Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.16, 2013 www.iiste.org

Conformal Liouville Theory from D=10 Parallelizable type IIB supergravity with a 5-form Ramond-Ramond.

Abderrahman EL Boukili[1]*, Abdellah EL Boukili[§] and Moulay Brahim Sedra*

[*]*Université Ibn Tofail, Faculté des Sciences, Département de Physique,*

Laboratoire des Hautes énergies, Sciences de l'Ingénierie et Réacteurs (LHESIR), Kénitra, Morocco.

[§]*Université Mohamed V, Faculté des Sciences Agdal*

Département de Chimie (LECA), Rabat, Morocco.

Abstract

We investigate the effect of parallelizable property on supergravity equations of motion, therefore we will extract the exact solutions of Conformal Liouville Theory from D=10 Parallelizable IIB

Keywords: Conformal Liouville Theory, Parallelizability, supergravity.

1. Introduction

The conformal two-dimensional Liouville field theory is related string fields theories [1] and provide an effective theory of 2D quantum gravity[2]. However , the parallelizable manifold is one of different categories of manifolds[3], which has a lot of applications in physics namely, Parallelizable supergravity[4], Parallelizable supersymmetry[5] and Parallelizable plane waves[6]. In our studies we will introduce the parallelizability condition in the action of D=10 supergravity in presence of a 5 form Ramon-Ramon and we will see under which conditions we get a conformal Liouville equations.

The outline of our paper is as follows. In section 2, we review the parallelizability according to different definition. In section 3, we introduce the parallelizability in NS-NS part of the supergravity action. In section 4, we will extract the Conformal Liouville theory from Parallelizable IIB supergravity in the presence of a 5-form Ramond-Ramond. A brief conclusion is reported in Sec. 5.

[1]Corresponding author: aelboukili@gmail.com

2. Review on Parallelizability

According to a definitions given in the references [7], a manifold M is said to be parallelizable if there exists a smooth section of the frame bundle, or equivalently, if there exist n smooth sections of the tangent bundle T(M), such that they are linearly independent at each point of M.
More intuitively, a manifold is parallelizable if one can cover the whole manifold with a single non-degenerate coordinate system.
There is another definition of parallelizable manifolds due to Cartan-Schouten [8]: A manifold is called parallelizable if there exists a torsion which ``flattens" the manifold, i.e. makes the Riemann curvature tensor vanish.
Let us make explicit the decomposition of the connection into a Christoffel piece and a torsion contribution[8;5] :

$$\hat{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + T^{\lambda}_{\mu\nu} , \tag{1}$$

where $\Gamma^{\lambda}_{\mu\nu}$ is symmetric in $\mu\nu$ indices and $T^{\lambda}_{\mu\nu}$ (torsion) is anti-symmetric. The curvature $\hat{R}_{\mu\nu\alpha\beta}$ may be decomposed in a similar way, into a piece which comes only from the Christoffel connection and the torsional contributions:

$$\hat{R}_{\mu\nu\alpha\beta} \equiv R_{\mu\nu\alpha\beta} + \nabla_{\alpha} T_{\mu\nu\beta} - \nabla_{\beta} T_{\mu\nu\alpha} + T_{\mu\beta\rho} T^{\rho}_{\nu\alpha} - T_{\mu\alpha\rho} T^{\rho}_{\nu\beta} . \tag{2}$$

The parallelizability condition simply becomes $\hat{R}_{\mu\nu\alpha\beta} = 0$. If the modified Ricci tensor

$$\hat{R}_{\mu\nu} \equiv R_{\mu\nu} + \nabla_{\alpha} T^{\alpha}_{\mu\nu} - T_{\mu\lambda\rho} T^{\lambda\rho}_{\nu} \tag{3}$$

is zero, the manifold is said to be Ricci-parallelizable . Note that the generalized Ricci tensor $\hat{R}_{\mu\nu}$ is not symmetric in its $\mu\nu$ indices. For a manifold to be Ricci-parallelizable, the symmetric and anti-symmetric parts of $\hat{R}_{\mu\nu}$ should both vanish, namely $\nabla_{\alpha} T^{\alpha}_{\mu\nu} = 0$ and $R_{\mu\nu} - T_{\mu\lambda\rho} T^{\lambda\rho}_{\nu} = 0$.
With the above definitions, the parallelizing torsion for a group manifold is given by the structure constants of the group algebra [8].

3. Parallelizable supergravity

Since in most string theories, a torsion field naturally arises, one may look for implications of parallelizability for supergravities and their solutions [9]. Here we mainly focus on type II theories, however, most of our arguments can be used for heterotic theories as well.

