f H'' - EP_r f''^2 - E\, \sigma\, P_r \varepsilon_0^2 \left(\frac{(1 + d^* e^{-\eta})^2}{1 + de^{-\eta}}\right) f'^2 - \frac{P_r}{2} e^{-\eta} f', \tag{13}$$

$$H'' + S_c f\, H' - r\, S_c f' H = -S_c S_r \theta'', \tag{14}$$

where all the dimensionless parameters are defined in nomenclature section and the corresponding transformed boundary conditions in non-dimensional form are:

$$f(0) = -\frac{v_w}{(bv)^{\frac{1}{2}}} = -m, \quad f'(0) = 1, \quad \theta = 1, \quad H = 1 \quad at\ y = 0, \tag{15}$$

$$f' = 0, \quad \theta = 0, \quad H = 0 \quad as\ y \to \infty. \tag{16}$$

(ii) Prescribed Wall Heat and Concentration Flux (PWHCF)

In this case a dirichelet conditions on temperature is maintained at the wall which is assumed in the form $-k\frac{\partial T}{\partial y} = q_w E_0 x^s$ and $-D\frac{\partial C}{\partial y} = m_w E_1 x^s$. The following are the expressions for boundary conditions on velocity, temperature and concentration fields:

$$u = bx,\ v = v_w,\ -k\frac{\partial T}{\partial y} = q_w E_0 x^s,\ -D\frac{\partial C}{\partial y} = m_w E_1 x^s \quad at\ y = 0 \tag{17}$$

$$u = 0,\ T = T_\infty,\ C = C_\infty \quad as\ y \to \infty \tag{18}$$

where s is the heat flux parameter and the accelerating sheet is subject to uniform heat flux when $s = 0$ and E_0, E_1 are choosing constants. Since the Eqs. (1)-(4) are coupled highly non-linear partial differential equations, so it is difficult to get the solution of these equations using analytical methods or techniques. So we are adopting the numerical technique to solve them. Acharya et al [23] introduced the following similarity variable η and dimensionless parameters f, g, h to solve these equations numerically.

$$\psi = (\upsilon b)^{\frac{1}{2}} x f(\eta), \quad \eta = \left(\frac{b}{\upsilon}\right)^{\frac{1}{2}} y, \quad T - T_\infty = \frac{E_0 x^s}{k}\left(\frac{\upsilon}{a}\right)^{\frac{1}{2}} g(\eta), \quad C - C_\infty = \frac{E_1 x^s}{D}\left(\frac{\upsilon}{a}\right)^{\frac{1}{2}} h(\eta), \tag{19}$$

the velocity equation (1) will satisfy automatically when the stream function $\psi(x, y)$ is defined in such way that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ and the corresponding velocity components are given by Eq. (11). Substituting Eqs. (11) & (19) in Eqs. (2)-(4) and using Eqs. (6) & (7), we get the following transformed coupled non-linear ordinary differential equations

$$f''' - \left(\frac{1}{g - g_r}\right) g' f'' - \left(\frac{g}{g_r} - 1\right)\left(ff'' - f'^2\right) + \alpha^* \varepsilon_0 \sigma \left(\frac{1 + d^* e^{-\eta}}{1 + de^{-\eta}}\right)\left(\frac{g}{g_r} - 1\right) f' = \left(\frac{g}{g_r} - 1\right)\left(G_s g - G_c h\right), \tag{20}$$

$$g'' - P_r f g' - s P_r f' g = -P_r D_f h'' - EP_r f''^2 - E\,\sigma P_r \varepsilon_0^2 \left(\frac{(1 + d^* e^{-\eta})^2}{1 + de^{-\eta}}\right) f'^2 - \frac{P_r}{2} e^{-\eta} f', \tag{21}$$

$$h'' + S_c f h' - s S_c f' h = -S_c S_r g'', \tag{22}$$

where all the dimensionless parameters are defined in nomenclature section. The corresponding transformed boundary conditions in non-dimensional form are:

$$f(0) = -\frac{v_w}{(bv)^{\frac{1}{2}}} = -m, \quad f'(0) = 1, \quad g = -1, \quad h = -1 \quad at \;\; y = 0, \tag{23}$$

$$f' = 0, \quad g = 0, \quad h = 0 \quad as \;\; y \to \infty. \tag{24}$$

3. Numerical solution

For the PSTC case the Eqs. (12) - (14) along with the boundary conditions (15) & (16) are the functions of $f(\eta), \theta(\eta)\ \&\ H(\eta)$ and similarly for the PWHCF case the Eqs. (20) - (22) along with the boundary conditions (23) & (24) are the functions of $f(\eta),\ g(\eta)\ \&\ h(\eta)$ and which are to be determined. Since the ODE's are highly non-linear, coupled and involved variable coefficient like transcendental terms as well as the corresponding boundary conditions so that the known methods pertaining to analytical nature of the solution cannot be applied. To overcome the difficulty raised in the analytical method here we adopt the numerical method to find the solution by transforming the higher order coupled non-linear ODE's into system of first order ODE's and these first order ODE's can be solved by Runge-Kutta-Fehlberg explicit method and second order accuracy of Newton-Raphson method. The combination of these two numerical techniques to estimate an approximate solution of the problem is referred as shooting technique. By this we stop the iterative procedure for evaluating the numerical results for the above system of equations for the given boundary conditions. We developed these computations using the software like Mat Lab in which the computational work is carried out for an accuracy of 10^{-6} i.e. the difference between the two iterative successive values is less than the prescribed accuracy of 10^{-6}.

4. Results and discussion

The importance of this paper is to understand the effect of internal heat generation (IHG) along with the variation of viscosity parameter, variation of permeability and porosity is seen in the Darcy term in momentum equation and ohmic effect in energy equation respectively. Using Mat Lab software we have drawn the numerical results for the system of ODE's under two different conditions, which are (i) Prescribed Surface Temperature and Concentration (PSTC) (ii) Prescribed Wall Heat and Concentration Flux (PWHCF). The non-linear coupled ODE's with variable co-efficients are araising from the momentum, energy and specious equations and which involves important non-dimensional parameters like prandtl number, Eckert number, porous parameter, viscosity parameter, Soret and Dufour numbers and all other parameters involved in the physical problem are assumed to be fixed value throughout the computation. The numerical results are carried out to discuss the variation of velocity or momentum of the fluid flow, temperature variation and solutal changes for the above non-dimensional parameters and are depicted from the following Figures [2-19].

Effect of Prandtl number (P_r): Figs. 2-4 gives the effects of prandtl number over the velocity, temperature and concentration are observed, the variation of the prandtl number is due to changes of viscosity or thermal diffussivity i.e. the increase in the prandtl number of the fluid is due to decrease in the thermal diffusivity of the fluid or enhancement of the viscosity of the fluid, this will result a gradual decrease in the flow of velocity of the fluid. The percentage of decrement of the velocity is less in the case of PWHCF compared to that of PSTC case because of boundary conditions maintained at the accelerating surface, which depicted in the Fig. 2. For the same enhancement of the prandtl number we can observe that the gradual decrease of temperature in the Fig. 3, but the rate decreases of temperature of the fluid flow is more in the case of PSTC compare to that of PWHCF case because of adiabation nature of temperature is maintained at the accelerating wall. An opposite behavior is seen in the Fig.4 with the case of concentration profile for different values of prandtl number, the acceleration of the concentration will gradually increase due to less movement of fluid flow but the effect of prandtl number on concentration for both cases is very less compare to that of velocity and temperature. Our computational results have good agreement for the variation of prandtl number with the previous works.

Effect of Eckert number (E): Figs. 5-7 show the variation of Eckert number over the velocity, temperature and concentration profiles, it plays very important role in understanding the ohmic effect. With an enhancement of Eckert number will increase the non-dimensional velocity and temperature profiles in both the cases of PSTC and PWHCF, which is due to the increase of kinetic energy of the fluid flow in the physical model, which is dipicted in the Fig. 5 and Fig. 6 respectively. An opposite behavior can observe in the case of dimensionlee concentration profile in the Fig. 7, i.e. with an enhancement of Eckert number will reduce the concentration, but it is observe that the effect of Eckert number are more in case of PWHCF and compared with that of PSTC for velocity, temperature and concentration profiles.

