Superposed Two-mode Squeezed Laser Light Coupled to Squeezed Vacuum Reservoir

Sitotaw Eshete[1*] Misrak Getahun (PhD)[2]
1.Department of Physics, Oda Bultum University, Pobox.226, Chiro Ethiopia
2.Department of Physics, Hawassa University, Hawassa, Ethiopa

Abstract
The squeezing and statistical properties of a pair of superposed non-degenerate three-level lasers coupled to a squeezed vacuum reservoir could be analyzed with the help of the density operator for a pair of superposed two-mode laser light. We have determined the quadrature variances of the cavity modes. The results show that the light produced by the system under consideration is in squeezed state. According to our work, the quadrature squeezing for a pair of superposed laser light beams is found to be the average of the quadrature squeezing of the separate lasers. The presence of squeezed vacuum reservoir enhances the quadrature squeezing significantly.
Keywords: Density Operator, Laser, Reservoir, Squeezing, Two-mode light

1. Introduction

There are several quantum optical systems that could generate light with non-classical features such as squeezing, entanglement, anti-bunching etc.. Squeezing is one of the interesting features of light that has attracted a great deal of interest. In squeezed light, the nose in one-quadrature is below the vacuum level at the expense of enhanced fluctuation in the conjugate quadrature, with the product of the uncertainties in the two quadratures satisfies the uncertainty relation [1-13]. In addition to exhibiting a non-classical feature, squeezed light has application in detection of weak signals and low-noise communications [3-6].

A three-level laser is one of the quantum optical systems that can produce squeezed light under a certain condition. It may be defined as a quantum optical system in which the three-level atoms are injected into a cavity coupled to a vacuum reservoir via a single port mirror. The idea of generating squeezed and entangled light from various schemes of three-level atomic system via coherent superposition has been under study recently [7-13]. When a three-level atom in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, two photons are emitted. If the two photons have the same frequency, the three-level atom is called degenerate otherwise it is called non-degenerate. In this paper, we investigate the squeezing and statistical properties of two-mode laser light beams. It is very important to know about the squeezing properties of light which is more applicable for increment of material sensitivity to detect weak signal and gravitational wave detection.

2. Density operator

We seek to obtain the density operator for the superposition of the light beams produced by a pair of non-degenerate three-level lasers. Suppose $\rho'(t)$ be the density operator for the first two-mode light beam, upon expanding in power series in normal ordering, we have

$$\rho'(t) = \sum_{pqrs} C_{pqrs}\hat{a}_1^{+p}\hat{b}_1^{+q}\hat{a}_1^{r}\, b_1^{r} \tag{1}$$

where C_{pqrs} is the expansion coefficient. Employing the completeness relation for a two-mode coherent state

$$\rho'(t) = \int \frac{d^2\alpha_1}{\pi}\frac{d^2\beta_1}{\pi}\sum_{pqrs} C_{pqrs}|\alpha_1,\beta_1\rangle\langle\beta_1,\alpha_1|\hat{a}_1^{+p}\hat{b}_1^{+q}\hat{a}_1^{r}\, b_1^{r} \tag{2}$$

Moreover, applying the relation

$$|\alpha_1,\beta_1\rangle\langle\beta_1,\alpha_1|\hat{a}_1^{+p}\hat{b}_1^{+q} = \alpha_1^{*p}\beta_1^{*q}|\alpha_1,\beta_1\rangle\langle\beta_1,\alpha_1| \tag{3}$$

and

$$|\alpha_1,\beta_1\rangle\langle\beta_1,\alpha_1|\hat{a}_1^{r}\hat{b}_1^{s} = \left(\alpha_1+\frac{\partial}{\partial\alpha_1^*}\right)^r\left(\beta_1+\frac{\partial}{\partial\beta_1^*}\right)^s|\alpha_1,\beta_1\rangle\langle\beta_1,\alpha_1| \tag{4}$$

