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The Effects of Car Density on the Overall Interaction of the Vehicles Current Traffic Flow Models Case Study at Wolaita Sodo Town

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Abstract
Congestion of vehicular traffic within urban areas is a problem experienced worldwide. It has adverse effects on people quality of life due to delays, accidents and environmental pollution. Congestion is gaining popularity in the **Wolaita Sodo** and quantifying the effects of an additional vehicle joining the traffic stream is a critical issue to determine the toll rates. When additional vehicles enter a crowded roadway they increase travel time for all vehicles. The effect of additional vehicles worsens when the flow is near the capacity of the traffic stream. The speed and flow of the traffic stream are considered as the major variables to quantify these effects. Hence to better understand and quantify this issue it is necessary to accurately model the traffic stream near capacity. A mathematical macroscopic traffic flow model known as Lighthill, Whitham and Richards model appended with a closure nonlinear velocity-density relationship yielding a quasi-linear first order (hyperbolic) partial differential equation as an initial boundary value problem was considered. The aims of this analysis are principally represented by the maximization of vehicles flow, and the minimization of traffic congestions, accidents and pollutions. We present numerical simulation of the IBVP by a finite difference scheme report on the stability and efficiency of the scheme by performing numerical experiments. The computed result satisfies some well known qualitative features of the solution.
Keywords: Density function , quasi-linear, Macroscopic Traffic flow, Numerical simulation.

1. Introduction:

Sodo or **Wolaita Sodo** is a town and separate woreda in south-central Ethiopia. The administrative center of the Wolaita Zone of the Southern Nations, Nationalities, and Peoples Region, it has a latitude and longitude of 6°54′N 37°45′E with an elevation between Template:Convrt above sea level. It was part of the former Sodo woreda which included Sodo Zuria which completely surrounds it; which located in Wolaita Sodo town, 315 km faraway from Addis Ababa via Butajira-Alaba to Wolaita Sodo. The Wolaita zone also represents one of the most densely populated parts of the country, with a high population growth rate.In 2006, rural density varied from 167 persons per square kilometre in the Humbo District in the lowlands, to 746 persons per square kilometre in the Damot Gale District in the highlands (CSA 1998, 2006). These densities exceeded the national density by 2.5 and 11 times respectively in the same year. From 1998 to 2006 alone, density increased by 79 more persons per square kilometre in the zone. This has increased pressure on agricultural land, in a situation in which there is little or no growth in non-farm income opportunities. For instance, the average farm size decreased from 1.59 hectares in 1990/91 (Eshete 1995) to 1.41 hectares in 2006 (own field work).

Road traffic accidents are a major public health concern. In developing countries road traffic accidents are among the leading cause of death and injury. Ethiopia experiences the highest rate of such accidents in Sub-Saharan Africa. Out of all the accidents registered in Ethiopia, Addis Ababa accounts for 60% on average especially in Wolaita zone more 25% on average. Nowadays traffic flow and congestion is one of the main societal and economical problems related to transportation in many Town. Traffic congestion is one of the greatest problems in **walaita sodo Town (sodo Town)** like some other countries of the world since the most densely populated parts of the country. In this respect, countries managing traffic in congested networks requires a clear understanding of traffic flow operations. Increasing attention has been devoted to the modeling, simulation, and visualization of traffic flows to investigate causes of traffic congestion and accidents, to study the effectiveness of roadside hardware, signs and other barriers, to improve policies and guidelines related to traffic regulation and to assist urban development and the design of highway and road systems. The transportation infrastructure is one of the major pillars supporting life in cities and regions. In many large cities the potentialities of extensive development of the transportation network were exhausted over the last decades or are approaching completion. That is why optimal planning of the transportation, improvement of traffic organization and optimization of the routes of public conveyances take on special importance. Solution of these problems cannot do without mathematical modeling of the transportation system.

Traffic phenomena are complex and nonlinear, depending on the interactions of a large number of vehicles. Due to the individual reactions of human drivers, vehicles do not interact simply following the laws of mechanics, but rather show phenomena of cluster formation and shock wave propagation, both forward and backward, depending on vehicle density in a given area. Some mathematical models in traffic flow make use of a vertical queue assumption, where the vehicles along a congested link do not spill back along the length of the link.

Axel [1] presented a hierarchy of multilane traffic flow models and described the derivation of macroscopic multilane traffic flow model. The dimensionless form of the macroscopic traffic flow model for single lane highway was studied [2]. The numerical experiments were performed in order to verify some qualitative traffic flow behaviors with respect to the traffic flow parameters, [3], [4], [5]. The fluid dynamic traffic flow model as an initial boundary value problem (IBVP) with two sided boundary conditions was studied and a new version of the LaxFriedrichs scheme for the fluid dynamic traffic flow model was also presented [6-7]. A computational study of the non-scaled multilane traffic flow model was presented [9].In this research article, we choose a numerical scheme named explicit upwind difference scheme to compute the numerical solution of the two lane traffic flow model as an IBVP. We discredited the scaled two lane traffic flow model by using finite difference formula which leads to the explicit upwind difference scheme. We build up a code in MatLab programming language of scientific computing for the Explicit Upwind difference scheme. We present the density profile as well as computed velocity and flux profiles of our considered model which is scaled(dimensionless) model without transformation back and the scaled model with transformation back by using the Explicit Upwind difference scheme. Some experimental results are also presented for the stability restriction of the numerical scheme for the dimensionless form of two lane traffic flow model.