Effect of Porous parameter (σ): Figs. 8-10 indicates the effects of porous parameter σ over the velocity, temperature and concentration respectively. Due to enhancement of the viscosity of the fluid or decrease in the stretching rate of the moving surface or decrease in the permeability at the edge of the boundary layer, the porous parameter σ will increase and this will result a gradual decrement of the velocity of the fluid flow, which is seen in the Fig. 8. An opposite behavior is seen temperature profile for different values of the porous parameter σ . The dimensionless temperature increased both in the case of PSTC and PWHCF because the decrement of the permeability of porous medium at the edge of the boundary layer or increment of fluid viscosity, which is depicted in Fig. 9. For the enhancement of viscosity will reduce the movement of fluid hence the acceleration of the concentration will gradually increase i.e. an enhancement of the porous parameter σ there is a gradual increase of concentration is observed in the Fig. 10. If $\sigma = 0$ i.e. in the absence of porous parameter σ our numerical results have good agreement with earlier works.

Effect of Viscosity parameter (θ_r): Figs. 11-13 illustrate the variations of viscosity parameter θ_r in terms of velocity, temperature and concentration respectively. With the changes of viscosity parameter θ_r there is a drastic change in velocity, which diminishes exponentially and the percentage of decrement of the velocity is more in the case of PSTC compare to that of PWHCF case. This is due to the fact that the enhancement of the viscosity diminishes the flow rate or in the other words the heat transfer between wall to far away from accelerating plate is decrease and which is depicted in the Fig. 11. There is an opposite behavior of heat transfer can be observed with the variation of viscosity parameter θ_r on the non-dimensional temperature as shown in Fig. 12. Here heat transfer is more in the case of PWHCF compare to that of PSTC case. Similarly a slight change is observed in the Fig. 13 in the case of specious or concentration profile with respect to the viscosity parameter θ_r . Since the concentration will not affects directly for the variations of viscosity parameter but indirectly the concentration will affect through temperature and momentum equations.

Effects of Soret number (S_r) and Dufour number (D_f): Figs. 14-16 indicates the variations of the Soret number over the velocity, temperature and concentration profiles respectively. With increase of the Soret number the dimensionless velocity, temperature and concentration will also increase, which is due to the effect of temperature gradient and the effect of mass diffusion. Similarly we can also plot the behavior of velocity, temperature and concentration profiles for various values of Dufour number in the Figs. 17-19 respectively. The increase of Dufour number leads to raise the non-dimensional velocity and as well as temperature of the fluid, which can be depict in the Fig. 17 and Fig. 18 respectively where as an opposite behavior can be gleaned from the Fig. 19 in the case of concentration profile i.e. an increase of Dufour number helps in reducing the dimensionless concentration which is due to the fact that the contribution of the species or concentration gradients to the thermal energy flux in fluid flow.

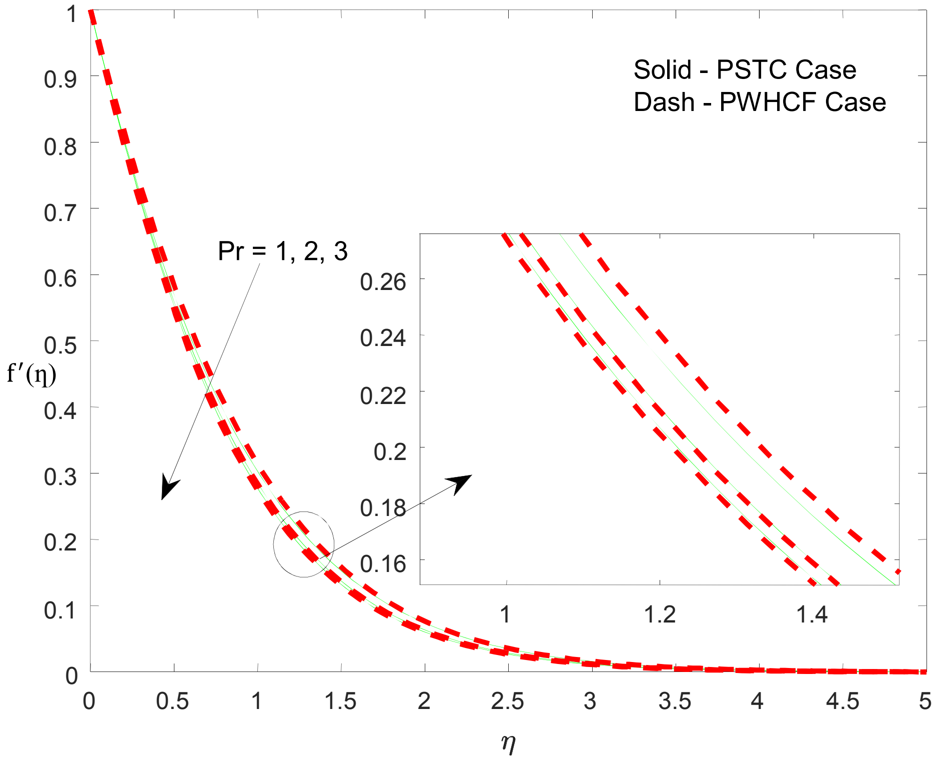

Fig. 2: Velocity profile for different values of Prandtl number.

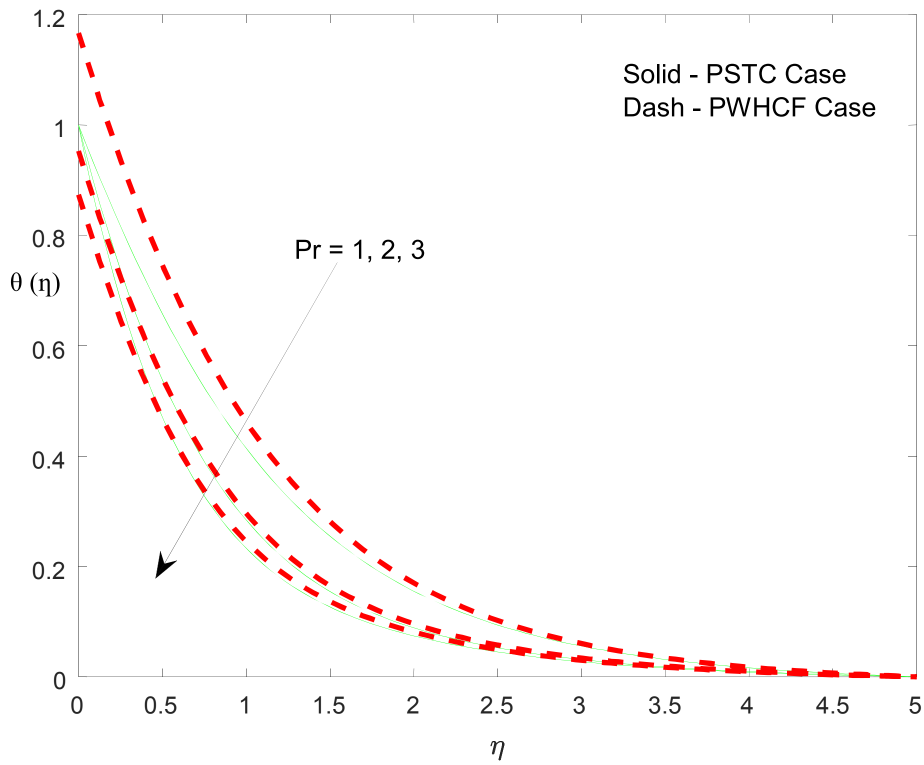

Fig. 3: Temperature profile for different values of Prandtl number.