Equation (2) can be expressed in terms of the Q function as

$$\rho'(t) = \int d^2\alpha_1 d^2\beta_1 Q_1\left(\alpha_1^*,\beta_1^*,\alpha_1+\frac{\partial}{\partial\alpha_1^*},\beta_1+\frac{\partial}{\partial\beta_1^*}\right)|\alpha_1,\beta_1\rangle\langle\beta_1,\alpha_1| \tag{5}$$

where

$$Q_1\left(\alpha_1^*,\beta_1^*,\alpha_1+\frac{\partial}{\partial\alpha_1^*},\beta_1+\frac{\partial}{\partial\beta_1^*}\right) = \frac{1}{\pi^2}\sum_{pqrs} C_{pqrs}\alpha_1^{*p}\beta_1^{*q}\left(\alpha_1+\frac{\partial}{\partial\alpha_1^*}\right)^r\left(\beta_1+\frac{\partial}{\partial\beta_1^*}\right)^s \ . \tag{6}$$

We note that coherent state can be generated from vacuum state by applying the displacement operator. In view of this, Equation (5) can be rewritten as

$$\rho'(t) = \int d^2\alpha_1 d^2\beta_1 Q_1\left(\alpha_1^*,\beta_1^*,\alpha_1+\frac{\partial}{\partial\alpha_1^*},\beta_1+\frac{\partial}{\partial\beta_1^*}\right)\hat{D}(\alpha_1)\hat{D}(\beta_1)\,\hat{\rho}_0\hat{D}(-\beta_1)\hat{D}(-\alpha_1) \tag{7}$$

In which

$$\hat{D}(\alpha_1) = \mathrm{Exp}[\alpha_1 \hat{a}_1^+ - \alpha_1^* \hat{a}_1], \tag{8}$$
$$\hat{D}(\beta_1) = \mathrm{Exp}[\beta_1 \hat{b}_1^+ - \beta_1^* \hat{b}_1], \tag{9}$$
and
$$\hat{\rho}_0 = |0_\alpha, 0_\beta\rangle\langle 0_\beta, 0_\alpha| \tag{10}$$
is the density operator for two-mode vacuum state at initial time.

On the basis of Eq. (7), the density operator for the superposition of the first light beam and another one is expressible as
$$\hat{\rho}(t) = \int d^2\alpha_2 d^2\beta_2 Q_2\left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}, \beta_2 + \frac{\partial}{\partial\beta_2^*}\right) \hat{D}(\alpha_2)\hat{D}(\beta_2)\, \hat{\rho}'(t)\hat{D}(-\beta_2)\hat{D}(-\alpha_2) \tag{11}$$
where
$$Q_2\left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}, \beta_2 + \frac{\partial}{\partial\beta_2^*}\right) = \frac{1}{\pi^2}\sum_{klmn} C_{klmn}\alpha_2^{*k}\beta_2^{*l}\left(\alpha_2 + \frac{\partial}{\partial\alpha_2^*}\right)^m\left(\beta_2 + \frac{\partial}{\partial\beta_2^*}\right)^n \tag{12}$$
We can write the Q function for the two non-degenerate three-level laser coupled to a squeezed vacuum as the i^{th} laser light beam as
$$Q_2(\alpha_i, \beta_i) = \frac{\mu_i v_i - h_i^2}{\pi^2}\mathrm{Exp}[-\mu_i\alpha_i^*\alpha_i - v_i\beta_i^*\beta_i + h_i(\alpha_i^*\beta_i^* + \alpha_i\beta_i)] \tag{13}$$
where
$$\mu_i = \frac{1+\langle\beta_i^*\beta_i\rangle}{1+\langle\alpha_i^*\alpha_i\rangle+\langle\beta_i^*\beta_i\rangle+\langle\alpha_i^*\alpha_i\rangle\langle\beta_i^*\beta_i\rangle-\langle\alpha_i\beta_i\rangle^2}, \tag{14}$$
$$v_i = \frac{1+\langle\alpha_i^*\alpha_i\rangle}{1+\langle\alpha_i^*\alpha_i\rangle+\langle\beta_i^*\beta_i\rangle+\langle\alpha_i^*\alpha_i\rangle\langle\beta_i^*\beta_i\rangle-\langle\alpha_i\beta_i\rangle^2}, \tag{15}$$
$$h_i = \frac{\langle\alpha_i\beta_i\rangle}{1+\langle\alpha_i^*\alpha_i\rangle+\langle\beta_i^*\beta_i\rangle+\langle\alpha_i^*\alpha_i\rangle\langle\beta_i^*\beta_i\rangle-\langle\alpha_i\beta_i\rangle^2}. \tag{16}$$

