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# Effect of Thermal Radiation on MHD Casson Fluid Flow over a Stretched Surface of Variable Thickness

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## Abstract

This study deals with the free convective heat transfer in a 2D magneto hydrodynamic flow of Casson fluid over a non-uniform thickness stretching sheet in the presence of thermal radiation and non-uniform heat source/sink effects. The governing equations of the flow and heat transfer are transformed as the asset of nonlinear ODEs and solved numerically using bvp4c Mat lab package. The effect of pertinent parameters, namely, magnetic field parameter, Casson parameter, thermal radiation parameter, non-uniform heat source/sink parameters on the flow and heat transfer is investigated with the assistance of graphs. Numerical results are computed for the friction factor and reduced Nusselt number. It is observed that the thermal radiation has tendency to enhance the temperature field of Casson fluid.

**Keywords:** MHD, Casson fluid, Thermal radiation, Slendering sheet, non-Uniform heat source/sink.

**Nomenclature:**

$u, v$ : Velocity components in $x$ and $y$ directions
$x$ : Direction along the surface
$y$ : Direction normal to the surface
$C_p$ : Specific heat capacity at constant pressure
$f$ : Dimensionless velocity
$A$ : Coefficient related to stretching sheet
$B(x)$ : Magnetic field parameter
$T$ : Temperature of the fluid
$Ra$ : Thermal radiation parameter
$A^*, B^*$ : non-Uniform heat source/sink parameters
$k$ : Thermal conductivity
$T_m$ : Mean fluid temperature
$T_\infty$ : Temperature of the fluid in the free stream
$h_1^*$ : Dimensional velocity slip parameter
$h_2^*$ : Dimensional temperature jump parameter
$a$ : Thermal accommodation coefficient
$b$ : Physical parameter related to stretching sheet
$m$ : Velocity power index parameter
$\Pr$ : Prandtl number
$M$ : Magnetic interaction parameter
$h_1$ : Dimensionless velocity slip parameter
$h_2$ : Dimensionless temperature jump parameter
$C_f$ : Skin friction coefficient
$\sigma^*$ : Stefan-Boltzmann constant
$k^*$ : mean absorption coefficient

$Nu_x$ : Local Nusselt number

$\mathrm{Re}_x$ : Local Reynolds number

**Greek Symbols**

$\eta$ : Similarity variable

$\sigma$ : Electrical conductivity of the fluid

$\theta$ : Dimensionless temperature

$\rho$ : Density of the fluid

$\beta$ : Casson fluid parameter

$\upsilon$ : Kinematic viscosity

$\lambda$ : Wall thickness parameter

## 1. Introduction

In recent days, the investigations on non-Newtonian fluids are significantly improved owing to their immense pragmatic usages in the arena of science and engineering. Viscosity of the non-Newtonian fluids pivots on shear rate. Several stuffs in the physical world such as, paints, shampoos, blood, and tooth paste are specimens of non-Newtonian fluids. The constituting equations are highly nonlinear in nature in these fluids. No single model exists in the literature to describe every asset of non-Newtonian fluids. The induced magnetic field effect on the peristaltic transport of Carreau Liquid was numerically investigated by Hayat et al. [1]. Thermal radiation effect on free convective nanofluid flow over a vertical plate was theoretically investigated by Sandeep et al. [2]. Sulochana and Sandeep [3] studied the flow and thermal transport in MHD dusty flow over a stretching/shrinking cylinder by considering the various temperatures. Cattaneo-Christov heat flux model for flow of variable thermal conductivity generalized Burgers fluid was studied by Waqas [4]. Inclined magnetic field effect on MHD nanofluid flow was theoretically studied by Sandeep [5].

The theoretical investigation of magnetohydrodynamic nanofluid flow embedded with the magnetite nanoparticles Sandeep et al. [6]. Jayachandra Babu and Sandeep [7] studied the upper convected Maxwell fluid flow over a melting surface with double stratification. Analysis of boundary layer formed on an upper horizontal surface of a paraboloid of revolution within a nanofluid flow in the presence of thermophoresis and Brownian motion was studied by Koriko et al. [8]. Magnetohydrodynamic Oldroyd-B fluid flow with Soret and Dufour effects was numerically explored by Sandeep and Gnaneswara Reddy [9]. Mohan Krishna et al. [10] investigated the parabolic flow of MHD Carreau fluid with buoyancy effects. The frictional heating effect on ferrofluid flow over a slendering sheet was reported by Ramana Reddy et al. [11]. MHD stagnation flow of Casson fluid over a stretched surface with convective boundary conditions was numerically studied by Ibrahim and Makinde [12]. Hayat et al. [13] studied the 3D boundary layer flow of Sisko nanofluid with transverse magnetic field effect.

The mixed convection flow of Maxwell nanofluid in the presence of heat generation and obsorption was studied by Abbasi et al. [14]. 3D flow of radiative nanofluid with magnetic field and internal heat source effects was studied by Abbasi et al. [15]. Hayat et al. [16] studied the chemical reaction effects on radiative MHD flow. Analytical solution for heat and mass transfer of the mixed hydrodynamic/thermal slip flow over a stretching sheet was studied by Turkyilmazoglu [17]. Dual solutions for unsteady mixed convection flow of micropolar fluid over a stretching and shrinking sheet was studied by Sandeep and Slochana [18]. The effects of radiation on MHD convective flow over a permeable stretching surface with suction and heat generation was studied by Mohan Krishna et al. [19]. The researchers [20-23] studied the heat transfer in magnetic non-Newtonian fluid flows over various flow geometries. The effect os Thermophoresis and Brownian moment on nanofluid flow over a microchannel was studied by Fani et al. [24]. The boundary layer flow of a nanofluid past a stretching sheet with a convective boundary conditions was examined by Makinde and Aziz [25]. The MHD three-dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with the convective boundary condition Nadeem et al. [26]. Very recently, the researchers [27-30] studied the heat transfer in MHD flows.

