

Quantum Nature of Light in Displaced Squeezed Chaotic State

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Abstract
In this work the squeezing and statistical properties for single-mode displaced squeezed chaotic state is analyzed with the help of the density operator for a single-mode light. It is found that the effect of the thermal light is to reduce the quadrature squeezing but increases the mean photon number. According to the generated results of analyzing the quadrature squeezing, the system has 98.17% of squeezing (in squeezed state) in which the system initial to be in vacuum state (no photon at all) and occurs at the squeeze parameter ($r = 2$). In addition, the squeeze parameter enhances both the quadrature squeezing and the mean photon number.
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1. Introduction

In squeezed light, the nose in one-quadrature is below the vacuum level at the expense of enhanced fluctuation in the conjugate quadrature, with the product of the uncertainties in the two quadratures satisfies the uncertainty relation [1-4]. In addition to exhibiting a non-classical feature, squeezed light has application in detection of weak signals and low-noise communications [5-11]. In this chapter we seek to analyze the squeezing and statistical properties of signal-mode displaced squeezed chaotic state. We here study the quantum analysis of the system under consideration by employing the density operator for a chaotic state and taking the degenerate parametric amplifier which is a typical source of a squeezed light. Squeezed state is a state of light in which the quadrature fluctuations in one quadrature is below the vacuum level at the expense of enhanced noise in the conjugate quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation.

1.1. Single-mode Light in Squeezed State

Squeezed states of light have been observed in a variety of quantum optical systems. A degenerate parametric amplifier, consisting of a nonlinear crystal pumped by coherent light, is a prototype source of a single-mode squeezed light. In this system a pump photon of frequency 2ω down converted into a pair of highly correlated signal photons each of frequency ω. A degenerate parametric amplifier is described by the Hamiltonian

$$H = \epsilon(\hat{a}^2 - \hat{a}^{+2}), \tag{1}$$

where ϵ is proportional to the amplitude of the pump light mode and $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators, respectively.

Mathematically, squeezed state can be generated by applying the squeezing operator on a coherent state, defined by

$$|\alpha, r\rangle = \hat{s}(r)|\alpha\rangle\ . \tag{2}$$

In which $\hat{s}(r)$ is the squeeze operator and the squeeze parameter r is taken for convenience to be real and positive. For $\alpha = 0$, squeezed coherent state reduces to a squeezed vacuum state or in short a squeezed vacuum. Displaced squeezed vacuum state is another ideal state which is generated from the vacuum state by applying the displacement operator on the vacuum state followed by the squeezed operator and is defined mathematically by

$$|\gamma, r\rangle = \hat{D}(\gamma)\hat{S}(r)|0\rangle\ , \tag{3}$$

where $\hat{D}(\gamma)$ is the displacement operator. We see from Eq. (3) that displaced squeezed state can be generated from a vacuum state by applying the displacement and squeeze operators. Here, it is interesting to justify that from nothing (Vacuum state) we can generate something (system with squeezed state). In this notion we can have develop an idea that explain such phenomena in quantum world of any system. According to Equation (2) and Equation (3), it is possible to generate squeezed sate and later on displaced squeezed state by applying operators that alter the first state of a system (vacuum) to the newly developed system (Squeezed vacuum or displaced squeezed vacuum that is not an ordinary vacuum).

1.2. Light in Chaotic State

Chaotic states are not pure states; instead they are given by a density matrix. They correspond very well to the states of radiating black body. With their help one describes thermal light, light of a bulb, etc. The density operator for chaotic light in general and for thermal light in particular is given in [1]. Here, we incorporate such sates of light into the squeezed states (simple squeezed state or displaced squeezed states). After having the quantum mixed states, we just proceed to which state of the system predominantly affects the nature of light which generated from the combined system. The density operator for a single-mode light in a chaotic (sometimes called thermal) state is given by the mathematical expression:

$$\hat{\rho}_T = \sum_{n=0}^{\infty} \frac{\bar{n}_T^n}{(1+\bar{n}_T^n)^{n+1}} |n\rangle\langle n| \quad (4)$$

in which n_T is the mean number of photons in such state. We infer from Equation (4) that the density matrix for a chaotic state of is expressed in terms of the number (fock) state. Taking into consideration both Equation (3) and the expression for an arbitrary quantum state with $|\varphi\rangle$ having density of

$$\hat{\rho} = |\varphi\rangle\langle\varphi| \,, \quad (5)$$

the density operator for the system that encompasses both the squeezed and the chaotic (thermal) states would have the following ultimate expression which holds the overall properties of the given system. In such systems, we offer both classical and quantum nature rather than quantum or classical natures only because of the system obeys' both features simultaneously. Thus, consider the light mode initially in a chaotic state of light and then density operator for the resulted system would be

$$\hat{\rho}_s = \hat{S}(r)\hat{D}(\gamma)\hat{\rho}_T\hat{D}^+(\gamma)\hat{S}^+(r) \quad (6)$$

where $\hat{\rho}_T$ is defined in Equation (4).