The macroscopic traffic model developed first by Lighthill and Whitham (1955) and Richard (1956) shortly called LWR model is most suitable for correct description of traffic flow; details can be seen in (Haberman 1977). In this model, vehicles in traffic flow are considered as particles in fluid: further the behaviour of traffic flow is modeled by the method of fluid dynamics and formulated by hyperbolic partial differential equation (PDE).

2. Conservation of the number of vehicles

Considering a traffic flow on section of a road, if there is no on-ramp and off-ramp(i.e. no sinks or sources) within the interval, then the number of cars coming in equals the number of cars going out of the segment (conservation law).

We choose an interval on any particular roadway between say $x = x_1$and $x = x_2$

As shown in Fig1.1

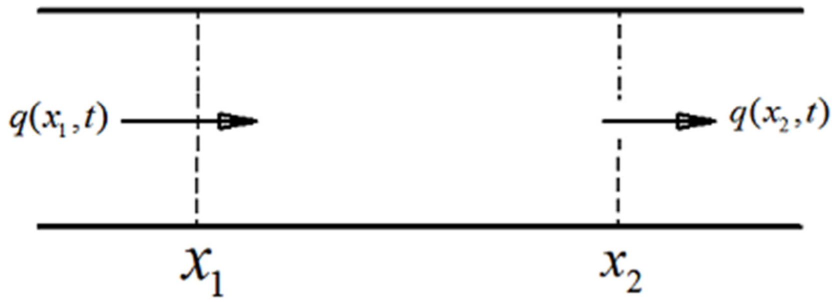

Fig 1.1 Cars entering and leaving a segment of roadway

Source: Richard Habeman, 1977

Our main aim is to take the statement that cars are conserved and turns it into a Partial Differential Equation (PDE). As said earlier we adopt a continuum model of traffic flow rather than modeling individual cars and their flow. We will assume the vehicles to be sufficiently numerous that they can be considered to be distributed continuously from x_1 to x_2.Accordingly, we defined the continuous and differentiable function $\rho(x,t)$ to be the number of cars per unit length of the road at time t and x position and it is called the vehicle density. We also define the Flux of the vehicle,$q(x,t)$as the number of cars passing a position x per unit time at time,t.The number of cars which are in the interval, (x_1 , x_2) denoted by N can be computed from the sum of vehicles in the segment and is equal to.:

$$N(t) = \int_{x_1}^{x_2} \rho(t,x)dx \qquad (1)$$

If more cars flow into the segment (x_1 , x_2) than flow out of it, the number of cars within the segment will increase, and similarly if more cars flow out than in, the number of cars will decrease. Assuming no vehicles are created or destroyed within the segment then mathematically the rate of change of the number of cars in the given segment of the road with respect to time,$\frac{dN}{dt}$, is equal to the difference between the cars entering and those leaving the section through its two ends, as illustrated in the equation below since the rate of change of the number of cars per unit time is the traffic flow at position x_1 minus the traffic flow at position x_2 both at time ,t:

$$\frac{dN}{dt}=q(x_1,t)-q(x_2,t) \quad (2)$$

Taking the derivative of both sides of equation (1) with respect to time gives the following:

$$\frac{dN}{dt}=\frac{d}{dt}\int_{x_1}^{x_2}\rho(x,t)dx \quad (3)$$

By equating equation (2) and equation (3), you get the result:

$$\frac{d}{dt}\int_{x_1}^{x_2}\rho(x,t)dx=q(x_1,t)-q(x_2,t) \quad (4)$$

In order to carry out thorough analysis of traffic, a conservation law in partial
Differential form is required. Now, taking the partial derivative of the right hand side of equation (4) with respect to,x and then taking the integral from $x=x_2\ to\ x=x_1$ gives the following equation:

$$\frac{d}{dt}\int_{x_1}^{x_2}\rho(x,t)dx=-\int_{x_1}^{x_2}\frac{\partial q(x,t)}{\partial x}dx \quad (5)$$

Moving the negative sign inside of the integral gives:

$$\frac{d}{dt}\int_{x_1}^{x_2}\rho(x,t)dx=\int_{x_1}^{x_2}\frac{-\partial q(x,t)}{\partial x}dx \quad (6)$$

We can now move the $\frac{d}{dt}$ inside of the integral to get the following equation; we can do this because derivatives and integrals are interchangeable. If you move the derivative inside the integral and it has a function of two variables, then the derivative becomes a partial derivative:

$$\int_{x_1}^{x_2}\frac{\partial}{\partial t}\rho(x,t)dx=\int_{x_1}^{x_2}\frac{-\partial q(x,t)}{\partial x}dx \quad (7)$$

Equation (7) implies:

$$\int_{x_1}^{x_2}\left(\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}\right)dx=0 \quad (8)$$

3. Velocity as a function of density

There are many factors that have an effect on the speed at which a car can go since it is operated by an individual. The person operating one car may want to drive faster than another person in a different vehicle. Once the traffic becomes a lot heavier, however, lane changing and speed are at a minimum for every driver on the road since it is difficult to change lanes when there are more vehicles on the road and it is not always possible to go the speed you want when there are more vehicles on the road. A lot of times you get stuck going.