3. Quadrature Fluctuations

(i) Quadrature Variances of Cavity Modes

Here we seek to determine the cavity quadrature variance for a superposed two-mode light beam. We define the quadrature variances for a superposed two-mode cavity light beams by
$$\Delta c_\pm^2 = \langle\hat{c}_\pm(t), \hat{c}_\pm(t)\rangle \tag{17}$$
where
$$\hat{c}_\pm = \sqrt{\pm 1}(\hat{c}^+ \pm \hat{c}) \tag{18}$$
The operators $\hat{c}$ and $\hat{c}^\dagger$ are the annihilation and creation operators for the superposed two-mode light with the modified commutation relation
$$[\hat{c}, \hat{c}^+] = 4. \tag{19}$$
On the other hand, we can easily verify that
$$[\hat{c}_+, \hat{c}_-] = 8i. \tag{20}$$
In view of these relations, the superposed two-mode light is said to be in squeezed state if either $\Delta c_+ < 4$ or $\Delta c_- < 4$ such that $\Delta c_+ \Delta c_- \geq 4$. With the aid of Equations (17), (18) and (19), we have
$$\Delta c_\pm^2 = 4 + 2\langle\hat{c}^+(t)\hat{c}(t)\rangle \pm \langle\hat{c}^{+2}(t)\rangle \pm \langle\hat{c}^2(t)\rangle - 2\langle\hat{c}^+(t)\rangle\langle\hat{c}(t)\rangle \mp \langle\hat{c}^+(t)\rangle^2 \mp \langle\hat{c}(t)\rangle^2 \tag{21}$$
We next proceed to determine the expectation values involved in Equation (21). Using the density operator, we write as
$$\langle\hat{c}^+(t)\hat{c}(t)\rangle = Tr(\hat{\rho}(t)\hat{c}^+\hat{c}) \tag{22}$$
Thus employing Equation (11), we have
$$\begin{aligned}\langle\hat{C}^+(t)\hat{C}(t)\rangle &= \int d^2\alpha_1 d^2\beta_1 Q_1\left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}, \beta_1 + \frac{\partial}{\partial\beta_1^*}\right)(\alpha_1^*\alpha_1 + \beta_1^*\beta_1)\\
&+ \int d^2\alpha_2 d^2\beta_2 Q_2\left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}, \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)(\alpha_2^*\alpha_2 + \beta_2^*\beta_2)\\
&+ \int d^2\alpha_1 d^2\beta_1 Q_1\left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}, \beta_1 + \frac{\partial}{\partial\beta_1^*}\right)(\alpha_1^*\beta_1 + \beta_1^*\alpha_1)\\
&+ \int d^2\alpha_2 d^2\beta_2 Q_2\left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}, \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)(\alpha_2^*\beta_2 + \beta_2^*\alpha_2)\\
&+ \int d^2\alpha_1 d^2\beta_1 d^2\alpha_1 d^2\beta_1 Q_1\left(\alpha_1^*, \beta_1^*, \alpha + \frac{\partial}{\partial\alpha_1^*}, \beta + \frac{\partial}{\partial\beta_1^*}\right)\\
&\times Q_1\left(\alpha_1^*, \beta_1^*, \alpha + \frac{\partial}{\partial\alpha_1^*}, \beta + \frac{\partial}{\partial\beta_1^*}\right)(\alpha_1^*\alpha_2 + \alpha_2^*\alpha_1 + \alpha_1^*\beta_2 + \beta_1^*\beta_2 + \beta_2^*\alpha_1 + \beta_2^*\beta_1 + \alpha_2^*\beta_1\\
&+ \beta_1^*\alpha_2) \ldots\ldots (23)\end{aligned}$$
The quadrature variance at steady state found to be