First we recall that the NS-NS part of the supergravity action is of the form[5]

$$S = \frac{1}{l_p^8}\int d^{10}x\sqrt{-g}e^{-2\phi}\left(R + 4(\nabla_\mu\phi)^2 - \frac{1}{12}H^2\right). \tag{4}$$

Since we are interested in solutions involving only the metric and torsion, we set the dilaton field ϕ to a constant. Then the supergravity equations of motion for the metric and $B_{\mu\nu}$ field are

$$\begin{aligned} &R_{\mu\nu} - \frac{1}{4}H_{\mu\rho\lambda}H_\nu^{\rho\lambda} = 0, \\ &\nabla_\alpha(\sqrt{-g}H^\alpha_{\mu\nu}) = 0, \\ &H = dB. \end{aligned} \tag{5}$$

If we define $\frac{1}{2}H_{\alpha\mu\nu}$ as torsion $T_{\alpha\mu\nu}$, the supergravity equations for the metric and $B_{\mu\nu}$ fields are nothing but the Ricci-parallelizability condition. Hence all parallelizable manifolds (which are obviously also Ricci-parallelizable) satisfying the constant dilaton constraint [10]

$$R = \frac{1}{12}H^2 \tag{6}$$

and have a closed torsion (i.e. $dH = 0$), are solutions of supergravity .

For any supergravity solution one may wonder about g_s and α' exactness as well as (classical) stability. The parallelizable solutions which we are interested in are non-dilatonic and hence they are g_s independent. The α'-exactness of these supergravity solutions were studied long ago in reference [11]. Computing the second α' contributions to the string theory β-functions, it was shown that such contributions are zero for parallelizable manifolds. Therefore the parallelizable solutions of supergravity are exact up to order α'^2. It has been argued that this property is expected to remain to all orders in α' [12].

4. Conformal Liouville theory from Parallelizable supergravity

Let's consider the effective action of D=10 type IIB supergravity with a Ramond-Ramond 5-form field strength [13]:

$$\mathbf{S}_{eff} = \int d^{10}x\sqrt{-g}e^{-2\Phi}\left(R + 4(\nabla_\mu \Phi)^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{e^{2\Phi}}{4.5!}\mathbf{F}_{\mu\nu\rho\sigma\tau}\mathbf{F}^{\mu\nu\rho\sigma\tau} \right), \tag{7}$$

where g is the metric of spacetime, Φ a dilaton, the field strength H=dB with B an anti-symmetric NS- NS field and F=∂C(4) where C(4) is the corresponding R-R 4-form.

The derived supergravity equations of motion in the σ-model frame

$$(I)\begin{cases} R_{\mu\nu} = -2\nabla_\mu\nabla_\nu\Phi + \frac{1}{4}H_{\mu\rho\sigma}H_\nu^{\cdot\rho\sigma} + \frac{e^{2\Phi}}{4.4!}\left(\mathbf{F}_{\mu\kappa\lambda\rho\sigma}\mathbf{F}_\nu^{\cdot\kappa\lambda\rho\sigma} - \frac{1}{10}g_{\mu\nu}\mathbf{F}_{\kappa\lambda\rho\sigma\tau}\mathbf{F}^{\kappa\lambda\rho\sigma\tau}\right), \\ 0 = \nabla_\mu\nabla^\mu\Phi - 2(\nabla_\mu\Phi)(\nabla^\mu\Phi) + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}, \\ 0 = \nabla_\mu(e^{-2\Phi}H^{\mu\nu\rho}), \\ 0 = \nabla_\mu\mathbf{F}^{\mu\nu\kappa\lambda\rho}, \end{cases} \tag{8}$$

introducing parallelizability conditions equ.(6) we obtain the following system of equations

$$(II)\begin{cases} 0 = -2\nabla_\mu\nabla_\nu\Phi + \frac{e^{2\Phi}}{4.4!}\left(\mathbf{F}_{\mu\kappa\lambda\rho\sigma}\mathbf{F}_\nu^{\cdot\kappa\lambda\rho\sigma} - \frac{1}{10}g_{\mu\nu}\mathbf{F}_{\kappa\lambda\rho\sigma\tau}\mathbf{F}^{\kappa\lambda\rho\sigma\tau}\right), \\ 0 = \nabla_\mu\nabla^\mu\Phi - 2(\nabla_\mu\Phi)(\nabla^\mu\Phi) + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}, \\ 0 = H^{\mu\nu\rho}\nabla_\mu e^{-2\Phi}, \\ 0 = \nabla_\mu\mathbf{F}^{\mu\nu\kappa\lambda\rho}, \end{cases} \tag{9}$$

As for the R-R 5-form, we assume that

$$\mathbf{F} = du \wedge \varphi(x^i) \tag{10}$$

where $\varphi(x^i)$ is an anti-self-dual closed 4-form in the transverse eight-dimensional space, i.e.,

$$\varphi(x^i) = -^*\varphi(x^i), \quad d\varphi(x^i) = 0 \tag{11}$$

Such as $\mathbf{F} =^* \mathbf{F}$.