With all of these types of observations, we can make a simplifying assumption that at any point along the road the velocity of a car only depends on the density of cars. This is illustrated in the equation below, which was mentioned above in the explanation of the conservation of cars: the same speed at which the flow of traffic is moving.

The equation we now have for vehicle conservation,

$$\frac{\partial\rho}{\partial t}+\frac{\partial q}{\partial x}=0$$

is one relation involving two unknowns. Conventionally, we would need another relation to close the system in two unknowns. A major assumption that is often made by traffic modelers is that **velocity may be reasonably assumed to be a function of the density alone.** That is, we can assume $u=u(\rho)$ and our equation becomes a relation in ρ and its derivatives:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u(\rho))}{\partial x}=0. \quad (9)$$

Such an equation is called partial differential equation (PDE) of first order. It is a PDE because of the two variables involved, t, and the partial differentiations with respect to these variables. It is a first-order equation because only first partials are involved. To see this set $F(\rho)=\rho u(\rho)$. Then the equation (9) can be written

$$\frac{\partial\rho}{\partial t}+F'(\rho)\frac{\partial\rho}{\partial x}=0 \quad (10)$$

We will focus later on how to solve this equation once $u(\rho)$ is given. For the moment our concern is whether or not this assumption is justified, and then what the function $u(\rho)$ should be. On a single-lane open road this assumption seems to be fairly reasonable. An isolated car tends to have a maximum velocity of travel, either the result of speed limits or road conditions or driver caution, call it u_{max} .Then for our function $u(\rho)$ we should take u(0) = u_{max}. We know that traffic speeds tend to go down with increasing traffic density, so we should assume that

$$\frac{du}{d\rho}<0,\rho>0.$$

Also there is surely a density, bumper to bumper traffic say, where the speed is essentially zero. Call this density ρ_{max} If L is average car length, we could take $\rho_{max} = 1/L$. One widely used relation is $u(\rho) = u_{max}(1 - \frac{\rho}{\rho_{max}})$.

$$u(\rho) = u_{max}(1 - \frac{\rho}{\rho_{max}}) \quad (11)$$

Most of our discussion will concern the model of traffic flow which results from using (11).

Numerical simulations

Result

Vehicle Factors: Vehicle type was found to be an important factor which affects human injury/fatality caused by traffic density. Even if buses and taxies/ minibuses play an essential role in public transportation, our results showed that these vehicles (in addition to automobiles, cargo vehicles and buses) pose a significantly greater fatal/serious injury risk to pedestrians.

Note that cargo vehicles have larger mass, greater momentum and longer stopping distance. For any given speed, the greater the mass of the vehicle, the greater would be its force of impact at collision with the pedestrians leading to higher injury severities. Furthermore, it is possible that drivers of small vehicles but high speedy are more likely to weave around in traffic, change lanes, dart ahead of others or even take corners and curves faster. This finding seemed to be in accordance with other studies (Tewolde, 2007).

The study of this paper is used to understand traffic behavior in Wolaita sodo Zuria context in order to answer to several questions: where to install traffic lights or stop signs; how long the cycle of traffic lights should be; where to construct entrances, exits, increasing number of lane and overpasses in Wolaita sodo Town.

There is no personal preference for any observed station used in this paper; the selected stations include on-ramps, off-ramps, and basic freeway segments with varying number of lanes and traffic conditions. The fact that deterministic models have deficiencies over a certain portion of density ranges is well-known [17]. This can be verified from the empirical results here in Fig. 1 and 3. A. D. May pointed out in his book [17] that a disconcerting feature of deterministic models is their inability to track the empirical data in the vicinity of capacity condition. From our results Fig. 1 and 3, we find that the stochastic speed-density model tracks the empirical data faithfully and works consistently well over the whole range of densities. Though in progress, the proposed stochastic speed-density model strives to overcome some of the well-known drawbacks of deterministic models.

Fig.2 Shows that the proposed stochastic speed-density model is also capable of capturing the dynamics at on-ramps and off-ramps. 4005005 is a single lane ,while 4005006 is a single lane off-ramp .From the empirical results of 22 on-ramps and off-ramps, the fundamental relationship has a similar shape but with distinguishable features from that of the basic freeway segments. One observation is that the speed-density curve on basic freeway segments is location based rather than freeway based. The speed-density curves at on-ramps and off-ramps are ramp geometry and characteristic based including, but not limited to, the ramp speed limit, number of lanes, ramp elevation,