$$\Delta C_{\pm}^2 = 4 + \frac{A_1(1-\eta)(2k + A_1 + 2A_1\eta) - 2A_1^2\eta^2 N}{(2k + A_1\eta)(k + A_1\eta)} \pm \frac{A_1\sqrt{1-\eta^2}\left(2k + A_1 + A_1\eta + 2A_1(N \mp M)\right)}{(2k + A_1\eta)(k + A_1\eta)} + \frac{2[(2k + A_1\eta)^2(N \pm M) + A_1^2(N \mp M)]}{(2k + A_1\eta)(k + A_1\eta)} + \frac{A_2(1-\eta)(2k + A_2 + 2A_2\eta) - 2A_2^2\eta^2 N}{(2k + A_2\eta)(k + A_2\eta)} \pm \frac{A_2\sqrt{1-\eta^2}\left(2k + A_2 + A_2\eta + 2A_2(N \mp M)\right)}{(2k + A_2\eta)(k + A_2\eta)} + \frac{2[(2k + A_2\eta)^2(N \pm M) + A_2^2(N \mp M)]}{(2k + A_2\eta)(k + A_2\eta)} \quad \ldots (24)$$

Where A_i $(i = 1, 2)$ is the linear gain coefficient for the i^{th} laser, and for squeeze vacuum reservoir ($N = \sinh 2r$ and $M = \sinh r \cosh r$) with the probability difference be η.

We now consider some special cases; we first consider the case in which the two-lasers have the same linear gain coefficients. Hence setting $A_1 = A_2 = A$, Equation (67) reduced to

$$\Delta C_{\pm}^2 = 4 + \frac{2A(1-\eta)(2k + A + 2A\eta) - 2A\eta^2 N}{(2k + A\eta)(k + A\eta)} \pm \frac{2A\sqrt{1-\eta^2}\left(2k + A + A\eta + 2A(N \mp M)\right)}{(2k + A\eta)(k + A\eta)} + \frac{4[(2k + A\eta)^2(N \pm M) + A^2(N \mp M)]}{(2k + A\eta)(k + A\eta)} \quad \ldots (25)$$

We note that Equation (25) represents a pair of superposed identical laser light beams. For the case in which the three-level atoms are not injected into the cavity, thus upon setting $A_1 = A_2 = 0$, the quadrature variances described by Equation (25) turns out to be

$$\Delta C_{\pm}^2 = 4 + 8(N \pm M) \quad (26)$$

Equation (26) represents the quadrature variances of a pair of superposed two-mode squeezed vacuum state. Furthermore, setting r = 0, Equation (26) reduced to the quadrature variance for a pair of superposed two-mode vacuum state,

