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Non-classical Analysis of Output Superposed Laser and Signal Twin Light Beams

Sitotaw Eshete
Department of Physics, Oda Bultum University, Po.box. 226, chiro, Ethiopia

Abstract
In this paper, we have shown that the mathematical expressions for a new integrated quantum optical system which encompass laser light and twin light beams. Our finding shows that the quadrature variance and quadrature squeezing for the superposed light beam is the sum of the quadrature variance and the average of the quadrature squeezing of the component systems respectively. We also found that the presence of the twin light beam improves the non-classicality of the system.
Keywords: Laser Light, Quadrature Fluctuation, Squeezing, Twin Light Beams

1. Introduction

Quantum analysis of different optical systems have been attracted a great deal of interest. As a result, different remarkable innovations are now available regarding with quantum optical systems. Several authors have been studied on quantum analysis of degenerate as well as non-degenerate optical parametric oscillators [1-3]. On the other hand, there are a lot of authors that pay their attention towards the quantum properties of light generated by laser under a certain condition [4-15]. Taking this as a scenario, in this paper we seek to investigate the quantum properties of superimposed output light generated by laser whose cavity is coupled to a vacuum reservoir and optical parametric oscillator whose cavity is coupled to vacuum reservoir. We arrange our system in such a way that the output light (LB1) from the laser is incident on a side of a perfectly transmitting mirror while the output light beam (LB2) from optical parametric oscillator is incident on a side of perfectly reflecting mirror as depicted in the figure 1.

2. The Density Operator

Here we seek to determine the density operator for superposed light beams. Let $\hat{\rho}'$ be the density operator for a certain single mode light. Upon expanding this density operator in power series in normal ordering

$$\hat{\rho}' = \sum_{nm} C_{nm} \hat{a}_1^{+n} \hat{a}_1^{m} \tag{1}$$

and employing the completeness relation for a single-mode coherent state

$$\hat{I} = \frac{1}{\pi} \int d^2\alpha_1 |\alpha_1\rangle \langle\alpha_1|, \tag{2}$$

we easily finds

$$\hat{I} = \frac{1}{\pi} \int d^2\alpha_1 \sum_{nm} C_{nm} |\alpha_1\rangle \langle\alpha_1| \hat{a}_1^{+n} \hat{a}_1^{m} \tag{3}$$

in which $\hat{a}_1$ is the annihilation operator for the first light beam. Employing the relations

$$\langle\alpha_1| \hat{a}_1^{+} = \alpha_1^{*} \langle\alpha_1| \tag{4}$$

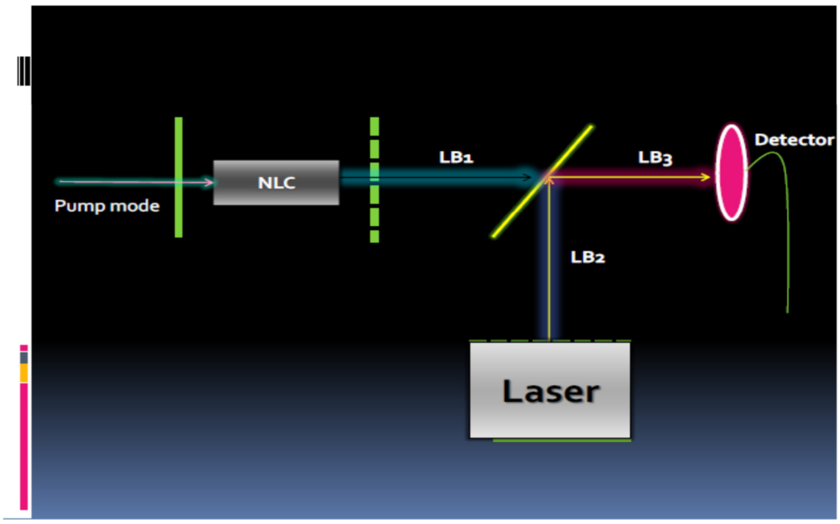

Figure 1 Schematic Design for the system.

and

$$\langle\alpha_1|\hat{a}_1 = \left(\alpha_1 + \frac{\partial}{\partial\alpha_1^*}\right)\langle\alpha_1| \tag{5}$$

Equation (3), can be rewritten as

$$\hat{\rho}' = \frac{1}{\pi}\int d^2\alpha_1 \sum_{nm} C_{nm}\alpha_1^{*n}\left(\alpha_1 + \frac{\partial}{\partial\alpha_1^*}\right)^m |\alpha_1\rangle\langle\alpha_1| \tag{6}$$

there follows

$$\hat{\rho}' = \int d^2\alpha_1 Q\left(\alpha_1^*,\ \alpha_1 + \frac{\partial}{\partial\alpha_1^*}\right)|\alpha_1\rangle\langle\alpha_1| \tag{7}$$

in which

$$Q\left(\alpha_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}\right) = \frac{1}{\pi}\sum_{nm} C_{nm}\alpha_1^{*n}\left(\alpha_1 + \frac{\partial}{\partial\alpha_1^*}\right)^m. \tag{8}$$