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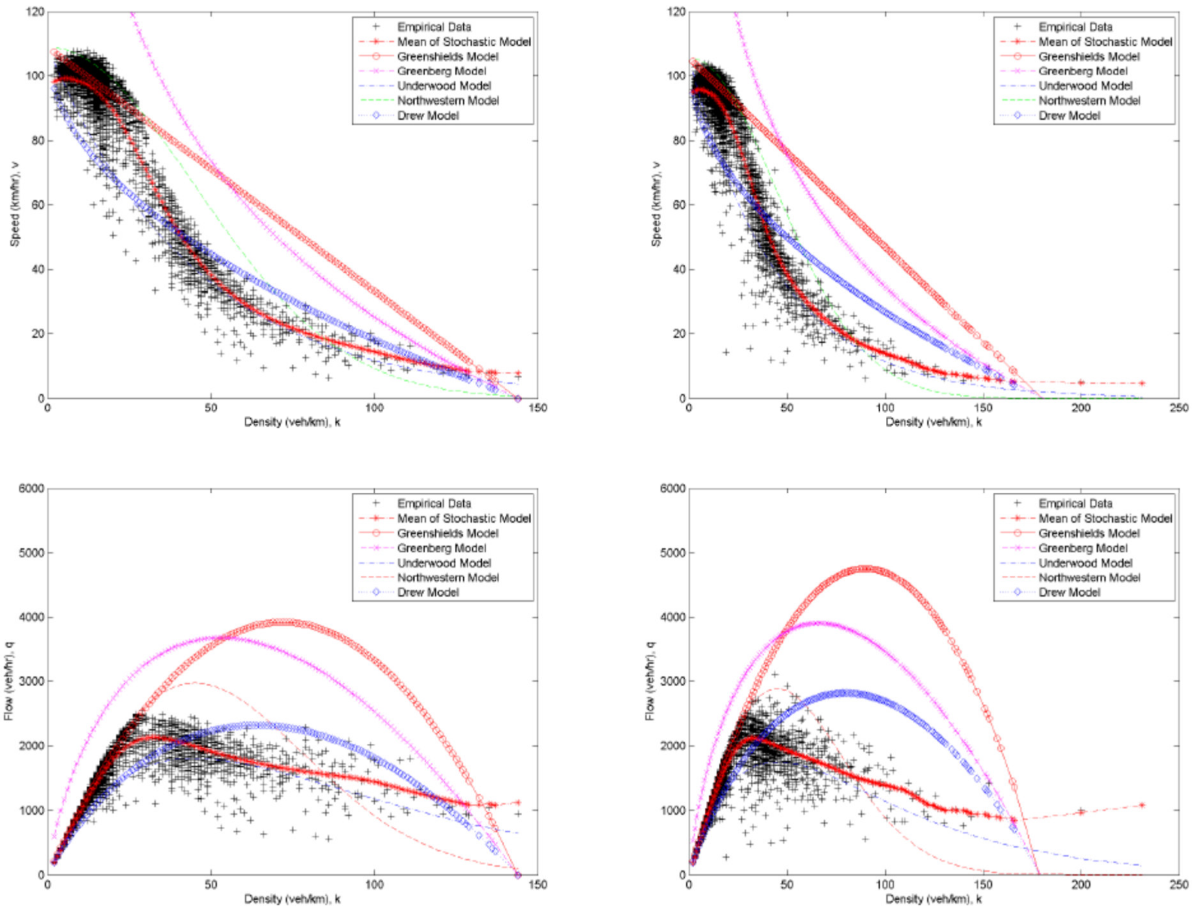

Fig 1Performance Comparison of Different Speed-Density Models at 4001118 (four lanes) and 4001119 (four lanes)