$$\left(\Delta c_{\pm}^2\right)_v = 4. \quad (27)$$

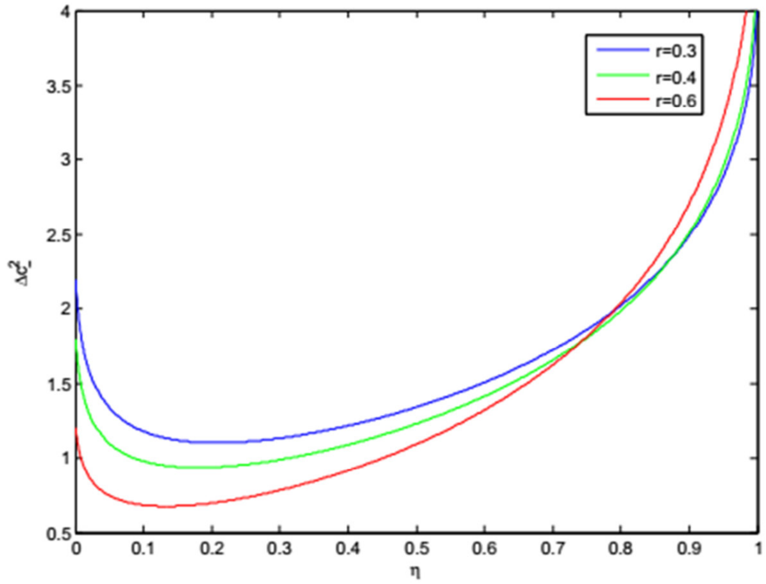

Figure 1: Plots of the minus quadrature variance [Eq. (25)] versus η for $A_1 = 10$, $A_2 = 100$, κ = 0.8 for different values of r.

The plots in Figure 1 represent the quadrature variance versus η for $A_1 = 10$, $A_2 = 100$, k = 0, 8, r = 0.3 (blue curve), r = 0.4 (green curve) and r = 0.6 (red curve). We clearly see from Fig. 1 that the quadrature variance gets more minimum value for larger squeezing parameter, r for small values of η, but decreases for larger values. In general the quadrature squeezing increases as the squeezing parameter, r increases for slightly small values of η.

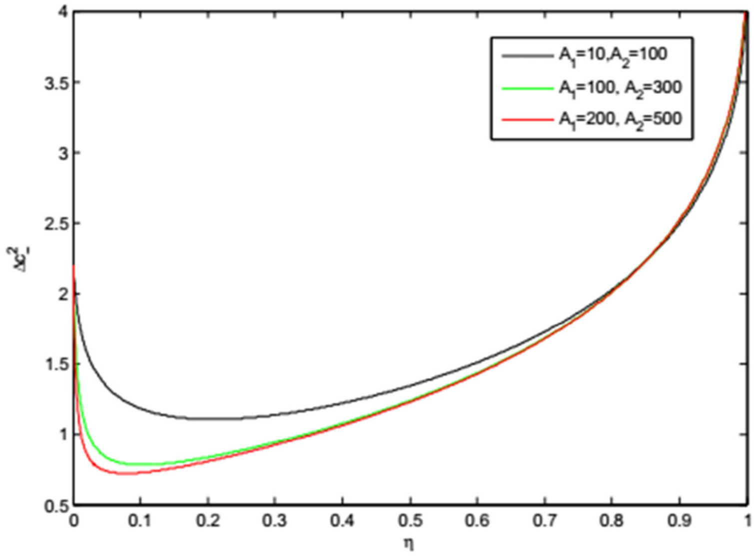

Figure 2: Plots of the minus quadrature variance [Eq. (67)] versus η for κ = 0.8, r = 0.2 and for different values of linear gain coefficients.

The plots in Figure 2 represent the quadrature variance for different values of linear gain coefficients, r = 0.2 and κ = 0.8. As depicted in the figure we note that the quadrature variance attained its small values for larger gain coefficients and slightly small values of η. This implies that the quadrature squeezing increases with linear gain coefficients for η between 0 and 0.85.

