Statistical and Squeezing Proprieties of Superposed Single-Mode Squeezed Chaotic State

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Abstract
In this paper we have studied the statistical and squeezing proprieties of light produced by superposition of a pair of single-mode squeezed chaotic light beams. Applying density operator of single-mode squeezed chaotic state; we obtain the anti-normal order characteristics function which enables us to find the Q function. With the resulting Q function, we calculate the photon statistics and the Quadrature squeezing for single-mode squeezed chaotic light. Moreover applying Q function of single-mode squeezed chaotic state the superposed light beams would be driven. With the resulting Q function we calculated the photon statics and the quadrature squeezing for superposed light beams. To get the maximum squeezing to be 95%, for nth = 0 and r = 1.5.
Keywords: squeezed chaotic state, superposed state, fluctuations
DOI: 10.7176/APTA/78-03
Publication date:June 30th 2019

1 Introduction

The most important quantum states of light are chaotic state, coherent state and squeezed state. Chaotic state is one of the classical features of light with super-Poissonian photon statics. And its best example is thermal light. The coherent state is a specific superposition of number states which does not possess number of photons as well as it is known by minimum uncertainty and poissonian photon statistics. Squeezed state satisfies non-classical feature of light, with sub-poissonian photon statistics [1]-[4].

The quantum distribution of radiation is the core idea in quantum optics. This used to describe the quantum properties of light. Some of them are the P function, the Wigner function and the Q functions. The P-function is c-number function with the anti-normal order density operator over π. And used to describe the superposition of two light beams with different states but having the same frequency. The Wigner function is the c-number function corresponding to the symmetric order density operator over π. The Q function is the most widely used one because, it is used to describe the superposition of two light beams with the same frequency but may be in the same or different states. This is described in terms of normally ordered density operator divided by π [5].

2 Methods

Within density operator we calculate the anti-normal order characteristic function which is used to obtain the Q function. With the help of resulting Q function we calculate the density operator for superposed single-mode squeezed Chaotic state, the mean photon number, the variance of photon number, the photon number distribution and quadrature variance for both single-mode and superposed single-mode squeezed chaotic state.

3 Single-mode squeezed chaotic state

3.1 Single-mode chaotic state

Thermal light is the best example of light mode in a chaotic state, which is generated by the source in thermal equilibrium at the minimum temperature. This is not a pure state, instead it can be described by density matrix and has mean which is less than variance so, and we called as classical feature of light. It can be used to describe light of the bulb, Black-body radiation and etc. In this section we seek to determine the density operator for chaotic light in terms of the P function using Lagrangian multipliers and by maximizing entropy. The entropy of thermal light can be described as [6].

3.2 Single-mode Squeezed State

A degenerate parametric amplifier, consisting of nonlinear crystal pumped by coherent light, is a source of single-mode squeezed light. In this system a pump photon of frequency 2ω is down converted into two twines signal photons each of frequency ω as shown in Fig. (1) [6]-[10].

Figure 1: *A degenerate parametric amplifier with nonlinear crystal pumped by coherent light*

3.3 The Q function for single-mode squeezed chaotic state

The Squeezed chaotic state is obtained by performing squeezing operator on thermal state.Which have both classical and quantum nature rather than quantum or classical nature only because the system obeys both features simultaneously. Thus, consider the light mode initially in a chaotic state then the state vector for the squeezed chaotic light would be given by,

$$\hat{\rho}_{sc} = |\psi\rangle_{sc}\langle\psi| = \hat{S}(r)|\varphi\rangle_{th}\langle\varphi|\hat{S}^{+}(r) = \hat{S}(r)_{th}\hat{S}^{+}(r). \quad (1)$$

Eq. (1) describes the density operator for single-mode squeezed chaotic state. The Q function for single-mode squeezed chaotic state in terms of anti-normally ordered characteristics function is expressed in [9]-[12],

$$Q(\alpha^*,\alpha,r) = \frac{1}{\pi}\int d^2z\varphi_a(z^*,z,r)\mathrm{e}^{(z^*\alpha - z\alpha^*)}, \quad (2)$$

Where

$$\varphi_a(z^*,z,r) = \frac{1}{\pi}\mathrm{e}^{az^*z+\frac{b}{2}(z^2+z^{*2})} \quad (3)$$

And

$$a = (1+2\bar{n}_{th})sinh^2r + \bar{n}_{th} + 1$$
$$b = -(1+2\bar{n}_{th})coshrsinhr \quad (4)$$

Substituting Eq.(3)in Eq.(2), we see that

$$Q(\alpha^*,\alpha,r) = \frac{1}{\pi}\int \frac{d^2z}{\pi}\mathrm{e}^{\alpha z^*z + z^*\alpha - z\alpha^* + \frac{b}{2}(z^2+z^{*2})} \quad (5)$$