Present study deals with the free convective heat transfer in a 2D magnetohydrodynamic flow of Casson fluid over a non-uniform thickness stretching sheet in the presence of thermal radiation and non-uniform heat source/sink effects. The governing equations of the flow and heat transfer are transformed as the asset of nonlinear ODEs and solved numerically using bvp4c Matlab package. The effect of pertinent parameters on the flow and heat transfer is investigated with the assistance of graphs. Numerical results are computed for the friction factor and reduced Nusselt number.

## 2. Mathematical formulation

Consider a steady 2D flow of Casson fluid over a stretching sheet of variable thickness in such a way that the *x*-axis is along the sheet and the y-axis is perpendicular to it. It is assumed that $y = A\left(x+b\right)^{\frac{1-m}{2}}$, $u_w(x) = \left(x+b\right)^m U_0$, $v_w = 0$, $m \neq 1$. The thermal radiation and non-uniform heat source/sink effects are taken into account. A transverse magnetic field of strength $B_0$ is imposed along the flow direction as depicted in Fig.1.

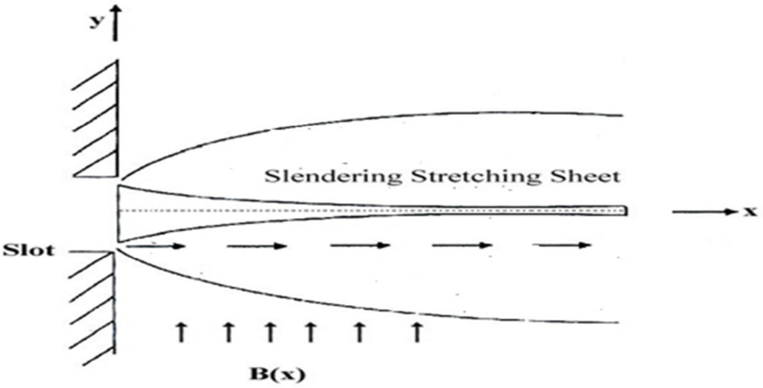

Fig.1 Physical Model

With the above assumptions, the governing equations can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho c_p}\frac{16\sigma^* T_\infty^{\ 3}}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho c_p}q''', \tag{3}$$

With the conditions

$$\left.\begin{aligned}&u=U_w(x)+h_1^*\left(\frac{\partial u}{\partial y}\right), v=0, T=T_w\left(x\right)+h_2^*\left(\frac{\partial T}{\partial y}\right),\\&and\ u(\infty)=0,\ T(\infty)=T_\infty\ ,\end{aligned}\right\} \tag{4}$$

where

$$h_1^*=\left[\frac{2-f_1}{f_1}\right]\xi_1\left(x+b\right)^{\frac{1-m}{2}},\ \xi_2=\left(\frac{2\gamma}{\gamma+1}\right)\frac{\xi_1}{\Pr},\ h_2^*=\left[\frac{2-a}{a}\right]\xi_2\left(x+b\right)^{\frac{1-m}{2}},$$

$$B(x)=B_0\left(x+b\right)^{\frac{m-1}{2}},\ T_w-T_\infty=T_0\left(x+b\right)^{\frac{1-m}{2}} \tag{5}$$

In Eq. (3) $q'''=\frac{\left(T_w-T_\infty\right)ku_w}{x\nu}\left(A^* f'+B^*\frac{\left(T-T_\infty\right)}{\left(T_w-T_\infty\right)}\right)$ is the non-uniform heat source/sink parameter.

We now introduce the similarity transformations as

we now suggest the following similarity transformations:

$$\psi = f(\eta)\left(\frac{2}{m+1}\nu U_0 (x+b)^{m+1}\right)^{0.5}, \eta = y\left(\frac{m+1}{2}U_0\frac{(x+b)^{m-1}}{\upsilon}\right)^{0.5}, \theta(T_w(x)-T_\infty) = T - T_\infty \quad (6)$$

If stream function $\psi$ be described as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

$$u = U_0 (x+b)^m f'(\eta) \text{ and } v = -\sqrt{(m+1)\frac{\nu U_0}{2}(x+b)^{m-1}}\left[f'(\eta)\ \eta\ \left(\frac{m-1}{m+1}\right) + f(\eta)\right] \quad (7)$$

with the help of (6) and (7), the equations (2)-(3) converted as

$$\left(1+\frac{1}{\beta}\right)f''' + f''f - \frac{2m}{m+1}f'^2 - M f' = 0, \quad (8)$$

$$\left(1+\frac{4}{3}Ra\right)\theta'' + \Pr f\theta' - \Pr\frac{1-m}{m+1}f'\theta + A^* f' + B^*\theta = 0, \quad (9)$$

and the corresponding conditions are

$$\left.\begin{array}{l} f(0) = \lambda\left(\frac{1-m}{m+1}\right)\left[1+h_1 f''(0)\right], f'(0) = \left[1+h_1 f''(0)\right], \\ \theta(0) = \left[1+h_2\theta'(0)\right], \\ f' = 0, \theta = 0, \phi = 0 \quad as \quad \eta \to \infty \end{array}\right\} \quad (10)$$

where $M$, $\Pr$, $Ra$ are defined as

$$M = \frac{2\sigma B_0^2}{\rho U_0 (m+1)}, \Pr = \frac{\mu C_p}{k}, Ra = \frac{4\sigma^* T_\infty^3}{k^* k}, \quad (11)$$

The physical quantities of engineering interest, the friction factor and the local Nusselt number are given by

$$C_f = 2\frac{\mu\frac{\partial u}{\partial y}}{\rho U_w^2}, \quad Nu_x = \frac{(x+b)\frac{\partial T}{\partial y}}{T_w(x) - T_\infty}, \quad (12)$$

By using (4), (12) becomes

$$C_f\left(\mathrm{Re}_x\right)^{0.5} = 2\left(\frac{m+1}{2}\right)^{0.5}\left(\left(1+\beta^{-1}\right)f''(0)\right), \quad Nu_x = -\left(\frac{m+1}{2}\right)^{0.5}\left(\mathrm{Re}_x\right)^{0.5}\theta'(0), \quad (13)$$

Where $\mathrm{Re}_x = \frac{U_w X}{\nu}$ and $X = (x+b)$ .