Taking the (uu)-component of the equation for the Ricci tensor(ie. $u = x^\mu$, $\mu = \nu$), we obtain

$$0 = -2\nabla_u\nabla_u\Phi + \frac{e^{2\Phi}}{4.4!}\varphi_{ijkl}\varphi^{ijkl} \tag{12}$$

where $\varphi_{ijkl}\varphi^{ijkl} \equiv \varphi^2$. The covariant derivative ∇_u is defined as follows [5;6;13]:

$$\nabla_u = \partial_u + \frac{1}{4}w_u^{ab}\Gamma_{ab} = \partial_u + \frac{1}{4}\hat{w}_u \tag{13}$$

where $\hat{w}_u = w_u^{ab}\Gamma_{ab}$ is the spin connection. By inserting the expression of covariant derivative in equation (12) we find the following equation

$$\partial_u^2.\Phi + (\frac{1}{4}\hat{w}_u).\partial_u\Phi + \frac{1}{4}(\partial_u\hat{w}_u + \frac{1}{4}\hat{w}_u^2).\Phi = \chi(\varphi)\exp(2\Phi) \tag{14}$$

where $\chi(\varphi) \equiv \frac{\varphi^2}{4!.8}$.

Case 1: For $\hat{\mathbf{w}}_u = \mathbf{0}$, equation (14) became

$$\partial_u^2.\Phi = \chi(\varphi)\exp(2\Phi) \tag{15}$$

this equation has been well studied in references [6;13] which corresponds to a Liouville equation whose solution is given by

$$\exp(2\beta\Phi) = \frac{2}{\chi(\varphi)}.\left(\frac{\partial_u f(u)}{1 - f^2(u)}\right)^2, \tag{16}$$

where f is an arbitrary analytic function.

Case 2: If we set $\hat{\mathbf{w}}_u \neq \mathbf{0}$ with the following constraints: with the following constraints:

$$(\tfrac{1}{4}\hat{w}_u).\partial_u\Phi + \tfrac{1}{4}(\partial_u\hat{w}_u + \tfrac{1}{4}\hat{w}_u^2).\Phi = 0 \tag{17}$$

equation (14) becomes

$$(III)\begin{cases} (\tfrac{1}{4}\hat{w}_u).\partial_u\Phi + \tfrac{1}{4}(\partial_u\hat{w}_u + \tfrac{1}{4}\hat{w}_u^2).\Phi = 0 \\ \\ \partial_u^2.\Phi = \chi(\varphi)\exp(2\Phi) \end{cases} \tag{18}$$

by integrating the system (III), we find the following solution

$$\begin{cases} \Phi(u) = K.\hat{w}_u e^{-\frac{1}{4}\int\hat{w}_u} \\ \exp(2\beta\Phi) = \frac{2}{\chi(\varphi)}\left(\frac{\partial_u f(u)}{1-f^2(u)}\right)^2 \end{cases} \tag{19}$$

where K and β are two arbitraries constants. if we combine the two equations of (19) we obtain the following expression for the function f

$$f(u) = 1 - 2\left(T\exp\left[2\int\left[\frac{\chi(\varphi)}{2}\exp[2\beta K.\hat{w}_u e^{-\frac{1}{4}\int\hat{w}_u}]\right]^{\frac{1}{2}}\right] + 1\right)^{-1} \tag{20}$$

Case 3: If we set $\hat{\mathbf{w}}_u \neq \mathbf{0}$ with the following constraints:

$$(\frac{1}{4}\hat{w}_u).\partial_u\Phi + \frac{1}{4}(\partial_u\hat{w}_u + \frac{1}{4}\hat{w}_u^2).\Phi = c(u).\partial_u^2\Phi \tag{21}$$

with this constraint, equation (14) is transformed to the following system

$$(IV)\begin{cases}\partial_u^2\Phi + a(u)\partial_u\Phi + b(u)\Phi = 0\\ \\ \partial_u^2\Phi = \zeta(\varphi).\exp 2\varphi\end{cases} \tag{22}$$

with $a(u) = -\frac{\hat{w}_u}{4.c(u)}$, $b(u) = -\frac{1}{4}(\frac{\partial_u\hat{w}_u}{c(u)} + \frac{1}{4}\frac{\hat{w}_u^2}{c(u)})$ and $\zeta(\varphi) = \frac{\chi(\varphi)}{1+c(u)}$.

We Φ_1 consider a particular solution of the system (22), for example the solution given in the reference [13]

$$\Phi_1(u) = -\log(|\cos(a.u)|) \tag{23}$$

where $a^2 = \zeta(\varphi)$.