We now realize that the density operator for the superposition of the first light beam of optical parametric oscillator and the light beam from the laser is expressible in terms of displacement operator as

$$\hat{\rho} = \frac{1}{\pi}\int d^2\alpha_2 \sum_{n'm'} C_{n'm'}\alpha_2^{*n'}\left(\alpha_2 + \frac{\partial}{\partial\alpha_2^*}\right)^{m'} \hat{D}(\alpha_2)\,\hat{\rho}'\hat{D}(-\alpha_2) \tag{9}$$

This density operator represents the quantum density of light beams (LB3) as shown in figure 1. This quantum density operator holds all the necessary information about light beam (LB3). We then will use this density operator to study about the statistical and squeezing nature of the superposed light (LB3).

3. Variance of Quadrature's Fields

In this section, we seek to determine the quadrature variance and then squeezing in quadrature fields. To study the squeezing and the quadrature variance of the superposed light, we first define the quadrature operators for single mode light as

$$\hat{a}_\pm = \sqrt{\pm 1}(\hat{a}^+ \pm \hat{a}) \tag{10}$$

where $\hat{a}$ is annihilation operator for superposed light beams and satisfying the commutation relation

$$[\hat{a}_+, \hat{a}_-] = 4i. \tag{11}$$

We define variance of the quadrature for single-mode light by

$$\Delta a_\pm^2 = \langle\hat{a}_\pm, \hat{a}_\pm\rangle \tag{12}$$

so that employing Equation (11), we readily obtain

$$\Delta a_\pm^2 = 1 + 2\langle\hat{a}^+, \hat{a}\rangle \pm (\langle\hat{a}^+, \hat{a}^+\rangle + \langle\hat{a}, \hat{a}\rangle) \tag{13}$$

using the relation

$$\langle\hat{S}, \hat{R}\rangle = \langle\hat{S}\hat{R}\rangle - \langle\hat{S}\rangle\langle\hat{R}\rangle \tag{14}$$

We see that

$$\Delta a_\pm^2 = 2 + 4\langle\hat{a}^+\hat{a}\rangle - \langle\hat{a}^+\rangle\langle\hat{a}\rangle \pm (\langle\hat{a}^{+2}\rangle + \langle\hat{a}^2\rangle) \mp (\langle\hat{a}^+\rangle^2 - \langle\hat{a}\rangle^2). \tag{15}$$

The expectation value of an operator $\hat{A}$ can be written in terms of the density operator as

$$\langle\hat{A}\rangle = Tr(\hat{\rho}\hat{A}) \tag{16}$$

Taking this fact to write the expectation values involved in Equation (15), we have get

$$\langle\hat{a}\rangle = \langle\hat{a}_1\rangle + \langle\hat{a}_2\rangle \tag{17}$$

This reveals that the annihilation operator for the superposed light is the sum of the annihilation operators of the twin light beam $\hat{a}_1$ and the annihilation operator for laser light beams $\hat{a}_2$.

Here we are interested in dealing the quantum phenomena of light beam (LB3). To do this, employing the input-output relation since a fraction of cavity light is available at the detector. Thus,

$$\hat{a}_{out} = \sqrt{k}\hat{a} - \hat{a}_{in} \tag{18}$$

in which k is cavity damping constant and $\hat{a}_{in}$ is the input field in to the cavity. Following a wonderful steps, we have found the quadrature variance for the superposed light (LB3) found to be

$$\left(\Delta a_\pm^2\right)_{out} = k_1\left(\Delta a_\pm^2\right)_1 + k_2\left(\Delta a_\pm^2\right)_2 \tag{19}$$

Or

$$\left(\Delta a_\pm^2\right)_{out} = \left(\Delta a_\pm^2\right)_1^{out} + \left(\Delta a_\pm^2\right)_2^{out} \tag{20}$$

where k_1 is the cavity damping constant for light beams (LB1) and k_2 is the cavity damping constant for light beams (LB2). Equation (20) represents the expression of quadrature variance for the superposed light beam(s). In this equation, we are noting that quadrature variance of superposed light happens to be the sum of the quadrature variance of output lights from the laser and twin light beams. Since the values of the damping constants (k_1 and k_2) are between 0 and 1, the output quadrature variance(s) becomes less when compared to the corresponding cavity mode quadrature variance(s). Here, the mathematical expression of squeezing relative to the quadrature squeezing of the superposed output vacuum state:

$$S = \frac{\left(\Delta a_\pm^2\right)_V^{out} - \left(\Delta a_\mp^2\right)_{out}}{\left(\Delta a_\pm^2\right)_V^{out}} \tag{21}$$

in which $\left(\Delta a_{\pm}^2\right)_V^{out}$ is the quadrature variance representing the output fields. There follows

$$S = \frac{k_1 + k_2 - (\Delta a_{\mp}^2)_{out}}{k_1 + k_2}. \tag{22}$$

Substituting Equation (19) into Eq. (22), we have

$$S = \frac{k_1 - k_1(\Delta a_{\mp}^2)_1}{k_1 + k_2} + \frac{k_2 - k_2(\Delta a_{\mp}^2)_2}{k_1 + k_2}. \tag{23}$$

the quadrature squeezing can be written in terms of the component quadrature squeezing as

$$\mathrm{S} = \frac{\mathrm{k_1 S_1 + k_2 S_2}}{\mathrm{k_1 + k_2}} \tag{24}$$

where $S_1 = 1 - e^{-2r}$ and

$$S_2 = \frac{\mathrm{A}}{A\eta + \mathrm{k_2}}\left[\sqrt{1 - \eta^2} - (1 - \eta)\right] \tag{25}$$

in which A is the linear gain coefficient of a three-level laser and η is the probability difference of an atom to be at the lower energy level and top energy level. For the special case $\mathrm{k_1} = \mathrm{k_1} = \mathrm{k}$, the quadrature squeezing is found to be

$$S = \frac{S_1 + S_2}{2}. \tag{26}$$

It is not difficult to see from the figure 2 that squeezing is highly dependent on the probability difference (η).

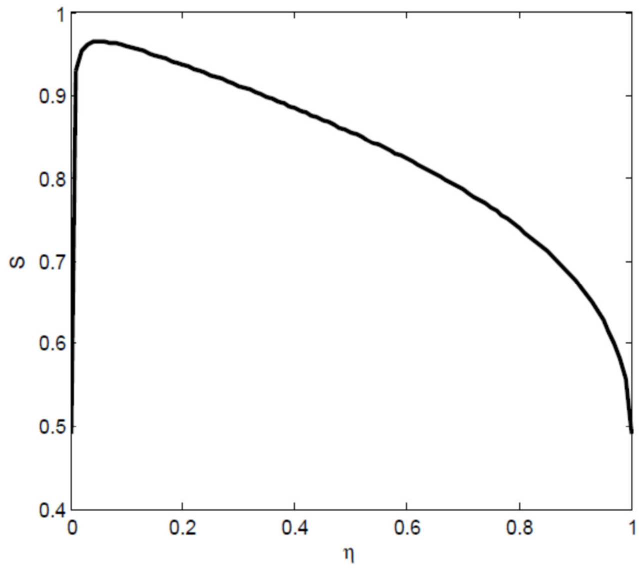

Figure 2: A plot of quadrature squeezing [Eq. (26)] versus _ for the linear gain coefficient of a laser, A=600, the squeeze parameter of which is a contribution of system one, r=2 (dimension less) and the damping constants assumed to be the same, $\mathrm{k_1} = \mathrm{k_1} = 0.8$.

When we run the maltha programme code for the mathematical expression given in Eq. (26), we generate the above figure and we have fixed the maximum value of squeezing found to be 0.9657 occurs at the probability difference, $\eta = 0.05$ which corresponds that the maximum degree of squeezing for the above given values is to be 96.57%. We see that from the figure, when the probability difference increases from 0.05 to 1, the quadrature squeezing decreases. This must be due to the influences of the atomic coherence since probability difference affects it. When η increases, the atomic coherence is not significance and results in decrement of quadrature squeezing. But it is important to noting that when slightly more atoms occupied the lower energy state at the initial preparation time, there is a production of light in a squeezed state and the degree of squeezing is very high. This is our finding for the constructed system as shown in the figure 1. So the system gives the desired result under a certain conditions.

4. Conclusion

In this paper we have studied that the non-classical nature of light for a superposed output two-mode squeezed laser light and tin light beams. It is found that the presence of twin light beams (light beams in squeezed quantum state) improves the degree of squeezing. We have shown that the degree of squeezing is highly depending on the probability difference. The maximum amount of squeezing is found to be 96.57% when the linear gain coefficient A = 600, the damping constant η = 0.8 and the squeeze parameter r = 2 at the probability difference η = 0.05. We have also mentioned that quadrature squeezing is depending on the atomic coherence. The quantum optical system, the we have proposed is significance because it gives the desired quantum properties light for the superposed of two-mode laser light and twin light beams whose cavities are coupled to a vacuum reservoir. It is expected this system to contribute one step in the area of quantum optics. It is proved in this paper that all mathematical expressions are valid by setting the certain constraints conditions.

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