Carrying out integration with the help of

$$\int \frac{d^2z}{\pi}\mathrm{e}^{(az^*z + bz - cz^* + Az^2 + Bz^{*2})} = \sqrt{\frac{1}{a^2-4AB}}\exp^{[\frac{abc+Ac^2+Bb^2}{a^2-4AB}]} \quad (6)$$

For $a > 0$

We get

$$Q(\alpha^*,\alpha,r) = \frac{M^2-N^2}{\pi}exp^{\left[-M\alpha\alpha^* + \frac{N}{2}(\alpha^2+\alpha^{*2})\right]}. \quad (7)$$

Where

$$M = \frac{a}{a^2-b^2}\ , \quad (8)$$

And

$$N = \frac{b}{a^2-b^2}\,. \quad (9)$$

3.4 Photon statistics

Here we wish to calculate the mean photon number; the variance of photon number and then photon number distribution of the light generated by single-mode squeezed chaotic light employing the Q function.

3.4.1 The mean photon number

The mean photon number in terms of the Q function is given by

$$\bar{n} = \int d^2\alpha Q(\alpha^*,\alpha,r)\mathrm{e}^{A\alpha^*\alpha}. \quad (10)$$

Up on expanding the exponential function $\mathrm{e}^{A\alpha^*\alpha}$ in power series as

$$\mathrm{e}^{A\alpha^*\alpha} = \left(1 + A + \frac{1}{2}A^2 + \cdots\right)\alpha^*\alpha,$$
$$= (1-\mathrm{M})\alpha^*\alpha + \mathrm{N}\alpha^2. \quad (11)$$

Interims of Eqs. (4), (8) and Eq. (9) the mean photon number for single -mode squeezed chaotic state take the form

$$\bar{n} = \bar{n}_{th} + (1+2\bar{n}_{th})sinh^2r \quad (12)$$

Fig. (2), shows that as the mean photon number of thermal light and squeezed parameter(r) increase the mean photon number of single-mode squeezed chaotic state increases. So the use of thermal light is to increase the mean photon number of single-mode squeezed chaotic state.

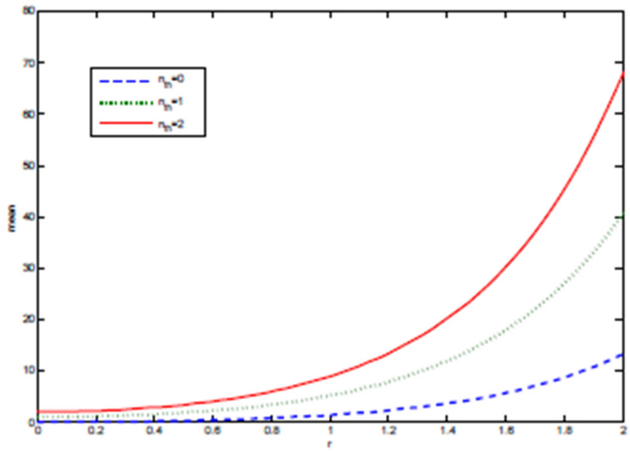

Figure 2: *Plots of mean photon number Eq.* (10) *versus squeeze parameter(r) for different value of* $\bar{n}_{th}$.

3.4.2 The Variance of Photon Number

The variance of photon number for single-mode squeezed chaotic light in anti-normal order form is putted as,

$(\Delta n)^2 =< \hat{a}^{+2}\hat{a}^2 > +2\bar{n} - \bar{n}^2.$ (13)

Where

$< \hat{a}^{+2}\hat{a}^2 > = \int d^2\alpha Q(\alpha^*,\alpha,r)\mathrm{e}^{A\alpha^{*2}\alpha^2}.$ (14)

we can expand $\mathrm{e}^{A\alpha^{*2}\alpha^2}$ in power series form as,

$$\mathrm{e}^{A\alpha^{*2}\alpha^2} = \left[1 - (M\alpha^* - N\alpha)\frac{\partial}{\partial\alpha^*} + \frac{N}{2}\frac{\partial^2}{\partial\alpha^{*2}} + \frac{1}{2!}\left(-(M\alpha^* - N\alpha)\frac{\partial}{\partial\alpha^*} + \frac{N}{2}\frac{\partial^2}{\partial\alpha^{*2}}\right)^2 + \cdots\right]\alpha^{*2}\alpha^2, \quad (15)$$

Using Eq. (15) in Eq. (14) and applying the differentiation, we see that

$< \hat{a}^{+2}\hat{a}^2 >= 2a^2 + b^2$ (16)

On account of Eqs. (4), (16) and Eq. (12) the variance of photon number described as

$$\Delta n^2 = 2\bar{n}_{th} - \bar{n}_{th}{}^2 + 3(1+2\bar{n}_{th})^2 sinh^2 r + (1+2\bar{n}_{th})^2 sinh^4 r - (1+2\bar{n}_{th}) coshr sinhr. \quad (17)$$

Fig. (3) Shows that the variance of photon number (broken line) is greater than mean photon number (solid line) which indicates the radiation has super-Poissonian photon stastics.