## 3. Results and discussion

The set of ODEs (8)- (9) with the boundary conditions (10) is solved numerically by employing the bvp4c Matlab package. The non-dimensional parameter values chosen as $\Pr = 6$, $m = 0.5$, $M = 1$, $Ra = 0.5$, $h_1 = h_2 = A^* = B^* = 0.5$, $\lambda = 0.2$ . The values of these parameters remain invariable in the present discussion unless they are allowed to vary as in graphs.

Figs. 2 and 3 depict the impact of external magnetic field on the flow and temperature fields of Casson fluid. It is observed that the rising value of the transverse magnetic field parameter boosts the thermal field and decline the velocity field. Physically, increasing values of the magnetic field parameter strengthen the Lorentz force, which acts opposite to the flow field. This leads to decline the velocity profiles. The similar results has been observed for increasing values of the Casson parameter as depicted in Figs. 4 and 5. Generally, increasing values of the Casson parameter suppresses the viscous nature of the flow.

Figs. 6-8 illustrate the influence of thermal radiation and non-uniform heat source/sink parameters on temperature profiles of the Casson fluid. It is noticed that the rising values of the thermal radiation and non-uniform heat source/sink parameters enhances the temperature profiles of the flow. Physically, the positive values of the uneven heat source/sink parameters act like heat generators. These causes to increase the temperature field. An opposite trend to above has been observed for rising value of the wall thickness parameter as shown in Fig. 9.

The effect of dimensionless velocity slip parameter on flow and thermal fields is displayed in Figs. 10 and 11. It is clear that the increasing value of the velocity slip declines the flow field and boosts the thermal field. A reverse trend in thermal field is observed for increasing values of the temperature jump parameter and Prandtl number. Which is displayed in Figs. 12 and 13.

The variations in the skin friction coefficient and the local Nusselt number at different pertinent parameters is displayed in Table 1. It is observed that the rising values of the magnetic field parameter, thermal radiation parameter, velocity, thermal slip parameters and Casson parameter reduces the local Nusselt number. The increasing value of the wall thickness parameter enhances the heat transfer rate.