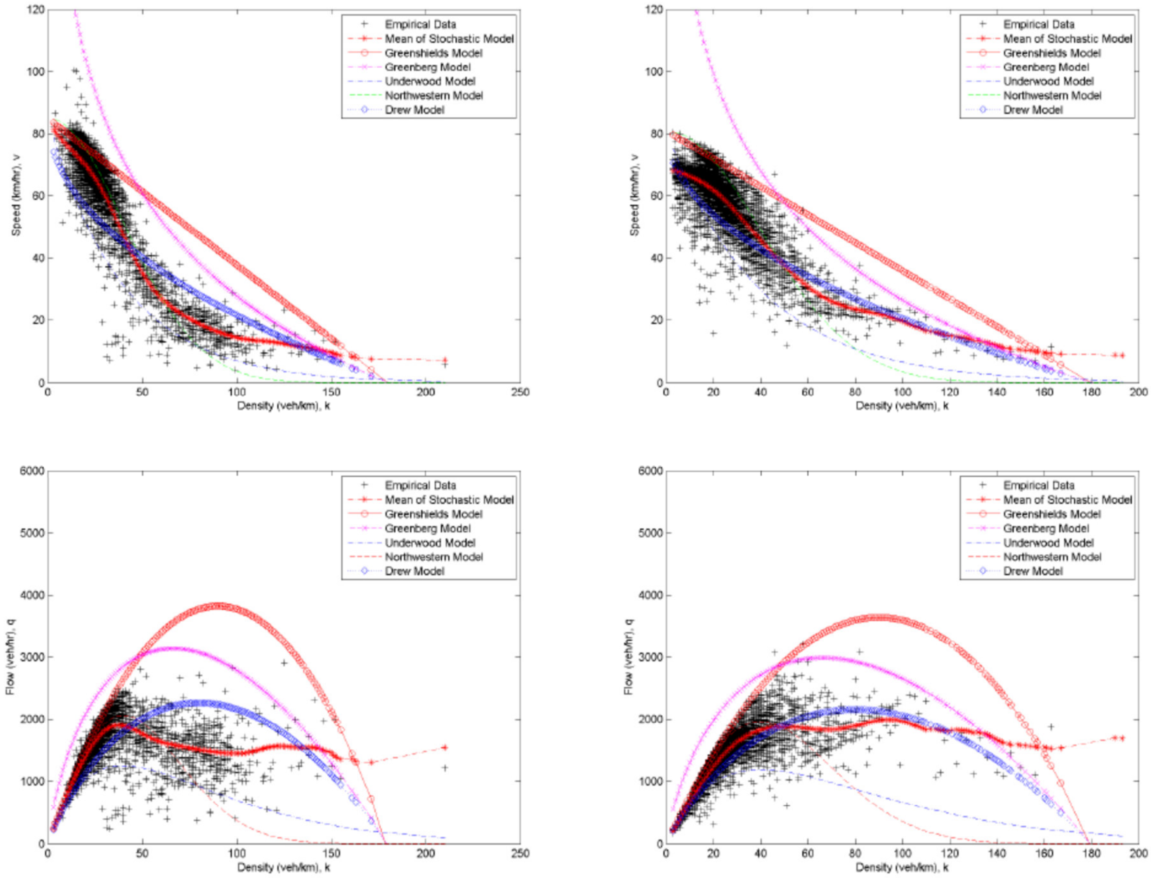

Fig 2 Comparison of Different Speed-Density Models at 4005005 (on-ramp) and 400600 (off-ramp)

Fig. 4 compares the simulated speed-density model with empirical speed-density relationships at two stations (4001118 and 4001119). Simultaneously, the mean and variance of the simulated speed-density models and the empirical speed-density data are plotted. The results show a fairly good match between the simulation of the stochastic speed-density models and the empirical speed-density observations. By doing this, we want to demonstrate the performance of the proposed stochastic speed-density model. Most importantly, we want to demonstrate the model's capability to track empirical data, and its robustness to work consistently at varying traffic conditions. The point we want to conclude is that the proposed stochastic speed-density model potentially performs better than deterministic models by taking care of second-order statistics. Fig. 5 took two stations,Similarly, the results further verified the arguments and assumptions underlying the proposed stochastic speed-density model.

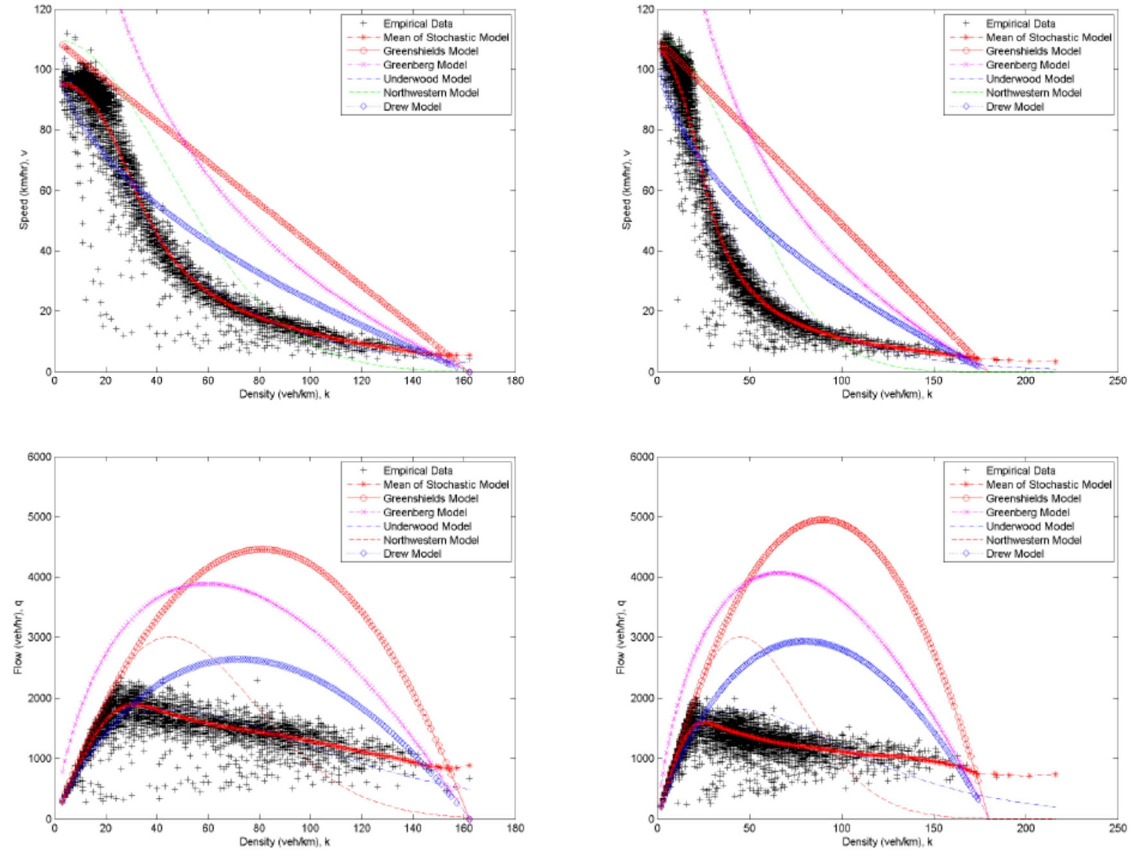

Fig 3 Comparison of Different Speed-Density Models and Corresponding Flow-Density Models at 4000058 (two lanes) and 4000059 (three lanes)
Models at 4000058 (two lanes) and 4000059 (three lanes)