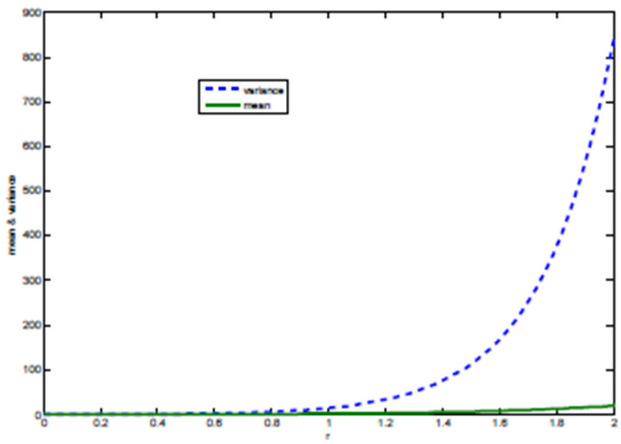

Figure 3: *Plots of the mean photon number Eq.* (12) *and the variance of photon number Eq.* (17) *versus squeeze parameter(r) for* $\bar{n}_{th}$= 0.25.

3.4.3 Photon number distribution

The photon number distribution for single-mode light is expressible in terms of the Q function as,

$$P(n,r) = \frac{(M^2-N^2)^{\frac{1}{2}}}{n!}\frac{\partial^{2n}}{\partial\alpha^{*n}\partial\alpha^n} exp^{\left[(1-M)\alpha^*\alpha + \frac{N}{2}(\alpha^2+\alpha^{*2})\right]}|_{\alpha=\alpha^*=0} \quad (18)$$

Up on expanding the exponential part in a power series, we have

$$exp^{[(1-M)\alpha^*\alpha)]} = \sum_v \frac{(1-m)^v \alpha^{*v}\alpha^v}{v!},$$

$$exp^{\left[\frac{N}{2}\alpha^2\right]} = \sum_u \frac{\left(\frac{N}{2}\right)^u \alpha^{2u}}{u!}$$

$$exp^{\left[\frac{N}{2}\alpha^{*2}\right]} = \sum_x \frac{\left(\frac{N}{2}\right)^x \alpha^{*2x}}{x!}. \quad (19)$$

Substituting Eq. (19) into Eq. (18), we find

$$P(n,r) = \frac{(M^2-N^2)^{\frac{1}{2}}}{n!}\sum_{v!u!x!}\frac{\left(\frac{N}{2}\right)^{x+u}(1-M)^{v}}{v!u!x}\frac{\partial^{2n}}{\partial\alpha^{*n}\partial\alpha^{n}}\left(\partial^{2u+v}\alpha^{*2x+v}\right)|_{\alpha=\alpha^*=0}, \quad (20)$$

Performing some mathematical rules we find

$$P(n,r) = (M^2 - N^2)^{\frac{1}{2}}\sum_{v=0}^{[n]}\frac{n!(1-m)^{n-2u}N^{2v}}{2^{2v}(u!)^2(n-2u)!}. \quad (21)$$

On account of Eq. (8), (9) and Eq. (4) we can put the photon number distribution as,

$$P(n,r) = \frac{1}{((1+2\bar{n}_{th})sinh^2r+(1+2\bar{n}_{th})^2)^{\frac{1}{2}}}\left(\sum_{x=0}^{[n]}\frac{n!\left(\frac{\bar{n}_{th}{}^2-\bar{n}_{th}}{(1+2\bar{n}_{th})sinh^2r+(1+\bar{n}_{th})^2}\right)^{n-2x}\left(\frac{-(1+2\bar{n}_{th})coshrsinhr}{(1+2\bar{n}_{th})sinh^2r+(1+\bar{n}_{th})^2}\right)^{2x}}{2^{2x}(x!)^2(n-2x)!}\right). \quad (22)$$

Fig. (4) Shows that the photon number distribution increase with number of photons. But photon number distribution decrease as mean photon number of thermal light and squeezed parameter(r) increase.