Next we determine the quadrature squeezing for a pair of superposed two-mode light beams relative to the quadrature variance for a pair of superposed two-mode vacuum state. The quadrature squeezing of a pair of superposed two-mode cavity light beam is defined by [13].

$$S = \frac{(\Delta C_\pm^2)_v - \Delta C_-^2}{(\Delta C_\pm^2)_v} \tag{28}$$

Thus on account of (27), we get

$$S = 1 - \frac{\Delta C_-^2}{4} \tag{29}$$

Or

$$S = \frac{S_1 + S_2}{2} \tag{30}$$

Where

$$S_i = 4 + \frac{A_i(1-\eta)(2k + A_i + 2A_i\eta) - 2A_i^2\eta^2 N}{(2k + A_i\eta)(k + A_i\eta)} \pm \frac{A_i\sqrt{1-\eta^2}\left(2k + A_i + A_i\eta + 2A_i(N \mp M)\right)}{(2k + A_i\eta)(k + A_i\eta)} + \frac{2[(2k + A_i\eta)^2(N \pm M) + A_i^2(N \mp M)]}{(2k + A_i\eta)(k + A_i\eta)}, \quad i = 1,2 \tag{31}$$

is the two-mode cavity quadrature squeezing of the ith laser light beam. We see from Equation (31) and Figure 3 that the quadrature squeezing of the superposed laser light beams (red curve) is the average of the quadrature squeezing of the component laser light for all values of η between zero and one. We also notice that the degree of squeezing takes higher values for small values of η.

(ii) Quadrature variance of the output modes

The quadrature variances of the output light beams produced by the superposition of a two-mode light beams produced by a pair of non-degenerate three-level lasers is defined by

$$(\Delta C_\pm^2)_{out} = 2\left[(\Delta C_{\pm\,out}^2)_1 + (\Delta C_{\pm\,out}^2)_2\right] \tag{32}$$

with

$$(\Delta C_{\pm\,out}^2)_i = k(\Delta C_\pm^2)_i + (1-k)\,(\Delta C_\pm^2)_{in} \tag{33}$$

and

$$(\Delta C_\pm^2)_{in} = 1 + 2(N \pm M)\,. \tag{34}$$

Using Equations. (33) and (34), the quadrature variance for the superposed output light found to be

$$\left(\Delta C_{\pm}^2\right)_{out} = 2k\Delta C_{\pm}^2 + (1-k)(4+2(N \pm M)) \quad (35)$$

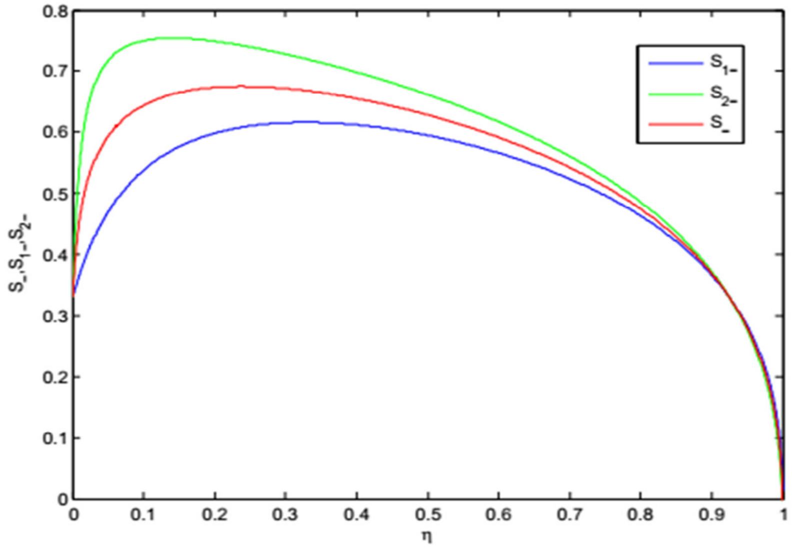

Figure 3: Plots of the quadrature squeezing S [Eq. (30)] and S_{i-} [Eq. (31)] versus η for $A_1 = 10$, $A_2 = 100$, $r = 0.2$ and $\kappa = 0.8$.

4. Photon Statistics

In this section, we seek to study the statistical properties of a pair of superposed two-mode light beams generated by a pair of non-degenerate three-level lasers coupled to a squeezed vacuum reservoir.