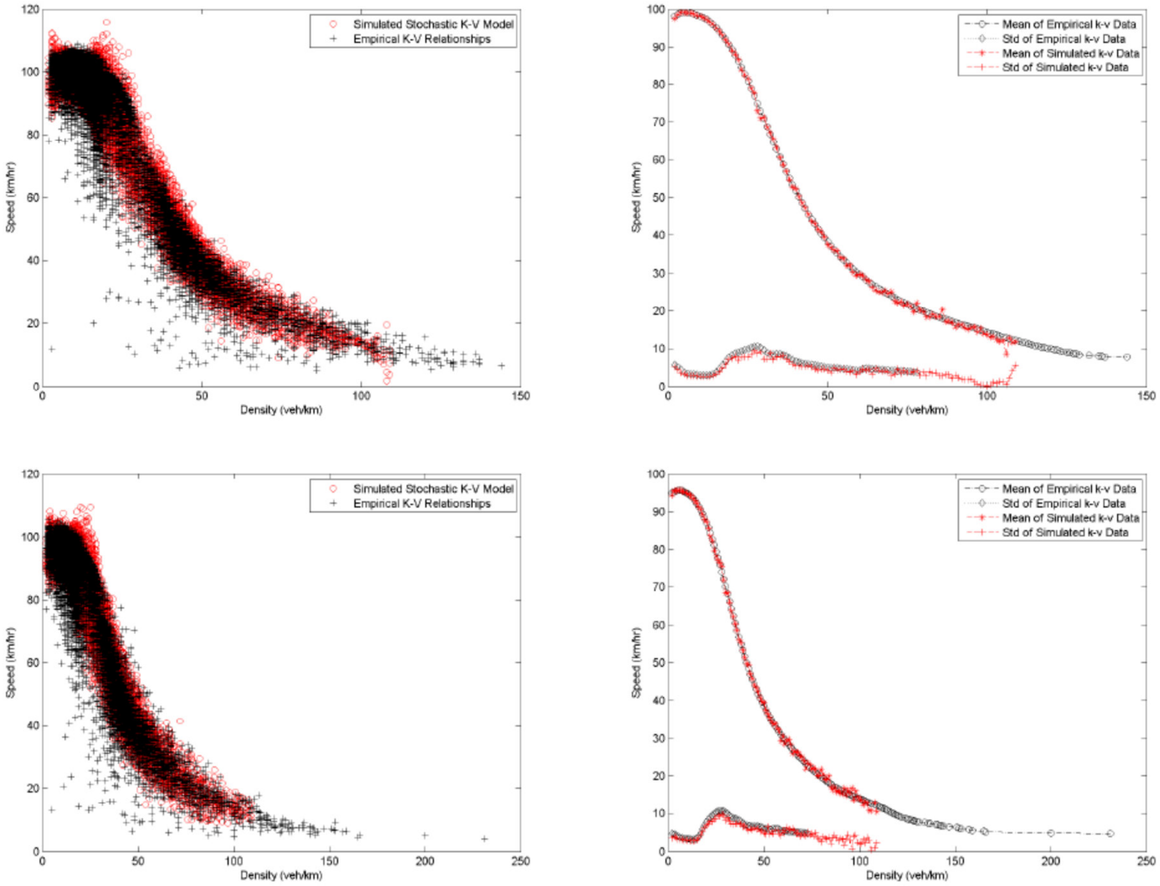

Fig 4 Stochastic Simulation of Speed-Density Relationship and its Corresponding Simulated and Empirical Mean, Standard Deviation at 4001118 (four lanes) and 4001119 (four lanes)