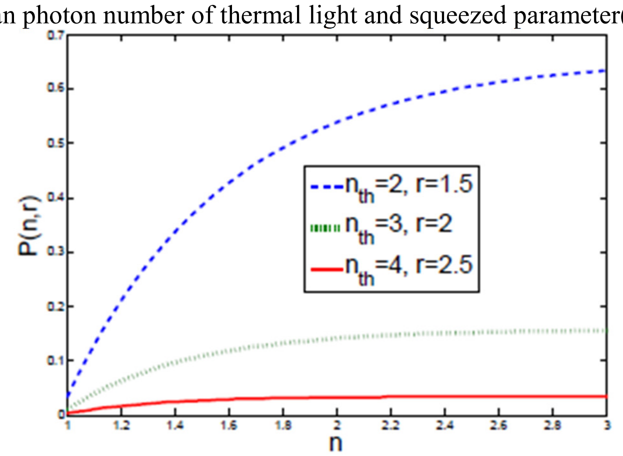

Figure 4: *Plots of photon number distribution Eq. (14) versus number of photons (n)*

3.5 Quadrature squeezing (fluctuation)

The squeezing properties of single-mode light are described by two quadrature operators defined by [13],

$\Delta a_{\pm}{}^2 =< \hat{a}_{\pm}, \hat{a}_{\pm} >$. (23)

Using $< U, V >=< UV > -< U >< V >$ Eq.(23) can be rewritten as

$\Delta a_{\pm}{}^2 = 1+< \hat{a}^2 > \pm< \hat{a}^{+2} > +< \hat{a}^{+2} > +2 < \hat{a}^{+}\hat{a} > \mp< \hat{a} >^2 - 2 < \hat{a} >< \hat{a}^{+} >$, (24)

Where the expectation values are

$< \hat{a} > =< \hat{a}^{+} \geq 0$. (25)

And

$< \hat{a}^2 > =< \hat{a}^{+2} >$=b (26)

On account of Eq. (4), and expanding trigonometric function in exponential form the plus and minus quadrature is rewritten as

$\Delta a_{\pm}{}^2 = (1 + 2\bar{n}_{th})e^{\mp 2r}$. (27)

Fig. (5), shows dependence of the system on its initial state. As we see from the figure while squeezing parameter increase the corresponding quadrature variance decreases. But the quadrature variance increases with mean photon number of thermal light.

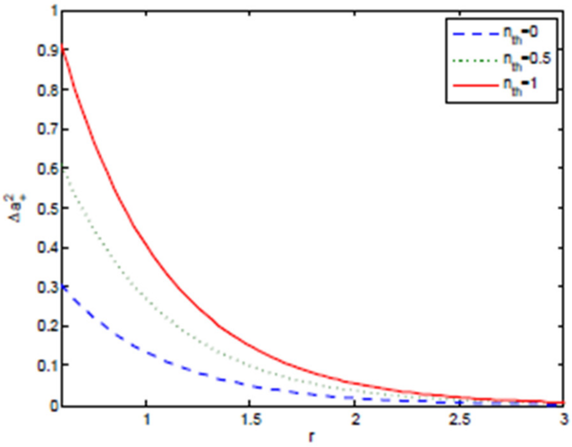

Figure 5: *Plots of* $\Delta a_{\pm}{}^{2}$ *Eq.* (27) *versus* r *for different values of* $\bar{n}_{th}$

Furthermore the quadrature squeezing for single-mode squeezed chaotic state relative to the coherent state is given by,

$$S = \frac{(\Delta a_{\pm}{}^{2})_{c} - \Delta a_{+}{}^{2}}{(\Delta a_{\pm}{}^{2})_{c}}. \quad (28)$$

Where the quadrature variance for coherent state is

$$(\Delta a_{\pm}{}^{2})_{c} = 1, \quad (29)$$

In view of this, we get

$$S = 1 - \Delta a_{+}{}^{2}, \quad (30)$$

Substituting Eq. (27) in (30), we get

$$S = 1 - (1 + 2\bar{n}_{th})e^{-2r}. \quad (31)$$

4 Superposed single-modes squeezed chaotic state

To describe the superposition we use two single-mode squeezed chaotic light beams which are emitted from different sides onto a mirror. We assume that one side of the mirror is totally trans missive and the other one is totally reflective. At the back of the mirror we see pair of light which is called as the superposed single-mode squeezed chaotic light as shown in Fig. (6)

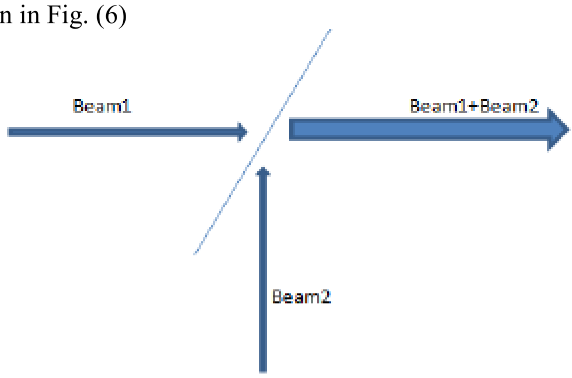

Figure 6: *The schematic representation for the superposition of two light beams*