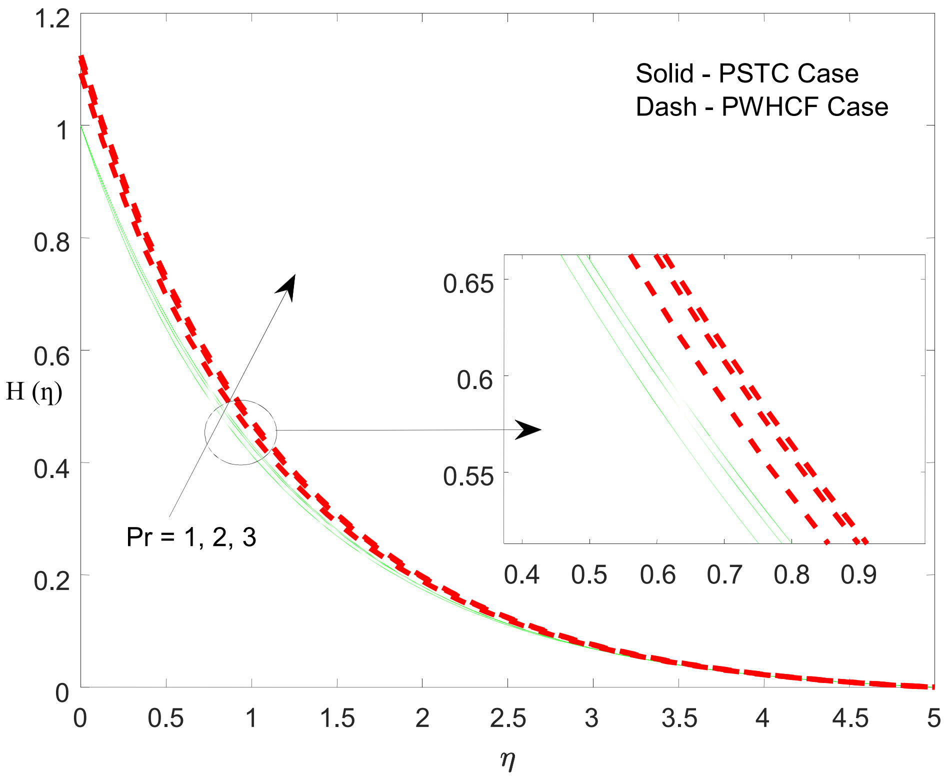

Fig. 4: Concentration profile for different values of Prandtl number.

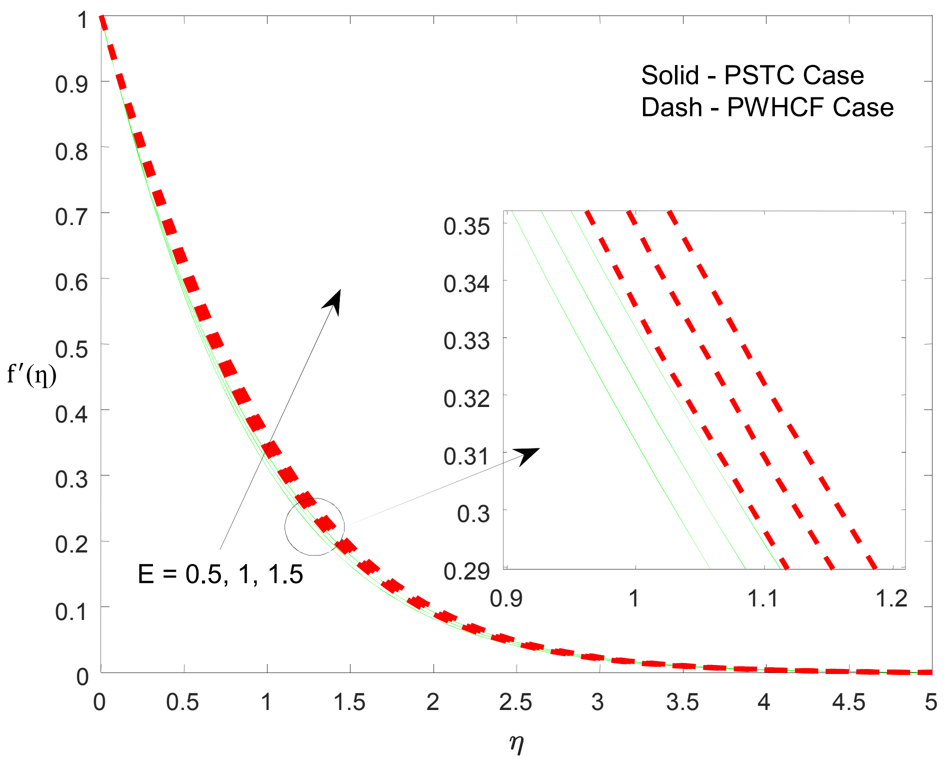

Fig. 5: Velocity profile for different values of Eckert number.

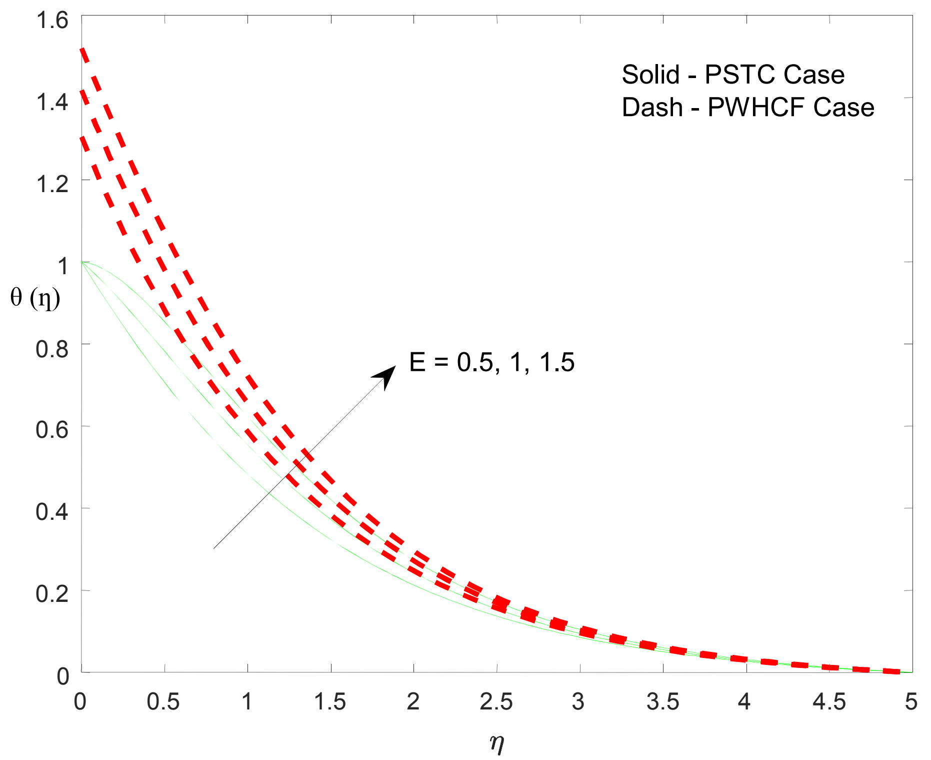

Fig. 6: Temperature profile for different values of Eckert number.

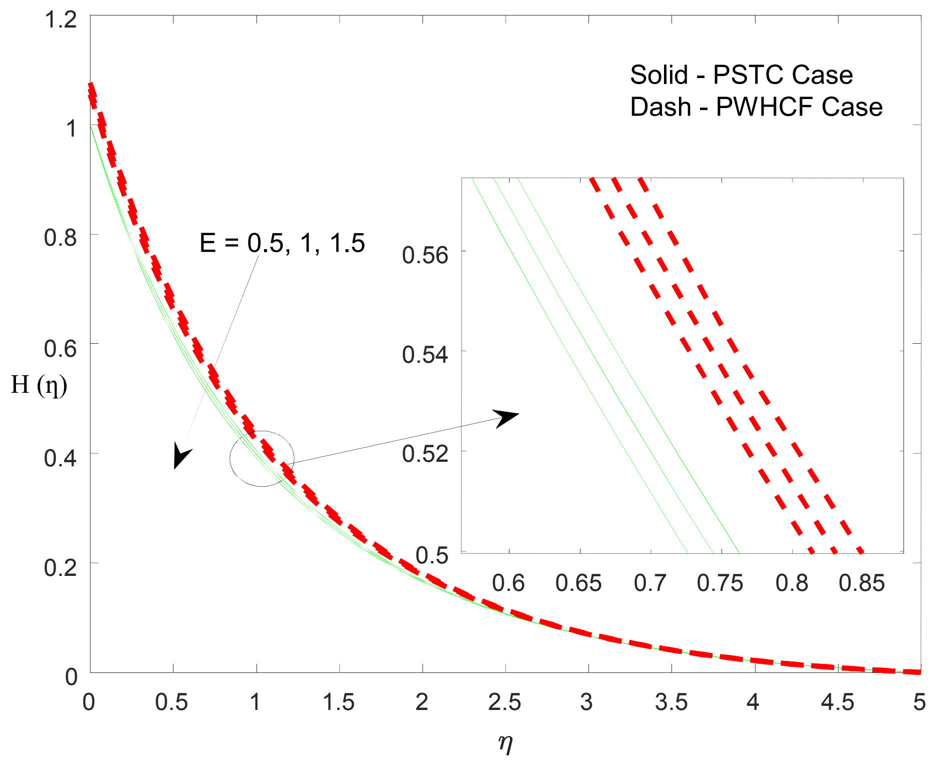

Fig. 7: Concentration profile for different values of Eckert number.

