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The electric field strength above atmospheric surface duct

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Abstract

The paper presents a method which allows the calculation of the atmospheric distortion of radar pules, provided that the influence of the atmosphere is to transfer the transmitted signal through a duct. The polarization of the primary sources, whose moment varies arbitrarily in time, is chosen in such a way that it allows the exact determination of the electric field strength at some field point above the duct layer. We can determinate the transient behavior of the electric field strength at any distance above the duct.

Keywords: Electromagnetic field; Atmospheric surface duct

1. Introduction

Historically, in the problem of electromagnetic radiation from a vertical dipole situated at a certain height h above a plane earth, all field quantities are usually assumed to vary harmonically in time. Sommerfeld (1909), calculated the electromagnetic radiation from an electric vertical dipole, located above the plane interface of two media. Many writers, Wait (1970), Moore & Blair (1961) and Durrani (1964) have considered this problem; the aim of the present work is to extend the study-state to transient excitation when no restrictions on the distance between receiving and transmitting ends are made. Two integral transforms are applied to analyse the transient field of vertical electric dipole above a dielectric layer. The distinction of different cases where the distance between the receiving and transmitting ends is great and lesser than the total reflection distance studies by Abo-Seliem (1998). The problem has been studied by Arutaki & chiba (1980) and Abo-Seliem (2003). This integral is estimated by using the steepest descent method, along the couture **r** and around the branch-cuts, from the obtained result. The Saddle point method show that the reflected waves and integrals (Abo-Seliem 2004), the component of the electric field strength is also arbitrary for the excitation function $\mathbf{F(t) = t}$ at some fixed but arbitrary position from the point of observation in the half-space.

2. Formulation of the problem

As show in Fig.1, the duct model of Kahan & Eckart (1950). A dielectric layer is assumed of relative permittivity over laying an infinitely conducting plane earth which is confined by the plane of a rectangular coordinate system.

The source of the field is assumed to be a vertical electric dipole in the medium 1 at the point $\mathbf{x = y = 0}$, $\mathbf{z = d > 0}$ whose moment is given by $\tilde{\boldsymbol{\pi}}_e = \mathbf{(0,0,F(t)\delta(x,y,z-d))}$, t being the time variable and the three dimensional-distributions. Regarding, we make the assumptions $\mathbf{F(t) = 0}$ for $\mathbf{t \le 0}$ and $\mathbf{dF(t)/dt = 0}$ for $\mathbf{t = 0}$.

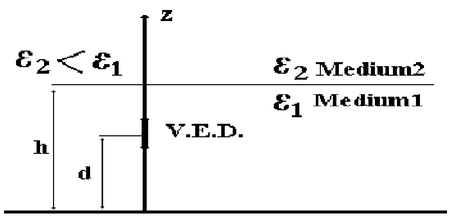

Fig.1: Geometric of the problem.

3. Method of solution

The starting point is the wave equation for the electrical field $\vec{\mathbf{E}} = (\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{t})$ in the two media:

$$[\nabla^2 - v_i^{-2}\partial_t^2]\vec{E} = \begin{cases} 0 & for\ i=2 \\ \mu_0 D_t^2 F(t)\delta(x,y,z-d)\vec{e}_z - \dfrac{F(t)}{\varepsilon_0\varepsilon_i}\nabla\partial_z\delta(x,y,z-d) & for\ i=1 \end{cases} \quad (1)$$