4.1 The Density operator for pair of superposed single-mode squeezed chaotic state

Density operator for superposed single-mode light beams can be written interims of Q function as;

$$\hat{\rho}'(r) = \sum_{k,m} \int \frac{d^2\alpha}{\pi} c_{km} \hat{a}^{+k} \hat{a}^{m} |\alpha><\alpha|. \quad (32)$$

Where

$$\hat{I} = \int \frac{d^2\alpha}{\pi} |\alpha><\alpha|, \quad (33)$$

Applying the fact that

$$|\alpha><\alpha| \hat{a}^{+k} = \alpha^{*k} |\alpha><\alpha|, \quad (34)$$

And

$$|\alpha><\alpha| \hat{a}^{m} = (\alpha + \frac{\partial}{\partial \alpha^{*}})^{m} |\alpha><\alpha|, \quad (35)$$

Employing the preceding two equations, one can write the density operator for first light beam as,

$$\hat{\rho}'(r) = \int d^2\,\alpha_1 Q_1 \left(\alpha_1{}^{*} + \alpha + \frac{\partial}{\partial \alpha_1{}^{*}}\right) \hat{D}(\alpha_1) \hat{\rho}(0) \hat{D}(-\alpha_1),$$

$$=\int d^2\alpha_1 Q_1\left(\alpha_1{}^* + \alpha + \frac{\partial}{\partial\alpha_1{}^*}\right)|\alpha_1 >< \alpha_1|. \tag{36}$$

In which

$$Q_1\left(\alpha_1{}^* + \alpha + \frac{\partial}{\partial\alpha_1{}^*}\right) = \frac{1}{\pi}\sum_{k,m} c_{km}\,\alpha_1{}^{*k}\left(\alpha_1 + \frac{\partial}{\partial\alpha_1{}^*}\right), \tag{37}$$

Is the Q function for first light beam.

$$\hat{D}\,(\alpha_1) = e^{\alpha_1\hat{a}^+ - \alpha_1\hat{a}}, \tag{38}$$

Is the displacement operator

$$\hat{\rho}(0) = |0 >< 0|, \tag{39}$$

Is the density operator at initial time.

Furthermore, the density operator at initial time corresponds to superposed light beams can be written as,

$$\hat{\rho}_{ss} = \int d^2\alpha_2 Q_2\left(\alpha_2{}^* + \alpha_2 + \frac{\partial}{\partial\alpha_2{}^*}\right)\hat{D}\,(\alpha_2)\hat{\rho}'(r)\hat{D}\,(-\alpha_2). \tag{40}$$

In which subscript ”ss” stand for superposed light beams, and the Q function in Eq. (40), takes the form,

$$Q_2\left(\alpha_2{}^* + \alpha_2 + \frac{\partial}{\partial\alpha_2{}^*}\right) = \frac{1}{\pi}\sum_{k,m} c_{km}\,\alpha_2{}^{*k}\left(\alpha_2 + \frac{\partial}{\partial\alpha_2{}^*}\right)^m, \tag{41}$$

Which represents the Q function corresponding to second light beams, Also using the fact that

$$\hat{D}\,(\alpha_1)\hat{D}\,(\alpha_2)|0 >< 0|\hat{D}\,(-\alpha_1)\hat{D}\,(-\alpha_2) = |\alpha_2 + \alpha_1 >< \alpha_1 + \alpha_2|, \tag{42}$$

And on account of Eqs. (36), (41) and Eq. (42) we can write the density operator for superposed single-mode light beams as,

$$\hat{\rho}_{ss} = \int d^2\alpha_1 Q_1\left(\alpha_1{}^* + \alpha_1 + \frac{\partial}{\partial\alpha_1{}^*}\right)\int d^2\alpha_2 Q_2\left(\alpha_2{}^* + \alpha_2 + \frac{\partial}{\partial\alpha_2{}^*}\right)|\alpha_2 + \alpha_1 >< \alpha_1 + \alpha_2|. \tag{43}$$

4.2 Photon statistics

Here we wish to calculate the mean photon number, the variance of photon number and the photon number distribution for the superposition of two light beams employing the normally ordered density operator.