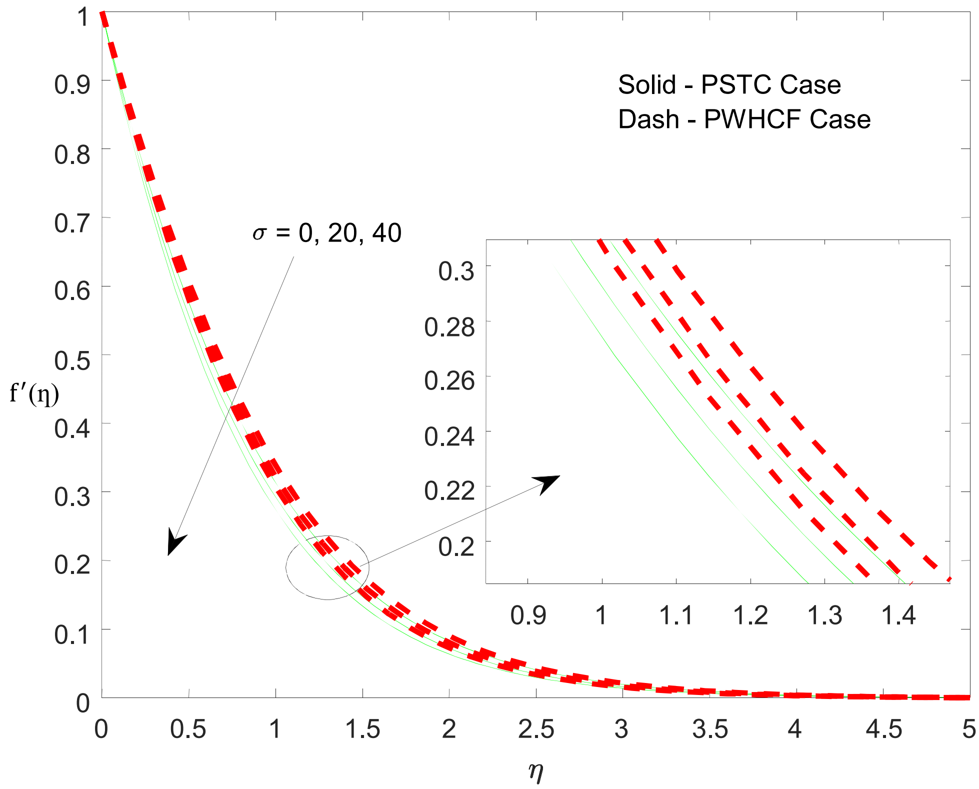

Fig. 8: Velocity profile for different values of Porous parameter.

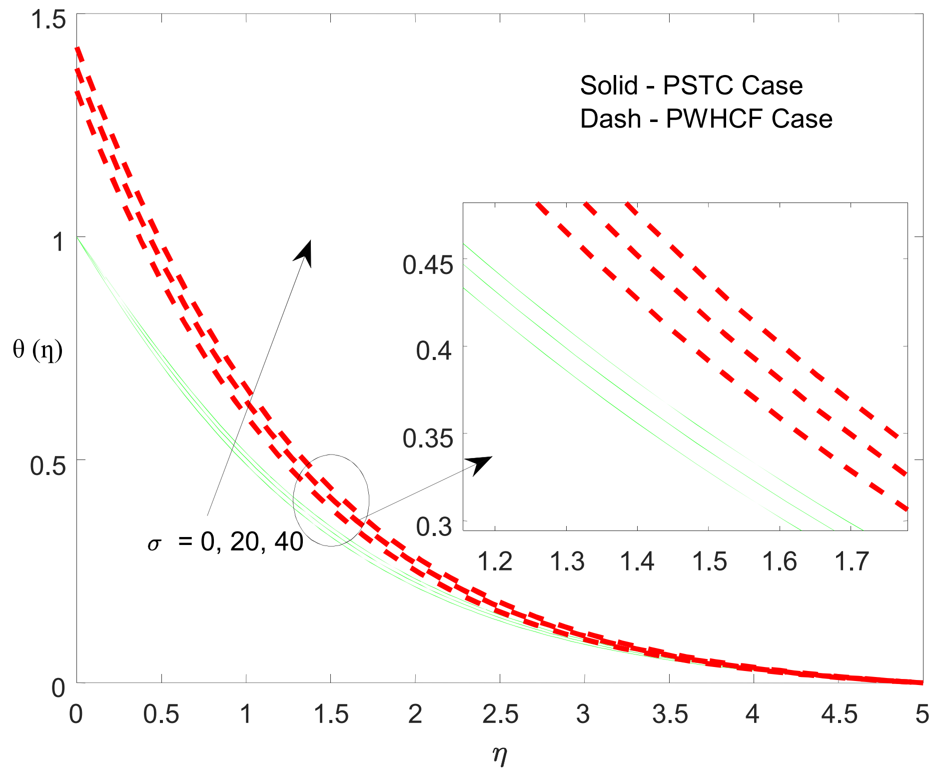

Fig. 9: Temperature profile for different values of Porous parameter.

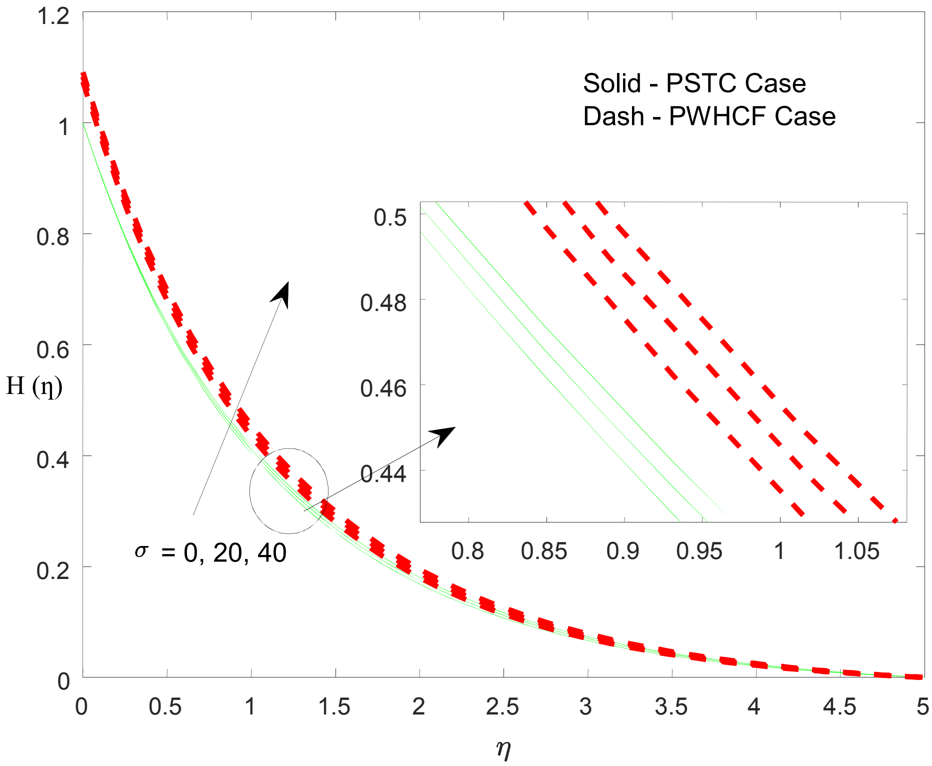

Fig. 10: Concentration profile for different values of Porous parameter.