2. System's Field Fluctuations

In this section we analysis the properties of field fluctuations of single-mode light whose state is displaced squeezed chaotic one. Here, we consider the squeezed light be generated from a system with components of light with squeezed state and considering the initially the state of the system is in a chaotic state. We now proceed to determine the variance of the quadrature operators for the single-mode displaced squeezed chaotic state. The quadrature operator be defined by

$$\hat{a}_{\pm} = \sqrt{\pm 1}(\hat{a}^+ \pm \hat{a}) \quad (7)$$

Satisfying the commutation relation

$$[\hat{a}_+, \hat{a}_-] = 2i \,. \quad (8)$$

Now, we define the variance of the quadrature fields as

$$\Delta a_{\pm}^2 = \langle a_{\pm}, a_{\pm}\rangle \,, \quad (9)$$

we have used the notation to represent the expression of $\langle u, v\rangle = \langle uv\rangle - \langle u\rangle\langle v\rangle$. Employing this relation along with Equation (8), the other face of Equation (9) is can be put in the form of

$$\Delta a_{\pm}^2 = 1 + 2\langle\hat{a}^+\hat{a}\rangle - \langle\hat{a}^+\rangle\langle\hat{a}\rangle \pm (\hat{a}^{+2}\rangle + \langle\hat{a}^2\rangle) \mp \left(\langle\hat{a}^+\rangle^2 - \langle\hat{a}\rangle^2\right) \quad (10)$$

We now, proceed to evaluate the expectation values involving in Equation (10) by employing the density operator and the general trace notation to evaluate the mean values:

$$\langle\hat{a}^+\hat{a}\rangle = Tr[\hat{\rho}_s\hat{a}^+\hat{a}] \quad (11)$$

After several elementary mathematical steps that are necessary for the problem it requires, we have obtain the quadrature variance of the system that constructed for a purpose would be found as:

$$\Delta a_{\pm}^2 = (1 + 2\bar{n}_T)e^{\mp 2r} \quad (12)$$

When we see explicitly the variance of the quadrature in the positive and negative quadrature's quantized fields, it can be explore that the negative quadrature blows up while the squeeze parameter (r) increases to infinity. On the other hand, for the same condition holds true for r, the positive filed quadrature terminates to a fixed value (approached to a constant value of zero). From these two sentences, it is important to note that the positive field converges while the negative filed will diverge.

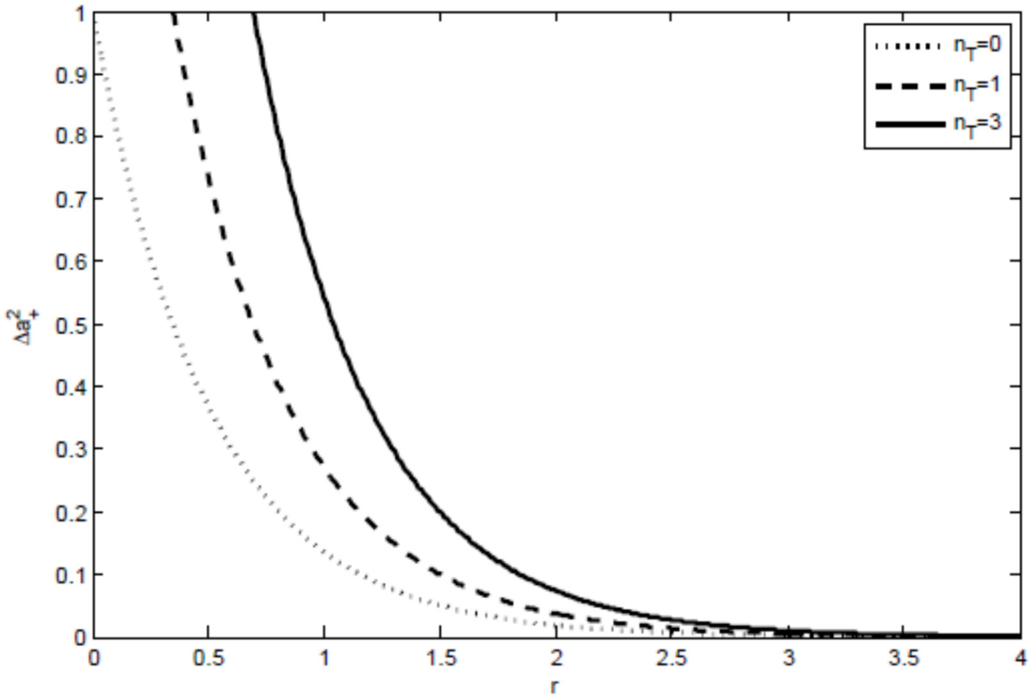

Figure 1. Plots of plus quadrature variance versus the squeeze parameter r for different values of the mean number of photons for the chaotic light [solid curve ($n_T = 3$), dashed curve ($n_T = 2$) and doted curve ($n_T = 0$)].