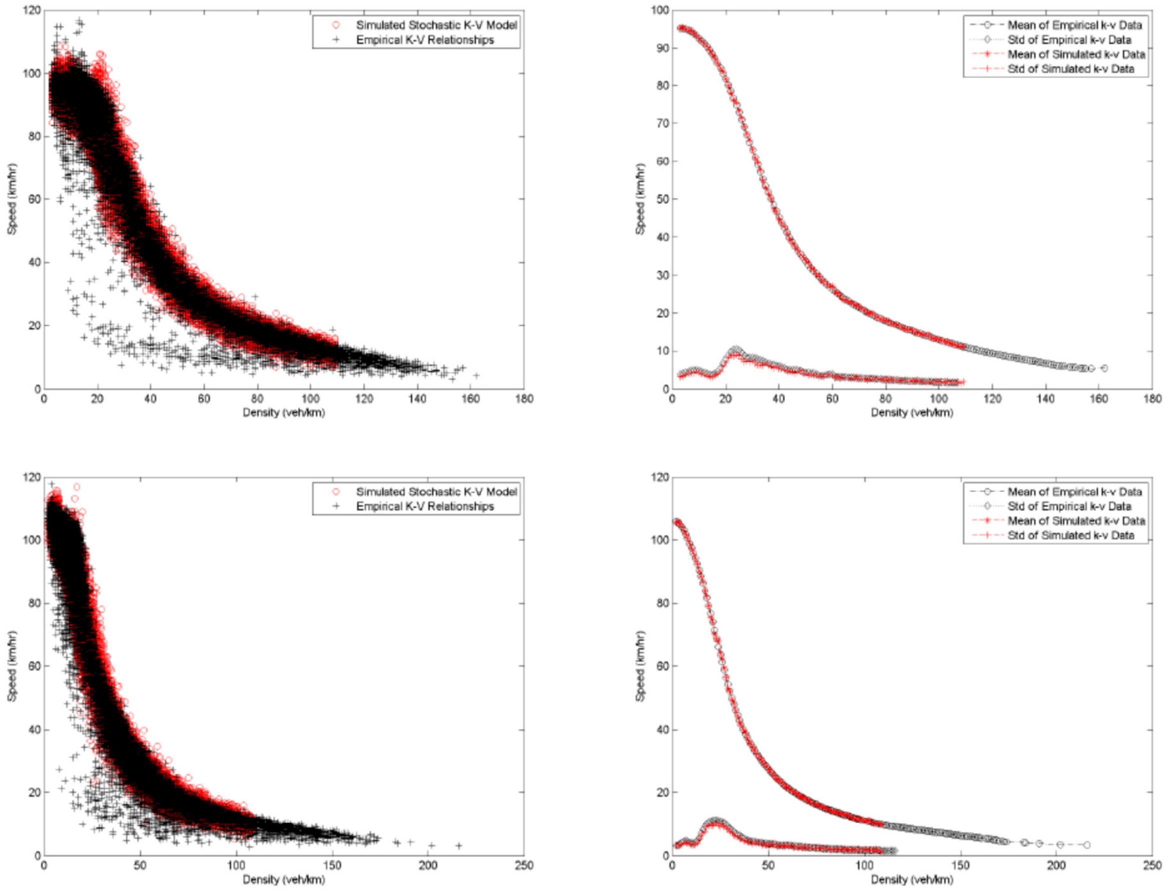

Fig 5 Stochastic Simulation of Speed-Density Relationship and Corresponding Simulated and Empirical Mean, Standard Deviation plotted for Comparison at 4000058 (two lanes) and4000059 (three lanes)

3.Conclusion

Though deterministic speed-density relationship models can explain physical phenomenon'sunderlying fundamental diagrams, the stochastic model is more accurate and more suitable to describe traffic phenomenon. From the results of stochastic model, we find out that a stochastic speed-density model matches the empirical observations better than deterministic ones. Following from this result, the LWR type conservation equation can be revisited in a stochastic setting of speed-density relationship. The LWR model in a stochastic setting is potentially capable to capture some interesting features (i.e., spontaneous congestion) where deterministic models fail. Another benefit from a stochastic speed-density model is its capability to perform real-time on-line prediction while deterministic models are claimed to be insufficient. The simulation model presented herein is useful to simulate the changes of traffic conditions with the employment of control strategies. Various Advanced Traffic Management Systems (ATMS) control strategies can be developed in conjunction with the simulation model. The freeway simulation model provides an essential requirement for the successful development of a comprehensive.

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References

[1] Jr., S. J. Y. Smith, J. M. Modeling Circulation Systems in Buildings Using State Dependent Queueing Models,Queueing Syst., 1989, 4, pages: 319-338.

[2] Rajat, J. Smith, J. Modeling Vehicular Traffic Flow using M/G/C/C State Dependent Queueing Models. Transportation Science, 1997, 31, pages: 324-335.