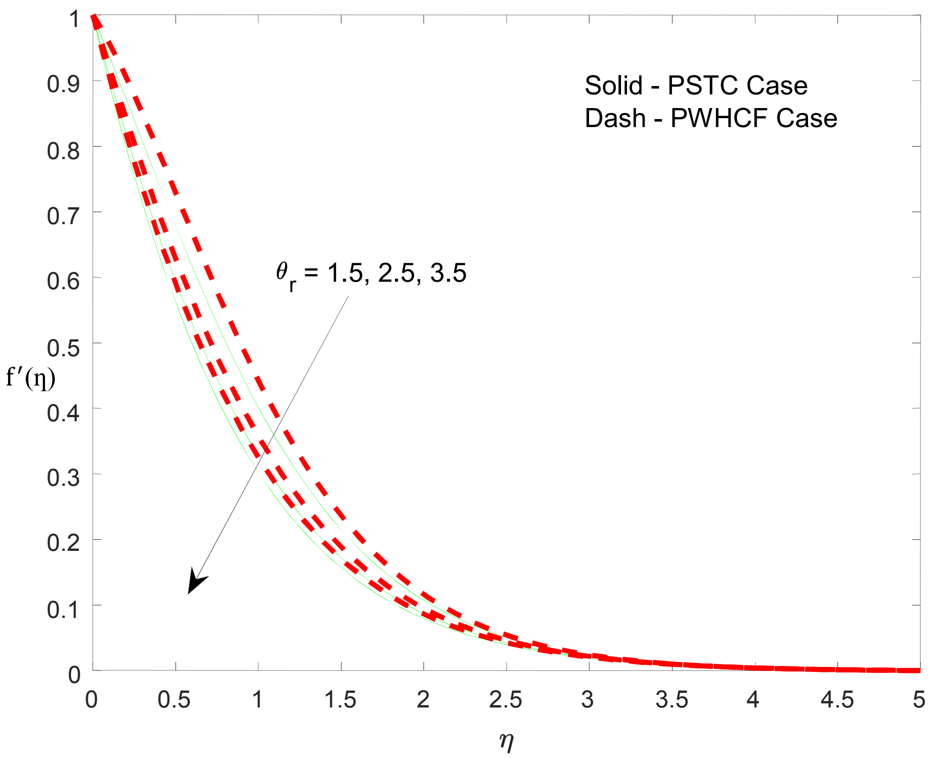

Fig. 11: Velocity profile for different values of viscosity parameter.

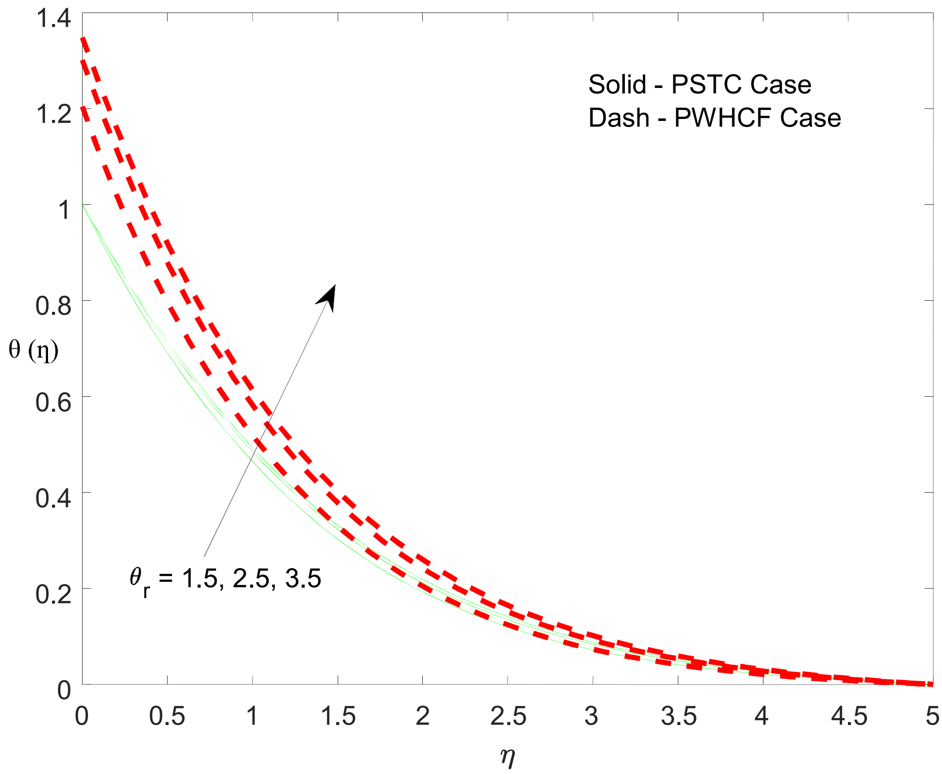

Fig. 12: Temperature profile for different values of viscosity parameter.

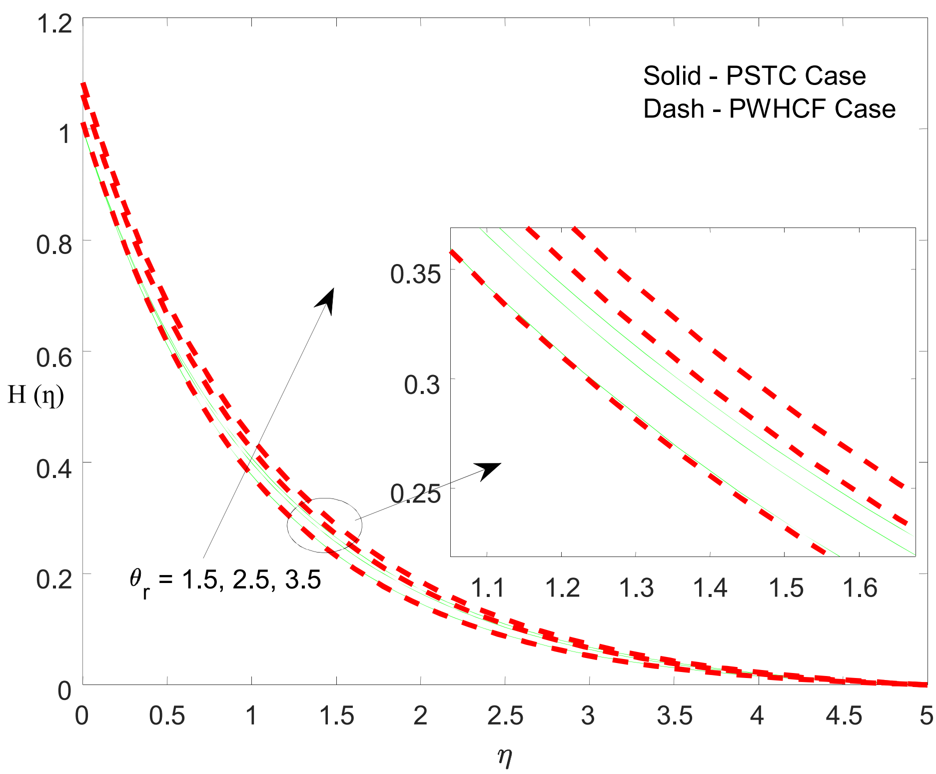

Fig. 13: Concentration profile for different values of viscosity parameter.