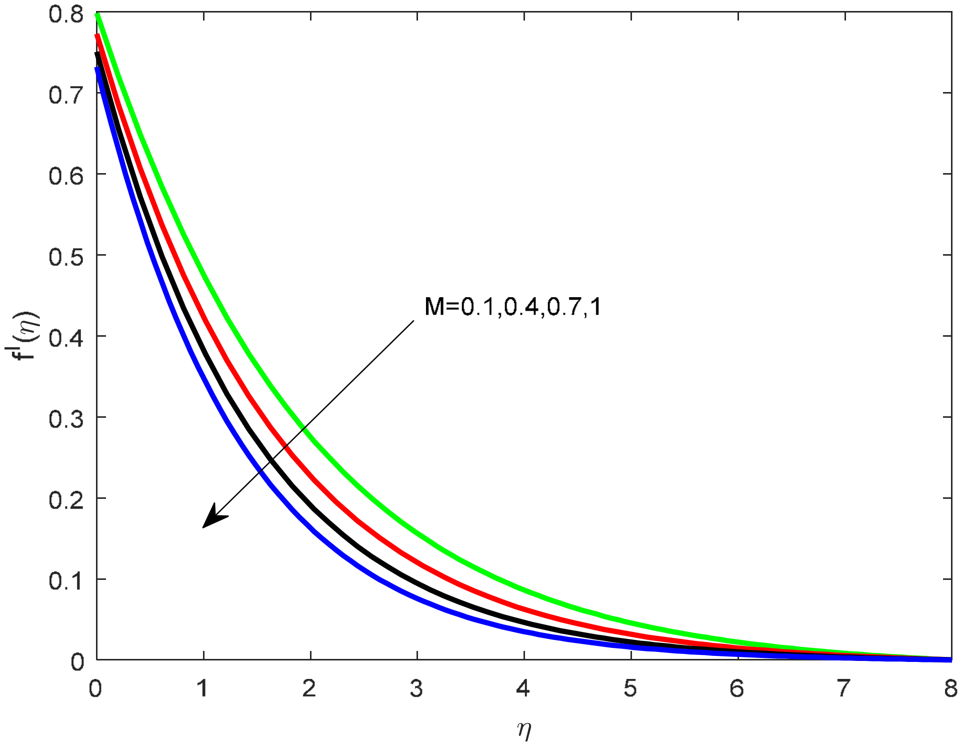

Fig.2 Effect of M on velocity field

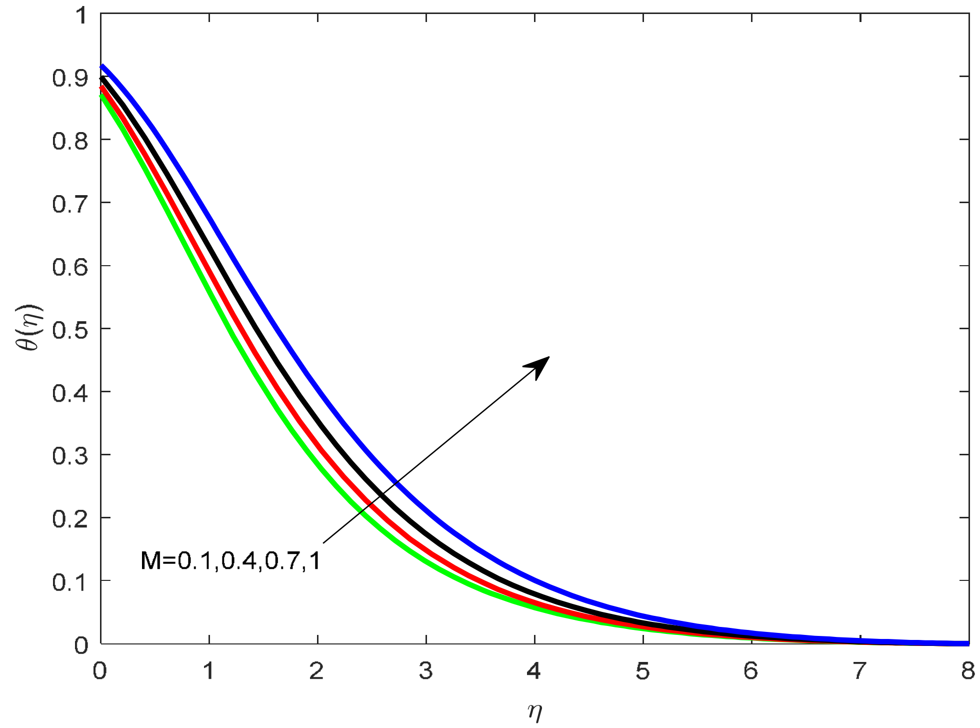

Fig.3 Effect of M on thermal field

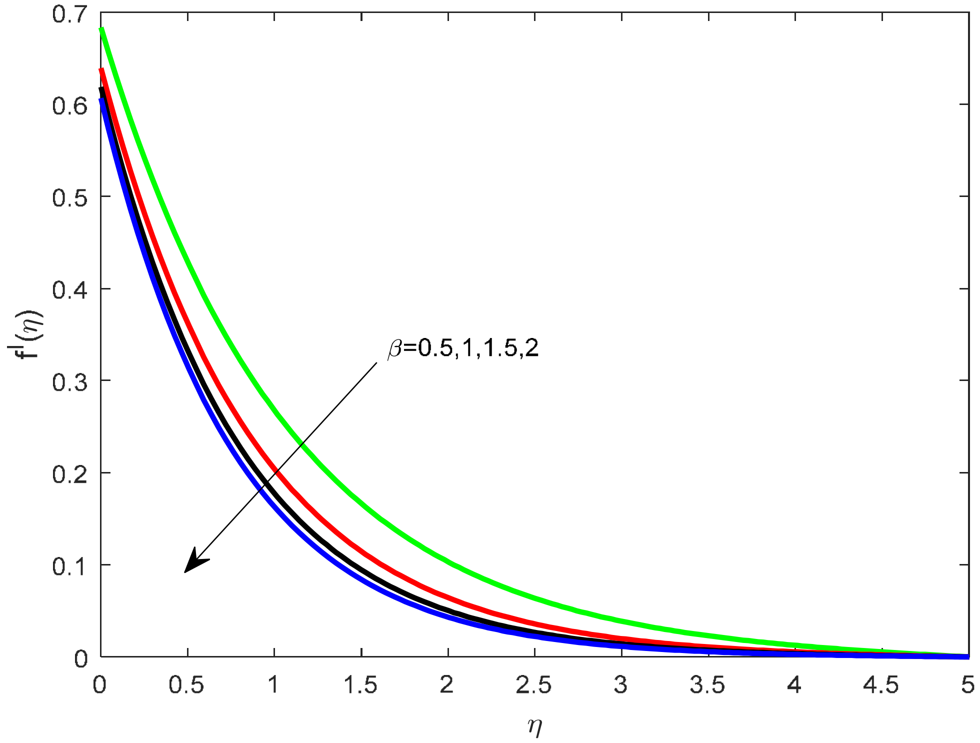

Fig.4 Effect of $\beta$ on velocity field

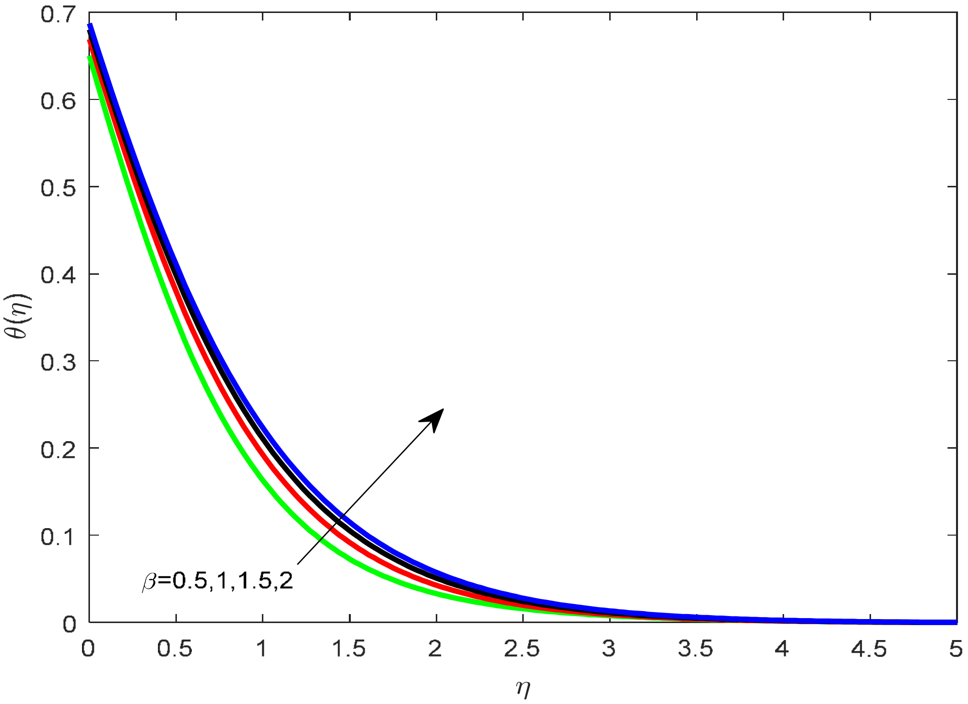

Fig.5 Effect of $\beta$ on thermal field

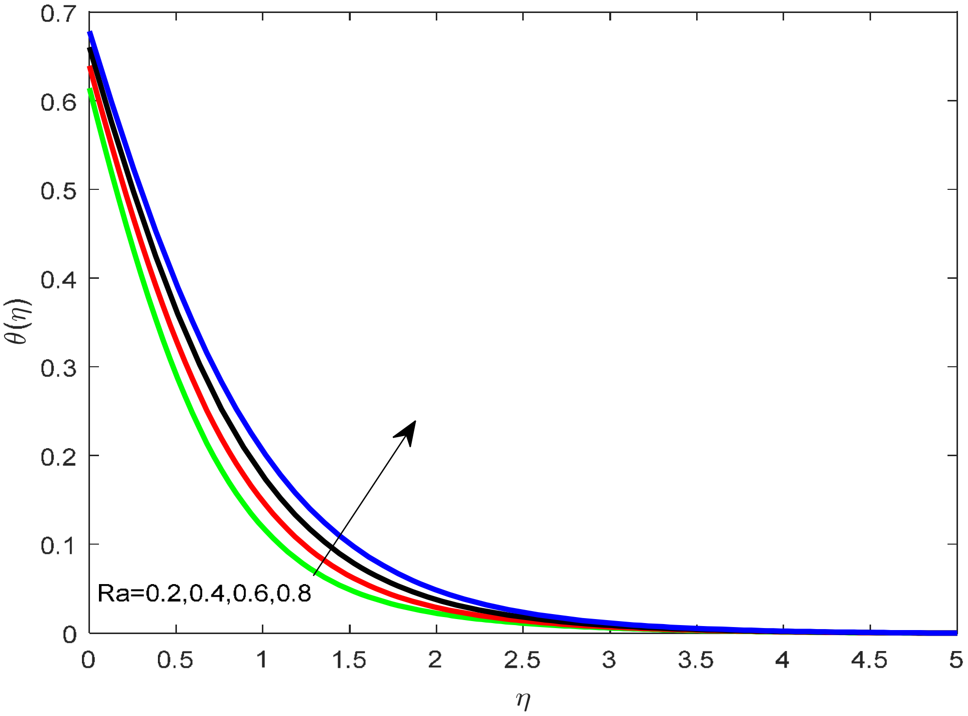

Fig.6 Effect of $Ra$ on thermal field

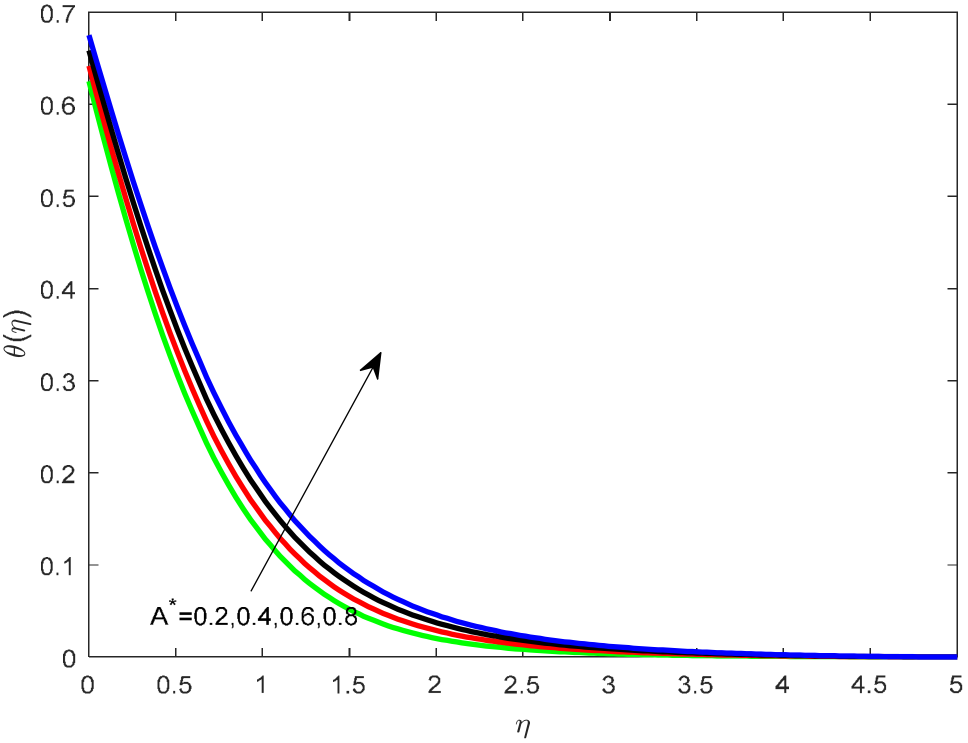

Fig.7 Effect of $A^*$ on thermal field

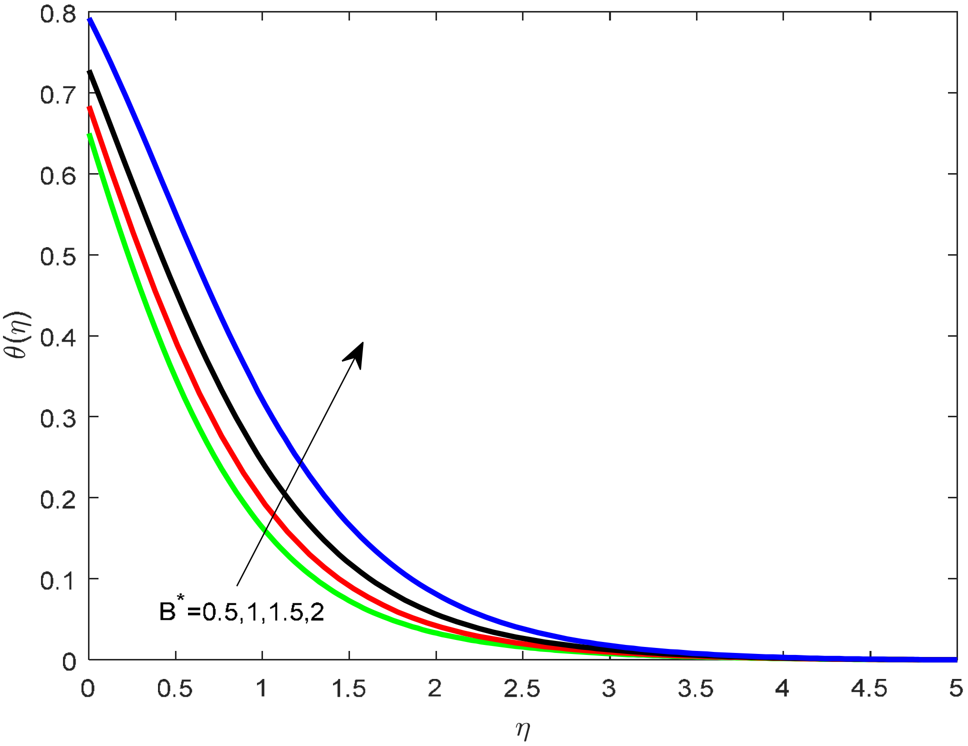

Fig.8 Effect of $B^*$ on thermal field

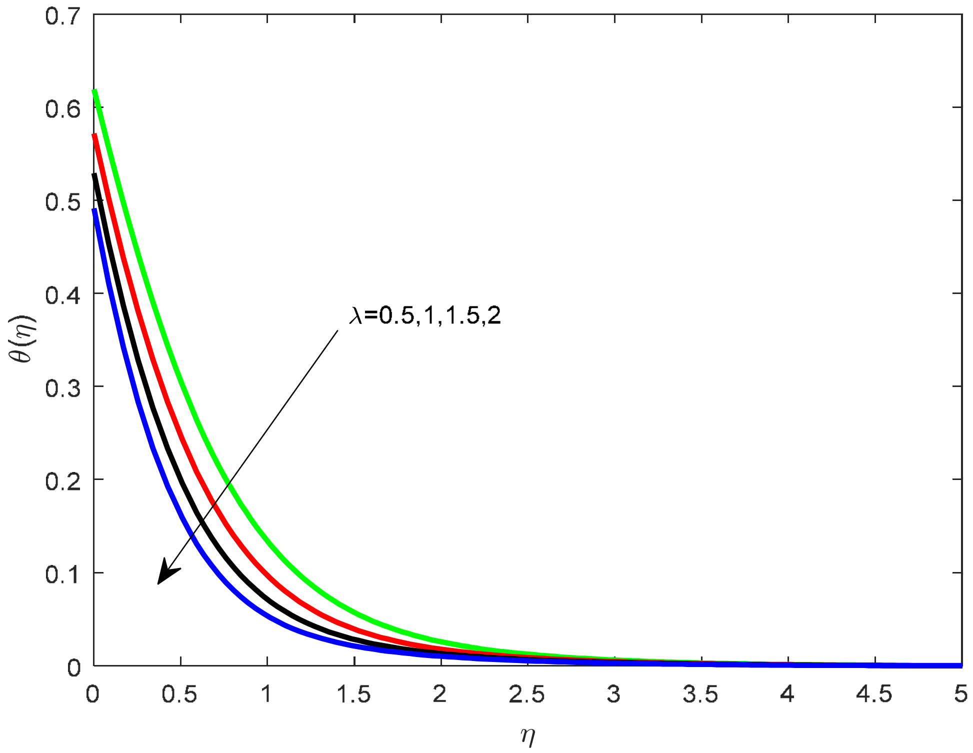

