

# Rayleigh-Bénard Convection in a Cell with Aspect Ratio of One

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## Abstract

A large-scale circulation velocity, often called the "wind", has been observed in turbulent convection in the Rayleigh-Benard apparatus with a container of an aspect ratio of unity. Temperature fluctuations measured in different positions inside the container for Rayleigh numbers of the order of 1011 when the flow is fully turbulent, are analyzed with auto and cross correlation. It is found that the trajectory of the mean wind has the shape of one convection cell with the size of the order of the container.

**Keywords:** thermal convection, convection cells, plumes, mean wind, autocorrelation

## 1. Introduction

Thermal turbulence is part of many large natural and engineering systems. These can range from stars to atmospheric and oceanic circulations, and even motion within the Earth's mantle and core. To reduce the problem to one that we can more easily attend to, we introduced a typically simplified model known as Rayleigh-Benard convection (RBC). In RBC, a thin layer of fluid, contained between two horizontal, rigid boundaries is heated from below and cooled from above so that the fluid becomes mechanically unstable to convection. For a Newtonian fluid, the dynamical state of convection is governed by the Rayleigh number  $Ra = g\alpha\Delta TL^3/\nu\kappa$  and the Prandtl number  $Pr = \nu/\kappa$ . Here  $g$  is the acceleration due to gravity,  $\Delta T$  is the vertical temperature difference across the fluid layer of height  $L$ , and  $\alpha$ ,  $\nu$  and  $\kappa$  are, respectively, the thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid. In a finite system with a lateral size  $D$ , the aspect ratio,  $\Gamma = D/L$ , could also be a relevant parameter.

In phenomenological models when RBC becomes turbulent, the fluid layer is divided into three distinct regions, a turbulent core and two thermal boundary layers at the top and the bottom of the heated surface (see Howard and Malkus). These boundary layers are periodic oscillators since a high heat flux initially applied causes their size to grow diffusively (as the square root of time) until they exceed a certain size where they become convectively unstable. The result is an emission of plumes which depletes the boundary layer, starting the process over again in a cyclic manner. This is a fundamental and periodic time-dependence of turbulence that manifests itself in other periodic signatures, such as the circulation frequency of the large scale flow or 'mean wind' which is indeed driven by the plumes. The evidence for the presence of wind is plentiful. The visualization of water flows (e.g., Qiu et al, Qiu and Tong 2001a and Qiu and Tong 2002) directly confirms its existence at Rayleigh numbers of the order of 109. Thus, the aim of this study is to show that the mean wind exists even when convection is more fully turbulent (higher  $Ra$  numbers) and to find the shape of its trajectory. This is done indirectly from low temperature experiments described in details below.

## 2. The apparatus and the measurement of the temperature fluctuations

The data analyzed in this article have been taken from the same apparatus used by Niemela et al, Niemela and Sreenivasan and Sreenivasan et al. Some salient features of the apparatus and the procedures used to obtain the data will be described below.

The apparatus was a closed cylindrical container of 50 cm diameter and 50 cm height, with the aspect ratio of one. The top and bottom walls of the container were made of copper annealed under oxygen-free conditions, having a high thermal conductivity of about 2 kW/mK near 5 K (hence approximating well the condition of an isothermal surface). The thickness of these plates was 3.8cm, and the surface finish was better than 10  $\mu\text{m}$ . Specially designed thin metal film heaters were attached to the back of each copper plate by dilute varnish, and sandwiched by an

additional copper plate of 0.16 cm thickness. A constant heat flux occurred at the bottom plate (special efforts were made to heat it uniformly along its surface) whenever experimental conditions were altered, but measurements were begun in the steady state only after a constant temperature was reached. The top wall was connected to a liquid helium reservoir through a distributed and adjustable thermal link, and its temperature was maintained constant by means of a resistance bridge and servo. The lateral confinement of the fluid was achieved with type 304 stainless steel of thickness 0.267 cm. The conductivity of stainless steel at the operating temperature was approximately 0.25 W/m K. The container was insulated by three thermal shields at various graded temperatures, residing in a common vacuum space. The working fluid was cryogenic helium gas at a temperature of about 5 K.