The description for Figure 1 is stated as follow, we have drawn the notion of the statement that ass the dependences of a system on its initial state. As we see from the Figure, while the squeeze parameter increases the corresponding quadrature variance decreases. The other statement in connection with this is that as the squeeze parameter increases the fluctuation in the system would be decreases and gradually it seems attain no fluctuations ours in the system. Such phenomena are very important in the case of constructing gravitational wave detectors.

It is important to see the effect of the initial (former) state of the system on the later state of the system. One key factor that needs to be analyzed is the thermal state of the system which has adequate effect on it. To see the significance effect, we offer to notice the curve structures plotted in Figure 1. From this Figure we can write down a statement which is in line with the mathematical approach. As we notice, when the mean number of photons for thermal state of the system increases there is more fluctuations occur in the system. These fluctuations make the system unlikely to be selected for detectors. Therefore; the mean number of photons in thermal state causes more fluctuations to happen. So it is possible to reduce to quadrature variance by eliminating the thermal state of light. In such case the system is allowed to have predominantly the quantum nature rather than the classical one as expected. It is another way to treat chaotic state of light has also classical nature that results the system to have the classical properties besides its non-classical one.

2.1. Quadrature Squeezing

In this section, we seek to determine the quadrature squeezing for single-mode light beams relative to the coherent state. The quadrature squeezing to the vacuum state can be defined by

$$S = \frac{(\Delta a_{\pm}^2)_v - \Delta a_{\pm}^2}{(\Delta a_{\pm}^2)_v}. \quad (13)$$

The quadrature variance for vacuum state would be

$$\left(\Delta a_{\pm}^2\right)_v = 1. \quad (14)$$

Incorporating Equation (12) into (13), we have got

$$S = 1 - \Delta a_{\pm}^2. \quad (15)$$

The notion of Equation (15) is that there is an inverse relation between squeezing and variance (fluctuation), when the fluctuation of the system increases, the corresponding numerical values in the squeezing decreases.

Table 1. The relation between the mean photon number of chaotic light, quadrature variance and quadrature squeezing with the squeeze parameter taken to be ($r = 2$).

$\bar{n}_T$	$\Delta a_{\pm}^2$	S	Degree of squeezing
0	0.0183	0.9817	98.17%
1	0.0366	0.9634	96.34%
3	0.0733	0.9267	92.67%
4	0.0916	0.9084	90.84%
5	0.1099	0.8901	89.01%

According to the above table it is clear to see that the relations between mean photon number of chaotic light quadrature variance and quadrature squeezing. Look! The table carefully, we see that when the mean number of the chaotic light increases, the variance and squeezing of the system increases and decrease, respectively. The quadrature squeezing has been seen high when the initial state of the system is vacuum (no photon for convenience) which has corresponding degree of 98.17%. Analogous interpretation has to be holding true for the other values shown in the table.

Another important point that should be known here is the displaced sub state of the system has no any effect on the quadrature variance as well as quadrature squeezing. So that non-classical nature of the system is greatly depend on the other parameters like, the mean photon number for initial state of the system (chaotic state) and squeeze parameter, r.