Where v_i denotes the phase velocity of medium i, $\vec{\mathbf{e}}_\mathbf{z}$ is a unit vector in the z-direction. The application of a Laplace transform in time and two-dimensional. Fourier transform horizontal coordinates x,y leads under consideration of the initial boundary and transform of $\vec{\mathbf{E}} = (\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{t})$ being the variable in the transform space, we get for $\mathbf{h} < \mathbf{z} < \infty$

$$[\frac{\partial^2}{\partial z^2} - \gamma_i^2 s^2]F^{(i)}(\alpha,\beta,z,s) = \begin{cases} 0 & for\ i=2 \\ f(s)[\dfrac{-js(\alpha\vec{e}_x + \beta\vec{e}_y)}{\varepsilon_0\varepsilon_i}\dfrac{\partial}{\partial z}\delta(z-d) + \\ (\Gamma_0 s^2\delta(z-d) - \dfrac{\partial^2}{\partial z^2}\dfrac{\delta(z-d)}{\varepsilon_0\varepsilon_u})\vec{e}_z] & for\ i=1 \end{cases} \quad (2)$$

Where $\mathbf{j}^2 = -\mathbf{1}$, $\gamma^2 = (\alpha^2 + \beta^2 + \mathbf{v}_i^{-2})$, $\mathbf{i} = \mathbf{1},\mathbf{2}$ with $\Re\mathbf{e}(\gamma_i) \geq \mathbf{0}$ in the medium 1. This an integral representation result of the Laplace transform of electric field in terms of two-dimensional inverse Fourier integral.

$$E_z^{(i)}(x,y,z;s) = \frac{s^3 f(s)}{8\pi^2}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\alpha^2+\beta^2)[\frac{\exp(-s\gamma_i|z-d|)}{\gamma_i} + \frac{(1+c_{12}\exp(-2s\gamma_i(h-d)))\exp(-s\gamma_i(z+d))}{\gamma_i(1+c_{12}\exp(-2s\gamma_i h))} +$$
$$c_{12}\frac{(1+c_{12}\exp(-2s\gamma_i d))\exp(-s\gamma_i(2h-z-d))}{\gamma_i(1+c_{12}\exp(-2s\gamma_i h))}\}\exp(js(\alpha x+\beta y))d\alpha d\beta \quad (3)$$

With the reflection coefficient at the upper duct boundary: $\mathbf{c}_{12} = \dfrac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}$ here α and β are variables in the transform space of the two-dimensional Fourier transform $\mathbf{f}(\mathbf{s})$ is the Laplace transform $\mathbf{F}(\mathbf{t})$.

4. Discussion of the integrals

To discuss the function $\mathbf{E}_\mathbf{z}^{(i)}(\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{s})$, which is stated in (3) from mathematical and physical points of view. If follows that by using polar coordinates:

$\mathbf{x} = \rho\cos\varphi$, $\mathbf{y} = \rho\sin\varphi$ and $\alpha = \lambda\cos\varphi'$, $\beta = \lambda\sin\varphi'$

Where $\lambda^2 = (\alpha^2 + \beta^2)$, $\rho^2 = (\mathbf{x}^2 + \mathbf{y}^2)$ and $\mathbf{d}\alpha\mathbf{d}\beta = \lambda\mathbf{d}\lambda\mathbf{d}\varphi'$

The evaluation of the double integral (3) is a difficult task. Therefore, using Bessel integral representation:

$$\mathbf{E}_\mathbf{z}^{(i)}(\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{s}) = \frac{\mathbf{s}^3\mathbf{f}(\mathbf{s})}{8\pi^2}\int_0^\infty[\frac{\exp(-s\gamma_i|z-d|)}{\gamma_i} + \frac{(1+c_{12}\exp(-2s\gamma_i(h-d)))\exp(-s\gamma_i(z+d))}{\gamma_i(1-c_{12}\exp(-2s\gamma_i h))} +$$
$$c_{12}\frac{(1+c_{12}\exp(-2s\gamma_i d))\exp(-s\gamma_i(2h-z-d))}{\gamma_i(1-c_{12}\exp(-2s\gamma_i h))}\lambda^3 J_1(\lambda sp)d\lambda \quad (4)$$

Where $\mathbf{J_1(\lambda sp)}$ is the Bessel function of order one.