Temperature fluctuations on the large-scale circulation were measured by a pair of sensors with nearly identical temperature sensitivities, placed a radial distance  $w = 4.4$  cm from the sidewall on the horizontal mid-plane, aligned vertically and separated by a distance  $d=2.54$ cm (see Figure 1), and by another sensor placed on the same plane  $180^\circ$  around the circumference. To check the flow in the center of the cell a sensor was put there too. Our temperature signals were discrete time signals since the acquisition rate of the data was 50 Hz and the number of data points in a single run was  $2^{19}$ . The direction of the wind was arbitrary and presumably fixed by initial perturbations, or perhaps some small experimental artifact.

### 3. Analysis of Temperature Fluctuations

In Figure 2, are shown the temperature fluctuations measured simultaneously from one of the neighboring sensors near sidewall (upper plot) (temperature fluctuations from the other neighboring sensor, not shown, look very similar to the one shown) and the sensor mounted at  $180^\circ$  around the circumference (lower plot), for  $Ra=4 \times 10^{11}$  when the flow is fully turbulent. Clearly plumes of opposite signs are seen in the two sides of the flow: simultaneously one of these two sensors measures a warm plume going up and the other one a cold plume coming down which tells that large scale motion near sidewall has opposite sense. This means that the number of cells established inside the container is odd.

Temperature fluctuations measured at the geometrical center of the container, are analyzed by autocorrelation function which is able to show if a periodic feature is embedded in the measured signal. First the signal is brought in standard form with zero mean and unity standard deviation. The autocorrelation is performed afterwards. The result for almost the same Rayleigh number as above is shown in Figure 3. The lack of periodicity indicates the absence of a large scale motion at the geometrical centre of the cell (see e.g., Kuqali). This is in very good agreement with the result found from the raw temperature data measured near sidewall shown in Figure 2.

The autocorrelation of the signal measured for  $Ra=4 \times 10^{11}$  from one of the neighboring probes near sidewall (shown in the upper plot of figure 2) is presented in Figure 4. A period  $T$  of about 30 s corresponds to the period of the mean wind (see Qiu and Tong 2001b).

Say  $\theta_1(t)$  and  $\theta_2(t)$  are the temperature fluctuations measured simultaneously by the neighboring sensors. If the convective motion consisted only of a steady wind with some periodic modulation, the temperature signals from the sensors near the sidewall would be identical, with a well defined phase lag given by their separation and the magnitude of the wind velocity. Define a short time correlation between  $\theta_1(t)$  and  $\theta_2(t)$  as:

$$G(\tau) = \sum_{i=1}^N \theta_1(t_i) \theta_1(t_{i+n}) \quad (1)$$

(see [Niemela et al and Wu]) where  $t_i$  is the  $i$ -th sample of a digitized signal,  $\tau = t_{i+n} - t_i$ , and the window width  $N$  used for calculating the correlation is approximately equal to one convective turnover time for the large-scale flow (about 30 s) The delay time  $\tau_M$  for which  $G$  has a maximum defines the average phase lag between the two signals  $\theta_1(t)$  and  $\theta_2(t)$ . The mean value of wind velocity  $V$ , is then given by  $V = d/\tau_M$ . The direction of this vertical velocity can be decided by the sign of the delay time (which can be positive or negative) and the relative positions of the sensors. Clearly, the method cannot resolve fluctuations that oscillate more rapidly than  $N$ . Small value of  $N$  yields a large variability of  $G(\tau)$  while a large value diminishes the maximum of  $G(\tau)$ .

Traces of temperature signal measured with one of the two neighboring probes for  $Ra=4 \times 10^{11}$  and of the wind

velocity are shown in Figure 5. The two plots agree with each other. Temperature signal has groups of cold peaks which are cold plumes advected by the descending flow and warm plumes advected by the rising flow.