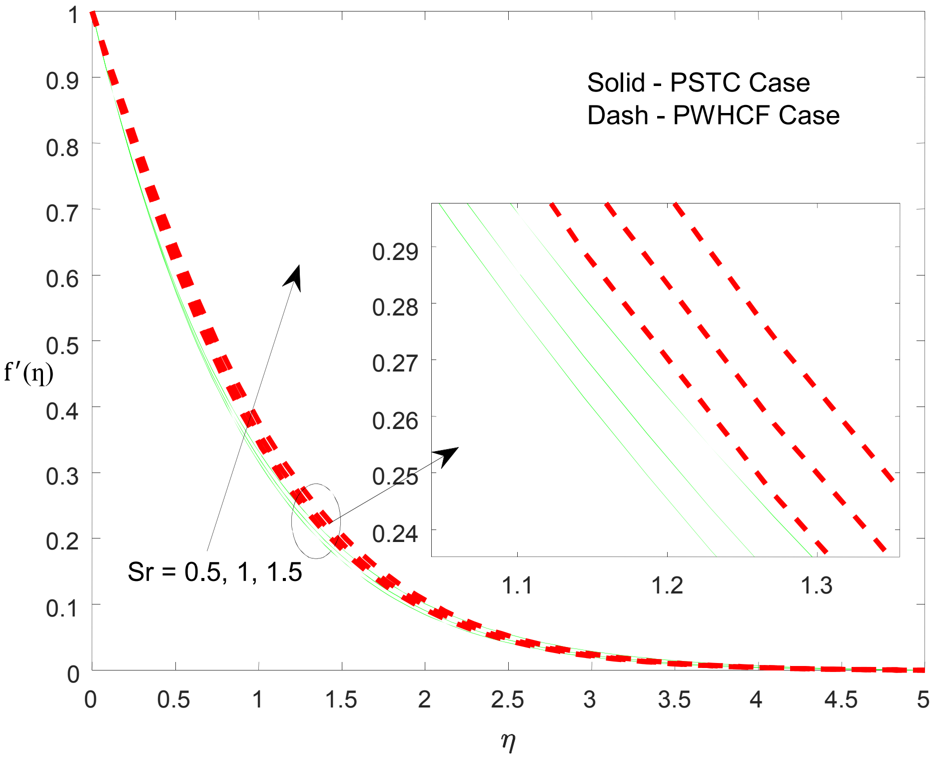

Fig. 14: Velocity profile for different values of Soret number.

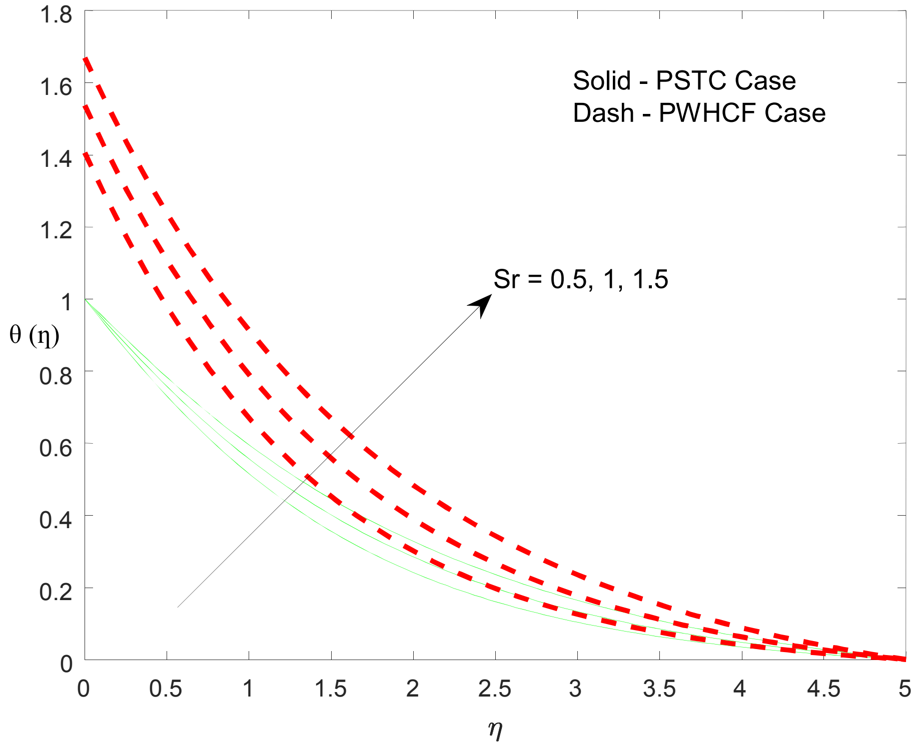

Fig. 15: Temperature profile for different values of Soret number.

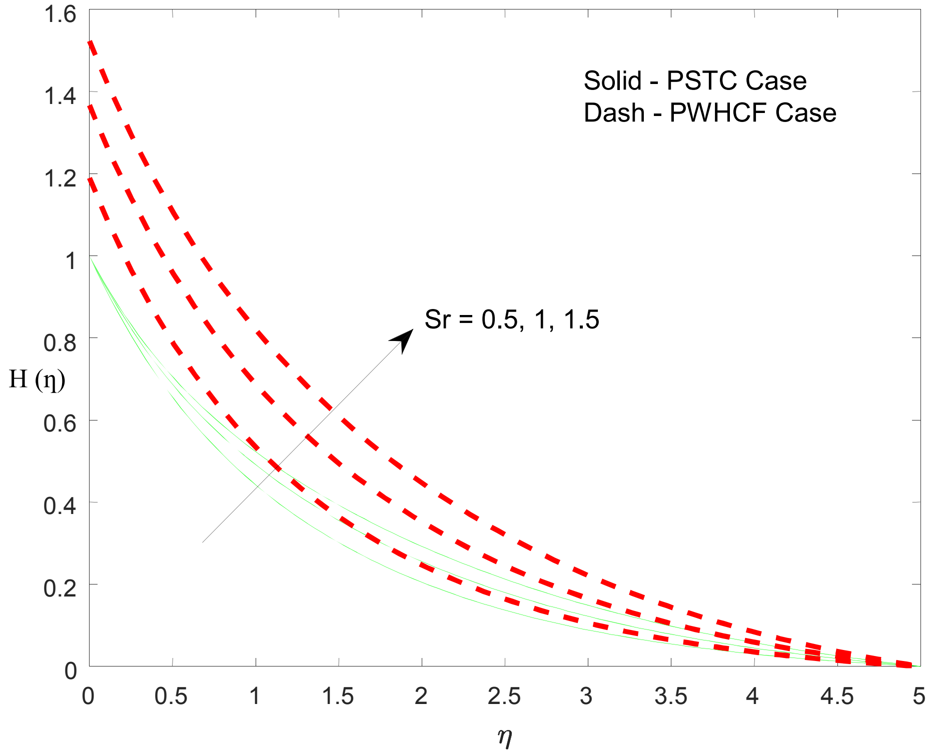

Fig. 16: Concentration profile for different values of Soret number.

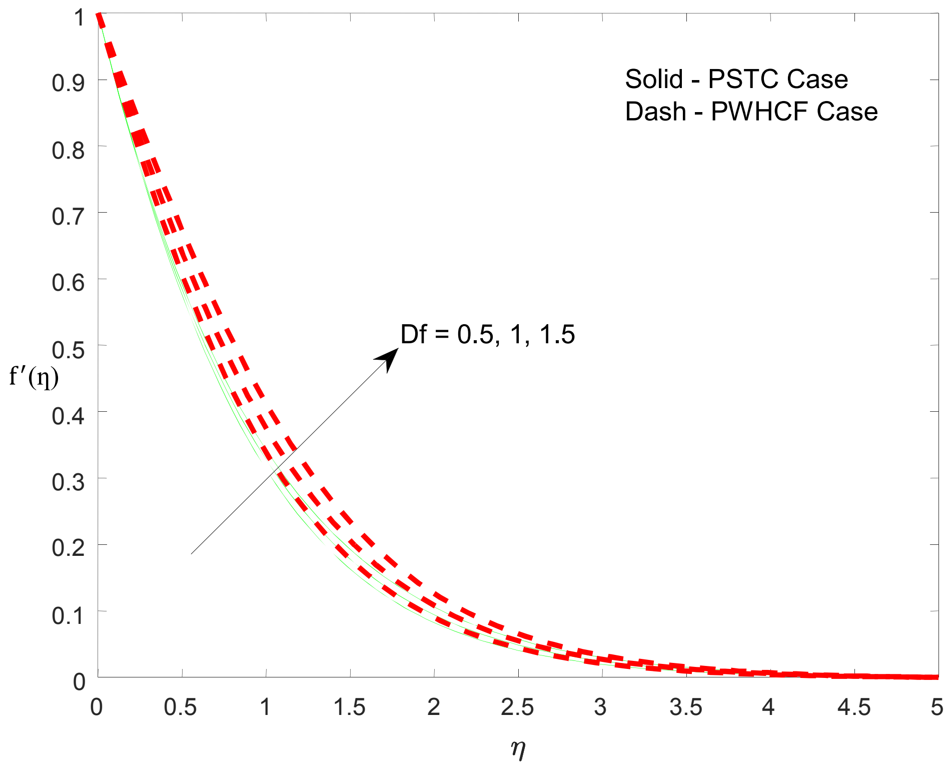

Fig. 17: Velocity profile for different values of Dufour number.