Fig.9 Effect of $\lambda$ on thermal field

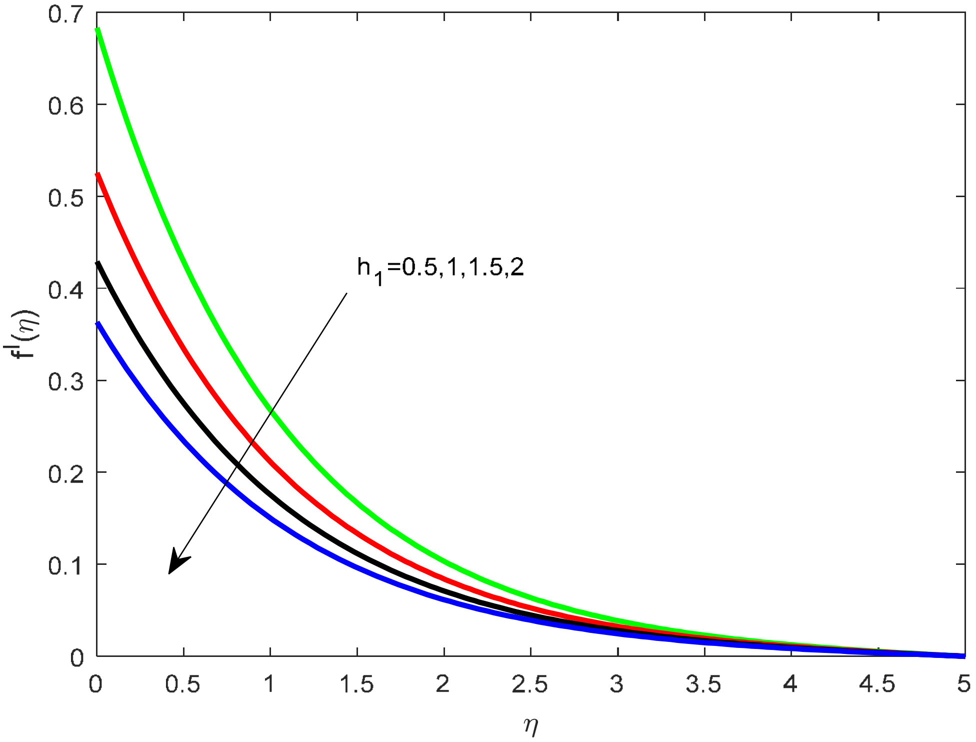

Fig.10 Effect of $h_1$ on velocity field

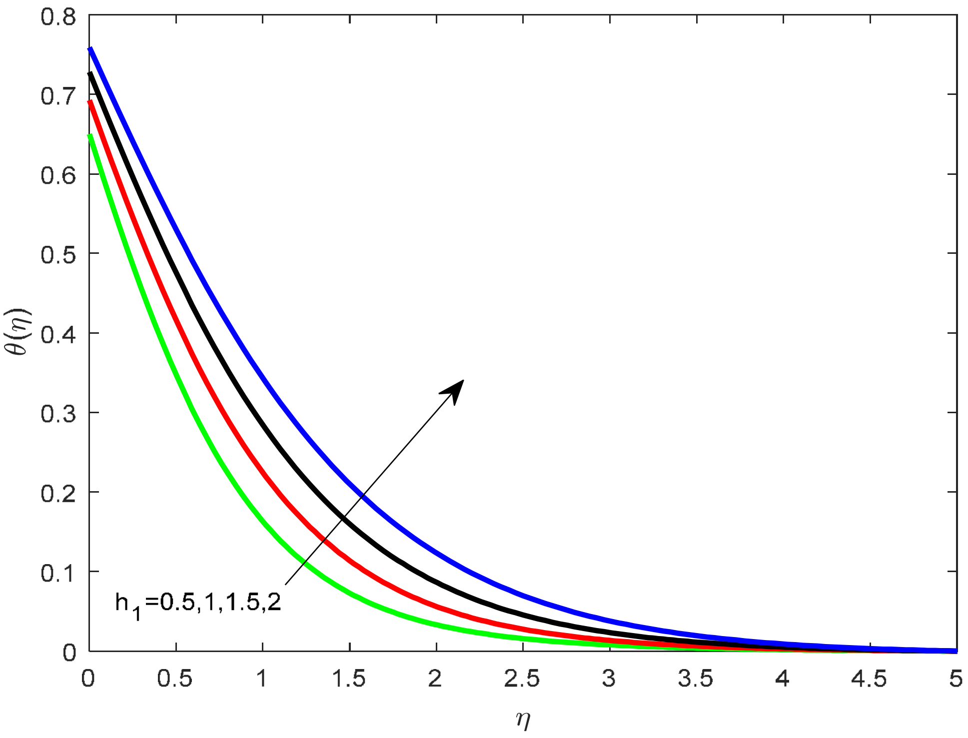

Fig.11 Effect of $h_1$ on thermal field

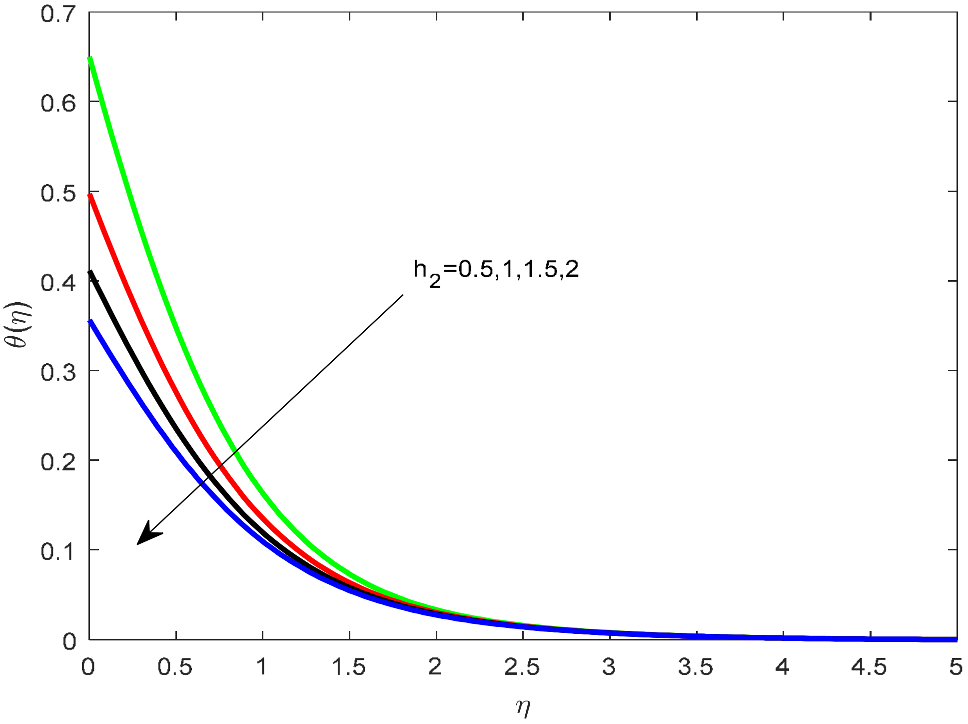

Fig.12 Effect of $h_2$ on thermal field

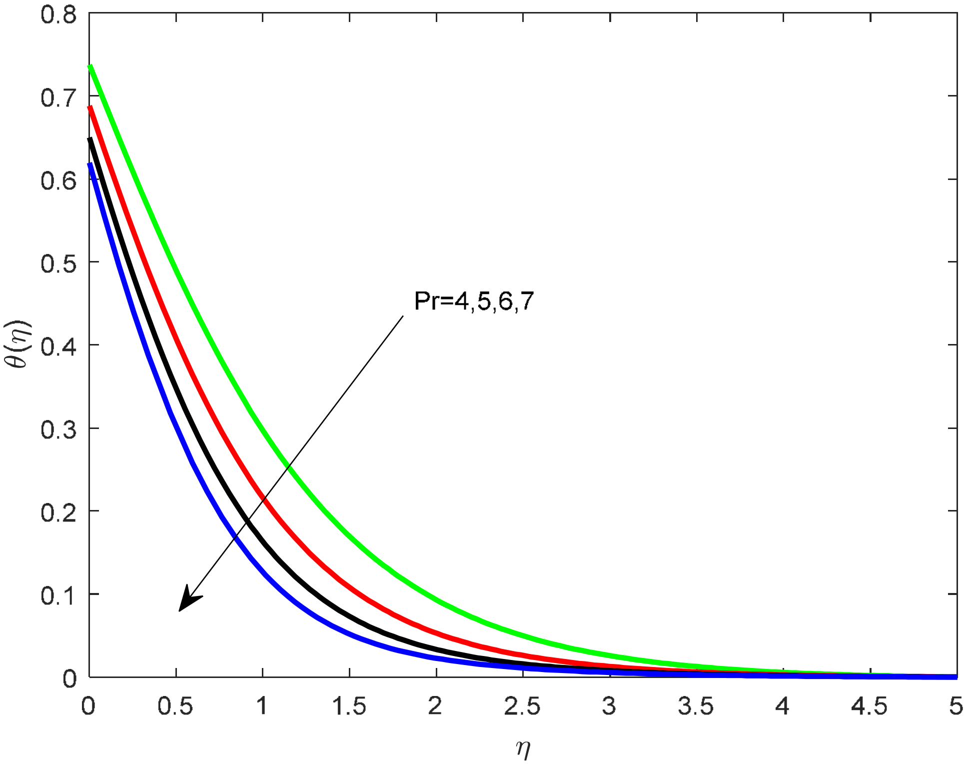

Fig.13 Effect of Pr on thermal field

Table 1 Variations in physical quantities at different pertinent parameters

| $M$ | $Ra$ | $\lambda$ | $h_1$ | $h_2$ | $A^*$ | $\beta$ | $C_f$ | $Nu_x$ |
|---|---|---|---|---|---|---|---|---|
| 0.1 | | | | | | | -0.402640 | 0.222246 |
| 0.4 | | | | | | | -0.454791 | 0.200086 |