4.2.1 The mean photon number

The mean photon number is expressed in terms of the density operator as;

$$\bar{n}_{ss} = Tr(\hat{\rho}_{ss}\hat{a}^+\hat{a}). \tag{44}$$

Using Eq. (43) in (44), we see that

$$\bar{n}_{ss} = \int d^2\alpha_1 Q_1\left(\alpha_1{}^* + \alpha_1 + \frac{\partial}{\partial\alpha_1{}^*}\right)\int d^2\alpha_2 Q_2\left(\alpha_2{}^* + \alpha_2 + \frac{\partial}{\partial\alpha_2{}^*}\right)$$
$$X(\alpha_1{}^*\alpha_1 + \alpha_2{}^*\alpha_2 + \alpha_2{}^*\alpha_1 + \alpha_1{}^*\alpha_2), \tag{45}$$

This can be seated in the form,

$$\bar{n}_{ss} =< \hat{a}_1{}^+(r)\hat{a}_1(r) > + < \hat{a}_1{}^+(r) >< \hat{a}_1(r) > + < \hat{a}_2{}^+(r)\hat{a}_2(r) > + < \hat{a}_2{}^+(r) >< \hat{a}_1(r) >, \tag{46}$$

Next we proceed to evaluate the expectation values evolved in Eq. (46).

$$< \hat{a}_i{}^+\hat{a}_i >= \int d^2\alpha_i Q_i\left(\alpha_i{}^* + \alpha_i + \frac{\partial}{\partial\alpha_i{}^*}\right)\alpha_i{}^*\alpha_i, i = 1,2. \tag{47}$$

In which

$$Q_i\left(\alpha_i{}^* + \alpha_i + \frac{\partial}{\partial\alpha_i{}^*}\right) = Q_i(\alpha_i{}^* + \alpha_i)e^{A_i}, \tag{48}$$

Where

$$A_i = -(M_i - N_i)\frac{\partial}{\partial\alpha_i{}^*} + \frac{N_i}{2}\frac{\partial^2}{\partial\alpha_i{}^{*2}}, \tag{49}$$

Substituting Eq. (48) in (47), we see that

$$< \hat{a}_i{}^+\hat{a}_i >= \int d^2\alpha_i Q_i(\alpha_i{}^* + \alpha_i)e^{A_i}\alpha_i{}^*\alpha_i. \tag{50}$$

where

$$e^{A_i}\alpha_i{}^*\alpha_i = \alpha_i{}^*\alpha_i - (M_i\alpha_i{}^* - N_i\alpha_i)\alpha_i,$$
$$=(1-M_i)\,\alpha_i{}^*\alpha_i + N_i\alpha_i{}^2. \tag{51}$$

Using Eq. (51) into Eq. (50) the mean photon number for superpose light beams takes the form,

$$\bar{n}_{ss} = 2\bar{n}_{th} + 2(1 + 2\bar{n}_{th})sinh^2r. \tag{52}$$

In view of Eqs. (12) and (52), we see that the mean photon number for identical superposed light beams are two times that of single-mode one.

4.2.2 The variance of photon number

The variance of photon number for the superposed light beams is expressible as

Advances in Physics Theories and Applications www.iiste.org
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online) DOI: 10.7176/APTA
Vol.78, 2019

$$\Delta n_{ss}{}^2 = <\hat{a}^{+2}\hat{a}^2> + 2\bar{n}_{ss} - \bar{n}_{ss}{}^2. \quad (53)$$

Where

$$<\hat{a}^{+2}\hat{a}^2> = <\hat{a}_1{}^{+2}\hat{a}_1{}^2> + <\hat{a}_2{}^{+2}\hat{a}_2{}^2> + 4<\hat{a}_1{}^{+}\hat{a}_1><\hat{a}_2{}^{+}\hat{a}_2> + <\hat{a}_1{}^{+2}><\hat{a}_1{}^2> + <\hat{a}_1{}^2><\hat{a}_1{}^{+2}>, \quad (54)$$

After calculating all expectation values and using Eq. (52) the variance of photon number given by,

$$\Delta n_{ss}{}^2 = 4\bar{n}_{th} - 2\bar{n}_{th}{}^2 + 6(1+2\bar{n}_{th})^2 sinh^2 r + 4(1+2\bar{n}_{th})^2 sinh^4 r - 2(1+2\bar{n}_{th}) coshrsinhr \quad (55)$$

From Eq. (17) and (55), we see that the variance of photon number for superposed light beams is twice that of single-mode one. But the variance of photon number for the superposed light beams is greater than single-mode squeezed chaotic state. Fig. (7) Indicates both the mean photon number and variance photon number increase with squeezing parameter. But the variance of photon number is greater than the mean photon number; so the photon statics for the superposed single-mode squeezed chaotic state satisfies super-Poissonian photon statistics.