(i) Mean of the Photon Number

The mean of the photon number for a pair of superposed two-mode light beams in terms of the density operator can be expressed as.

$$\bar{n} = Tr(\hat{\rho}(t)\hat{c}^+\hat{c}) \quad (36)$$

The mean photon number at steady state written as

$$\bar{n} = \frac{2kA_1(1-\eta)(2k+A_1\eta) + 2kA_1^2(1-\eta^2) + 2k^2N[4k^2+4kA_1\eta+A_1^2] - 4kA_1^2M\sqrt{1-\eta^2}}{4k(2k+A_1\eta)(k+A_1\eta)} + \frac{2kA_2(1-\eta)(2k+A_2\eta) + 2kA_2^2(1-\eta^2) + 2k^2N[4k^2+4kA_2\eta+A_2^2] - 4kA_2^2M\sqrt{1-\eta^2}}{4k(2k+A_2\eta)(k+A_2\eta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots . (37)$$

We notice from Equation (37) that the mean of photon number for a pair of superposed two-mode laser light beams is the sum of the mean photon number of the separate two-mode laser light beams. This implies that the mean photon number for a pair of superposed laser light beams is greater than the separate laser light. So that by superposing a pair of non-degenerate three-level lasers, it is possible to enhance the mean photon number.

(ii) Variances of The Photon Number

Here we calculate the variances of the photon number for the superposed two-mode light beams. We define the photon number variance for a pair of superposed two-mode light beams by

$$\Delta n^2 = \langle(\hat{c}^+(t)\hat{c}(t))^2\rangle - \langle\hat{c}^+(t)\hat{c}(t)\rangle^2 \quad (38)$$

Or

$$\Delta n^2 = \langle\hat{c}^{+2}(t)\hat{c}^2(t)\rangle + 4\langle\hat{c}^+(t)\hat{c}(t)\rangle - \langle\hat{c}^+(t)\hat{c}(t)\rangle^2 \quad (39)$$

We now recall that Gaussian operators with vanishing mean satisfies the relation

$$\langle\hat{A}\hat{B}\hat{C}\hat{D}\rangle = \langle\hat{A}\hat{B}\rangle\langle\hat{C}\hat{D}\rangle + \langle\hat{A}\hat{C}\rangle\langle\hat{B}\hat{D}\rangle + \langle\hat{A}\hat{D}\rangle\langle\hat{B}\hat{C}\rangle \quad (40)$$

Based on this relation, we have

$$\langle\hat{c}^{+2}(t)\hat{c}^2(t)\rangle = \langle\hat{c}^{+2}(t)\rangle\langle\hat{c}^2(t)\rangle + 2\langle\hat{c}^+(t)\hat{c}(t)\rangle^2 \quad (41)$$

Thus one can put Eq. (39) in the form

$$\Delta n^2 = \langle\hat{c}^{+2}(t)\rangle\langle\hat{c}^2(t)\rangle + \langle\hat{c}^+(t)\hat{c}(t)\rangle^2 + 4\langle\hat{c}^+(t)\hat{c}(t)\rangle \quad (42)$$

This shows that unlike that of the mean photon number, the variance of the photon number for a pair of superposed two-mode laser light beams is not the sum of the variances of photon number for the separate two-mode laser light beams.

Figure 4 represents the mean photon number (dashed curve) and the variance of the photon number (solid curve) for a pair of superposed two-mode cavity laser light beams. We clearly see from the figure that the variance of the photon number is greater than the mean photon number. Hence the photon statistics is super-Poissonian. Moreover, the mean photon number for a pair superposed laser light gets higher for small values of η where there is a better squeezing.