It can also be seen that the wind reverses its direction following the abrupt change of the temperature of fluid nearby the probes near sidewall. The mean value of velocity is found to be 7 cm/s. Then the mean path length of the wind is defined as:

$P \approx V \times T \approx 7 \text{ cm/s} \times 30 \text{ s} = 210 \text{ cm} \approx 4 \times 50 \text{ cm}$ , where  $4 \times 50 \text{ cm}$  is the approximate perimeter of the cell with aspect ratio of 1, of diameter and height 50 cm. Therefore we conclude that the large scale motion is essentially one roll like: up in one side and down in the other side similar to the sketch shown in figure 1.

#### 4. Conclusions

No clear evidence of periodicity in the autocorrelation function of the temperature fluctuations measured for  $Ra = 1 \times 10^{11}$  at the geometric center of the cell is observed. This indicates that the mean wind at the geometric center of the cell does not exist. On the other hand the autocorrelation of the temperature fluctuations measured near sidewall for almost same Rayleigh number has shown periodicity belonging to the period of mean wind. The wind velocity is found to be about 7 cm/s and the mean length of the wind path is about 200 cm indicating that the mean wind has the shape of one large cell of the order of the size of the container. This result is confirmed by plumes of opposite signs seen in the two sides of the flow measured simultaneously from two sensors placed near sidewalls at mid-height,  $180^\circ$  around the circumference.

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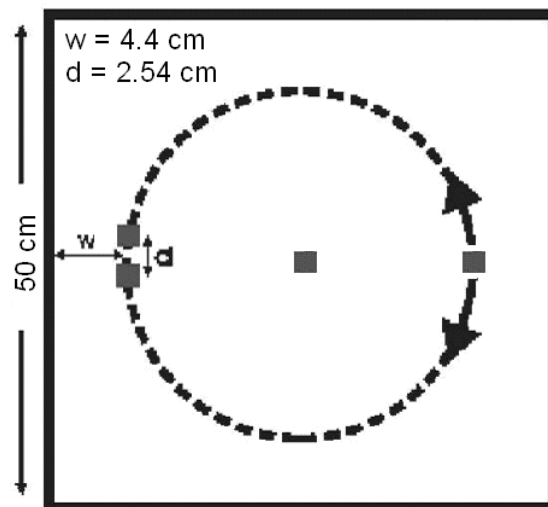


Figure 1. The probes used to measure the temperature fluctuations. They are small (250 micrometer on a side) cubes of doped germanium that have a very sensitive temperature dependent resistance.

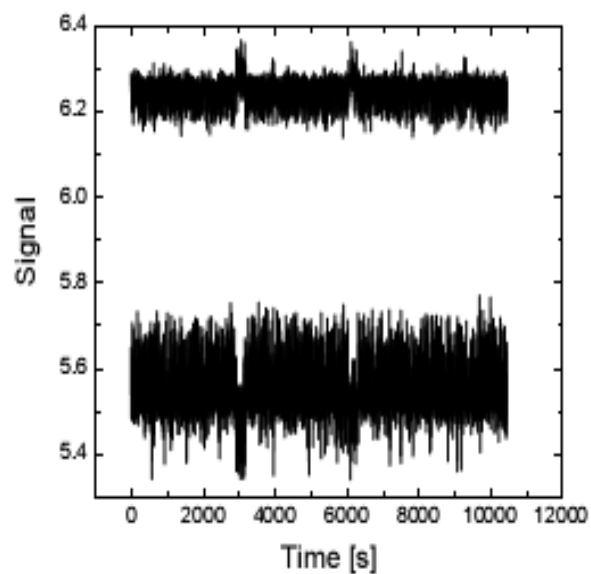


Figure 2. Temperature signal measured simultaneously from two sensors mounted at  $180^\circ$  about the azimuth, 4 cm from sidewall,  $Ra=4 \times 10^{11}$ . They indicate the opposite sense of flow and reversals.