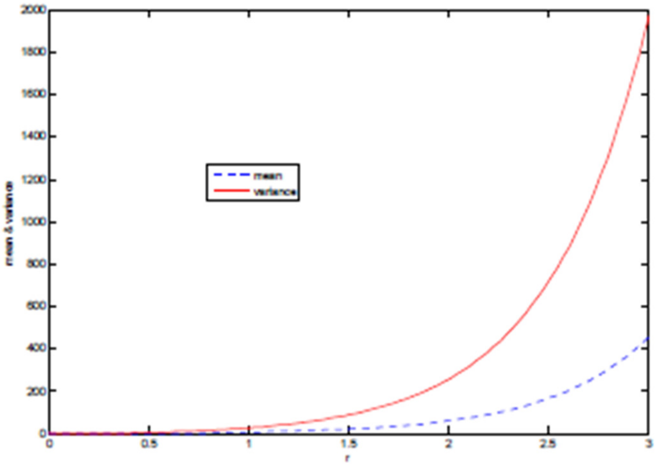

Figure 7: *Plots of mean (Eq.* (52)*) and variance of photon number (Eq.* (55)*) versus r for the superposed light beams taking* $\bar{n}_{th} = 0.25$

4.2.3 Photon number distribution

The photon number distribution for superposed light beams written as,

$$P(n,r)_{ss} = \frac{(M_i{}^2 - N_i{}^2)^{\frac{1}{2}}}{n!} \frac{\partial^{2n}}{\partial \alpha_i{}^{*n} \alpha_i{}^n} \left[exp^{\left[(1-M_i)\alpha_i{}^*\alpha_i + \frac{N_i}{2}(\alpha_i{}^{*2} + \alpha_i{}^2)\right]} \right] |_{\alpha_i{}^* = \alpha_i} = 0. \quad (56)$$

Applying the power series expansion, we find

$$exp^{[(1-M_i)\alpha_i{}^*\alpha_i]} = \sum_v \frac{(1-M_i)^v \alpha_i{}^* \alpha_i}{v!}, \quad (57)$$

$$exp^{\left[\frac{N_i}{2}(\alpha_i{}^2)\right]} = \sum_u \frac{(\frac{N_i}{2})^u \alpha_i{}^{2u}}{u!}, \quad (58)$$

And

$$exp^{\left[\frac{N_i}{2}(\alpha_i{}^{*2})\right]} = \sum_x \frac{(\frac{N_i}{2})^x \alpha_i{}^{*2x}}{x!}. \quad (59)$$

With these result, Eq. (56), is expressed as

$$P(n,r)_{ss} = \frac{2}{(A)^{\frac{1}{2}}} \left[\sum_{x=0}^{[n]} n! \frac{\left(\frac{B}{A}\right)^{n-2x} \left(\frac{-C}{A}\right)^{n-2x}}{2^{2x}(x!)^2(n-2x)!} \right]. \quad (60)$$

Eq. (60) represents the photon number distribution for identical superposed single-mode squeezed chaotic state.
Where

$$A = 2(1+2\bar{n}_{th}) sinh^2 r + 2(1+\bar{n}_{th})^2$$
$$B = 2\bar{n}_{th}(1+\bar{n}_{th})$$
$$C = 2(1+2\bar{n}_{th}) coshrsinhr$$

Fig. (8) Shows relation between the photon number distributions for single-mode squeezed chaotic state (solid line) and the superposed light beams (broken line). We observe that for both cases photon number distribution is increase with number of photons. But the photon number distribution for superposed light beams is greater than single-mode one.

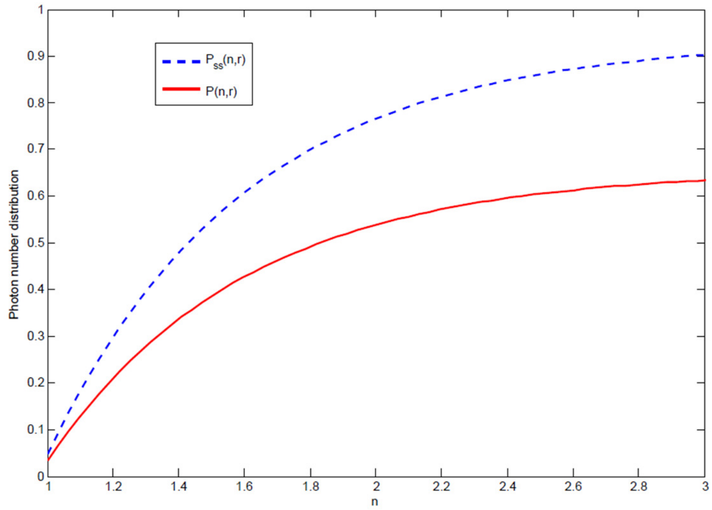

Figure 8: *Plots of photon number distribution (Eq.* (60)*) versus number of photons (n) for* $\bar{n}_{th}$= 2 *and* r = 1.5

4.3 Quadrature squeezing

The quadrature variance for the superposed light beams takes the form,

$$(\Delta a_{\pm}{}^2)_{ss} = 2 \pm [< \hat{a}^2 > + < \hat{a}^{+2} > +2 < \hat{a}^{+}\hat{a} >]. \tag{61}$$