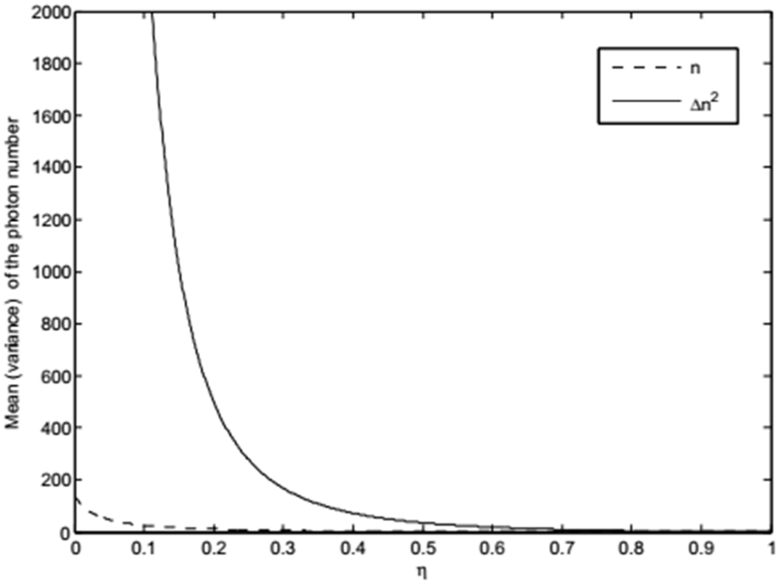

Figure 4: Plots of the mean [Eq. (37)] and variance [Eq. (42)] of the photon number difference for a pair of superposed laser light beams versus η for A1 = 10, A2 = 20, κ = 0.8 and r = 0.3.

5. Conclusion

In this paper, the squeezing and statistical properties of a pair of superposed non-degenerate three-level lasers coupled to a squeezed vacuum reservoir could be analyzed with the help of the density operator for a pair of superposed two-mode laser light. We have also presented slightly modified commutation relation for a pair of superposed two-mode cavity light. Applying the density operator, we have obtained the quadrature variances of a pair of superposed two-mode laser light beams and the superposed two-mode light beams are in squeezed state. It is found that the quadrature squeezing is found to be the average of the quadrature squeezing of the separate two-mode light beams.

We have obtained that the mean of the photon number of a pair of superposed two-mode laser light beams is the sum of the mean photon number of the separate laser light beams. However, the photon number variance of a pair of superposed two-mode laser light does not happen to the sum of the photon number variance of the separate laser light beams. And a pair of superposed two-mode laser light exhibits super-Poissonian photon statistics.

Finally, our result shows that it is possible to obtain bright light in squeezed regimes due to superposition of laser light. Even though, the squeezing for the superposed light be the average values of the separate light beams, the mean photon number for a pair superposed laser light gets higher for small values of η where there is a better squeezing.

References

[1] Fessehakassahun, Fundamentals of Fuantum Mechanics (Lulu, North carolina)(2008).
[2] D.F.Walls and G.J. Milburn, Quantum Optics, Springer-Verlag, Berlin (1991).
[3] Fesseha Kassahun, arxiv:1201.3712v1[quant-ph](2012).
[4] C.C.Gerry and P.L.Knight, Introductory Quantum optics (Cambridge University Press, Cambridge) (2005).
[5] M.S.Chapman et al., phys.Rev. let.**96** (2006) 09360.
[6] Sintayehu Tesfa, arxiv: 1105.287v1[quant-ph](2011).
[7] E.Alebachew, optics commun. **280** (2007) 133-141.
[8] E.Alebachew, optics commun. **281**(2008) 6124-61229.
[9] N.Luand S.Y.Zhu, Phys Rev. A **49**(1994) 48114.
[10] E.Alebachew and K.Fesseha, opt.commun.**265** (2006).
[11] Sintayehu Tesfa, arxiv:0708.2815 [quant-ph] (2007).
[12] T.Yirgashewa and F.Kassahun, PMC Physics B **3** (2010) 1.
[13] S.Getahun, Fundamental Journal of modern physics, Vol.**8** (2015)35-55.