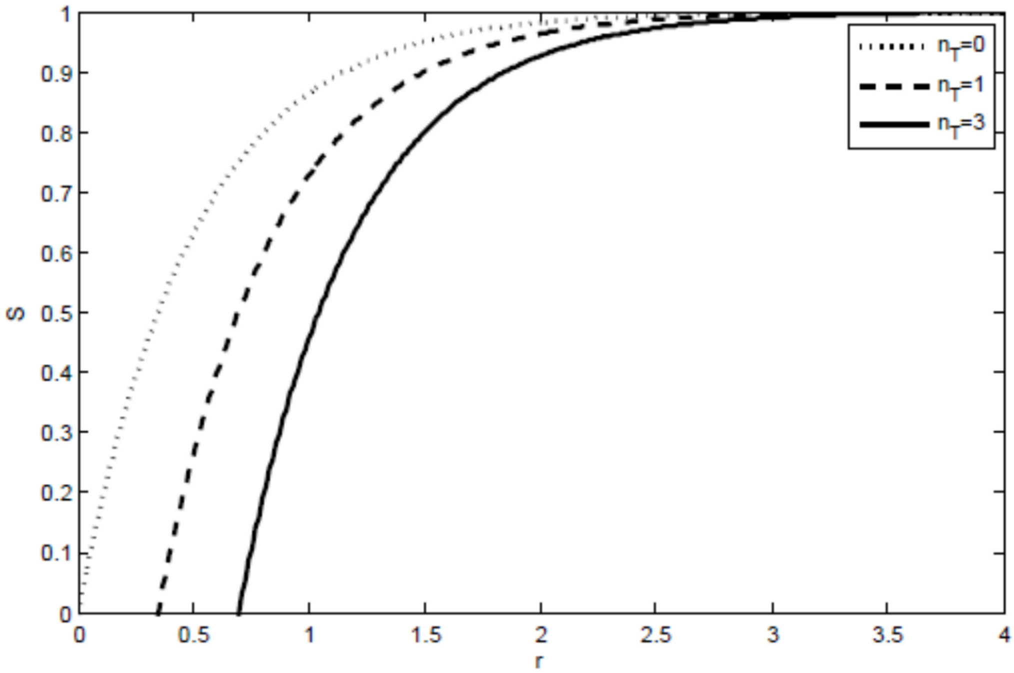

Figure 2. Plots of quadrature squeezing versus the squeeze parameter (r) for different values of chaotic light mean photon numbers [solid curve ($n_T = 3$), dashed curve ($n_T = 1$) and dotted curve ($n_T = 0$)].

As usual we need to some ideas which explain what Figure 2 tells us. Note the dotted curve ($n_T = 0$) tells us when the system initially in vacuum state rather the in a chaotic state have high degree of squeezing. On the other hand, the other two curves ($n_T = 1$ and $n_T = 3$) tells that the degree squeezing is not much in significant as the third curve ($n_T = 0$) which tells us setting the initial state of a given system is not important in enhancement of squeezing. The other pick point in connection with the figure is that; don't forget the squeeze parameter (r) enhances the squeezing of photons in the system and makes the system more valuable in the construction process of detectors and have invaluable contribution.

2.2. Statistical Analysis of the System

The mean photon number for the system is another area of concept in this paper which is to be calculated. We can calculate the mean photon number in normal order for the system using Equation (11) and employing Equation (6) up on following mathematical operations, we have get

$$\bar{n}_s = \langle \hat{a}^+ \hat{a} \rangle = \gamma^* \gamma + \bar{n}_T + (1 + \bar{n}_T) \sin^2 r. \quad (16)$$

In this Equation, we see that the mean photon number for the system is turn out to be the combination of the mean photon numbers of chaotic light, displaced state and squeezed vacuum state.

Table 2. Statistics of the mean photon number for the system with fix value of the squeeze parameter ($r = 2$).

$\gamma^*\gamma$	$\bar{n}_T$	$\bar{n}_s$
0	0	0.8268
1	1	3.6536
2	4	10.1341
6	5	15.9609
7	10	26.0950

As the above table generates how much the sub-states photon number affect the mean photon number for the whole system. Even though the displaced state of the system has not any effect on the fluctuations of the quadrature, it has a greater role in enhancing of the mean photon number of the system.

From this fundamental state, we can launch into different sub-states which are constituents to the larger system. For example it can be easily established using this result that the mean photon number for a displaced squeezed vacuum by setting $\bar{n}_T = 0$ is

$$\bar{n}_s = \langle \hat{a}^+ \hat{a} \rangle = \gamma^* \gamma + \sin^2 r \tag{17}$$

In the absence of displaced state, Equation (17) can be reduced to an expression written as

$$\langle \hat{a}^+ \hat{a} \rangle = \sin^2 r \, , \tag{18}$$

which is equal to the exact expression for the mean photon number for squeezed vacuum as expected.

3. Conclusion

In this paper the squeezing and statistical properties of single-mode displaced squeezed chaotic state. Moreover in our work the squeezing and statistical properties for a single-mode displaced squeezed chaotic state is analyzed with the help of the density operator. We calculate the quadrature variance and quadrature squeezing of the system under consideration and the system is in 98.17% squeezed state when its initial state is vacuum one occurs at the squeeze parameter $r = 2$. Our result shows that the system is in a squeezed state and the squeezing occurs in the plus quadrature. It is found that the effect of the thermal light is to reduce the quadrature squeezing but increases the mean photon number. In addition, the squeeze parameter increases both the quadrature squeezing and the mean photon number. In connection with this the system under consideration has significance in the area of quantum optics for the construction of detectors since it generates squeezed bright light.

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