After evaluating the expectation values of above terms the Variance for the superposed light beams can be,

$$(\Delta a_{\pm}{}^2)_{ss} = 2(1 + 2\bar{n}_{th})e^{\mp 2r}, \tag{62}$$

From above equation we can understand that squeezing occurs in plus quadrature. And when we compare Eq. (27) with Eq. (62) one can understand that the plus quadrature for superposed light beams for identical light are two times that of single-mode one. Quadrature squeezing can be written in terms of quadrature variance as

$$S_{ss} = \frac{(\Delta a_{\pm}{}^2)_v - \Delta a_{+}{}^2}{(\Delta a_{\pm}{}^2)_v}. \tag{63}$$

Where quadrature variance for vacuum state is

$$(\Delta a_{\pm}{}^2)_v = 2. \tag{64}$$

On account of Eq. (64), one can describe quadrature squeezing superposed light beams as

$$S_{ss} = 1 - (1 + 2\bar{n}_{th})e^{-2r}. \tag{65}$$

This is the same with single-mode light beam. Fig (9), shows relation between the mean photon numbers of thermal light, squeeze parameter and quadrature squeezing. As mean photon number of thermal light increases quadrature squeezing decreases and as squeezed parameter increase, the quadrature squeezes increases.

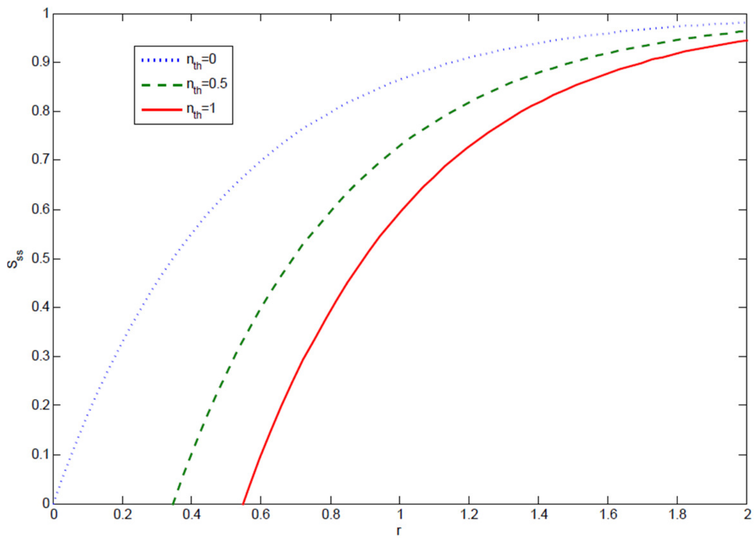

Figure 9: *Plots of quadrature squeezing for superposed light beams versus squeeze parameter Eq.* (65)*) for different values of* $\bar{n}_{th}$.

5 Results

In this paper we have study the statistical and squeezing properties of both single-modes squeezed chaotic state and superposed single-mode squeezed chaotic state. In order to carry out analysis, we have obtained the Q function from density operator of single-mode squeezed chaotic state. With Q function we calculate the variance of photon number, the photon number distribution, the quadrature variance and quadrature squeezing for single-mode squeezed chaotic light. And we get;

$\bar{n}_{th}$	$\Delta a_+{}^2$	S_{ss}	Degree of squeezing
0	0.049	0.950	95%
5	0.248	0.751	75.1%
10	0.547	0.452	45.2%
15	0.746	0.253	25.3%

Table (1) Which shows the relation between mean, plus quadrature, quadrature squeezing and degree of squeezing.

Table (1), summarize dependence of quadrature variance, quadrature squeezing and degree of squeezing on mean photon number of thermal light. And when the mean photon number of thermal light increases, the variance and squeezing of the system increases and decrease, respectively. The quadrature squeezing is observed to be high when the initial state of the system is vacuum (no photon for convenience) which has corresponding degree of 95% for r = 1.5.

And With Q function of single-mode squeezed chaotic state we calculate the density operator for superposed light beams. The result shows the mean photon number increase with increment of the mean photon number of thermal light and squeeze parameter, but quadrature variance are decreases as both mean photon number of thermal light and squeeze parameter increase. And also the variance of photon number is greater than the mean photon number and the radiation has super-poissonian photon statics.

With density operator for superposed single-mode squeezed chaotic light and respective Q function I have calculate the mean photon number, the variance of photon number, the quadrature variance and quadrature squeezing. We have found that the mean photon number for superposed single-mode squeezed chaotic light is two times that of single-mode squeezed chaotic light with identical light beams. But the quadrature squeezing does not affected by the superposition.

The other one is that the probability of getting n-number of photon for superposed light beams are greater than single-mode squeezed chaotic state. And the variance for superposed light beams are greater than variance of single-mode one so the system still satisfies super-poissonian photon statics.

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