| 0.7 | | | | | | | -0.498500 | 0.174230 |
| | 0.2 | | | | | | -0.633381 | 0.668461 |
| | 0.4 | | | | | | -0.633381 | 0.624995 |
| | 0.6 | | | | | | -0.633381 | 0.588461 |
| | | 0.5 | | | | | -0.638598 | 0.659612 |
| | | 1.0 | | | | | -0.647254 | 0.742026 |
| | | 1.5 | | | | | -0.655855 | 0.815724 |
| | | | 0.5 | | | | -0.633381 | 0.605993 |
| | | | 1.0 | | | | -0.474382 | 0.532111 |
| | | | 1.5 | | | | -0.380660 | 0.470960 |
| | | | | 0.5 | | | -0.633381 | 0.605993 |
| | | | | 1.0 | | | -0.633381 | 0.435435 |
| | | | | 1.5 | | | -0.633381 | 0.339799 |
| | | | | | 0.2 | | -0.633381 | 0.649458 |
| | | | | | 0.4 | | -0.633381 | 0.620481 |
| | | | | | 0.6 | | -0.633381 | 0.591505 |
| | | | | | | 0.5 | -0.633381 | 0.605993 |
| | | | | | | 1.0 | -0.721482 | 0.572876 |
| | | | | | | 1.5 | -0.762779 | 0.554049 |

## 4. Conclusions

In this study, we analyzed the convective heat transfer in a 2D magnetohydrodynamic flow of Casson fluid over a non-uniform thickness stretching sheet in the presence of thermal radiation and non-uniform heat source/sink effects. The governing equations of the flow and heat transfer are transformed as the asset of nonlinear ODEs and solved numerically using bvp4c Matlab package. The effect of pertinent parameters on the flow and heat transfer is investigated with the assistance of graphs. Numerical results are computed for the friction factor and reduced Nusselt number. The numerical observations are as follows:

- Nonuniform heat source/sink parameters regulates the thermal boundary layer.
- Rising values of the thermal radiation decline the heat transfer rate and enhances the temperature field.
- Increasing the wall thickness enhances the heat transfer rate.
- Casson parameter has a tendency to reduce the heat transfer rate.
- Slip parameters control the thermal filed.

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