Taking $\Phi = \Phi_1.z$ and $\theta = \partial_u z$ then equation (22) becomes

$$\Phi_1.\partial_u\theta + [a(u)\Phi_1 + 2\partial_u\Phi_1]\theta = 0 \tag{24}$$

which has as solution

$$\theta(u) = A\Phi_1^{-2}(u)\exp(\alpha u) \tag{25}$$

where $\alpha = \int a(u)$ and A is a real constant. Consequently, the solution Φ is given by

$$\Phi(u) = -\log\left[\cos(\sqrt{\zeta(\varphi)}.u)\right]\left(\mathsf{A}.\int\frac{\int a(u)}{\log^2(\cos(\sqrt{\zeta(\varphi)}.u)}+\mathsf{B}\right) \tag{26}$$

Case 4: If we set $\hat{\mathbf{w}}_u \neq \mathbf{0}$ and $(\frac{1}{4}\hat{w}_u).\partial_u\Phi + \frac{1}{4}(\partial_u\hat{w}_u + \frac{1}{4}\hat{w}_u^2).\Phi = \gamma(\varphi).\exp(2\Phi)$

In this case we have the system of equations

$$\begin{cases}(\frac{1}{4}\hat{w}_u).\partial_u\Phi + \frac{1}{4}(\partial_u\hat{w}_u + \frac{1}{4}\hat{w}_u^2).\Phi = \gamma(\varphi)\exp(2\Phi)\\ \partial_u^2.\Phi = (\chi(\varphi) - \gamma(\varphi))\exp(2\Phi)\end{cases} \tag{27}$$

The second equation of this system corresponds to a Liouville equation which has the following solution

$$\exp(2\beta\Phi) = \frac{2}{|\chi(\varphi) - \gamma(\varphi)|} \cdot \left(\frac{\partial_u f(u)}{1 - f^2(u)} \right)^2, \tag{28}$$

Where

$$f(u) = 1 - 2\left(T \exp\left[2 \int \left[\frac{\chi(\varphi) - \gamma(\varphi)}{2} \exp[2\beta K.\hat{w}_u e^{-\frac{1}{4}\int \hat{w}_u}] \right]^{\frac{1}{2}} \right] + 1 \right)^{-1} \tag{29}$$

and

$$\gamma(\varphi) = \frac{1}{4}\left[\hat{w}_u.\partial_u \Phi + (\partial_u \hat{w}_u + \frac{1}{4}\hat{w}_u^2).\Phi \right] \exp(-2\Phi) \tag{30}$$

5. Conclusion

In our studies we have study some applications of parallelizability in physics. Such property is introduced as an ansatz in the action of D=10 supergravity in presence of a 5 form Ramon-Ramon. We have proposed and discussed the different cases of analytic expression for the solutions of conformal Liouville equation obtained from the background of parallelizable supergravity.

References

[1] A. Polyakov, Quantum Geometry of Bosonic Strings, Phys. Lett., B103 (1981) 207-210.

[2] E. D' Hoker and R. Jackiw, Classical and quantal Liouville field theory, Phys. Rev., D26 (1982) 3517

[3] T. Lance, Differentiable structures on manifolds, Surveys on Surgery Theory, vol. 1, Ann. of Math. Stud. 145, Princeton Univ. Press, NJ, 2000, pp. 73-104. 38

[4] M.J. Duff, B.E.W. Nilsson and C.N. Pope , Kaluza-Klein Supergravity, Phys.Rept. 130 (1986) 1-142

[5] Darius Sadri and M. M. Sheikh-Jabbari, String Theory on Parallelizable PP-Waves, JHEP 0306:005,2003

[6] M. Hssaini and M.B.Sedra, Type IIB String Backgrounds on Parallelizable PP-Waves and Conformal Liouville Theory, Afr.J.Math.Phys. 1 (2004) 91-97

[7] R.L. Bishop and S.I. Goldberg, Tensor Analysis on Manifolds, Dover, New York, 1980.

QA433 B54 1968.

[8] E. Cartan and J. Schouten, Proc. K. Ned. Akad. Wet. Amsterdam 29 (1926) 803, 933.

[9] Friedemann Brandt, Lectures on supergravity, , Fortsch.Phys. 50 (2002) 1126-1172

[10] L. Pando-Zayas, “Ricci-Parallelizable Spaces in the NS-NS Sector: $AdS_3 \times S^7$, hep-th/0007134.

[11] R. R. Metsaev and A. A. Tseytlin, “Order Alpha-Prime (Two Loop) Equivalence Of The String Equations Of Motion And The Sigma Model Weyl Invariance Conditions: Dependence On The Dilaton And The Antisymmetric Tensor,” Nuc l . P h y s . B293, 385 (1987).

[12] I. Bena and R. Roiban, Supergravity pp-wave solutions with 28 and 24 supercharges, hep-th/0206195.

[13] I. Bakas and J. Sonnenschein, On integrable models from pp-wave string backgrounds, JHEP 0212 (2002) 049