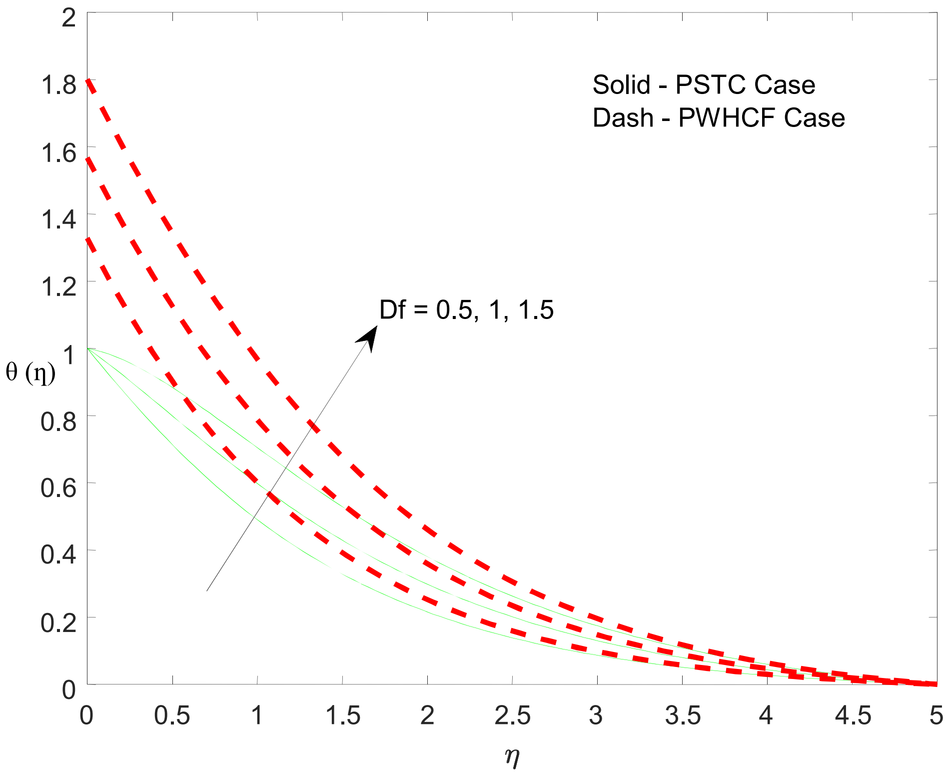

Fig. 18: Temperature profile for different values of Dufour number.

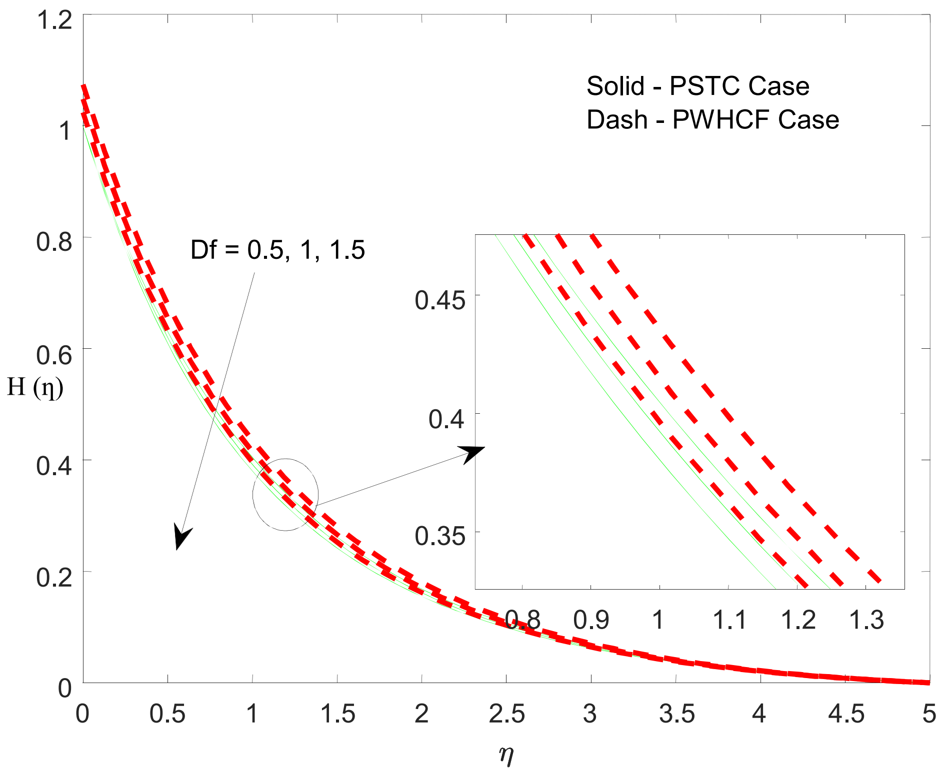

Fig. 19: Concentration profile for different values of Dufour number.

Table 1. The numerical results for $f^{''}(0),\ \theta^{'}(0)\ and\ \phi^{'}(0)$ for $E = 0.1, \varepsilon_0 = 0.4, Sc = 0.22, P_r = 0.71$ for variable porosity and permeability and effect of internal heat generation.

σ^*	$\frac{G_r}{\mathrm{Re}^2}$	$\frac{\alpha^*}{\sigma \mathrm{Re}}$	Nalinakshi et al[31]			Present value		
			$f^{''}(0)$	$\theta^{'}(0)$	$\phi^{'}(0)$	$f^{''}(0)$	$\theta^{'}(0)$	$\phi^{'}(0)$
2	0.0	0.0	0.363800	0.282750	0.280750	0.364000	0.284760	0.280340
		0.1	0.435870	0.325750	0.328590	0.435660	0.326753	0.328689
		0.5	0.687800	0.400580	0.400780	0.686700	0.410080	0.400790
	0.2	0.0	0.425800	0.400990	0.401205	0.426700	0.400678	0.403405
		0.1	0.538500	0.541456	0.545672	0.539500	0.542456	0.547672
		0.5	0.775600	0.561578	0.567652	0.777800	0.560478	0.565552
	2.0	0.0	1.346070	0.781453	0.794323	1.345570	0.780353	0.796723
		0.1	1.379100	0.881132	0.901256	1.378500	0.881130	0.901306
		0.5	2.004900	0.980023	0.988976	2.005300	0.980076	0.989000
4	0.2	0.1	0.552345	0.584573	0.571562	0.552465	0.584784	0.571775

Table 1 shows our computational results and which are compared with the earlier work of Nalinakshi et al [31] and found to be an exllent agreement for the accuracy of 10^{-6} for the constant viscosity of the fluid.