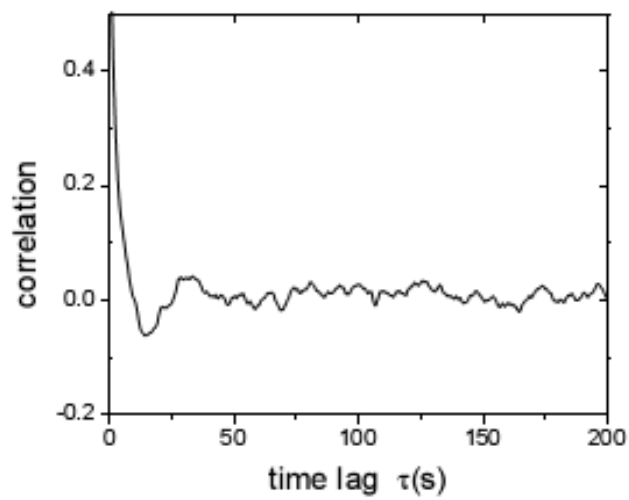


Figure. 3 The autocorrelation as a function of the delay time, of the temperature fluctuations data for  $Ra=1 \times 10^{11}$  taken from the probe placed in the geometrical center of the cell.

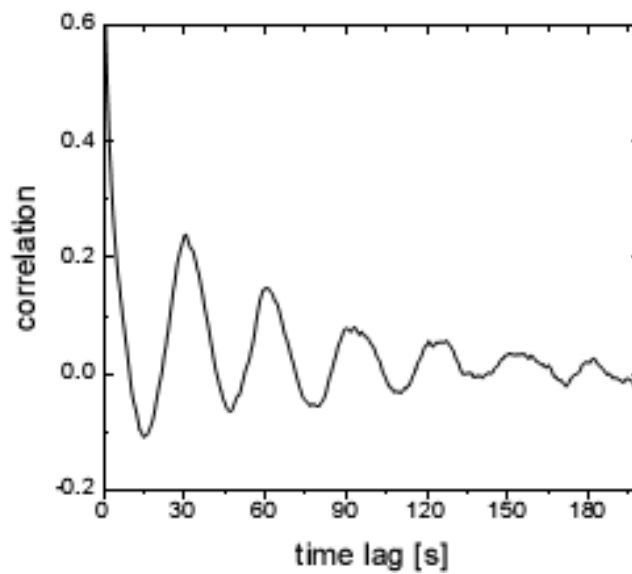


Figure 4. Autocorrelation of the temperature signal for  $Ra=4 \times 10^{11}$ .

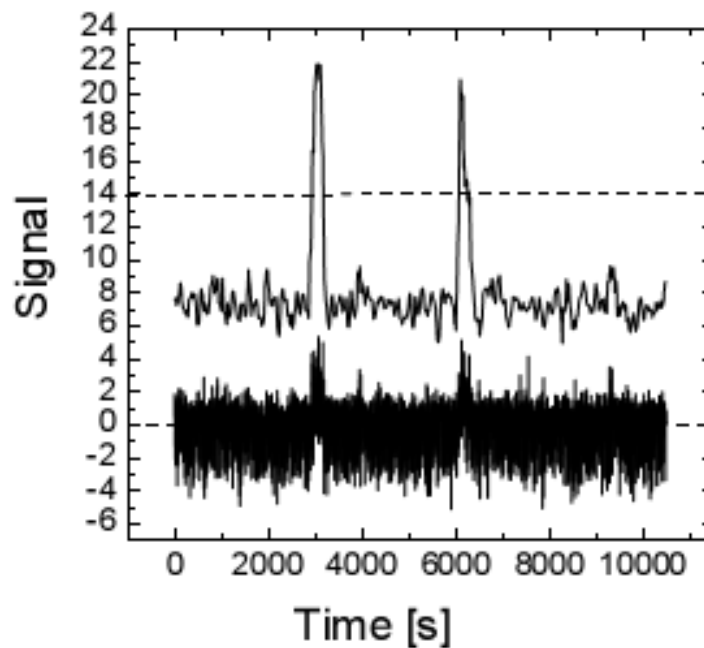


Figure 5. The upper plot shows the wind velocity,  $V(t)$ , smoothed. The lower plot shows the temperature fluctuations.

The plot of velocity is shifted upward by 14 cm/s to avoid overlapping. The two upward spikes in the velocity measurement plot show that the wind changes direction and it corresponds to the relatively warmer fluid passing by the sensors.

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