[3] Gabor S, C. I. The analogies of highway and computer network traffic. Physica A, 2002, 307, pages: 516-526.
[4] Helbing, D. Traffic and Related Self-Driven Many-Particle Systems. Rev. Mod. Phys., 2001,73,1067-1141.
[5] Chowdhury, D. Schadschneider, A., Nishinari, K. Physics of Transport and Traffic Phenomena in Biology: from molecular motors and cells to organisms. Physics of Life Reviews, 2005, 2, 318.
[6] Roger G. Ghanem, P. D. S. Stochastic Finite Elements: A Spectral Approach. Springer-Verlag, New York, 1990.
[7] Jia Li, Qian-Yong Chen, H. Wang. D. Ni. Investigation of LWR model with flux function driven by random free flow speed. Symposium on The Greenshields Fundamental Diagram: 75 Years Later, 2008.
[8] Gazis, D. C. The Origins of Traffic Theory,Operation Research, INFORMS, 2002, 50, pages: 69-77.
[9] Greenshields, B. D. A Study in Highway Capacity. Highway Research Board Proceedings, 1935, 14, pages: 448-477.
[10] Greenberg, H. An analysis of traffic flow. Operation Research, 1959, 7, pages: 79-85.
[11] Underwood, R. T. Speed, Volume, and Density Relationship. Quality and Theory of Traffic Flow, Yale Bur. Highway Traffic, New Haven, Connecticut, 1961, pages: 141-188.
[12] J. S. Drake, J. L. Schofer, A. D. May. A Statistical Analysis of Speed Density Hypotheses.Third International Symposium on the Theory of Traffic Flow Proceedings, Elsevier North Holland, Inc. New York, 1967.
[13] Pipes, L. A. Car-Following Models and the Fundamental Diagram of Road Traffic. Transportation Research, 1967, 1, pages: 21-29.
[14] Drew, D. R. Traffic Flow Theory and Control, McGraw-Hill Book Company, 1968, Chapter 12.
[15] Douglas I. Rouse. Stochastic Modeling of Plant Disease Epidemic Processes, Handbook of Applied Mycology Sail and Plants, Vol. 1 edited by Dilip K. Arora etc, 1991.
[16] Lu Sun, J. Z. Development of Multiregime Speed-Density Relationship by Cluster Analysis. Transportation Research Record, 2005, Volume 1934, pages: 64-71.
[17] May, A. D. Traffic Flow Fundamentals, Prentice Hall, Englewood Cliffs, 1990.
[18] Hall, F.L., Hurdle, V.F., Banks, J.H. A synthesis on recent work on the nature of speedflow and flow-occupancy (or density) relationships on freeways. Transportation Res. Rec. 1365, pp. 12-18, 1992.
[18] J. S. Drake, J. L. Schofer, D. A. May, A statistical analysis of speed-density hypotheses, Highway Research Record 156 (1967) 53{87.
[19] A. Ceder, A deterministic tra_c ow model for the two-regime approach 567 (1976) 16.
[20] A. D. May, Tra_c Flow Fundamentals, Prentice Hall, 1990.
[21] H. V. B. J. Hall, F.L., A synthesis on recent work on the nature of speed-ow and ow-occupancy (or density) relationships on freeways, Transportation Res. Rec. 1365 (1992) 12{18.
[22] M. J. Cassidy, Bivariate relations in nearly stationary highway tra_c, Transportation Research Part B: Methodological 32 (1998) 49{59 (11).
[23] D. I. Rouse., Stochastic Modeling of Plant Disease Epidemic Processes, Handbook of Applied Mycology Sail and Plants, Vol. 1, 1991.
[24] J. H. Banks., Review of empirical research on congested freeway ow, Transportation Res. Rec. 1802 (2002) 225{232.
[25] D. C. Gazis, R. Herman, R. B. Potts, Car-following theory of steady-state tra_c ow, Operations Research 7 (4) (1959) 499{505. URL http://www.jstor.org/stable/166948
[26] D. C. Gazis, R. Herman, R. W. Rothery, Nonlinear follow-the-leader models of tra_cow, Oper. Res. 9 (1961) 545.
[27] L. A. Pipes, Car following models and the fundamental diagram of road tra_c, Transportation Research 1 (1967) 21{29. doi:doi:10.1016/0041-1647(67)90092-5.
[28] M. V. Aerde, Single regime speed-ow-density relationship for congested and uncongested highways, in: The 74th TRB Annual Conference, Washington D.C., Transportation Research Board, 1995.
[29] G. F. Newell, Nonlinear e_ects in the dynamics of car following, Operation Research 9 (1961) 209.
[30] M. J. MacNicholas, A simple and pragmatic representation of tra_c ow, in: Symposium on The Fundamental Diagram: 75 years, Transportation Research Board, Woods Hole, MA, 2008.
[31] B. S. Kerner, P. Konh auser, Structure and parameters of clusters in tra_c ow, Phys. Rev. E 50 (1) (1994) 54{83. doi:10.1103/PhysRevE.50.54.
[32] J. Yi, H. Lin, L. Alvarez, R. Horowitz, Stability of macroscopic tra_c ow modeling through wavefront expansion, American Control Conference, 2002. Proceedings of the 2002 2 (2002) 1484{1490 vol.2. doi:10.1109/ACC.2002.1023231.
[33] J. M. D. Castillo, F. G. Ben__tez, On the functional form of the speed-density relationship{i: General theory, Transportation Research Part B: Methodological 29B (1995) 373{389.
[34] J. M. D. Castillo, F. G. Ben__tez, On the functional form of the speeddensity relationship{ii: Empirical investigation, Transportation Research Part B:Methodological 29B (1995) 391{406.

[35] Z. F. M. Li, A generic characterization of equilibrium speed-ow curves, Transportation Science 42 (2008) 220{235.
[36] L. C. Edie, Car-following and steady-state theory for noncongested tra_c, Operation Research. 9 (1961) 66.
[37] L. Sun, J. Zhou, Development of multiregime speed-density relationship by cluster analysis., Transportation Research Record 1934 (2005) 64{71.
[38] D. S. Berry, D. M. Belmont, Distribution of vehicle speeds and travel times, in: Proc. Second Berkeley Symp. on Math. Statist. and Prob. (Univ. of Calif. Press), 1951, pp.589{602.
[39] A. L. Soyster, G. R. Wilson, A stochastic model of ow versus applied to tra_c ow on hills, Highway Research Record 456 (1973) 28{39.
[40] J. P. Kharoufeh, N. Gautam, Deriving link travel-time distribution via stochastic speed processes, Transportation Science 38 (1) (2004) 97{106. doi:10.1287/trsc.1030.0048.
[41] D. Ghosh, G. Iaccarino, Applicability of the spectral stochastic _nite element method in time-dependent uncertain problems, Tech. rep., Center for Turbulance Research, Standford University (2007).
[42] G. Stefanou, The stochastic _nite element method: Past, present and future, Comput. Methods Appl. Mech. Engrg 198 (2009) 1031{1051. doi:10.1016/j.cma.2008.11.007.
[43] C. Paciorek, Nonstationary gaussian processes for regression and spatial modelling, Ph.D. thesis, Carnegie Mellon University, Pittsburgh, Pennsylvania (2003).