5. Conclusions

In this study we analyzed the effects of internal heat generation (IHG), combined effects of Soret and Dufour with variable fluid properties like variable porosity, permeability and viscosity on thermal and diffusion mixed (free and forced) convection flow fluid through porous media over moving surface. The numerical computation has been carried out for other additional terms like internal heat generation (IHG), viscous dissipation and ohmic effect. We have drawn the numerical results and presented graphically for the flow characteristics like velocity, temperature and concentration for the effect of IHG, Soret and Dufour effects on mixed (free and forced) convective flow with variable fluid properties for various non-dimensional parameters namely prandtl number, Eckert number, porous parameter, viscosity parameter, Soret number and Dufour number of the problem using the shooting technique. In the light of present study the main conclusions are drawn:

- The velocity and temperature profiles decreased for an increase in prandtl number P_r whereas the concentration profile is increased for an increase prandtl number P_r.
- The velocity profile decreased for an increase in viscosity parameter θ_r whereas temperature and concentration profiles increased for an increase in viscosity parameter θ_r.
- As an increase in porous parameter σ results decrease the velocity profile but increase the temperature and concentration profiles.
- Soret number helps to enhance the velocity, temperature and concentration.

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References

[1] B.C. Sakiadis, Boundary-layer behavior on continuous solid surface: Boundary-layer equations for two-dimensional and axisymmetric flow, AIChE J. 7 (1961) 26–28.
[2] B.C. Sakiadis, Boundary-layer behavior on continuous solid surface: The boundary layer on a continuous flat surface, AIChE J. 7 (1961) 221–225.
[3] E.M. Sparrow, R.D. Cess, Temperature dependent heat sources or sink in a stagnation point flow, Appl. Sci. Res. A 10 (1961) 185–197.
[4] F.K. Tsou, E.M. Sparrow, R.J. Goldstein, Flow and heat transfer in the boundary layer on a continuous moving

surface, Int. J. Heat Mass Transfer. 10 (1967) 219–235.
[5] L.J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys (ZAMP). 21 (1970) 645–647.
[6] J. Vleggaar, Laminar boundary-layer behavior on continuous accelerating surface, Chem. Engng Sci. 32 (1977) 1517–1525.
[7] P.S. Gupta, A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, Can. J. Chem. Engng. 55 (1977) 744–746.
[8] A. Bejan, K.R. Khair, Heat and mass transfer by natural convection in a porous medium, Int. J. Heat Mass Transfer. 28 (1985) 909–918.
[9] F.C. Lai, Coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium, Int. commun. Heat Mass Transfer. 18 (1991) 93–106.
[10] A.R. Bestman, Natural convection boundary layer with suction and mass transfer in a porous medium, Int. J. Energy Res. 14 (1990) 389–396.
[11] V.M. Soundalegekar, Effects of mass transfer on flow past a uniformly accelerated vertical plate, Lett. Heat Mass Transfer. 9 (1977) 499-501.
[12] A.K. Singh, J. Singh, Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux, Astrophys. Sp. Sci. 97 (1983) 57-61.
[13] H.T. Chen, C.K. Chen, Free convection of non-Newtonian fluids along a vertical plate embedded in a porous medium, ASME J. Heat Transf. 110 (1988) 257-260.
[14] R.Y. Jumah, A.S. Mujumdar, Free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in a saturated porous media, Int. Commun. Heat Mass Transf. 27 (2000) 485-494.
[15] S. Abel, P.H. Veena, Visco-elastic fluid flow and heat transfer in aporous medium over a stretching sheet, Int. J. Non-Linear Mech. 33 (1998) 531-540.
[16] E.R.G. Eckert, R.M.Drake, Analysis of heat and mass transfer, New York: McGraw-Hill, 1972.
[17] N.G. Kafoussias, E.W. Williams, Thermal-diffusion and diffusion-thermo effect on mixed freeforced convective and mass transfer boundary-layer flow with temperature dependent viscosity, Int. J. Engng Sci. 33 (1995) 1369–1384.
[18] M. Anghel, H.S. Takhar, I. Pop, Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium, J. Heat and Mass Transfer. 43 (2000) 1265-1274.
[19] C.K. Chen, M. Char, Heat transfer of a continuous stretching surface with suction or blowing, J. Math. Anal. Appl. 135 (1988) 568–580.
[20] F.C. Lai, F.A. Kulacki, The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, Int. J. Heat Mass Transfer. 33 (1990) 1028–1031.
[21] M.A. Alabraba, A.R. Bestman, A. Ogulu, Laminar convection in binary mixture of hydromagnetic flow with radiative heat transfer, Astrophys. Space Sci. 195 (1992) 431–439.
[22] M.A. Seddeek, Effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation, Int. Comm. Heat Mass Transfer. 27 (2000) 1037–1064.
[23] M. Acharya, L.P. Singh, G.C. Dash, Heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing, Int. J. Eng. Sci. 37 (1999) 189–211.
[24] M.A. Seddeek, Thermal - diffusion and diffusion thermo effects on mixed free-forced convective flow and mass transfer over an accelerating surface with a heat source in the presence of suction and blowing in the case of variable viscosity, Acta Mechanica. 172 (2004) 83-95.
[25] B.C. Chandrasekhara, P.M.S. Namboodiri, Influence of variable permeability on combined vertical surfaces in porous medium, International Journal of Heat Mass Transfer 28 (1985) 199-206.
[26] A.A. Mohammadein, N.A. El-Shaer, Influence of variable permeability on combined free and forced convection flow past a semi-infinite vertical plate in a saturated porous medium, Heat Mass Transfer. 40 (2004) 341-346.
[27] P.A. Dinesh, N. Nalinakshi, D.V. Chandrashekar, Effects of Variable Fluid Properties and MHD on Mixed Convection Heat Transfer from a Vertical Heated Plate Embedded in a Sparsely Packed Porous Medium, IOSR Journal of Mathematics.7 (2013) 20-31.
[28] P.A. Dinesh, N. Nalinakshi, D.V. Chandrashekar, Soret and Dufour effects on mixed convection heat and mass transfer with variable fluid properties, Int. J. Mathematical Archive. 4 (2013) 203-215.
[29] P.A. Dinesh, N. Nalinakshi, N. Sandeep, Double Diffusive Mixed Convection in a Couple Stress Fluids With Variable Fluid Properties, Advances in Physics Theories and Applications. 41 (2015) 30-42.
[30] M.A. Seddeek, Thermal radiation and buoyancy effects on MHD free convection heat generation flow over an accelerating permeable surface with temperature dependent viscosity, Can. J. Phys. 79 (2001) 725–732.
[31] N. Nalinakshi, P.A. Dinesh, D.V. Chandrashekar, Effects of Internal Heat Generation and Variable Fluid Properties on Mixed Convection Past a Vertical Heated Plate, Fluid Dynamics and Material Processing. 10 (2014) 465-490.

[32] N. Nalinakshi, P.A. Dinesh, D.V. Chandrashekar, Shooting method to study mixed convection past a vertical heated plate with variable fluid properties and internal heat generation, Mapana J. Sci. 13 (2014) 31-50.
[33] P.A. Dinesh, B. Rushi Kumar, R. Suresh Babu, Combined effects of internal heat generation and viscous dissipation for double diffusive with Forchheimer fluid model, Sixth International Conference on Porous Media and Its Applications in Science, Engineering and Industry, Eds, ECI Symposium Series, ECI Digital Archives, (2016).
[34] N. Nalinakshi, P.A. Dinesh, D.V. Chandrashekar, Internal heat generation effect on mixed convection heat and mass transfer over a vertical heated plate with Soret and Dufour effects, Sixth International Conference on Porous Media and Its Applications in Science, Engineering and Industry, Eds, ECI Symposium Series, ECI Digital Archives, (2016).
[35] J.V. Ramana Reddy, K.A. Kumar, V. Sugunamma, N. Sandeep, Effect of cross diffusion on MHD non-Newtonian fluids flow past a stretching sheet with non-uniform heat source/sink: A comparative study, Alexandria Engineering Journal. (2017).
[36] B. Ramandevi, J.V. Ramana Reddy, V. Sugunamma, N. Sandeep, Combined influence of viscous dissipation and non-uniform heat source/sink on MHD non-Newtonian fluid flow with Cattaneo-Christov heat flux, Alexandria Engineering Journal. (2017).
[37] K. Pushpalatha, J.V. Ramana Reddy, V. Sugunamma, N. Sandeep, Numerical study of chemically reacting unsteady Casson fluid flow past a stretching surface with cross diffusion and thermal radiation, Open Engineering. 7(1) (2017) 69-76.