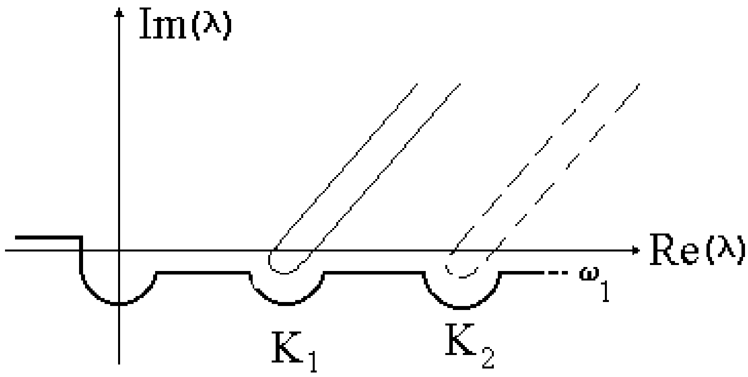

Fig.2: Branch cuts, the steepest descent paths and poles in the appositive λ-plane.

The second term in (4) solved in Abo-Seliem (2004), the third term will be dealt with as it represents the secondary field. Therefore:

$$\mathbf{E_z^{(i)}(\rho,\theta,z;s) = \frac{-sf(s)}{\pi}\int_0^{\infty}\frac{F_1(\lambda s)}{M_1(\lambda s)}\lambda^3 J_0(\lambda sp)d\lambda} \qquad \mathbf{(5)}$$

Where $\mathbf{F_1(\lambda s) = (\gamma_1 - \gamma_2)\exp(-s\gamma_1(z-d)) + (\gamma_1 - \gamma_2)\exp(-s\gamma_1(z+2h-d))}$ **(6)**

$$\mathbf{M_1(\lambda s) = \gamma[(\gamma_1+\gamma_2)-(\gamma_1-\gamma_2)\exp(-2s\gamma_1 h)]} \qquad \mathbf{(7)}$$

To discuss the integral (5) we have to investigate the singularities in the denominator of the integral. The integral has four values that correspond to the four combinations of signs of γ_1 and Riemann surface also has four sheets. To insure the convergence of our integrals, we require that the path of integration, at infinity, should be on the permissible

Sheet only as previously demonstrated by Kahan & Eckart (1950), the poles, the branch cuts on the branch point which are suitable for operating the integration will be also determinate. We find two branch at $\mathbf{\lambda = k_1}$ and $\mathbf{\lambda = k_2}$ where $\mathbf{v_i^{-1} = jk}$, an infinity number of poles on the upper Riemann sheets Fig. (2). illustrates these two branch cuts and the steepest descent paths.

Next, $\mathbf{J_1(\lambda sp)}$ is written in terms of Hankel function $\mathbf{H_1^{(1)}(\lambda sp)}$ to change the semi-infinite integral (5) into a fully infinite integral.

$$\mathbf{E_z^{(i)}(\rho,\theta,z;s) = \frac{-sf(s)}{\pi}\int_{-\infty}^{\infty}\frac{F_1(\lambda s)}{M_1(\lambda s)}\lambda^2 H_1^{(1)}(\lambda sp)d\lambda} \qquad \mathbf{(8)}$$

In (8) can be evaluated along the contour $\boldsymbol{\omega}$ from $-\infty$ to ∞, and its values go around the poles and the branch cut Eq. (8) then takes the form:

$$\mathbf{E_z^{(i)}(\rho,\theta,z;s) = -2jsf(s)[\sum\frac{F_1(\lambda s)}{M_1(\lambda s)}H_1^{(1)}(\lambda sp)\lambda^2;\lambda_k] + [-\frac{sf(s)}{2\pi}\int_{\infty}\frac{F_1(\lambda s)}{M_1(\lambda s)}H_1^{(1)}(\lambda sp)\lambda^2 d\lambda]} \qquad \mathbf{(9)}$$

Where, $\mathbf{n'(\alpha)}$ are the eigenvalues of the poles of the integral and $\boldsymbol{\lambda_k}$ is the solution of the poles equation.

$$\mathbf{M_1(\lambda s) = \gamma_1(\lambda_k s)[(\gamma_1(\lambda_k s)+\gamma_2(\lambda_k s)) - (\gamma_1(\lambda_k s)-\gamma_2(\lambda_k s))\exp(-2s\gamma_1(\lambda_k s)h)] = 0} \qquad \mathbf{(10)}$$

In the first term of (9), substituting the value $\boldsymbol{\lambda = \lambda_k}$.

$$\mathbf{D^k(\lambda_k s) = -2jsf(s)[\frac{H_1^{(1)}(\lambda sp)\lambda^2 F_1(\lambda_k s)}{M_1(\lambda_k s)}; \lambda_k]} \quad \textbf{(11)}$$

Where $\mathbf{M_1(\lambda_k s) = [\frac{\partial M(\lambda_k s)}{\partial \lambda}]_{\lambda=\lambda_k}}$

Next, we can estimate the second term in (9) by using Saddle-point method. The Hankel function can be transformed into the asymptote expansions as is well known (Abo-Seliem 2004)

$$H_n^{(1)}(p) = \sqrt{\frac{2}{\pi\rho}}\exp(j(p - \pi(2n-1)/4)) \quad (12)$$

From (6) and (7), we can get the following

$$\mathbf{I = \int \sqrt{\frac{2\lambda}{\pi p \gamma_1}} \exp(j(sp - \pi/4))\, A(\lambda s) \exp(-s\gamma_1 (z+d)) d\lambda} \quad \textbf{(13)}$$

Where

$$\mathbf{A(\lambda s) = \frac{\lambda^2[(\gamma_1 - \gamma_2)\exp(-s\gamma_1(z-d)) + (\gamma_1 - \gamma_2)]\exp(-s\gamma_1(z+2h-d)}{\gamma_1[(\gamma_1+\gamma_2) - (\gamma_1-\gamma_2)\exp(-2s\gamma_1 h)]}} \quad \textbf{(14)}$$

We let $\mathbf{\rho = R\sin\alpha}$, $\mathbf{(z+d) = -R\cos\alpha}$ and $\mathbf{\gamma_i^2 + k_i^2 = \lambda^2}$,

$$\mathbf{I = \int \sqrt{\frac{2}{\pi s R \sin\alpha}} \exp(j(Rsg(\lambda s) - \pi/4))\, \Phi(\lambda s) d\lambda} \quad \textbf{(15)}$$

Where $\mathbf{g(\lambda s) = \lambda\sin\alpha - j\sqrt{\lambda^2 - k_i^2 \cos\alpha}}$ and $\mathbf{\Phi(\lambda s) = A(\lambda s)\sqrt{\frac{\lambda}{\gamma_1^2}}}$

Thus, the Saddle-point $\mathbf{\lambda = \lambda_s}$ for the integral is determinate by (Kahan & Eckart 1950).

$$\mathbf{g'(\lambda s) = \sin\alpha - j\frac{\lambda\cos\alpha}{\sqrt{\gamma_1^2}}} \quad \textbf{(16)}$$

Therefore $\mathbf{\lambda = k_1 \sin\alpha}$ **(17)**

Then, the integral (15) is evaluated as follows:

$$I = A(\lambda_k s)\frac{2\exp(isk_i R)}{sR} \quad (18)$$

5. Conclusion

A theoretical study for computing the electromagnetic field from a Hertizan vector in the ionosphere is presented. The solution is valid for arbitrary distances between receiving and transmitting ends for a source position. The Saddle point method is used to compute the problem, where distance measuring can lead to some-what erroneous results if the "wrong" polarization is used, since it may happen that the travel time is different for each polarization. In this method we chose a vertical polarization of the primary source.

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